



**GAUTENG PROVINCE**

EDUCATION  
REPUBLIC OF SOUTH AFRICA

**PROVINCIAL EXAMINATION**  
**NOVEMBER 2023**  
**GRADE 11**

**MATHEMATICS**

**PAPER 2**

**TIME: 3 hours**

**MARKS: 150**

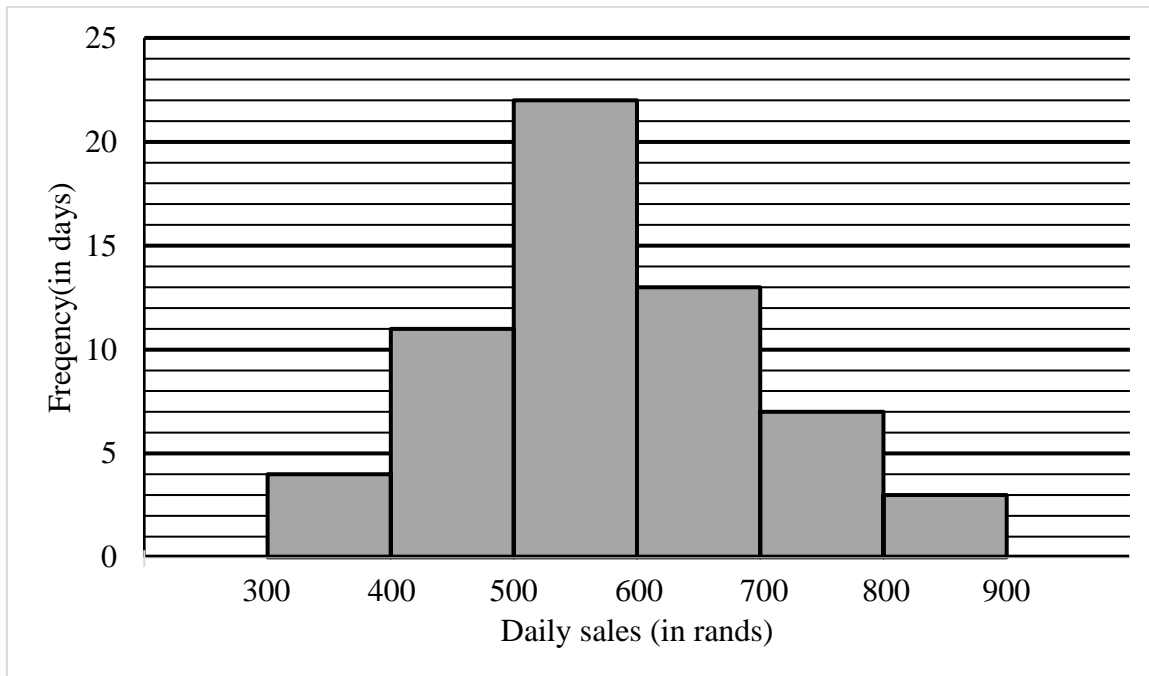
**12 pages + 1 information sheet and 3 answer sheets**

**INSTRUCTIONS AND INFORMATION**

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
7. Answer sheets for answering QUESTIONS 1.1, 1.2 and 9.1 are provided at the end of the question paper. Write your name in the spaces provided on the ANSWER SHEETS and submit them together with your ANSWER BOOK.
8. Diagrams are NOT necessarily drawn to scale.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

**QUESTION 1**

A small tuck shop displayed a record of daily sales in rands, for the past two months (60 days) using the following histogram.



1.1 Complete the following table. Use the table provided on ANSWER SHEET A.

Class interval	Frequency	Cumulative frequency
$300 < x \leq 400$	4	4
$400 < x \leq 500$		
$500 < x \leq 600$		
$600 < x \leq 700$		
$700 < x \leq 800$		
$800 < x \leq 900$		

(2)

1.2 Draw a cumulative frequency curve for the sales over the past two months. Use the graph sheet provided on ANSWER SHEET B.

(3)

1.3 Use the graph in QUESTION 1.2 and determine the estimated median value for the daily sales.

(2)

1.4 The tuck shop must make R475 in sales per day to break-even. On how many days did the tuck shop make a profit?

(2)

1.5 On the first day of the following month, the tuck shop made R725 in sales. Does this day lie within the top 25% of sales from the previous two months?

(2)

**[11]**

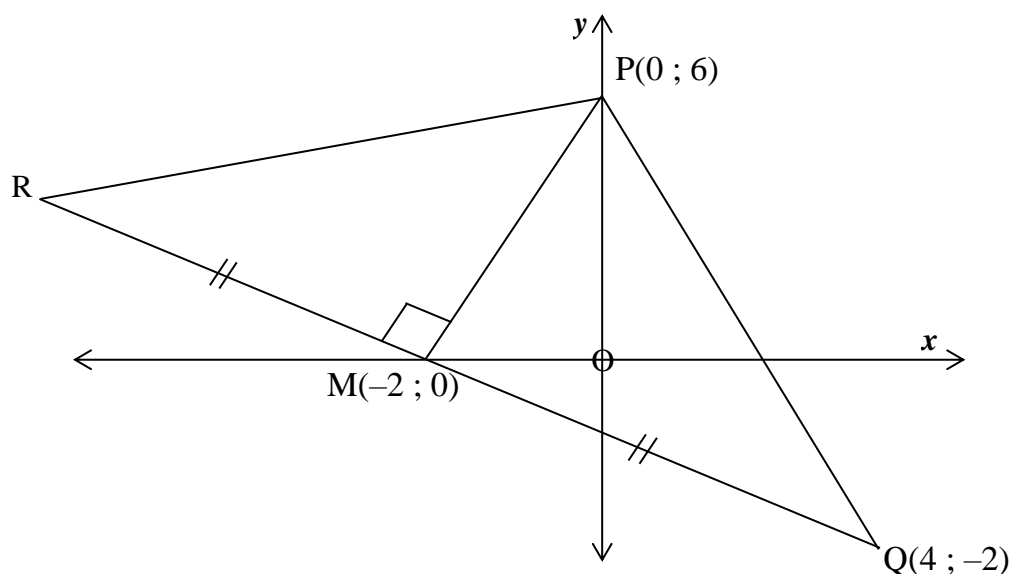
**QUESTION 2**

Five data values are represented as follows:  $2k ; k + 1 ; k + 2 ; k - 3 ; 2k - 2$

- 2.1 If the mean of the data set is 15, show that  $k = 11$ . (3)
- 2.2 Calculate the standard deviation ( $\sigma$ ) for this data, rounded-off to one decimal place. (2)
- 2.3 If  $t$  units are subtracted from each data value in the set, without further calculation, explain how the mean and standard deviation would be affected in terms of  $t$ . (2)
- [7]**

**QUESTION 3**

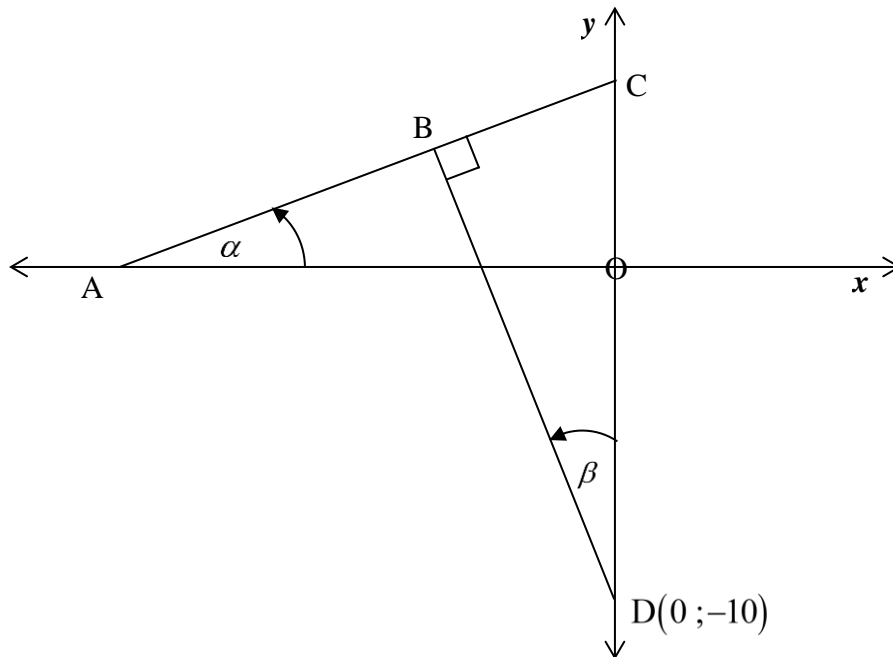
In the diagram below,  $\Delta RPQ$  is drawn, with  $P(0 ; 6)$ ,  $Q(4 ; -2)$ .  $M(-2 ; 0)$  is the midpoint of  $RQ$ .



- 3.1 Determine the gradient of the line  $MQ$ . (2)
- 3.2 Determine the equation of the line  $MP$ , in the form  $y = mx + c$ . (3)
- 3.3 Determine the coordinates of  $R$ . (3)
- 3.4 Calculate the length of  $PQ$ , in simplified surd form. (2)
- 3.5 Given that  $RPQT$  is a parallelogram, determine the coordinates of  $T$  if point  $T$  is in the third quadrant. (2)
- 3.6 Explain why  $RPQT$  is a rhombus. (2)
- [14]**

QUESTION 4

In the diagram below, lines AC and BD intersect at B, where  $AC \perp BD$ . C and D lie on the y-axis, while A lies on the x-axis. The equation of AC is  $py - x - 5p = 0$ , while  $\alpha$  is the angle of inclination for AC, with  $\widehat{CDB} = \beta$ .



- 4.1 Determine the coordinates of C. (3)
- 4.2 If the gradient of AC is  $\frac{1}{2}$ , show that  $p = 2$ . (2)
- 4.3 Calculate the coordinates of B. (5)
- 4.4 Determine the size of  $\alpha$ . (2)
- 4.5 Hence, or otherwise, prove that ABOD is a cyclic quadrilateral. (3)
- 4.6 Determine the coordinates of the centre of the circle which passes through D, B and C. (2)

[17]

**QUESTION 5**

5.1 If  $3\sin \beta = 2$ , and  $\cos \beta < 0$ , determine with the aid of a diagram and **without the use of a calculator**, the value of:

5.1.1  $3\cos^2 \beta - 1$  (4)

5.1.2  $\tan(-\beta - 180^\circ)$  (3)

5.2 Given:  $t \cos 15^\circ = 4$

Determine the following in terms of  $t$ , **without the use of a calculator**:

5.2.1  $\sin 15^\circ$  (3)

5.2.2  $\sin 75^\circ$  (2)

5.2.3  $1 - \tan^2 15^\circ$  (Give the answer as a single fraction.) (3)

5.3 Simplify the following to a single trigonometric function, without the use of a calculator.

$$\frac{\cos(90^\circ - \alpha) \sin(-\alpha - 540^\circ)}{\tan 225^\circ + \sin \alpha \cdot \sin(180^\circ + \alpha)} \quad (5)$$

5.4 Given:  $1 - \cos \theta = 2 \sin^2 \theta$

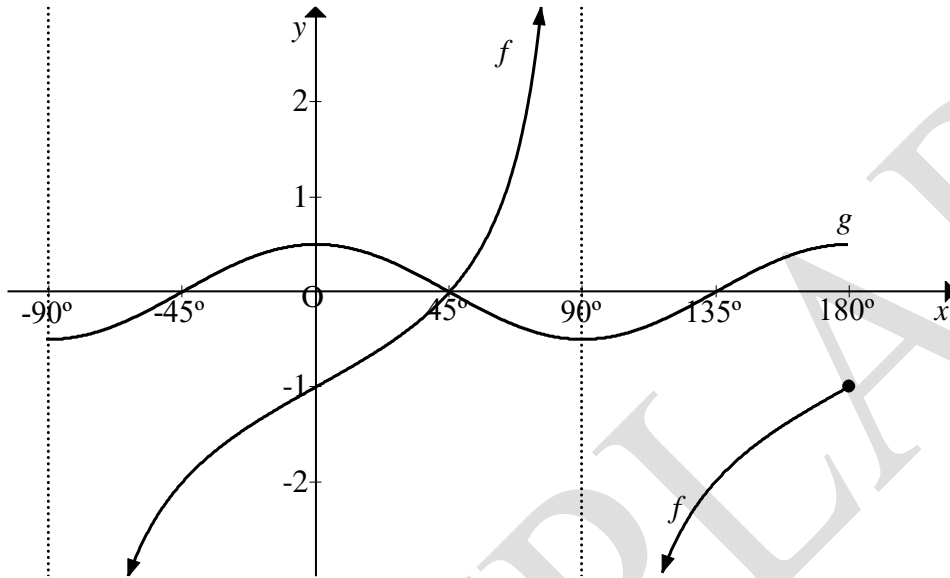
5.4.1 Show that the equation can be written as:  $(2\cos \theta + 1)(\cos \theta - 1) = 0$ . (2)

5.4.2 Hence, determine the general solution of  $(2\cos \theta + 1)(\cos \theta - 1) = 0$ . (5)

**[27]**

QUESTION 6

In the diagram below, the graphs of  $f(x) = \tan x - 1$  and  $g(x) = -\frac{1}{2} \cos 2x$  are drawn, where  $x \in [-90^\circ ; 180^\circ]$ .

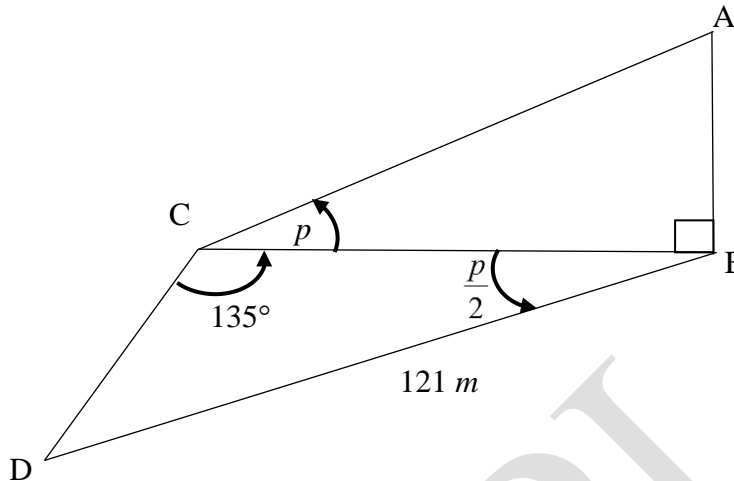


- 6.1 Write down the period of  $g$ . (1)
- 6.2 Determine the range of  $g(x)$ . (2)
- 6.3 Use the graphs to determine graphically the values of  $x$  where:
- 6.3.1  $f(x) \geq 0$  (2)
- 6.3.2  $f(x) \cdot g(x) > 0$  (2)
- 6.3.3  $2 \tan x + \cos 2x = 2$  (3)
- 6.4 If  $h(x) = \frac{\sin x + \cos x}{\cos x}$ , describe the vertical translation of  $h$  from  $f$ . (2)
- 6.5 Determine the maximum value of  $p(x) = 4g(x)$ . (2)

[14]

**QUESTION 7**

In the figure below,  $\triangle ABC$  is drawn where  $AB \perp BC$  and  $\hat{ACB} = p$ , with  $CB = 3AB$ .  $\triangle DCB$  is drawn such that  $\hat{DCB} = 135^\circ$ , and  $BD = 121 \text{ m}$

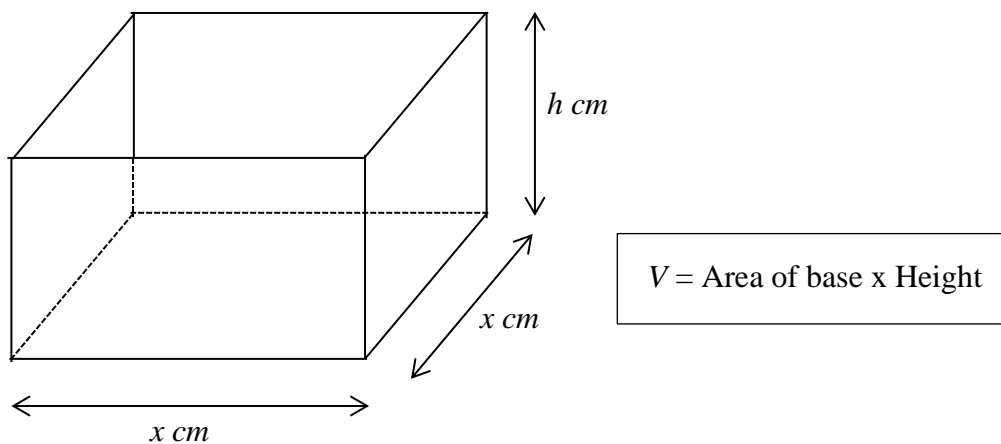


7.1 Determine the value of  $p$ , correct to 3 decimal places. (2)

7.2 If  $p = 18,4^\circ$ , determine the length of CD. (3)  
[5]

**QUESTION 8**

The diagram below represents an open tank with a square base (side dimensions of  $x \text{ cm}$ ) and a height of  $h \text{ cm}$ . The tank has a volume of  $490 \text{ cm}^3$ .



8.1 Determine the height ( $h$ ) of the tank in terms of  $x$ . (2)



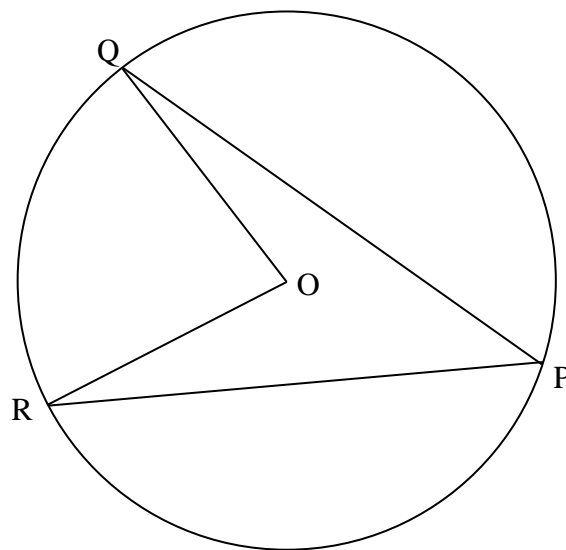
8.2 Show that A, the external surface area of the tank, is given by the formula:

$$A = x^2 + \frac{1\ 960}{x} \text{ cm}^2 \quad (2)$$

8.3 Given that the tank is 10 cm high, calculate the external surface area of the tank. (4)  
[8]

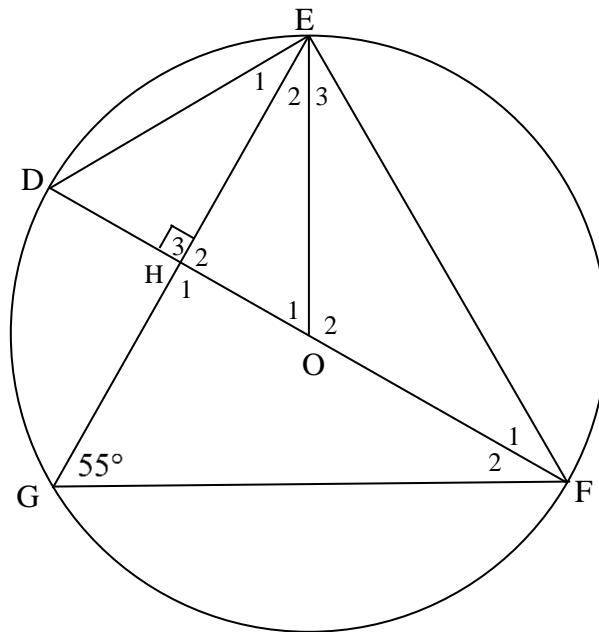
**QUESTION 9**

9.1 In the diagram below, O is the centre of the circle passing through P, Q and R. Chords PQ and PR are drawn, with OQ and OR joined.



Use the diagram provided in ANSWER SHEET C to prove the theorem that states that the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle, that is  $\hat{ROQ} = 2\hat{P}$  (6)

- 9.2 In the diagram below, DF is a diameter of the circle with centre O. Chord EG intersects DF at H such that  $DF \perp EG$ . Chords EF and GF are drawn.  $\hat{E}GF = 55^\circ$ .



- 9.2.1 Determine, giving reasons, the size of:

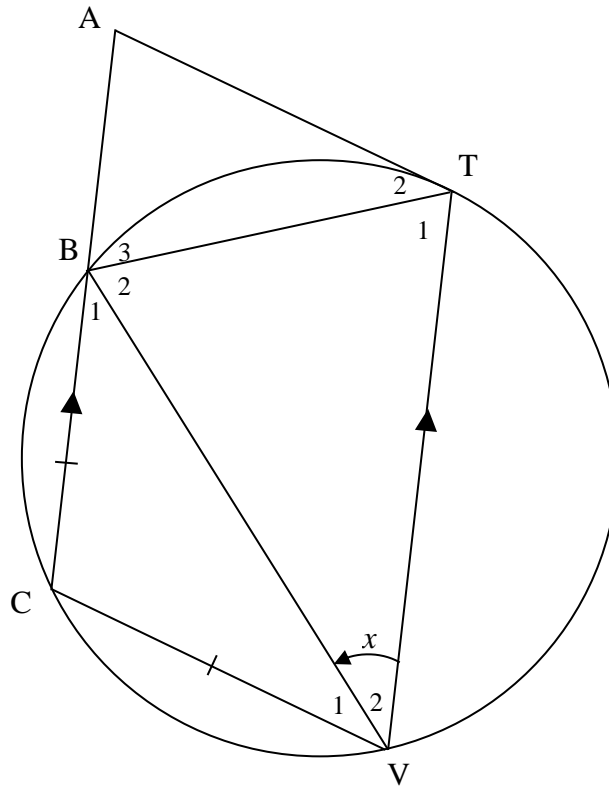
- (a)  $\hat{D}$  (2)
- (b)  $\hat{O}_2$  (2)
- (c)  $\hat{E}_2$  (2)
- (d)  $\hat{E}_3$  (3)

- 9.2.2 Determine the length of OH, if the diameter of the circle is 10 units and  $GE = 9,4$  units.

(4)  
[19]

QUESTION 10

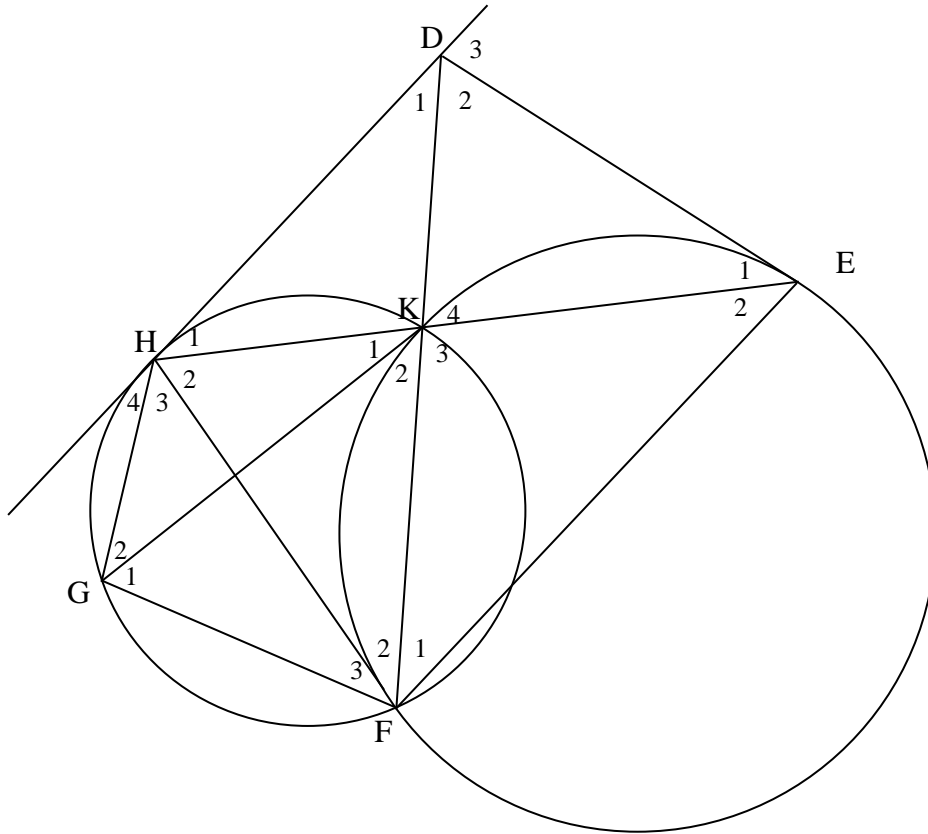
In the diagram below,  $AT$  is a tangent to the circle at  $T$ . Chords  $BT$ ,  $BV$  and  $VC$  are drawn.  $CB$  is extended to  $A$ , such that  $AC \parallel TV$ .  $BC = CV$  and  $\widehat{BVT} = x$ .



- 10.1 Determine, with reasons, 3 angles equal to  $x$ . (6)
- 10.2 If  $ATVC$  is a parallelogram, prove that  $AT = BT$ . (5)
- 10.3 Determine the size of  $x$ . (2)
- [13]

**QUESTION 11**

DE is a tangent to the larger circle at E. DH is a tangent to the smaller circle at H. Chord HK is extended to meet the larger circle at E. F and K are the points of intersections between the circles, with FK produced to D. GK is a chord of the smaller circle with FE a chord of the larger circle. HF, GH and GF are joined.



11.1 Complete the following:

$$\hat{D}_3 = \dots + \dots \quad (\text{ext } \angle \Delta) \quad (1)$$

11.2 Prove that DEFH is a cyclic quadrilateral. (4)

11.3 Prove that DF bisects  $\hat{HFE}$ . (3)

11.4 If  $\hat{K}_1 = \hat{E}_1$ , prove that GK is a tangent to the larger circle at K. (7)

**[15]**

**TOTAL: 150**

## INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1 \quad S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

Name and Surname: \_\_\_\_\_ Grade: \_\_\_\_\_

**ANSWER SHEET A**

**QUESTION 1**

1.1

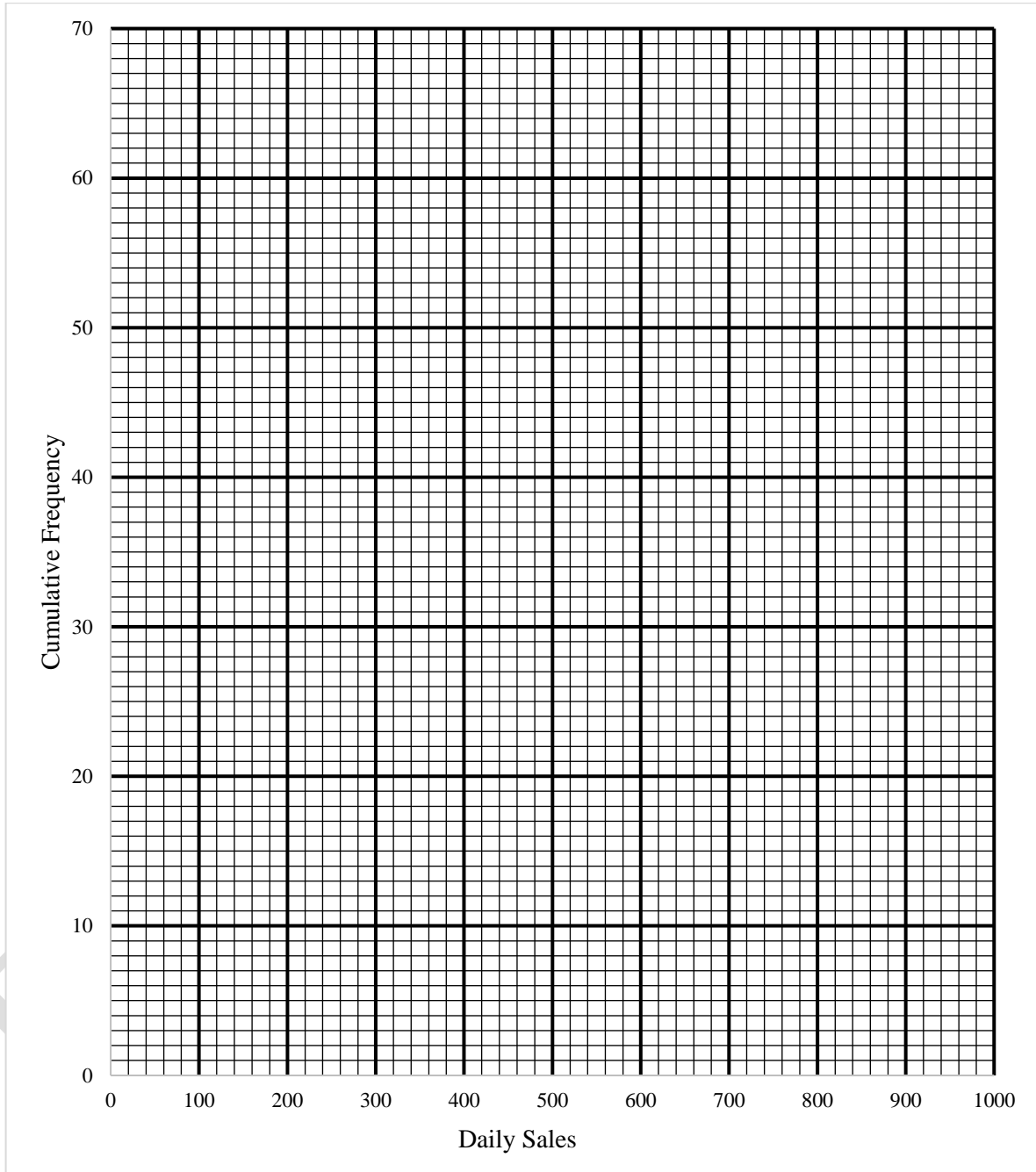
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$400 < x \leq 500$		
$500 < x \leq 600$		
$600 < x \leq 700$		
$700 < x \leq 800$		
$800 < x \leq 900$		

(2)

Name and Surname: \_\_\_\_\_ Grade: \_\_\_\_\_

**ANSWER SHEET B**

1.2

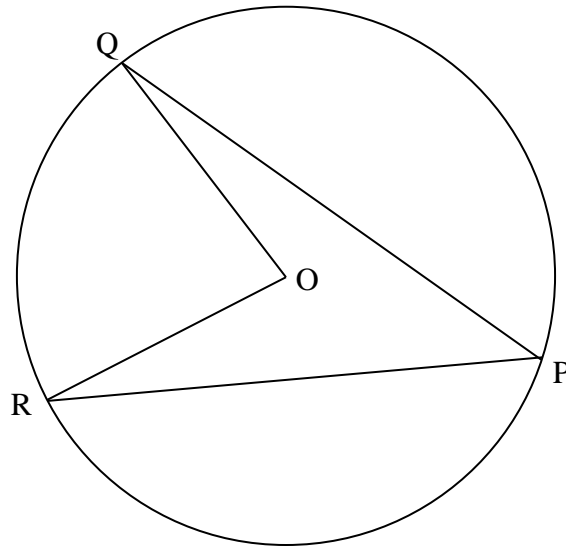


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**ANSWER SHEET C**

**QUESTION 9**

9.1



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