



GAUTENG PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINCIAL EXAMINATION
NOVEMBER 2023
GRADE 11
MARKING GUIDELINES

MATHEMATICS (PAPER 1)

18 pages

INSTRUCTIONS AND INFORMATION

A – ACCURACY

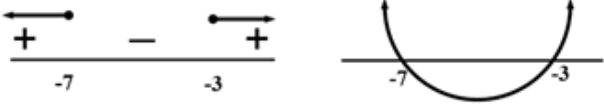
C.A. – CONTINUED ACCURACY

NOTE:

- If a candidate answered a question TWICE, mark only the first attempt.
- If a candidate crossed OUT an answer and did NOT redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answers in order to solve a question is UNACCEPTABLE.

QUESTION 1

1.1	1.1.1	$K = \sqrt{\frac{5}{p+2}} + \frac{p}{3}$ $K = \sqrt{\frac{5}{3+2}} + \frac{3}{3}$ $K = \sqrt{1} + 1$ $\therefore K = 2$	✓ simplification ✓ answer	(2)
	1.1.2	$K = \sqrt{\frac{5}{p+2}} + \frac{p}{3}$ $\therefore p + 2 > 0$ $\therefore p > -2$ <p>NOTE: Answer only, full marks.</p>	✓ answer	(1)
1.2	1.2.1	$3x^2 = 4x + 2$ $\therefore 3x^2 - 4x - 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} + 1$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)} + 1$ $\therefore x = 1,72 \text{ or } x = -0,39$ <p>NOTE: Penalise with 1 mark for rounding-off in this question ONLY. Answers only, 1 mark.</p>	✓ substitution ✓ answers	(2)

1.2.2	$x^2 + 10x > -21$ $x^2 + 10x + 21 > 0$ $(x + 7)(x + 3) > 0$  $x < -7 \text{ or } x > -3$ <p>NOTE: Penalise with 1 mark for the use of AND instead of OR.</p>	<ul style="list-style-type: none"> ✓ standard form ✓ factors ✓✓ answers 	(4)
1.2.3	$\sqrt{x^2 - 5} = 2\sqrt{x}$ $(\sqrt{x^2 - 5})^2 = (2\sqrt{x})^2$ $x^2 - 5 = 4x$ $x^2 - 4x - 5 = 0$ $(x - 5)(x + 1) = 0$ $\therefore x = 5 \text{ or } x = -1(N/A)$	<ul style="list-style-type: none"> ✓ squaring both sides ✓ simplification ✓ standard form ✓ factors ✓ both answers with exclusion 	(5)
1.2.4	$3^{x+1} + 3^{x-1} = \frac{10}{9}$ $3^x \cdot 3^1 + 3^x \cdot 3^{-1} = \frac{10}{9}$ $\therefore 3^x \left(3^{-1} + \frac{1}{3} \right) = \frac{10}{9}$ $\therefore 3^x \cdot \frac{10}{3} = \frac{10}{9}$ $\therefore 3^x = \frac{1}{3}$ $\therefore 3^x = 3^{-1}$ $x = -1$	<ul style="list-style-type: none"> ✓ factorisation ✓ simplification ✓ same bases ✓ answer 	(4)

1.3	1.3.1	$x + y = 2$ and $y + 1 = \frac{-3}{x-1} \dots(2)$ $x = 2 - y \dots(1)$ sub (1) into (2) $y + 1 = \frac{-3}{2 - y - 1}$ $y + 1 = \frac{-3}{1 - y}$ $(y + 1)(1 - y) = -3$ $1 - y^2 = -3$ $y^2 = 4$ $y = \pm 2 \dots$ sub into (1) $x = 2 - 2$ of $x = 2 - (-2)$ $x = 0$ $x = 4$ NOTE: Any other valid method.	✓ x as subject in (1) ✓ substitution in (2) ✓ value of y^2 ✓ values for y ✓ values for x	(5)
	1.3.2	$\left(\frac{1}{x} + \frac{1}{y}\right)$ $\left(\frac{1}{4} + \frac{1}{-2}\right)$ $= -\frac{1}{4}$	✓ correct selection for x and y values ✓ answer	(2)
1.4	1.4.1	$2mx^2 - (m - 2)x + m + 1 = 0$ $\Delta = b^2 - 4ac$ $\Delta = [-(m - 2)]^2 - 4(2m)(m + 1)$ $\Delta = (m - 2)^2 - 8m(m + 1)$ $\Delta = m^2 - 4m + 4 - 8m^2 - 8m$ $\Delta = 7m^2 - 12m + 4$ For real roots, $\Delta \geq 0$ $\therefore 7m^2 - 12m + 4 \geq 0$ $\therefore 7m^2 + 12m - 4 \leq 0$ $\therefore (7m - 2)(m + 2) \leq 0$ $\therefore -2 \leq m \leq \frac{2}{7}$	✓ substitution ✓ simplification ✓ standard form ✓ condition for Δ ✓ factors ✓ answers	(6)

	1.4.2	<p>For roots to have opposite signs, the value of 'c' must be positive.</p> <p>$\therefore c > 0$ $\therefore m + 1 > 0$ $\therefore m > -1$</p> <p>\therefore Real roots opposite in sign for: $-1 < m < 0$</p>	<p>✓ condition of 'c'</p> <p>✓ answer</p>	(2)
[33]				

QUESTION 2

2.1	2.1.1	$\left\{ \frac{512x^3}{64x^{-3}} \right\}^{-\frac{1}{3}}$ $= \left\{ \frac{2^9 x^3}{2^6 x^{-3}} \right\}^{-\frac{1}{3}}$ $= \{2^3 x^6\}^{-\frac{1}{3}}$ $= \{2^{-1} x^{-2}\}$ $= \frac{1}{2} \times \frac{1}{x^2}$ $= \frac{1}{2x^2}$ <p>NOTE: Any correct alternative method.</p>	<p>✓ prime bases</p> <p>✓ simplification</p> <p>✓ answer</p>	(3)
	2.1.2	$\frac{x^{n+2} + x^{n+1} - x^n - x^{n-1}}{x^2 - 1}$ $= \frac{x^n(x^2 - 1) + x^n(x - x^{-1})}{x^2 - 1}$ $= \frac{x^n(x^2 - 1) + x^n\left(\frac{x^2 - 1}{x}\right)}{x^2 - 1}$ $= x^n + x^n \frac{1}{x}$ $= x^n + x^{n-1}$ <p>NOTE: Any correct alternative method.</p>	<p>✓ factorisation</p> <p>✓ simplification</p> <p>✓ answer</p>	(3)

2.1.3	$\sqrt{a + \sqrt{2a-1}} \cdot \sqrt{a - \sqrt{2a-1}}$ $= \sqrt{(a + \sqrt{2a-1})(a - \sqrt{2a-1})}$ $= \sqrt{a^2 - (\sqrt{2a-1})^2}$ $= \sqrt{a^2 - 2a + 1}$ $= \sqrt{(a-1)^2}$ $= a - 1$ <p>NOTE: Any correct alternative method.</p>	<ul style="list-style-type: none"> ✓ write as A single surd ✓ quadratic expression ✓ factors ✓ answer 	(4)
2.2	$(1 - \sqrt{2})(x + y\sqrt{z}) = -3 + \sqrt{2}$ $\therefore x + y\sqrt{z} = \frac{(-3 + \sqrt{2})}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}}$ $\therefore x + y\sqrt{z} = \frac{(-3 + \sqrt{2})(1 + \sqrt{2})}{1 - 2}$ $\therefore x + y\sqrt{z} = \frac{-3 - 2\sqrt{2} + 2}{-1}$ $\therefore x + y\sqrt{z} = 1 + 2\sqrt{2}$ $\therefore x = 1$ $\therefore y = 2$ $\therefore z = 2$	<ul style="list-style-type: none"> ✓ isolating variables and rationalising ✓ simplification ✓ simplification ✓ answers 	(4)
[14]			

QUESTION 3

3.1	3.1.1	Multiply the previous term by $\cos x$.	✓ answer	(1)
	3.1.2	$\sin x \cdot \cos^2 x$	✓ answer	(1)
3.2	3.2.1	$-1; 2; 5; 8; \dots$ $T_2 - T_1 = T_3 - T_2$ $T_2 - T_1 = 2 - (-1) = 3$ $T_3 - T_2 = 5 - 2 = 3$ $\therefore d = 3$ $\therefore T_n = dn + a$ $\therefore T_n = 3n + a$ $\therefore -1 = 3(1) + a$ $\therefore a = -4$ $\therefore T_n = 3n - 4$ <p>NOTE: Answer only, full marks.</p>	<ul style="list-style-type: none"> ✓ value of d ✓ substitution ✓ value of a ✓ answer 	(4)

	3.2.2	$T_n = 3n - 4$ $161 = 3n - 4$ $165 = 3n$ $\therefore n = 55$	✓ substitution ✓ answer	(2)
				[8]

QUESTION 4

4.1	4.1.1	25 squares	✓ answer	(1)
	4.1.2	64 dots	✓ answer	(1)
	4.1.3	$4 ; 9 ; 16 ; 25$ $5 \quad 7 \quad 9$ $2 \quad 2$ $2a = 2$ $\therefore a = 1$ $3a + b = 5$ $\therefore b = 2$ $a + b + c = 4$ $\therefore c = 1$ $\therefore T_n = n^2 + 2n + 1$ $\therefore T_n = (n + 1)^2$	✓ second difference ✓ value of a ✓ value of b ✓ value of c	(4)
4.2	4.2.1	$17 ; 10 ; 5 ; 2 ; 1 ; 2$ $-7 \quad -5 \quad -3 \quad -1 \quad +1$ $2 \quad 2 \quad 2 \quad 2$ $\therefore r = 1$ $\therefore s = 2$	✓ answer ✓ answer	(2)

	4.2.2	$2a = 2$ $\therefore a = 1$ $d(n) = n^2 + bn + c$ $d(1) = 1^2 + b(1) + c$ $\therefore 17 = 1 + b + c$ $\therefore 16 = b + c \dots(2)$ $d(2) = 2^2 + b(2) + c$ $\therefore 10 = 4 + 2b + c$ $\therefore 6 = 2b + c \dots(1)$ $(1) - (2)$ $\therefore b = -10$ $\therefore 16 = -10 + c \dots(2)$ $\therefore c = 26$ NOTE: Any other valid method.	✓ 2^{nd} difference ✓ value of a ✓ value of b ✓ value of c	(4)
	4.2.3	$d(n) = n^2 - 10n + 26$ $\therefore d(8) = 8^2 - 10(8) + 26$ $\therefore d(8) = 10m$	✓ substitution ✓ answer	(2)
4.2	4.2.4	$d(n) = n^2 - 10n + 26$ \therefore Minimum turning point $\therefore n = \frac{-(-10)}{2(1)}$ $\therefore n = 5$ $\therefore n > 0 \dots n \in N$ \therefore minimum value \therefore Mikayla is CORRECT NOTE: No marks are awarded for correct name only. The explanation and the choice in name must correlate.	✓ substitution ✓ value of n ✓ explanation and choice of Mikayla	(3)
				[17]

QUESTION 5

5.1	$A = P(1 - i)^n$ $\frac{1}{3} = (1 - i)^2$ $i = 0,42\%$ NOTE: The substitution is for the correct formula and the interpretation of substituting for A and P .	<ul style="list-style-type: none"> ✓ correct substitution into correct formula ✓ answer 	(2)
5.2	5.2.1 $1 + i_{eff} = \left(1 + \frac{i_{nom}}{n}\right)$ $1 + i_{eff} = \left(1 + \frac{0,075}{12}\right)^{12}$ $i_{eff} = 0,07763\dots$ $i_{eff} = 7,76\%$	<ul style="list-style-type: none"> ✓ correct substitution into correct formula ✓ simplification ✓ answer 	(3)
5.2.2	$A = P(1 - i)^n$ $\therefore A = 10\,000\left(1 + \frac{0,075}{12}\right)^{12} + \left(1 + \frac{0,078}{4}\right)^8 + 5000\left(1 + \frac{0,078}{4}\right)^6$ $\therefore A = R18\,191,03$ YES. He will have sufficient funds. NOTE: The values must be substituted into the correct formula. Only award the conclusion mark if a valid calculation has been done.	<ul style="list-style-type: none"> ✓ $10\,000\left(1 + \frac{0,075}{12}\right)^{12}$ ✓ $\times \left(1 + \frac{0,078}{4}\right)^8$ ✓ $5000\left(1 + \frac{0,078}{4}\right)^6$ ✓ answer ✓ conclusion 	(5)

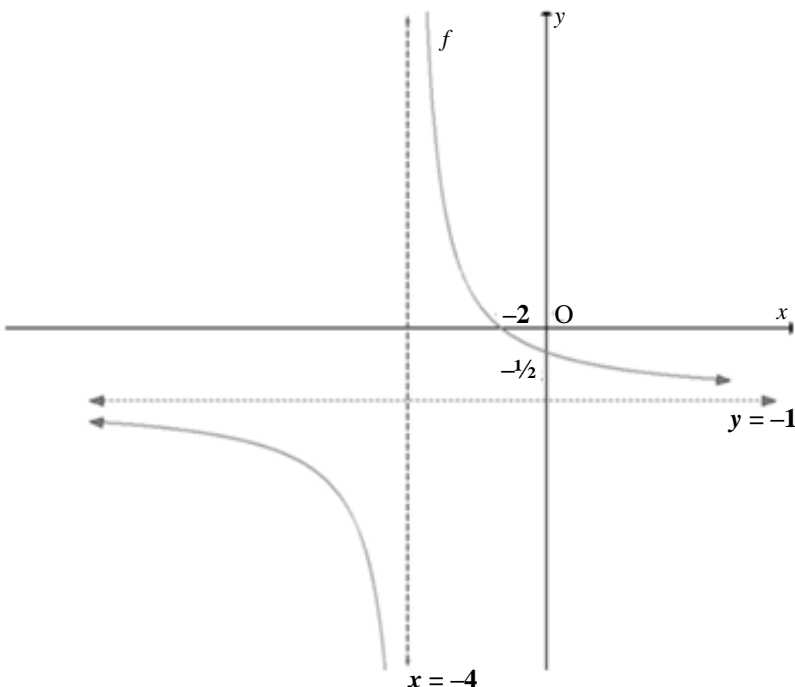
5.3	<p>Mervin:</p> $A = P(1 - ni)$ $A = 6\,000(1 + 5 \times 0,085)$ $A = R8\,550$ $\therefore 5\% \times R6\,000 = R300$ $\therefore \text{total} = R8\,550 + R300$ $A = R8\,850$ <p>Haley:</p> $i = 0,08 \div 4 = 0,02$ $n = 5 \times 4 = 20$ $A = P(1 - i)^n$ $A = 6\,000(1 + 0,02)^{20}$ $A = R8\,915,68$ <p>Haley will have the larger investment.</p> <p>NOTE: Only award the conclusion mark if a valid calculation has been done.</p>	<ul style="list-style-type: none"> ✓ correct substitution into correct formula ✓ value of A (Mervin) ✓ bonus value + final value of A (Mervin) ✓ correct substitution into correct formula ✓ value of A (Haley) ✓ conclusion 	(6)
[16]			

QUESTION 6

6.1	$x \in \mathfrak{R}$ <p style="text-align: center;">OR</p> $x \in (-\infty; \infty)$	<ul style="list-style-type: none"> ✓ answer 	(1)
6.2	$y = 0$	<ul style="list-style-type: none"> ✓ answer 	(1)
6.3	$f(x) = a \cdot b^x$ $144 = a \cdot \left(\frac{3}{4}\right)^2$ $a = 256$	<ul style="list-style-type: none"> ✓ substitution ✓ answer 	(2)

6.4	$g(x) = 256 \left(\frac{3}{4} \right)^{-x}$ <p>OR</p> $g(x) = 256 \left(\frac{4}{3} \right)^x$	✓ answer	(1)
6.5	Increasing function. If x increases, increases $f(x)$.	✓ answer ✓ explanation	(2)
6.6	$[f(x)]^2 - [f(-x)]^2 = a.f(2x) - a.f(-2x)$ <p>LHS</p> $[f(x)]^2 - [f(-x)]^2$ $[256 \left(\frac{3}{4} \right)^x]^2 - [256 \left(\frac{3}{4} \right)^{-x}]^2$ $= [256^2 \left(\frac{3}{4} \right)^{2x}] - [256^2 \left(\frac{3}{4} \right)^{-2x}]$ <p>RHS</p> $a.f(2x) - a.f(-2x)$ $= 256 [256 \left(\frac{3}{4} \right)^{2x}] - 256 [256 \left(\frac{3}{4} \right)^{-2x}]$ $= 256^2 \left(\frac{3}{4} \right)^{2x} - 256^2 \left(\frac{3}{4} \right)^{-2x}$	✓ substitution ✓ answer (LHS) ✓ answer (RHS)	(3)
			[10]

QUESTION 7

7.1	$x = -4$ $y = -4$	✓ answer ✓ answer	(2)
7.2	x-intercept: $f(x) = \frac{2}{x+4} - 1$ $0 = \frac{2}{x+4} - 1$ $x + 4 = 2$ $\therefore x = -2$ y-intercept: $f(x) = \frac{2}{x+4} - 1$ $y = \frac{2}{0+4} - 1$ $\therefore y = -\frac{1}{2}$	✓ make $y = 0$ ✓ x- value ✓ y-value	(3)
7.3	 <p>NOTE: The mark for shape is for the graph shifting from quadrants 1 towards 3.</p>	✓ shape ✓ asymptotes ✓ intercepts	(3)

7.4	$(-6 ; -2)$	✓ x-value ✓ y-value	(2)
7.5	$y \in \mathfrak{R}$ and $y \neq 1$ NOTE: Both conditions must be stated.	✓ answer	(1)
7.6	Reflection in the y-axis	✓ answer	(1)
			[12]

QUESTION 8

8.1	$y = a(x + p)^2 + q$ $\therefore 6 = a(0 - 1)^2 + 8$ $\therefore 6 = a + 8$ $\therefore a = -2$ $y = -2(x - 1)^2 + 8$ $y = -2(x^2 - 2x + 1) + 8$ $y = -2x^2 + 4x - 2 + 8$ $\therefore y = -2x^2 + 4x + 6$ NOTE: The mark for simplification is awarded either at the 3 rd or 2 nd last step.	✓ substitute points (1 ; 8) and (0 ; 6) ✓ value for a ✓ correct simplification	(3)
8.2	$j(x) = -2x^2 + 4x + 6$ $\therefore j(3) = 0 \dots(3;0)$ $\therefore j(1) = 8 \dots(1;8)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{8 - 0}{1 - 3}$ $\therefore m = -4$ NOTE: Candidates do not have to write the coordinates.	✓ coordinates of $j(3)$ and $j(1)$ ✓ correct substitution ✓ answer	(3)

<p>8.3</p>	<p>$j(x) = k(x)$</p> $-2x^2 + 4x + 6 = \frac{1}{3}x - 1$ $-6x^2 + 12x + 18 = x - 3$ $-6x^2 + 11x + 21 = 0$ $-6x^2 - 11x - 21 = 0$ $(x - 3)(6x + 7) = 0$ $x \neq 3 \text{ or } x = -\frac{7}{6}$ $k\left(-\frac{7}{6}\right) = -\frac{25}{18}$ $\therefore P\left(-\frac{7}{6}; -\frac{25}{18}\right)$ <p>NOTE: Answer need not be in coordinate form.</p>	<p>✓ equating</p> <p>✓ standard form</p> <p>✓ factors</p> <p>✓ x-values with exclusion</p> <p>✓ y-value</p>	<p>(5)</p>
<p>8.4</p>	$VW = -2x^2 + 4x + 6 - \frac{1}{3}x + 1$ $VW = -2x^2 + \frac{11}{3}x + 7$ $VW = -2\left(x^2 - \frac{11}{6}x - \frac{7}{2}\right)$ $VW = -2\left(x^2 - \frac{11}{6}x + \frac{121}{144} - \frac{121}{144} - \frac{7}{2}\right)$ $VW = -2\left(x - \frac{11}{12}\right)^2 + \frac{625}{72}$ $VW_{\max} = \frac{625}{72} \text{ of } 8,68 \text{ units}$ <p style="text-align: center;">OR</p>	<p>✓ method</p> <p>✓ simplification</p> <p>✓ method</p> <p>✓ answer</p> <p style="text-align: center;">OR</p>	

	$VW = -2x^2 + 4x + 6 - \left(\frac{1}{3}x - 1\right)$ $VW = -2x^2 + 4x + 6 - \frac{1}{3}x + 1$ $VW = -2x^2 + \frac{11}{3}x + 7$ <p>Turning point:</p> $x = \frac{\frac{11}{3}}{2(-2)}$ $\therefore x = \frac{11}{12}$ $VW_{\max} = \frac{625}{72} \text{ of } 8,68 \text{ units}$	<p>✓ method</p> <p>✓ simplification</p> <p>✓ method</p> <p>✓ answer</p>	(4)
8.5	8.5.1 $x < 1$	✓ answer	(1)
	8.5.2 $x \geq -1 \dots \text{but} \dots x \neq 3$ $\therefore -1 \leq x < 3 \dots \text{or} \dots x > 3$ NOTE: Both answers must be completely correct.	✓✓ answers	(2)
8.6	$k < 8$	✓✓ answer	(2)
			[20]

QUESTION 9

ANSWERS CAN EITHER BE IN FRACTION OR DECIMAL FORM.

<p>9.1</p>	<p>Tree 1</p>	<p>Tree 2</p>	<p>✓ Tree 1 ✓ Tree 2</p>	<p>(2)</p>
<p>9.2</p>	<p>$P(C \text{ and } C) = 0,7 \times 0,7$ $P(C \text{ and } C) = 0,49$ NOTE: Answer only, full marks.</p>	<p>✓ correct method ✓ answer</p>	<p>(2)</p>	
<p>9.3</p>	<p>$P(C) \text{ or } P(L \text{ and } C)$ $= 0,7 + (0,2 \times 0,9)$ $= 0,88$ NOTE: Answer only, full marks.</p>	<p>✓ 0,7 ✓ $+(0,2 \times 0,9)$ ✓ answer</p>	<p>(3)</p>	
			<p>[7]</p>	

QUESTION 10

10.1	10.1.1	$P(A \text{ and } C) = 0$	✓ answer	(1)
	10.1.2	$P(B \text{ and } C)$ $= P(B) \times P(C)$ $= 0,4 \times 0,2$ $= 0,08$ NOTE: Answer only, full marks.	✓ correct formula or correct substitution ✓ answer	(2)
	10.1.3	$P(A \text{ and } B)$ $= P(A) \times P(B)$ $= 0,3 \times 0,4$ $= 0,12$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $= 0,3 + 0,4 - 0,12$ $P(A \text{ or } B) = 0,58$	✓ correct substitution ✓ value of $P(A \text{ and } B)$ ✓ correct substitution ✓ answer	(4)
10.2	10.2.1	$8 + 10 + x + 65 + 3 + 5 + 5 + 2 = 103$ $x = 5$	✓ answer	(1)
	10.2.2	$P(\text{any2}) = \frac{n(\text{any2})}{n(S)}$ $P(\text{any2}) = \frac{10+65+5+5}{103}$ $P(\text{any2}) = \frac{85}{103}$ $P(\text{any2}) = 0,83$	✓ $\frac{n(\text{any2})}{n(S)}$ with correct substitution ✓ answer (any shape)	(2)
10.3		$\frac{9}{80} \times \frac{(17+8)}{79} + \frac{(17+8)}{80} \times \frac{9}{79}$ $\frac{9}{80} \times \frac{25}{79} + \frac{25}{80} \times \frac{9}{79}$ 0,0712 ... $\approx 0,07$	✓ correct substitution ✓ simplification ✓ answer	(3)
				[13]
TOTAL:				150