# WTS TUTORING

# WTS NUMBER PATTERNS

# PAST PAPERS

| | |
|---|---|
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# KWV 01

## QUESTION 2

2.1 Consider the sequence: $\frac{1}{2}; 4; \frac{1}{4}; 7; \frac{1}{8}; 10; ...$

2.1.1 If the pattern continues in the same way, write down the next TWO terms in the sequence. (2)

2.1.2 Calculate the sum of the first 50 terms of the sequence. (7)

2.2 Consider the sequence: 8 ; 18 ; 30 ; 44 ; …

2.2.1 Write down the next TWO terms of the sequence, if the pattern continues in the same way. (2)

2.2.2 Calculate the $n^{th}$ term of the sequence. (6)

2.2.3 Which term of the sequence is 330? (4)

**[21]**

## QUESTION 3

Given the geometric series: $8x^2 + 4x^3 + 2x^4 + ...$

3.1 Determine the $n^{th}$ term of the series. (1)

3.2 For what value(s) of $x$ will the series converge? (3)

3.3 Calculate the sum of the series to infinity if $x = \frac{3}{2}$. (3)

**[7]**

# KWV 02

## QUESTION 2

A cyclist, preparing for an ultra cycling race, cycled 20 km on the first day of training. She increases her distance by 4 km every day.

2.1 On which day does she cycle 100 km? (3)

2.2 Determine the total distance she would have cycled from day 1 to day 14. (3)

2.3 Would she be able to keep up this daily rate of increase in distance covered indefinitely? Substantiate your answer. (2)

**[8]**

**QUESTION 3**

Consider the following sequence: 5; 12; 21; 32; ...

3.1 Write down the next term of the sequence. (1)

3.2 Determine a formula for the $n^{\text{th}}$ term of this sequence. (5)
**[6]**

## QUESTION 4

4.1 Prove that $a + ar + ar^2 + \ldots$ (to $n$ terms) $= \dfrac{a(1-r^n)}{1-r}$ for $r \neq 1$

4.2 Given the geometric series $15 + 5 + \dfrac{5}{3} + \ldots$

4.2.1 Explain why the series converges.

4.2.2 Evaluate $\sum_{n=1}^{\infty} 5(3^{2-n})$

4.3 The sum of the first $n$ terms of a sequence is given by $S_n = 2^{n+2} - 4$

4.3.1 Determine the sum of the first 24 terms.

4.3.2 Determine the $24^{\text{th}}$ term.

4.3.3 Prove that the $n^{\text{th}}$ term of the sequence is $2^{n+1}$.

## KWV 03

**QUESTION 2**

2.1 Tebogo and Matthew's teacher has asked that they use their own rule to construct a sequence of numbers, starting with 5. The sequences that they have constructed are given below.

Matthew's sequence: 5 ; 9 ; 13 ; 17 ; 21 ; …
Tebogo's sequence: 5 ; 125 ; 3 125 ; 78 125 ; 1 953 125 ; …

Write down the $n^{\text{th}}$ term (or the rule in terms of $n$) of:

2.1.1 Matthew's sequence (3)

2.1.2 Tebogo's sequence (2)

2.2 Nomsa generates a sequence which is both arithmetic and geometric. The first term is 1. She claims that there is only one such sequence. Is that correct? Show ALL your workings to justify your answer. (5)
**[10]**

## QUESTION 3

Given: $\sum_{t=0}^{99}(3t-1)$

3.1 Write down the first THREE terms of the series. (1)

3.2 Calculate the sum of the series. (4)
**[5]**

## QUESTION 4

The following sequence of numbers forms a quadratic sequence:

$-3 ; -2 ; -3 ; -6 ; -11 ; \ldots$

4.1 The first differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)

4.2 Calculate the first difference between the $35^{th}$ and $36^{th}$ terms of the quadratic sequence. (2)

4.3 Determine an expression for the $n^{th}$ term of the quadratic sequence. (4)

4.4 Explain why the sequence of numbers will never contain a positive term. (2)
**[11]**

## QUESTION 5

Data regarding the growth of a certain tree has shown that the tree grows to a height of 150 cm after one year. The data further reveals that during the next year, the height increases by 18 cm. In each successive year, the height increases by $\frac{8}{9}$ of the previous year's increase in height. The table below is a summary of the growth of the tree up to the end of the fourth year.

| | First year | Second year | Third year | Fourth year |
|---|---|---|---|---|
| **Tree height (cm)** | 150 | 168 | 184 | $198\frac{2}{9}$ |
| **Growth (cm)** | | 18 | 16 | $14\frac{2}{9}$ |

5.1 Determine the increase in the height of the tree during the seventeenth year. (2)

5.2 Calculate the height of the tree after 10 years. (3)

5.3 Show that the tree will never reach a height of more than 312 cm. (3)
**[8]**

## QUESTION 2

Consider the series: $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+...$

2.1 Express each of the following sums as a fraction of the form $\frac{a}{b}$ :

2.1.1 The sum of the first two terms of the series (1)

2.1.2 The sum of the first three terms of the series (1)

2.1.3 The sum of the first four terms of the series (1)

2.2 Make a conjecture about the sum of the first $n$ terms of the given series. (2)

2.3 Use your conjecture to predict the value of the following:

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+...+\frac{1}{2008\times 2009}$$ (1)

**[6]**

## QUESTION 3

The following is an arithmetic sequence:

$$1-p \;;\; 2p-3 \;;\; p+5 \;;\; \ldots$$

3.1 Calculate the value of $p$. (3)

3.2 Write down the value of:

3.2.1 The first term of the sequence (1)

3.2.2 The common difference (1)

3.3 Explain why none of the numbers in this arithmetic sequence are perfect squares. (2)

**[7]**

## QUESTION 4

Consider the sequence: $6 \;;\; 6 \;;\; 2 \;;\; -6 \;;\; -18 \;;\; \ldots$

4.1 Write down the next term of the sequence, if the sequence behaves consistently. (1)

4.2 Determine an expression for the $n^{\text{th}}$ term, $T_n$. (5)

4.3 Show that $-6838$ is in this sequence. (4)

**[10]**

## QUESTION 5

A sequence of squares, each having side 1, is drawn as shown below. The first square is shaded, and the length of the side of each shaded square is half the length of the side of the shaded square in the previous diagram.

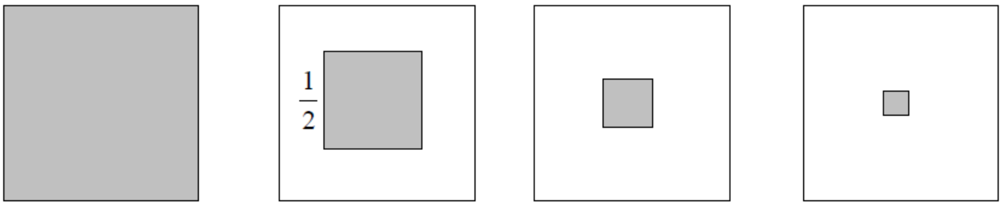



**DIAGRAM 1** **DIAGRAM 2** **DIAGRAM 3** **DIAGRAM 4**

5.1 Determine the area of the unshaded region in DIAGRAM 3. (2)

5.2 What is the sum of the areas of the unshaded regions on the first seven squares? (5)

**[7]**

## QUESTION 2

2.1 Evaluate: $\sum_{n=1}^{20} 3^{n-2}$ (4)

2.2 The following sequence forms a convergent geometric sequence: $5x \; ; \; x^2 \; ; \; \frac{x^3}{5} \; ; \ldots$

2.2.1 Determine the possible values of $x$. (3)

2.2.2 If $x = 2$, calculate $S_\infty$. (2)

2.3 The following arithmetic sequence is given: 20 ; 23 ; 26 ; 29; … ; 101

2.3.1 How many terms are there in this sequence? (2)

2.3.2 The even numbers are removed from the sequence.
Calculate the sum of the terms of the remaining sequence. (6)

**[17]**

## QUESTION 3

The sequence $4 \; ; \; 9 \; ; \; x \; ; \; 37; \ldots$ is a quadratic sequence.

3.1 Calculate $x$. (3)

3.2 Hence, or otherwise, determine the $n^{th}$ term of the sequence. (4)

**[7]**

**QUESTION 2**

Consider the following sequence: 399 ; 360 ; 323 ; 288 ; 255 ; 224 ; …

2.1 Determine the $n^{th}$ term $T_n$ in terms of $n$. (6)

2.2 Determine which term (or terms) has a value of 0. (3)

2.3 Which term in the sequence will have the lowest value? (1)
**[10]**

**QUESTION 3**

3.1 Prove that: $a + ar + ar^2 + \ldots$ (to $n$ terms) $= \dfrac{a(r^n - 1)}{r - 1}$ , $r \neq 1$ (4)

3.2 Given the geometric series: $3 + 1 + \dfrac{1}{3} + \ldots$

Calculate the sum to infinity. (3)
**[7]**

**QUESTION 4**

Matli's annual salary is R120 000 and his expenses total R90 000. His salary increases by R12 000 each year while his expenses increase by R15 000 each year. Each year he saves the excess of his income.

4.1 Represent his total savings as a series. (4)

4.2 If Matli continues to manage his finances this way, after how many years will he have nothing left to save? (3)

4.3 Matli calculates that if his expenses increase by $x$ rand every year (instead of R15 000 each year), he will spend as much as he earns in the $25^{th}$ year. Determine $x$. (2)
**[9]**

# KWV 07

## QUESTION 2

2.1 Given the sequence: $4\ ;\ x\ ;\ 32$

Determine the value(s) of $x$ if the sequence is:

2.1.1 Arithmetic (2)

2.1.2 Geometric (3)

2.2 Determine the value of P if $P = \sum_{k=1}^{13} 3^{k-5}$ (4)

2.3 Prove that for any arithmetic sequence of which the first term is $a$ and the constant difference is $d$, the sum to $n$ terms can be expressed as $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$. (4)
**[13]**

## QUESTION 3

The following sequence is a combination of an arithmetic and a geometric sequence:

$$3\ ;\ 3\ ;\ 9\ ;\ 6\ ;\ 15\ ;\ 12\ ;\ \ldots$$

3.1 Write down the next TWO terms. (2)

3.2 Calculate $T_{52} - T_{51}$. (5)

3.3 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)
**[9]**

## QUESTION 4

A quadratic pattern has a second term equal to 1, a third term equal to – 6 and a fifth term equal to – 14.

4.1 Calculate the second difference of this quadratic pattern. (5)

4.2 Hence, or otherwise, calculate the first term of the pattern. (2)
**[7]**

## QUESTION 2

The sequence 3 ; 9 ; 17 ; 27 ; … is a quadratic sequence.

2.1 Write down the next term. (1)

2.2 Determine an expression for the $n^{th}$ term of the sequence. (4)

2.3 What is the value of the first term of the sequence that is greater than 269? (4)
**[9]**

## QUESTION 3

3.1 The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$. Prove, without the use of a calculator, that the sum of the series to infinity is $16+8\sqrt{2}$. (4)

3.2 The following geometric series is given: $x = 5 + 15 + 45 + \ldots$ to 20 terms.

3.2.1 Write the series in sigma notation. (2)

3.2.2 Calculate the value of $x$. (3)
**[9]**

## QUESTION 4

4.1 The sum to $n$ terms of a sequence of numbers is given as: $S_n = \frac{n}{2}(5n+9)$

4.1.1 Calculate the sum to 23 terms of the sequence. (2)

4.1.2 Hence calculate the $23^{rd}$ term of the sequence. (3)

4.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three terms of the arithmetic sequence.

Determine TWO possible values for the common ratio, $r$, of the geometric sequence. (6)
**[11]**

# KWV 09

## QUESTION 2

2.1 $3x + 1$ ; $2x$ ; $3x - 7$ are the first three terms of an arithmetic sequence. Calculate the value of $x$. (2)

2.2 The first and second terms of an arithmetic sequence are 10 and 6 respectively.

2.2.1 Calculate the $11^{th}$ term of the sequence. (2)

2.2.2 The sum of the first $n$ terms of this sequence is –560. Calculate $n$. (6)

**[10]**

## QUESTION 3

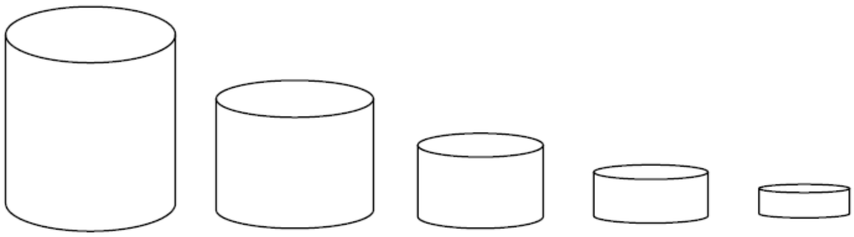
3.1 Given the geometric sequence: 27 ; 9 ; 3 …

3.1.1 Determine a formula for $T_n$, the $n^{\text{th}}$ term of the sequence. (2)

3.1.2 Why does the sum to infinity for this sequence exist? (1)

3.1.3 Determine $S_\infty$. (2)

3.2 Twenty water tanks are decreasing in size in such a way that the volume of each tank is $\frac{1}{2}$ the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water.



Would it be possible for the first water tank to hold all the water from the other 19 tanks? Motivate your answer. (4)

3.3 The $n^{\text{th}}$ term of a sequence is given by $T_n = -2(n-5)^2 + 18$.

3.3.1 Write down the first THREE terms of the sequence. (3)

3.3.2 Which term of the sequence will have the greatest value? (1)

3.3.3 What is the second difference of this quadratic sequence? (2)

3.3.4 Determine ALL values of $n$ for which the terms of the sequence will be less than $-110$. (6)

**[21]**

## QUESTION 2

Given the arithmetic series: $-7-3+1+\ldots+173$

2.1 How many terms are there in the series? (3)

2.2 Calculate the sum of the series. (3)

2.3 Write the series in sigma notation. (3)
**[9]**

## QUESTION 3

3.1 Consider the geometric sequence: $4\ ;\ -2\ ;\ 1\ \ldots$

3.1.1 Determine the next term of the sequence. (2)

3.1.2 Determine $n$ if the $n^{\text{th}}$ term is $\frac{1}{64}$. (4)

3.1.3 Calculate the sum to infinity of the series $4-2+1\ \ldots$ (2)

3.2 If $x$ is a REAL number, show that the following sequence can NOT be geometric:

$$1\ ;\ x+1\ ;\ x-3\ \ldots$$ (4)
**[12]**

## QUESTION 4

An athlete runs along a straight road. His distance $d$ from a fixed point P on the road is measured at different times, $n$, and has the form $d(n)=an^2+bn+c$. The distances are recorded in the table below.

| **Time (in seconds)** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Distance (in metres)** | 17 | 10 | 5 | 2 | $r$ | $s$ |

4.1 Determine the values of $r$ and $s$. (3)

4.2 Determine the values of $a$, $b$ and $c$. (4)

4.3 How far is the athlete from P when $n=8$? (2)

4.4 Show that the athlete is moving towards P when $n<5$, and away from P when $n>5$. (4)
**[13]**

**QUESTION 2**

2.1 Given the geometric sequence: $7 ; x ; 63 ; ...$

Determine the possible values of $x$. (3)

2.2 The first term of a geometric sequence is 15. If the second term is 10, calculate:

2.2.1 $T_{10}$ (3)

2.2.2 $S_9$ (2)

2.3 Given: $0 ; -\frac{1}{2} ; 0 ; \frac{1}{2} ; 0 ; \frac{3}{2} ; 0 ; \frac{5}{2} ; 0 ; \frac{7}{2} ; 0 ; ...$

Assume that this number pattern continues consistently.

2.3.1 Write down the value of the 191st term of this sequence. (1)

2.3.2 Determine the sum of the first 500 terms of this sequence. (4)

2.4 Given: $\sum_{k=2}^{20}(4x-1)^k$

2.4.1 Calculate the first term of the series $\sum_{k=2}^{20}(4x-1)^k$ if $x=1$. (2)

2.4.2 For which values of $x$ will $\sum_{k=1}^{\infty}(4x-1)^k$ exist? (3)
**[18]**

**QUESTION 3**

3.1 Given the arithmetic sequence: $-3 ; 1 ; 5 ; ... ; 393$

3.1.1 Determine a formula for the $n^{\text{th}}$ term of the sequence. (2)

3.1.2 Write down the $4^{\text{th}}$, $5^{\text{th}}$, $6^{\text{th}}$ and $7^{\text{th}}$ terms of the sequence. (2)

3.1.3 Write down the remainders when each of the first seven terms of the sequence is divided by 3. (2)

3.1.4 Calculate the sum of the terms in the arithmetic sequence that are divisible by 3. (5)

3.2 Consider the following pattern of dots:

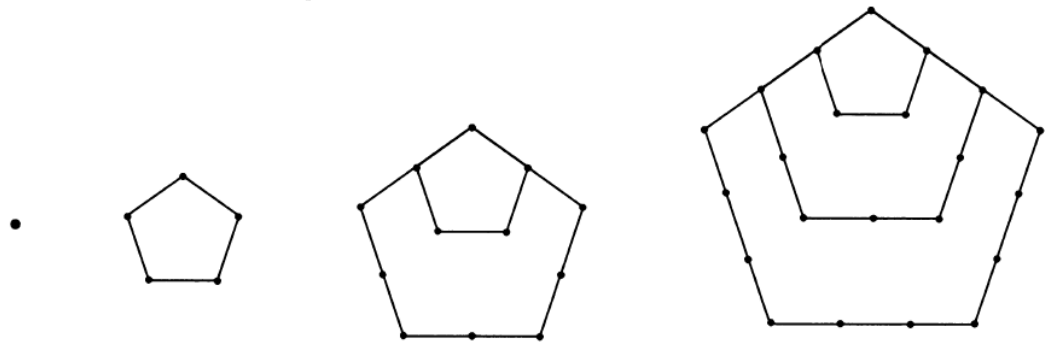


FIGURE 1 FIGURE 2 FIGURE 3 FIGURE 4

If $T_n$ represents the total number of dots in FIGURE $n$, then $T_1 = 1$ and $T_2 = 5$. If the pattern continues in the same manner, determine:

3.2.1 $T_5$ (2)

3.2.2 $T_{50}$ (5)
**[18]**

## QUESTION 2

2.1 Given the geometric series: $256 + p + 64 - 32 + \ldots$

2.1.1 Determine the value of $p$. (3)

2.1.2 Calculate the sum of the first 8 terms of the series. (3)

2.1.3 Why does the sum to infinity for this series exist? (1)

2.1.4 Calculate $S_{\infty}$ (3)

2.2 Consider the arithmetic sequence: $-8 ; -2 ; 4 ; 10 ; \ldots$

2.2.1 Write down the next term of the sequence. (1)

2.2.2 If the $n^{\text{th}}$ term of the sequence is 148, determine the value of $n$. (3)

2.2.3 Calculate the smallest value of $n$ for which the sum of the first $n$ terms of the sequence will be greater than 10 140. (5)

2.3 Calculate $\sum_{k=1}^{30}(3k+5)$ (3)

**[22]**

## QUESTION 3

Consider the sequence: $3 ; 9 ; 27 ; \ldots$

Jacob says that the fourth term of the sequence is 81.
Vusi disagrees and says that the fourth term of the sequence is 57.

3.1 Explain why Jacob and Vusi could both be correct. (2)

3.2 Jacob and Vusi continue with their number patterns.

Determine a formula for the $n^{\text{th}}$ term of:

3.2.1 Jacob's sequence (1)

3.2.2 Vusi's sequence (4)

**[7]**

**QUESTION 2**

Given the arithmetic series: $2 + 9 + 16 + ...$ (to 251 terms).

2.1 Write down the fourth term of the series. (1)

2.2 Calculate the $251^{st}$ term of the series. (3)

2.3 Express the series in sigma notation. (2)

2.4 Calculate the sum of the series. (2)

2.5 How many terms in the series are divisible by 4? (4)

**[12]**

**QUESTION 3**

3.1 Given the quadratic sequence: $-1 ; -7 ; -11 ; p ; \ldots$

3.1.1 Write down the value of $p$. (2)

3.1.2 Determine the $n^{th}$ term of the sequence. (4)

3.1.3 The first difference between two consecutive terms of the sequence is 96. Calculate the values of these two terms. (4)

3.2 The first three terms of a geometric sequence are: 16 ; 4 ; 1

3.2.1 Calculate the value of the $12^{th}$ term. (Leave your answer in simplified exponential form.) (3)

3.2.2 Calculate the sum of the first 10 terms of the sequence. (2)

3.3 Determine the value of: $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)$ ... up to 98 factors. (4)

**[19]**

**QUESTION 2**

2.1 Given the arithmetic series: $18 + 24 + 30 + \ldots + 300$

2.1.1 Determine the number of terms in this series. (3)

2.1.2 Calculate the sum of this series. (2)

2.1.3 Calculate the sum of all the whole numbers up to and including 300 that are NOT divisible by 6. (4)

2.2 The first three terms of an infinite geometric sequence are 16, 8 and 4 respectively.

2.2.1 Determine the $n^{\text{th}}$ term of the sequence. (2)

2.2.2 Determine all possible values of $n$ for which the sum of the first $n$ terms of this sequence is greater than 31. (3)

2.2.3 Calculate the sum to infinity of this sequence. (2)

**[16]**

**QUESTION 3**

3.1 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 1. The general term of the first differences is given by $4n + 6$.

3.1.1 Determine the value of $a$. (2)

3.1.2 Determine the formula for $T_n$. (4)

3.2 Given the series: $(1 \times 2) + (5 \times 6) + (9 \times 10) + (13 \times 14) + \ldots + (81 \times 82)$

Write the series in sigma notation. (It is not necessary to calculate the value of the series.) (4)

**[10]**

## KWV 15

**QUESTION 2**

2.1 A geometric sequence has $T_3 = 20$ and $T_4 = 40$.

Determine:

2.1.1 The common ratio (1)

2.1.2 A formula for $T_n$ (3)

2.2 The following sequence has the property that the sequence of numerators is arithmetic and the sequence of denominators is geometric:

$$\frac{2}{1}\,;\,\frac{-1}{5}\,;\,\frac{-4}{25}\,;\,\ldots$$

2.2.1 Write down the FOURTH term of the sequence. (1)

2.2.2 Determine a formula for the $n^{\text{th}}$ term. (3)

2.2.3 Determine the $500^{\text{th}}$ term of the sequence. (2)

2.2.4 Which will be the first term of the sequence to have a NUMERATOR which is less than –59? (3)

**[13]**

**QUESTION 3**

3.1 Given the arithmetic sequence: $w-3\ ;\ 2w-4\ ;\ 23-w$

3.1.1 Determine the value of $w$. (2)

3.1.2 Write down the common difference of this sequence. (1)

3.2 The arithmetic sequence 4 ; 10 ; 16 ; ... is the sequence of first differences of a quadratic sequence with a first term equal to 3.

Determine the $50^{\text{th}}$ term of the quadratic sequence. (5)

**[8]**

**QUESTION 4**

In a geometric series, the sum of the first $n$ terms is given by $S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$ and the sum to infinity of this series is 10.

4.1 Calculate the value of $p$. (4)

4.2 Calculate the second term of the series. (4)

**[8]**

## QUESTION 2

The following geometric sequence is given: 10 ; 5 ; 2,5 ; 1,25 ; ...

2.1 Calculate the value of the $5^{th}$ term, $T_5$, of this sequence. (2)

2.2 Determine the $n^{th}$ term, $T_n$, in terms of $n$. (2)

2.3 Explain why the infinite series $10 + 5 + 2{,}5 + 1{,}25 + \ldots$ converges. (2)

2.4 Determine $S_\infty - S_n$ in the form $ab^n$, where $S_n$ is the sum of the first $n$ terms of the sequence. (4)

**[10]**

## QUESTION 3

Consider the series: $S_n = -3 + 5 + 13 + 21 + \ldots$ to $n$ terms.

3.1 Determine the general term of the series in the form $T_k = bk + c$. (2)

3.2 Write $S_n$ in sigma notation. (2)

3.3 Show that $S_n = 4n^2 - 7n$. (3)

3.4 Another sequence is defined as:

$Q_1 = -6$
$Q_2 = -6 - 3$
$Q_3 = -6 - 3 + 5$
$Q_4 = -6 - 3 + 5 + 13$
$Q_5 = -6 - 3 + 5 + 13 + 21$

3.4.1 Write down a numerical expression for $Q_6$. (2)

3.4.2 Calculate the value of $Q_{129}$. (3)

**[12]**

## QUESTION 2

2.1 Prove that in any arithmetic series in which the first term is $a$ and whose constant difference is $d,$ the sum of the first $n$ terms is $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$. (4)

2.2 Calculate the value of $\sum_{k=1}^{50}(100-3k)$. (4)

2.3 A quadratic sequence is defined with the following properties:

$$T_2 - T_1 = 7$$
$$T_3 - T_2 = 13$$
$$T_4 - T_3 = 19$$

2.3.1 Write down the value of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

2.3.2 Calculate the value of $T_{69}$ if $T_{89} = 23\,594$. (5)

**[17]**

## QUESTION 3

Consider the infinite geometric series: 45 + 40,5 + 36,45 + …

3.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

3.2 Explain why this series converges. (1)

3.3 Calculate the sum to infinity of the series. (2)

3.4 What is the smallest value of $n$ for which $S_\infty - S_n < 1$? (5)

**[11]**

**QUESTION 2**

Given the finite arithmetic sequence: 5 ; 1 ; –3 ; ... ; –83 ; –87

2.1 Write down the fourth term ($T_4$) of the sequence. (1)

2.2 Calculate the number of terms in the sequence. (3)

2.3 Calculate the sum of all the negative numbers in the sequence. (3)

2.4 Consider the sequence: 5 ; 1 ; –3 ; ... ; –83 ; –87 ; … ; –4 187
Determine the number of terms in this sequence that will be exactly divisible by 5. (4)
**[11]**

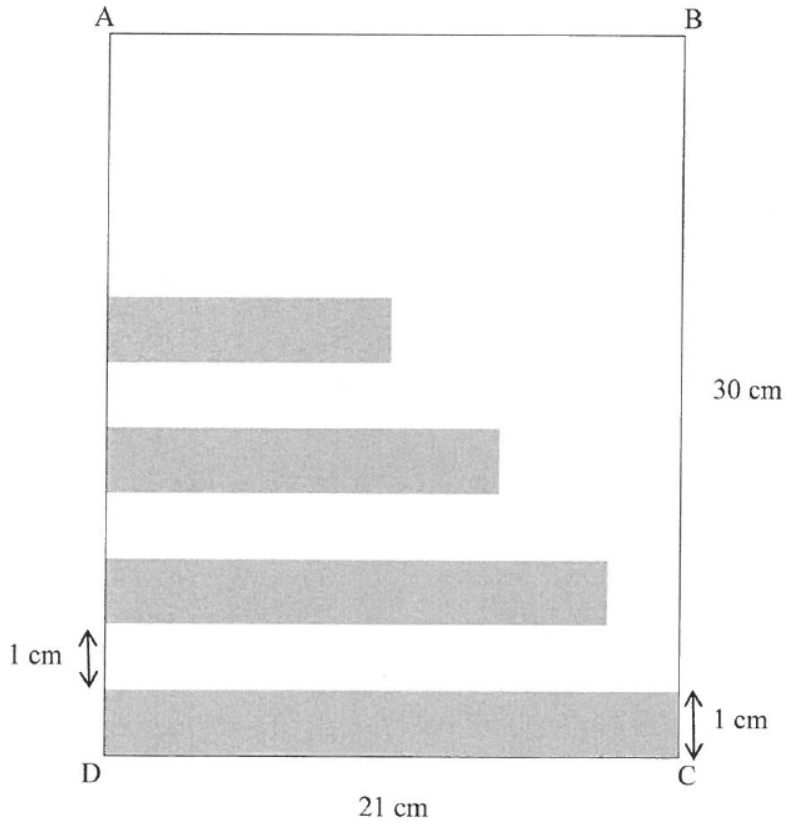
## QUESTION 3

3.1 The first four terms of a quadratic number pattern are $-1\ ;\ x\ ;\ 3\ ;\ x+8$

3.1.1 Calculate the value(s) of $x$. (4)

3.1.2 If $x = 0$, determine the position of the first term in the quadratic number pattern for which the sum of the first $n$ first differences will be greater than 250. (4)

3.2 Rectangles of width 1 cm are drawn from the edge of a sheet of paper that is 30 cm long such that there is a 1 cm gap between one rectangle and the next. The length of the first rectangle is 21 cm and the length of each successive rectangle is 85% of the length of the previous rectangle until there are rectangles drawn along the entire length of AD. Each rectangle is coloured grey.




3.2.1 Calculate the length of the 10th rectangle. (3)

3.2.2 Calculate the percentage of the paper that is coloured grey. (4)

**[15]**

## QUESTION 2

2.1 Given the following quadratic sequence: $-2 ; 0 ; 3 ; 7 ; ...$

2.1.1 Write down the value of the next term of this sequence. (1)

2.1.2 Determine an expression for the $n^{th}$ term of this sequence. (5)

2.1.3 Which term of the sequence will be equal to 322? (4)

2.2 Consider an arithmetic sequence which has the second term equal to 8 and the fifth term equal to 10.

2.2.1 Determine the common difference of this sequence. (3)

2.2.2 Write down the sum of the first 50 terms of this sequence, using sigma notation. (2)

2.2.3 Determine the sum of the first 50 terms of this sequence. (3)

**[18]**

## QUESTION 3

Chris bought a bonsai (miniature tree) at a nursery. When he bought the tree, its height was 130 mm. Thereafter the height of the tree increased, as shown below.

| INCREASE IN HEIGHT OF THE TREE PER YEAR | | |
|---|---|---|
| **During the first year** | **During the second year** | **During the third year** |
| 100 mm | 70 mm | 49 mm |

3.1 Chris noted that the sequence of height increases, namely 100 ; 70 ; 49 ..., was geometric. During which year will the height of the tree increase by approximately 11,76 mm? (4)

3.2 Chris plots a graph to represent the height $h(n)$ of the tree (in mm) $n$ years after he bought it. Determine a formula for $h(n)$. (3)

3.3 What height will the tree eventually reach? (3)

**[10]**

## KWV 20

**QUESTION 2**

2.1 Given the following quadratic number pattern: $5\ ;\ -4\ ;\ -19\ ;\ -40\ ;\ \ldots$

2.1.1 Determine the constant second difference of the sequence. (2)

2.1.2 Determine the $n^{th}$ term $(T_n)$ of the pattern. (4)

2.1.3 Which term of the pattern will be equal to $-25\,939$? (3)

2.2 The first three terms of an arithmetic sequence are $2k-7\ ;\ k+8$ and $2k-1$.

2.2.1 Calculate the value of the $15^{th}$ term of the sequence. (5)

2.2.2 Calculate the sum of the first 30 even terms of the sequence. (4)
**[18]**

**QUESTION 3**

A convergent geometric series consisting of only positive terms has first term $a$, constant ratio $r$ and $n^{\text{th}}$ term, $T_n$, such that $\sum_{n=3}^{\infty} T_n = \frac{1}{4}$.

3.1 If $T_1 + T_2 = 2$, write down an expression for $a$ in terms of $r$. (2)

3.2 Calculate the values of $a$ and $r$. (6)
**[8]**

## KWV 21

**QUESTION 2**

Given the geometric sequence: $-\frac{1}{4}\ ;\ b\ ;\ -1\ ;\ \ldots\ldots$

2.1 Calculate the possible values of $b$. (3)

2.2 If $b = \frac{1}{2}$, calculate the $19^{\text{th}}$ term $(T_{19})$ of the sequence. (3)

2.3 If $b = \frac{1}{2}$, write the sum of the first 20 positive terms of the sequence in sigma notation. (4)

2.4 Is the geometric series formed in QUESTION 2.3 convergent? Give reasons for your answer. (2)
**[12]**

## QUESTION 3

3.1 $6\ ;\ 6\ ;\ 9\ ;\ 15\ ;\ \ldots$ are the first four terms of a quadratic number pattern.

3.1.1 Write down the value of the fifth term $(T_5)$ of the pattern. (1)

3.1.2 Determine a formula to represent the general term of the pattern. (4)

3.1.3 Which term of the pattern has a value of 3 249? (4)

3.2 Determine the value(s) of $x$ in the interval $x \in [0^\circ ; 90^\circ]$ for which the sequence $-1\ ;\ 2\sin 3x\ ;\ 5\ ;\ \ldots$ will be arithmetic. (4)

**[13]**

# KWV 22

## QUESTION 2

2.1 Given the quadratic sequence: $2 ; 3 ; 10 ; 23 ; \ldots$

2.1.1 Write down the next term of the sequence. (1)

2.1.2 Determine the $n^{\text{th}}$ term of the sequence. (4)

2.1.3 Calculate the $20^{\text{th}}$ term of the sequence. (2)

2.2 Given the arithmetic sequence: $35 ; 28 ; 21 ; \ldots$

Calculate which term of the sequence will have a value of $-140$. (3)

2.3 For which value of $n$ will the sum of the first $n$ terms of the arithmetic sequence in QUESTION 2.2 be equal to the $n^{\text{th}}$ term of the quadratic sequence in QUESTION 2.1? (6)

**[16]**

## QUESTION 3

A geometric series has a constant ratio of $\frac{1}{2}$ and a sum to infinity of 6.

3.1 Calculate the first term of the series. (2)

3.2 Calculate the $8^{\text{th}}$ term of the series. (2)

3.3 Given: $\sum_{k=1}^{n} 3(2)^{1-k} = 5{,}8125$ Calculate the value of $n$. (4)

3.4 If $\sum_{k=1}^{20} 3(2)^{1-k} = p$, write down $\sum_{k=1}^{20} 24(2)^{-k}$ in terms of $p$. (3)

**[11]**

## QUESTION 2

2.1 Given the following geometric sequence: $30 \ ; \ 10 \ ; \ \frac{10}{3} \ ; \ldots$

2.1.1 Determine $n$ if the $n^{th}$ term of the sequence is equal to $\frac{10}{729}$. (4)

2.1.2 Calculate: $30+10+\frac{10}{3}+\ldots$ (2)

2.2 Derive a formula for the sum of the first $n$ terms of an arithmetic sequence if the first term of the sequence is $a$ and the common difference is $d$. (4)

**[10]**

## QUESTION 3

The first three terms of an arithmetic sequence are $-1 \ ; \ 2$ and 5.

3.1 Determine the $n^{th}$ term, $T_n$, of the sequence. (2)

3.2 Calculate $T_{43}$. (2)

3.3 Evaluate $\sum_{k=1}^{n} T_k$ in terms of $n$. (3)

3.4 A quadratic sequence, with general term $T_n$, has the following properties:

- $T_{11} = 125$
- $T_n - T_{n-1} = 3n - 4$

Determine the first term of the sequence. (6)

**[13]**

**QUESTION 2**

2.1 Given the quadratic sequence: 321 ; 290 ; 261 ; 234 ; ....

2.1.1 Write down the values of the next TWO terms of the sequence. (2)

2.1.2 Determine the general term of the sequence in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Which term(s) of the sequence will have a value of 74? (4)

2.1.4 Which term in the sequence has the least value? (2)

2.2 Given the geometric series: $\frac{5}{8} + \frac{5}{16} + \frac{5}{32} + \ldots = \text{K}$

2.2.1 Determine the value of $K$ if the series has 21 terms. (3)

2.2.2 Determine the largest value of $n$ for which $T_n > \frac{5}{8192}$ (4)

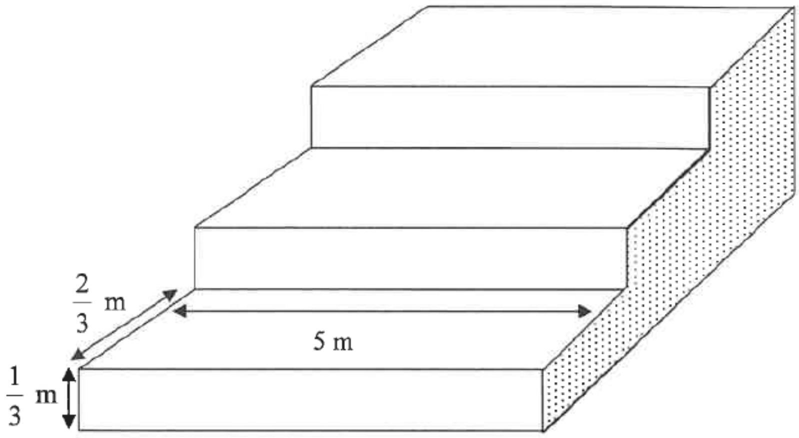
**[19]**

## QUESTION 3

3.1 Without using a calculator, determine the value of: $\sum_{y=3}^{10}\frac{1}{y-2}-\sum_{y=3}^{10}\frac{1}{y-1}$ (3)

3.2 A steel pavilion at a sports ground comprises of a series of 12 steps, of which the first 3 are shown in the diagram below.

Each step is 5 m wide. Each step has a rise of $\frac{1}{3}$ m and has a tread of $\frac{2}{3}$ m, as shown in the diagram below.




The open side (shaded on sketch) on each side of the pavilion must be covered with metal sheeting. Calculate the area (in $m^2$) of metal sheeting needed to cover both open sides. (6)

**[9]**

## KWV 25

**QUESTION 2**

2.1 7 ; $x$ ; $y$ ; $-11$ ; … is an arithmetic sequence. Determine the values of $x$ and $y$. (4)

2.2 Given the quadratic number pattern: $-3$ ; $6$ ; $27$ ; $60$ ; …

2.2.1 Determine the general term of the pattern in the form $T_n = an^2 + bn + c$. (4)

2.2.2 Calculate the value of the $50^{\text{th}}$ term of the pattern. (2)

2.2.3 Show that the sum of the first $n$ first-differences of this pattern can be given by $S_n = 6n^2 + 3n$. (3)

2.2.4 How many consecutive first-differences were added to the first term of the quadratic number pattern to obtain a term in the quadratic number pattern that has a value of 21 060? (4)

**[17]**

**QUESTION 3**

3.1 Prove that $\sum_{k=1}^{\infty} 4.3^{2-k}$ is a convergent geometric series. Show ALL your calculations. (3)

3.2 If $\sum_{k=p}^{\infty} 4.3^{2-k} = \frac{2}{9}$, determine the value of $p$. (5)

**[8]**

## KWV 26

**QUESTION 2**

Given the geometric series: $x + 90 + 81 + \ldots$

2.1 Calculate the value of $x$. (2)

2.2 Show that the sum of the first $n$ terms is $S_n = 1\,000\left(1 - (0,9)^n\right)$. (2)

2.3 Hence, or otherwise, calculate the sum to infinity. (2)

**[6]**

## QUESTION 3

Consider the quadratic number pattern: $-145 ; -122 ; -101 ; \ldots$

3.1 Write down the value of $T_4$. (1)

3.2 Show that the general term of this number pattern is $T_n = -n^2 + 26n - 170$. (3)

3.3 Between which TWO terms of the quadratic number pattern will there be a difference of $-121$? (4)

3.4 What value must be added to each term in the number pattern so that the value of the maximum term in the new number pattern formed will be 1? (3)

**[11]**

## QUESTION 4

Consider the linear pattern: $5 ; 7 ; 9 ; \ldots$

4.1 Determine $T_{51}$. (3)

4.2 Calculate the sum of the first 51 terms. (2)

4.3 Write down the expansion of $\sum_{n=1}^{5000}(2n+3)$. Show only the first 3 terms and the last term of the expansion. (1)

4.4 Hence, or otherwise, calculate $\sum_{n=1}^{5000}(2n+3) + \sum_{n=1}^{4999}(-2n-1)$.
ALL working details must be shown. (4)

**[10]**

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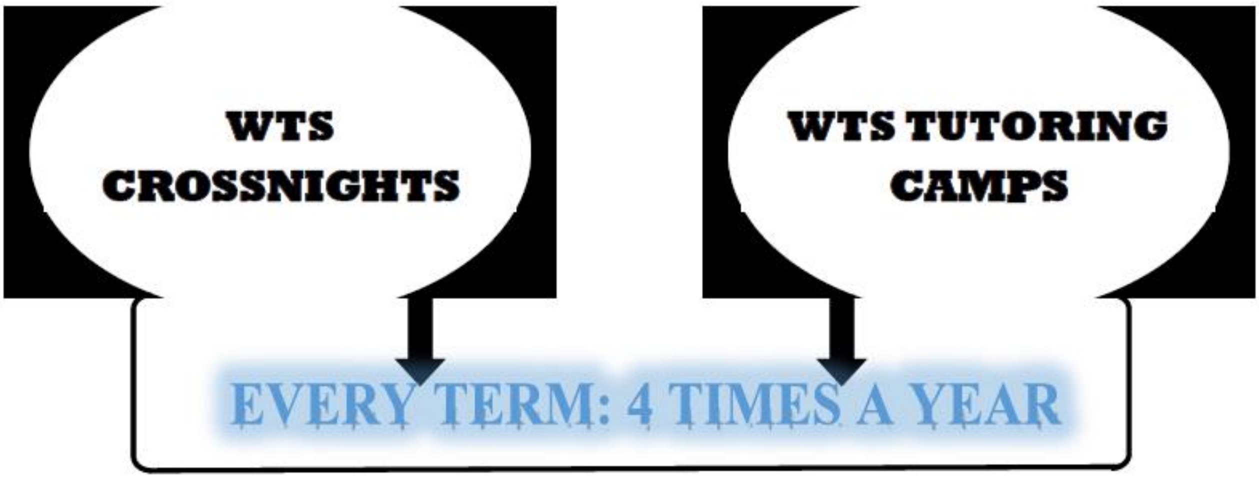

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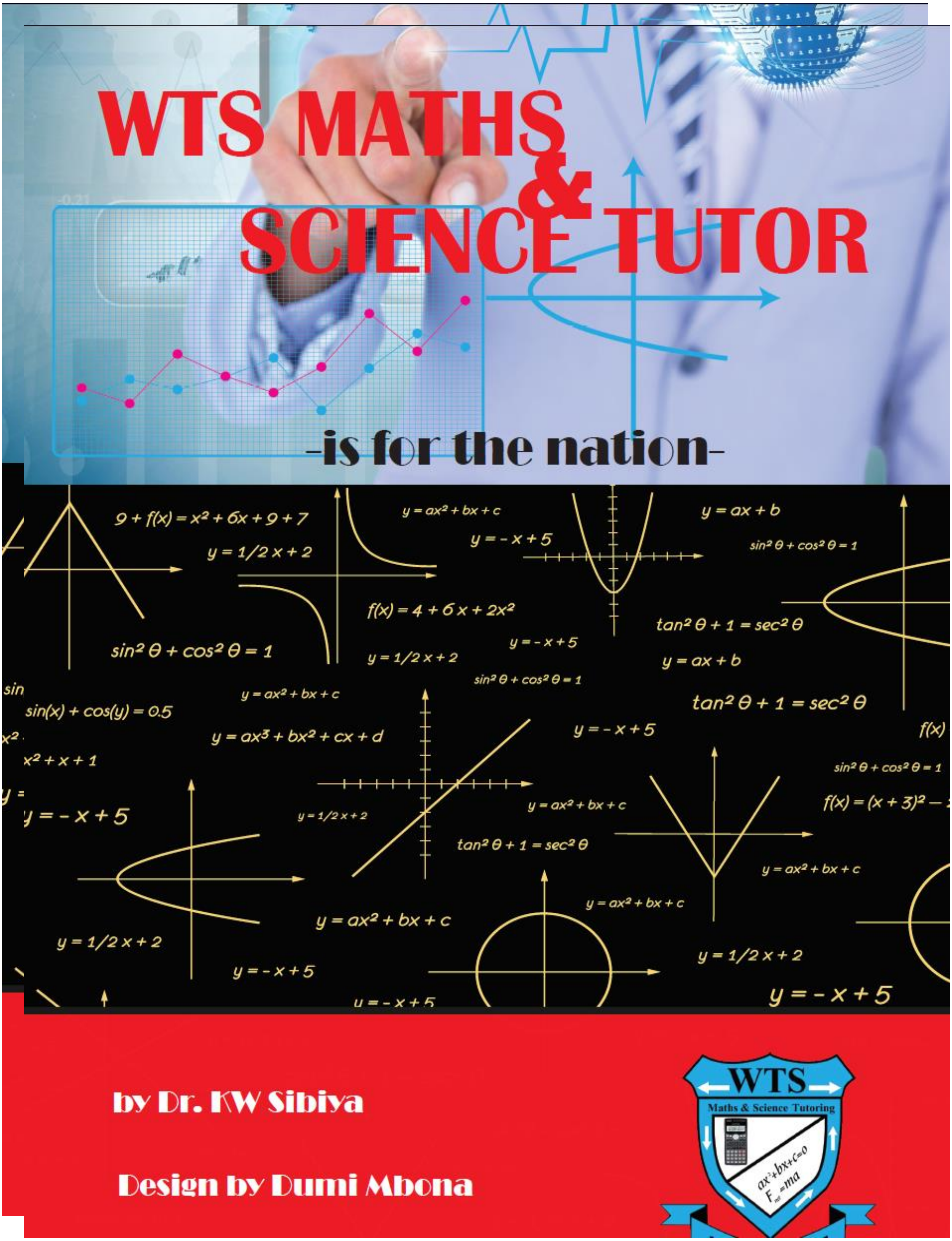

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