NATIONAL SENIOR CERTIFICATE

PUSH – ONE INTERVENTION PROGRAM

MATHEMATICS

GRADE 12

LAST PUSH

2022

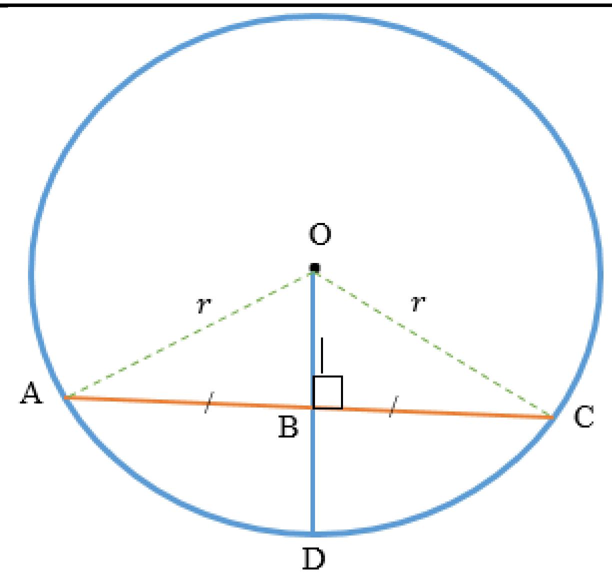
EUCLIDEAN GEOMETRY
CIRCLE GEOMETRY

<table>
<tr><th>CONCEPT</th><th>HOW TO LEARN IT?</th><th>RELEVANT FORMULAE AND KEYWORDS</th></tr>
<tr><td rowspan="3">Theorem 1:

Line segment through centre, mid-point and converse.

Theorem 2:

Perpendicular bisector of a chord.</td><td>Learn and use Congruency; $90°$; H; S or A; A; S

Lookout for the midpoint, then conclude $90°$ at the midpoint.</td><td>
</td></tr>
<tr><td>This theorem brings a right-angle triangle in the picture, so Pythagoras is important and the midpoint
(as in Analytical Geometry).</td><td>Look out for the centre or midpoint on a chord as well as mid-points of two sides of a triangle.

Look out for the centre</td></tr>
<tr><td>Congruency to prove the theorem (Theorem 1).

The use of Pythagoras as this theorem brings along $90°$ angle and a right-angled triangle.

Ensure that two sides of a triangle are bisected by the same line.

First Midpoint of a line, then $90°$ will follow.

You need to ensure that there is a centre and $90°$, then the line will pass through the centre.</td><td></td></tr>
</table>

<table>
<tr>
<td></td>
<td colspan="2">If $\hat{A} = \hat{B}$ and $\hat{B} = \hat{C}$, then $\hat{A} = \hat{C}$

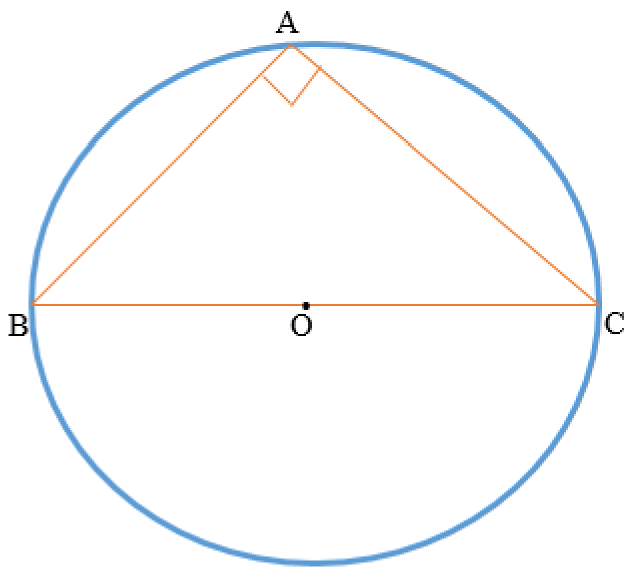
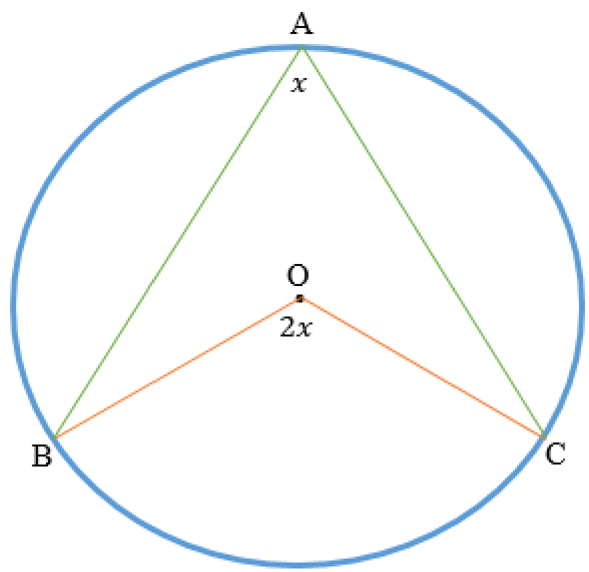
If $\hat{A} + \hat{B} = \hat{C}$ and $\hat{A} + \widehat{D} = \hat{C}$, then $\hat{B} = \hat{C}$.

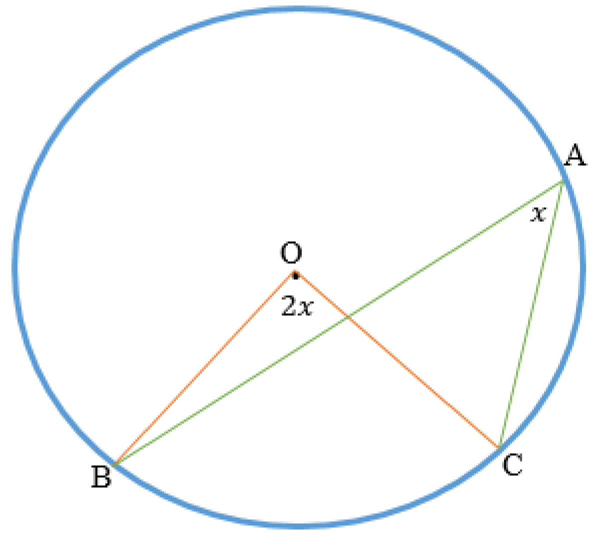
Applications of these rules without numerical values, that is given variables such as x or y.

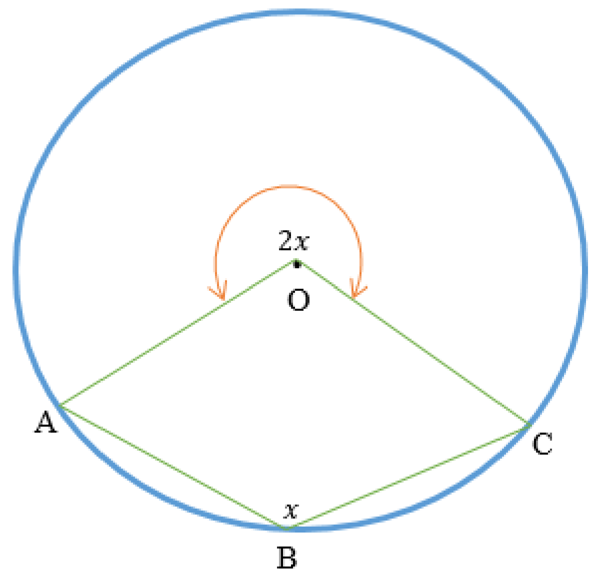
Expressing one variable in terms of the other information, that is changing the subject of the formula of the equation.

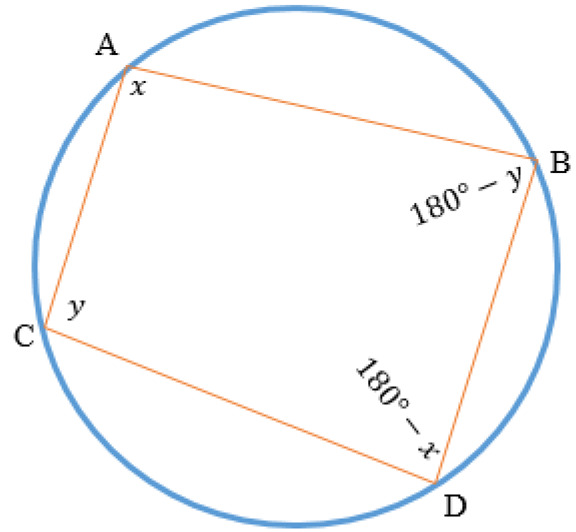
Substitution of equal quantities.</td>
</tr>
<tr>
<td>Theorem 3:

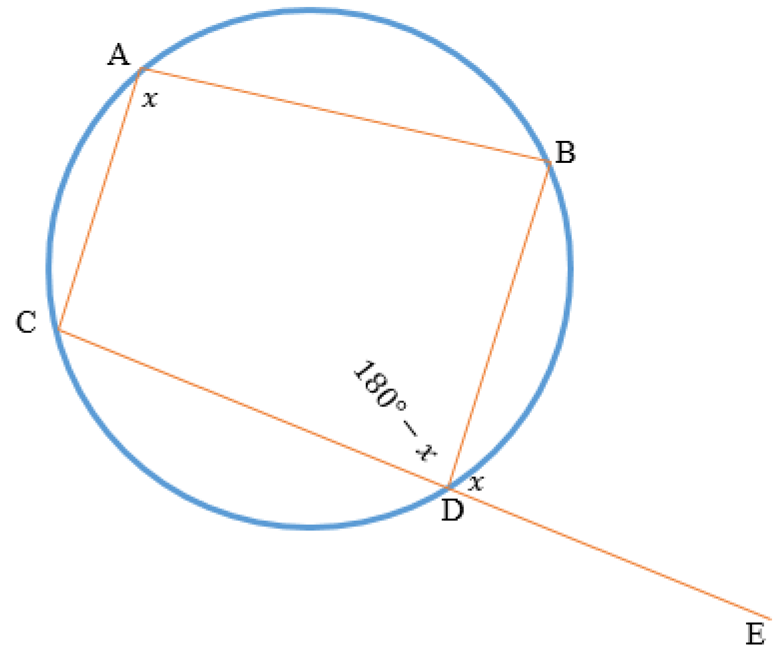
Angle at centre is twice angle at circumference.

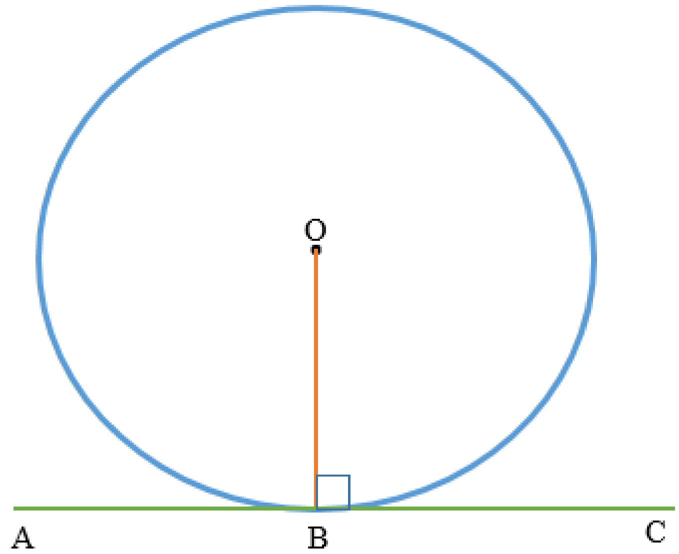
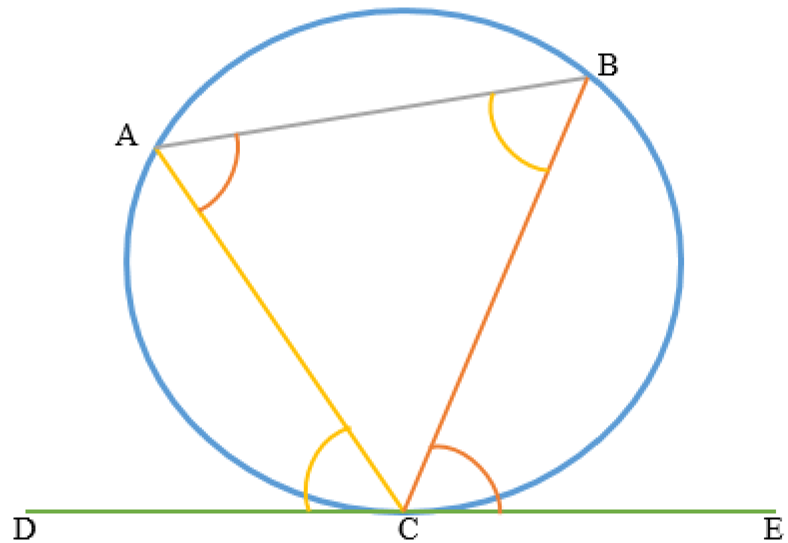
Diameter subtends right angle at circumference.</td>
<td>Learn exterior angle of a triangle and properties of an Isosceles triangle. The proof of this theorem is for examination purpose. (That is, it is a possible exam question).

Use the previous theorem or lookout for the centre, diameter and the angle subtended by the line that passes through the centre (diameter).</td>
<td></td>
</tr>
</table>

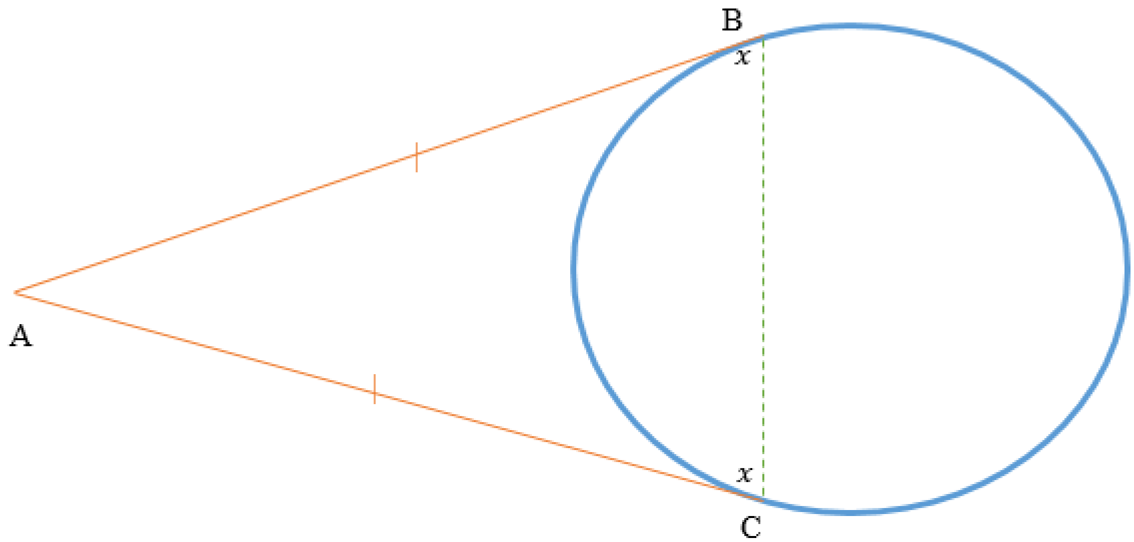
<table>
<tr><td rowspan="2"></td><td></td><td>


</td></tr>
<tr><td>Arc: The angle at the centre and the one on the</td><td>Centre</td></tr>
</table>

	circle must be on the same arc or chord. This is a special case of the above theorem; it also brings about a 90° angle into the picture a right-angled triangle.	Centre
	Always lookout for an arc that supports the angle at the centre, then look for the angle at the circle that is also supported by the arc. (Think about subtend or support, as a shouting mouth, shouting at the arc or chord).	
Opposite angles of a cyclic Quadrilateral Exterior angle of a cyclic quadrilateral. Proving that a quadrilateral is cyclic.	The proof of this theorem is for examination purpose. For the proof use the theorem about the angle at the centre and a revolution. Look out for the straight line coming from one of the vertices of the cyclic quad.	

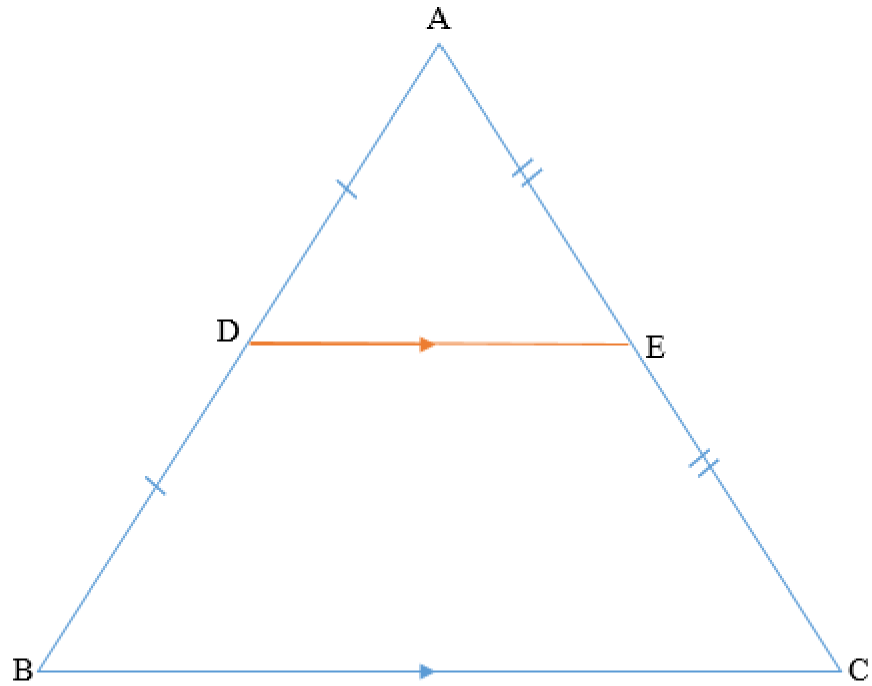
		
	This theorem brings = 180° into the picture. There are three options: Lookout to see if you can have opposite angles of a quad = 180°, then that quad will be cyclic. Lookout to see if you can have ext. angle of a quad = interior opposite angle, then the quad will be cyclic. Lookout to see if you can have two angles that are subtended by a line, then the four vertices will be a cyclic quad.	

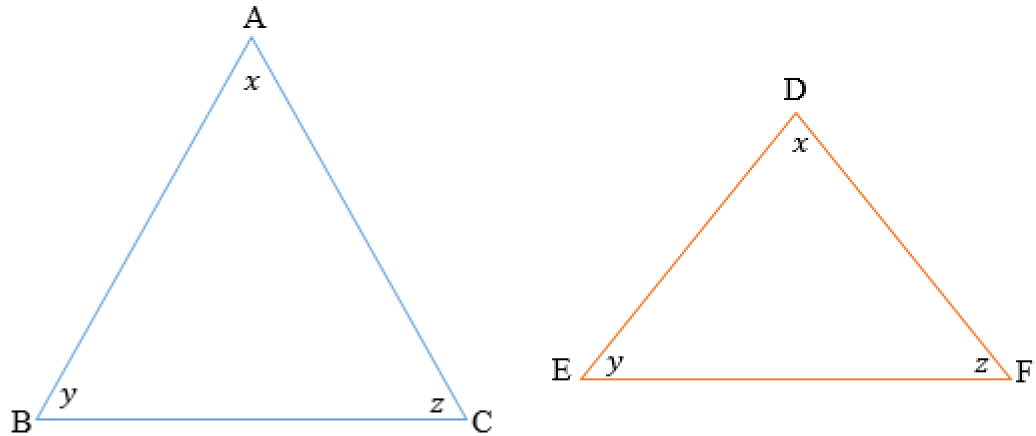
	Two properties of the cyclic quad.	
Tan-chord theorem Tangents from same point Tangent perpendicular to radius	Proof for examination purposed. To prove: use the diameter (angle at the centre), Tangent Radius Theorem (Tan – Radius), sum of angles of a triangle and second logical reason of addition.	 



B
x
A
x
C

	Divide the circle at the point of contact (between the chord and the tangent) to be able to identify the alternate segment, then focus on the chord and look for any angle that is supported by the chord in the other segment.	Tangent (see solving riders).
	Three properties of the tangent to a circle.	
Solving riders	Lookout for the key words connected to theorems in the explicitly given information, for instance, centre (connected theorems, include theorem 1, 2, angle at the centre together(diameter). Tangent (connected theorems include, (tan-chord, tan-radius and tangents drawn from the same point) and Cyclic Quad. (Connected theorems, include opposite angles are supplementary and ext. angle of a cyclic quad = to the opposite interior angle). Examine the implicitly given information from the diagram, then make conclusion on what you get from the diagram. Create a short list of statements and reasons, see if you cannot make logical conclusion from the list using important logical reasoning. Lookout for the angles on the same segment in the diagram. Lookout for angles on equal chords (the angles are	

	equal). Then answer the questions.	
	See above Solving riders.	
	The more you practice, the better you become.	
Theorem 1: Line drawn parallel to one side of triangle. The midpoint theorem	For examination purpose. Remember that a triangle can have its height inside the triangle or outside the triangle. The area of triangle between parallel lines and share a base have the equal area. After the constructions rotate the two sides that are not parallel to any of the lines so that it is horizontal, you will see the heights that are being shared by different triangles. Lookout for the midpoint of two sides of the triangle.	
	Properties (the conclusion of the theorem, the part that follows the word **then** in the theorem) of the theorem.	
Theorem 2:	Examination purpose. Use congruency S: A: S to create a line parallel to one side of a triangle, so that you can form proportions using one triangle.	

<table>
<tr><td>Equiangular triangles are similar.</td><td colspan="2">Application: Remember that this theorem requires that you use more than one triangle and the order in naming the triangles is also important as well.
Otherwise, lookout for corresponding sides to prove similarity.


ΔABC|||ΔDEF</td></tr>
<tr><td></td><td>Do plenty of examples.</td><td></td></tr>
<tr><td>Theorem of Pythagoras</td><td>Proof is not required for examination purpose. Otherwise, first write down the sum of the squares of the two side = the square of the hypotenuse.

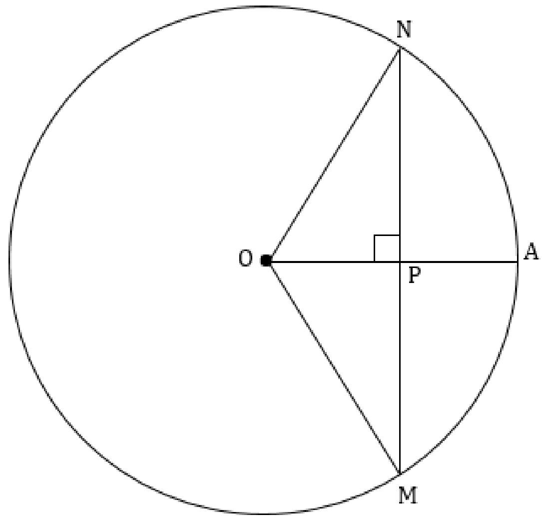
Then look out for either equal sides from the diagram or the sum of two sides making one side.</td><td></td></tr>
<tr><td></td><td>Do plenty of examples.</td><td></td></tr>
<tr><td>Proportion</td><td>Work with one triangle</td><td></td></tr>
<tr><td>Similarity</td><td>Work with two triangles. Remember after similarity (equiangular) you also get proportion, in this case use
two triangles. If asked to prove proportion look for the allocated marks, if the marks are at least 5,</td><td></td></tr>
</table>

	then start by proving similarity using (equiangular) by rearranging the proportion as follows, the first side on the right becomes the denominator on the left and the second side on the left becomes the denominator on the right, in that way you the numerators come from the same triangle and the denominators come from the second triangle. E.g. if asked to prove: $AB \times PQ = PR \times BC$ Then rearranging becomes: $\frac{AB}{PR} = \frac{BC}{PQ}$ so that the triangles are triangle ABC the numerators and triangle PQR from the denominators.	

TUTORIAL 1

Activity 1

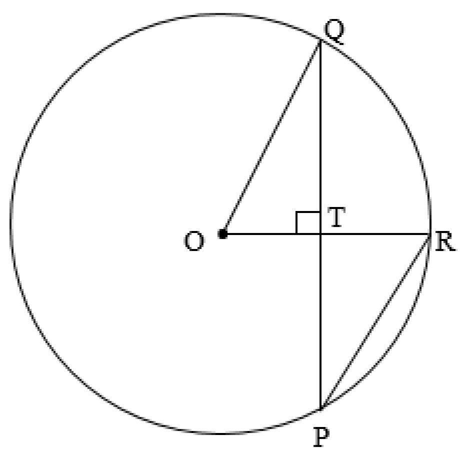
In the diagram, O is the centre of the circle NAM and $OPA \perp MPN$. $MN = 48$ units and $OP = 7$ units.



Calculate, with reason, the length of PA. **[5]**

Activity 2

In the diagram below, PQ is the chord of circle O. $OR \perp PQ$ and OR intersects PQ at T. If the radius of the circle is 13 cm and $PT = 12$ cm.



Calculate the length of:

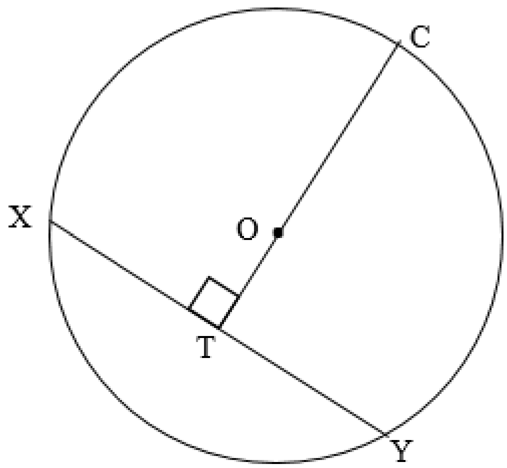
2.1 PQ (2)

2.2 PR (4)

Activity 3

In the diagram drawn below, O is the centre of the circle XCY. $\mathrm{COT} \perp \mathrm{XY}$.

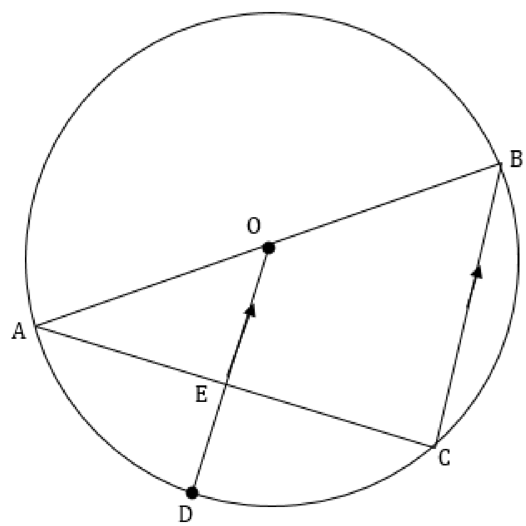
$\mathrm{OC} = r$ and $\mathrm{XY} = \frac{3}{2}r$.



Prove, stating reasons, that $\mathrm{CT} = \frac{4+\sqrt{7}}{4}r$. **[6]**

Activity 4

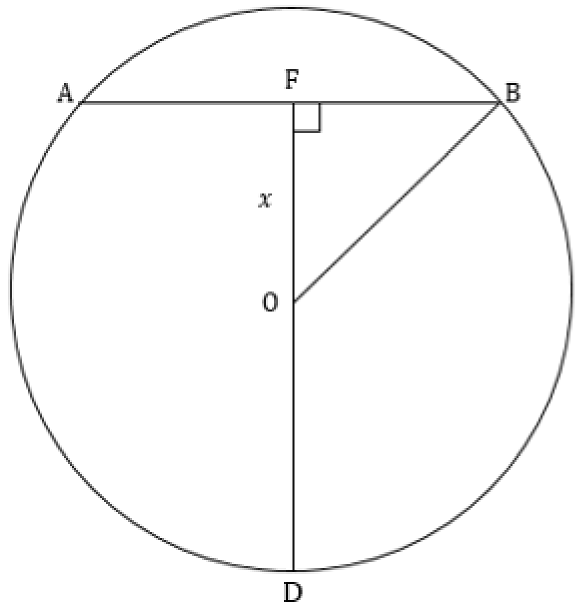
AB is a diameter of the circle ABCD. OD is drawn parallel to BC and meets AC at E.



If the radius is 10 cm and $\mathrm{AC} = 16$ cm, calculate the length of ED. **[5]**

Activity 5

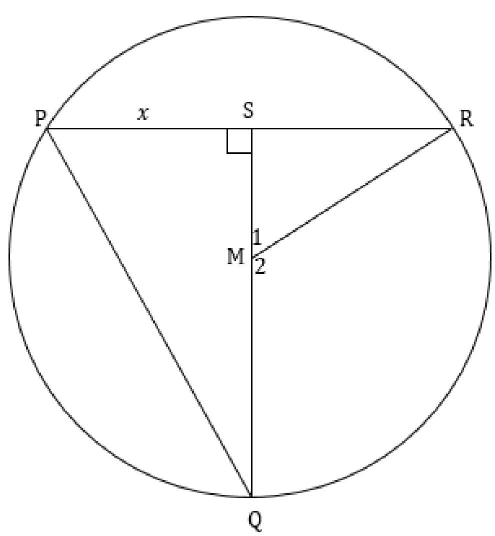
In the diagram, O is the centre of circle ABD. F is a point on chord AB such that $DOF \perp AB$. $AB = FD = 8$ cm and $OF = x$ cm.



Determine the length of the radius of the circle. **[5]**

Activity 6

In the diagram, PR and PQ are equal chords of the circle with centre M. QS is perpendicular to PR at S. $PS = x$ cm and MR is drawn.



6.1 Express, giving reasons, QS in terms of x. (5)

6.2 If $x = \sqrt{12}$ and $MS = 1$ unit, calculate the length of the radius of the circle. (2)

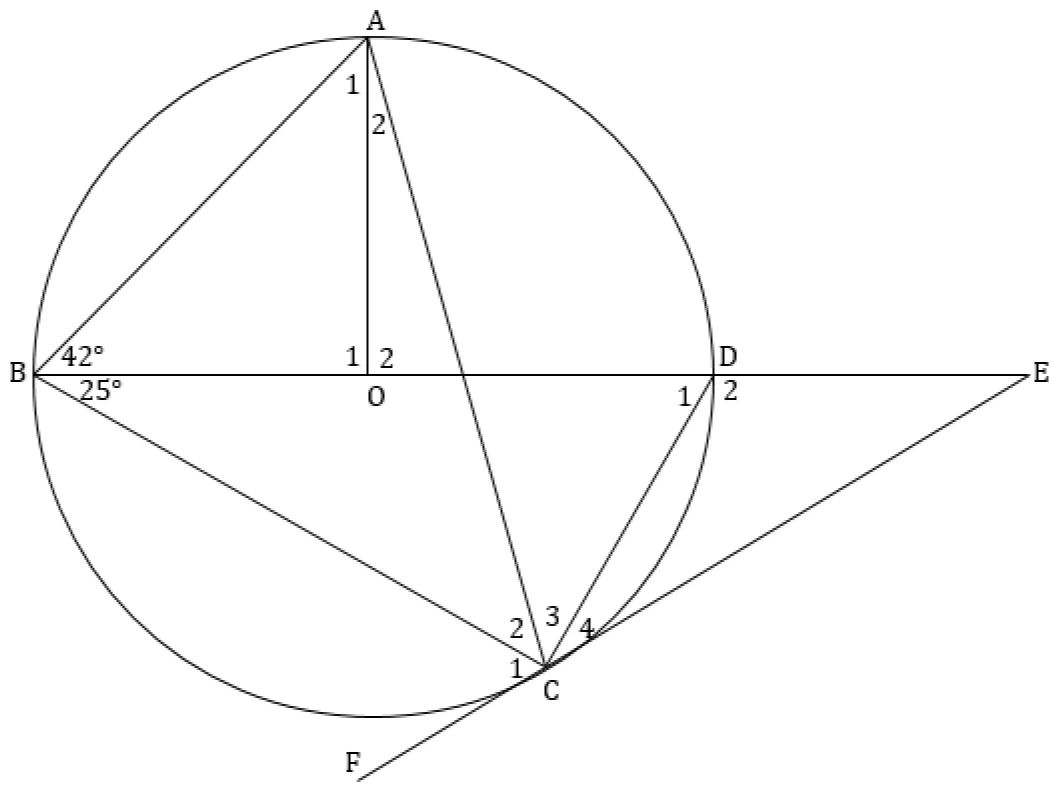
6.3 Calculate, giving reasons, the size of $\widehat{P}$. (5)

[12]

TUTORIAL 2

Activity 1

In the diagram below, the circle with centre O passes through A, B, C and D such that BOD is a diameter. BD is extended to E such that FCE is a tangent to the circle at C. $A\hat{B}E = 42°$ and $D\hat{B}C = 25°$



Calculate:

1.1 $B\hat{C}D$ (2)

1.2 $\hat{A}_1$ (2)

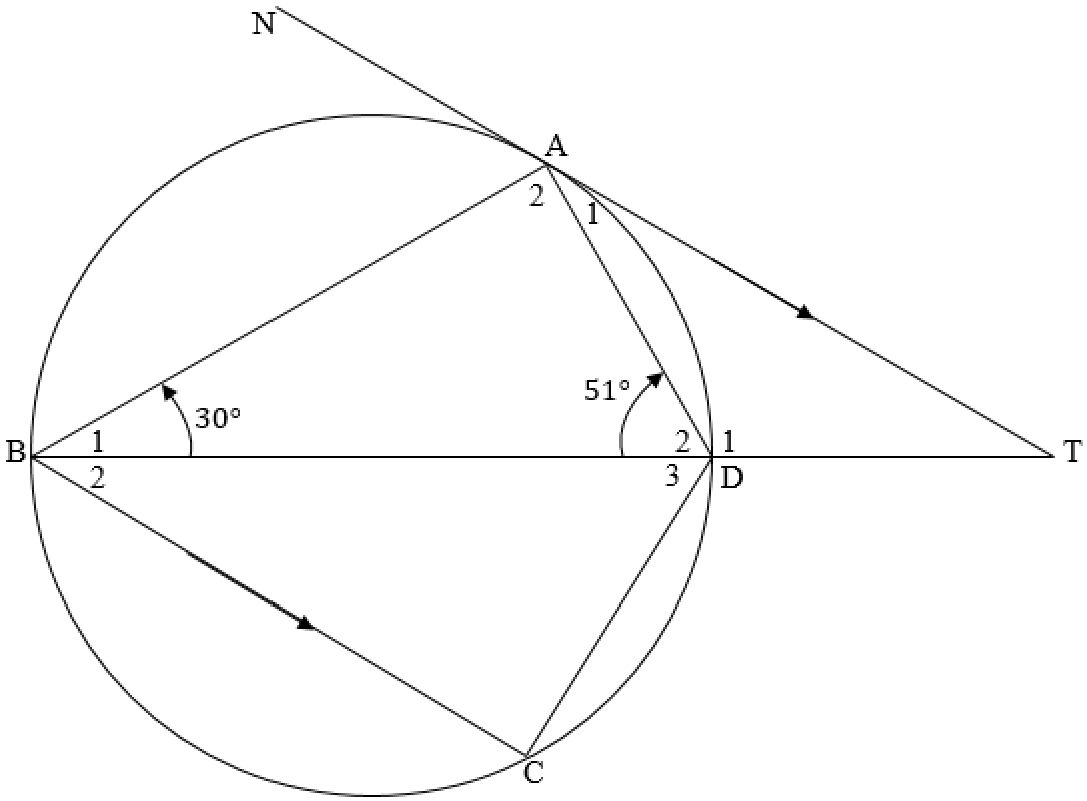
1.3 $\hat{O}_2$ (2)

1.4 $\hat{C}_4$ (2)

[8]

Activity 2

In the diagram, TAN is a tangent to the circle at A. ABCD is a cyclic quadrilateral. BD is drawn and produced to meet the tangent at T. $\hat{B}_1 = 30°$ and $\hat{D}_2 = 51°$. $TAN || CB$.



Calculate, giving reasons, the size of:

2.1 $\hat{A}_1$ (2)

2.2 $\hat{T}$ (2)

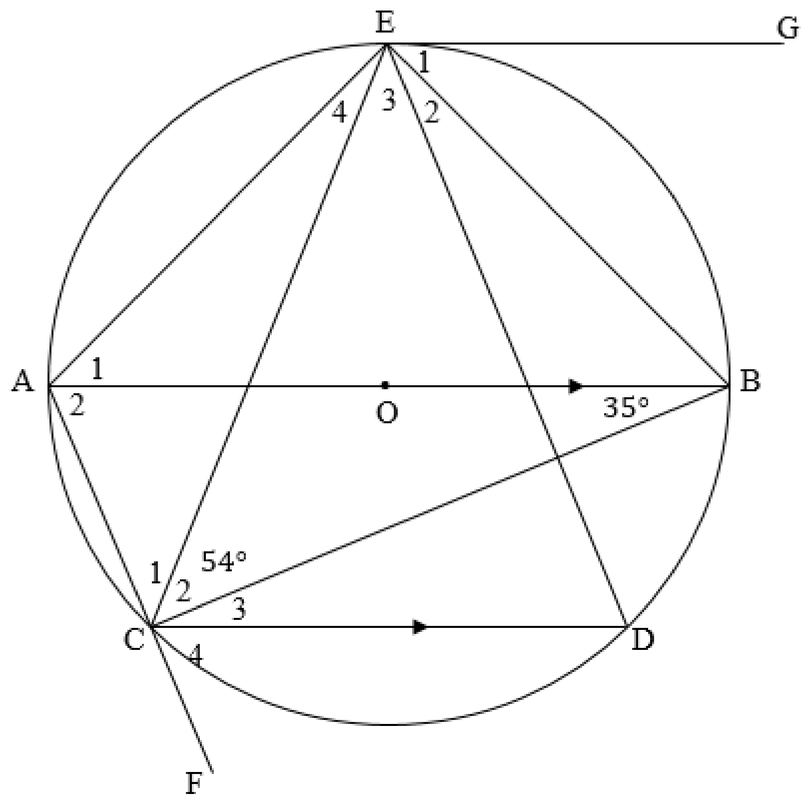
2.3 B_2 (2)

2.4 $\hat{C}$ (2)

[8]

Activity 3

O is the centre of the circle in the diagram with chord CD parallel to diameter AB. AC is produced to F and EG is a tangent to the circle. $A\hat{B}C = 35°$ and $\hat{C}_2 = 54°$.



Calculate, with reasons, the sizes of the following angles:

3.1 $\hat{E}_1$ (2)

3.2 $\hat{C}_1$ (2)

3.3 $\hat{C}_3$ (2)

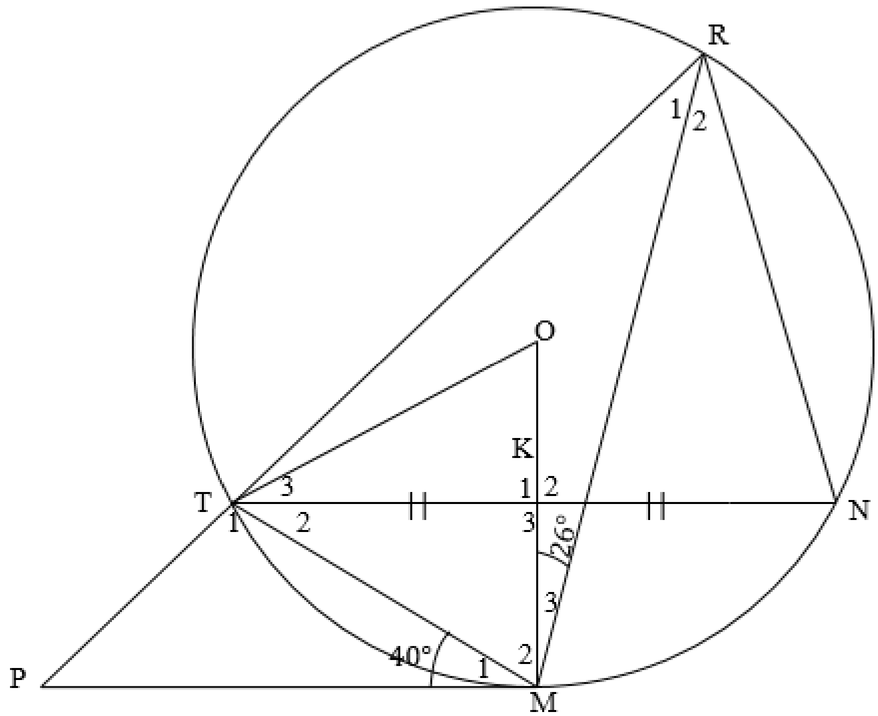
3.4 $A\hat{E}D$ (2)

3.5 $\hat{E}_3$ (3)

[11]

Activity 4

In the diagram below, O is the centre of circle TRNM. MP is a tangent to the circle at M such that RT produced meet MP at P. OM intersects TN at K. K is the midpoint of TN.
$P\hat{M}T = 40°$ and $\hat{M}_3 = 26°$.



Calculate, with reasons, the size of:

4.1 $T\hat{O}M$ (4)

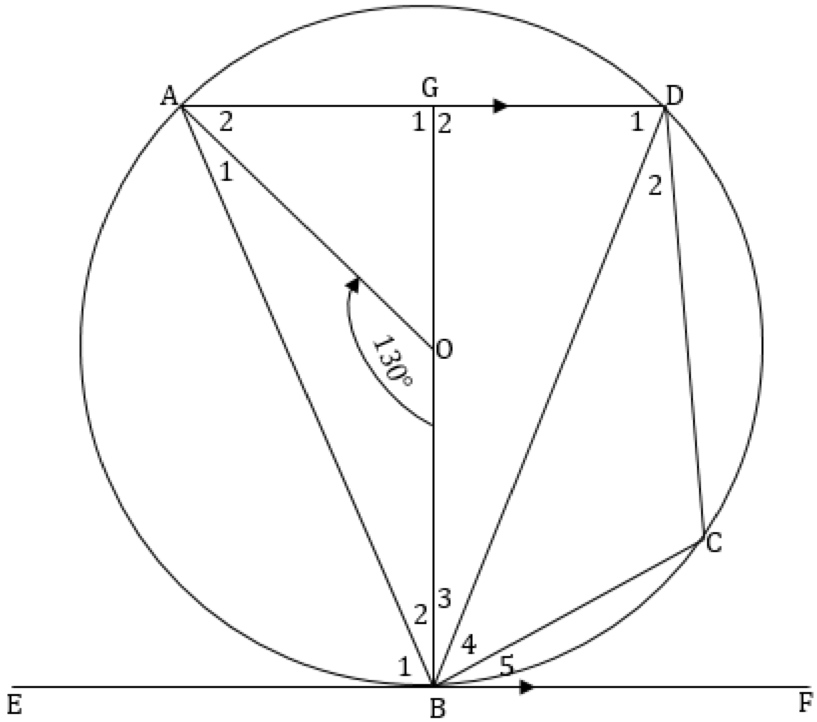
4.2 $\hat{N}$ (4)

4.3 $\hat{T}_3$ (3)

[11]

Activity 5

In the diagram below, the circle having centre O, passes through A, B, C and D, with $A\hat{O}B = 130°$. EBF is a tangent to the circle at B with $EF||AD$. BOG is a straight line.



Calculate, with reasons, the size of:

5.1 $\hat{D}_1$ (2)

5.2 $\hat{B}_1$ (2)

5.3 $B\hat{A}D$ (1)

5.4 $\hat{C}$ (2)

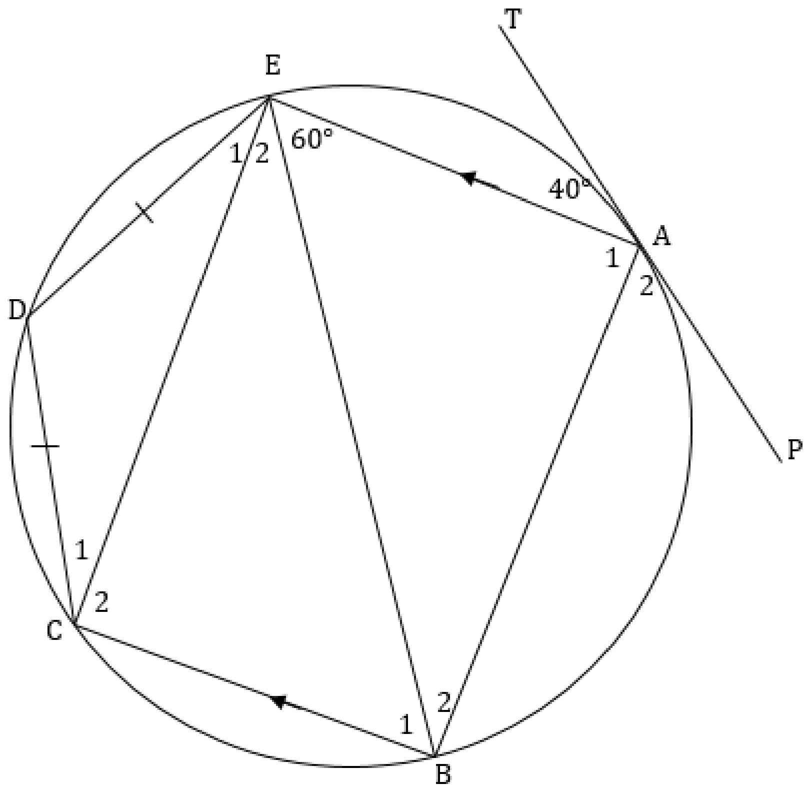
5.5 $\hat{B}_3$ (3)

5.6 Calculate the length of GD, if $AD = \frac{\sqrt{7}}{2}$ units. (3)

[13]

Activity 6

In the diagram below, TAP is a tangent to circle ABCDE at A. $AE||BC$ and $DC = DE$.
$T\hat{A}E = 40°$ and $A\hat{E}B = 60°$.



6.1	Identify TWO cyclic quadrilaterals.	(2)

Calculate, with reasons, the size of:

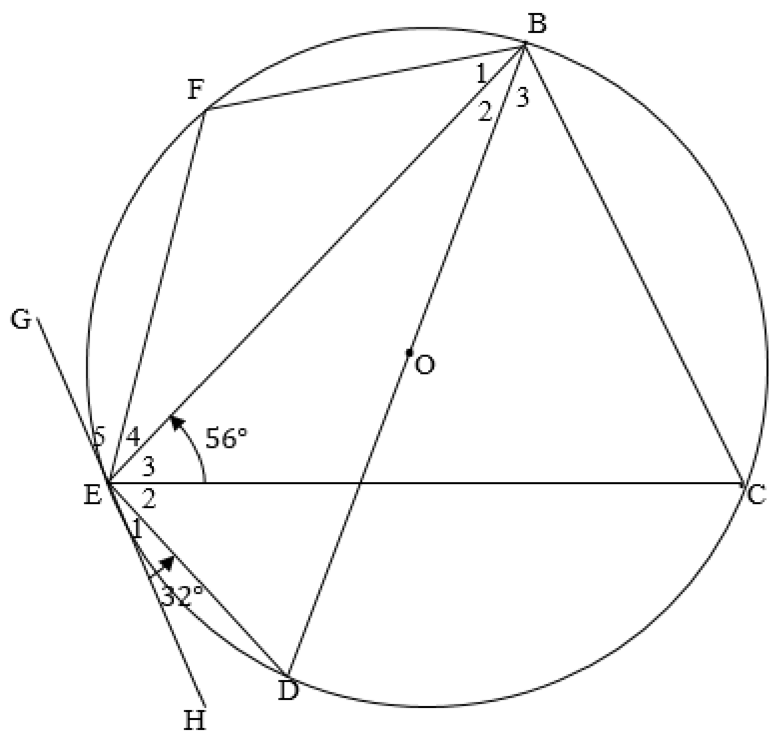
6.2	$\hat{B}_2$	(2)
6.3	$\hat{B}_1$	(2)
6.4	$\hat{D}$	(2)
6.5	$\hat{E}_1$	(3)

[11]

TUTORIAL 3

Activity 1

In the diagram below, O is the centre of the circle. BD is a diameter of the circle. GEH is a tangent to the circle at E. F and C are two points on the circle, then FB, FE, BC, CE and BE are drawn. $\hat{E}_1 = 32°$ and $\hat{E}_3 = 56°$.



Calculate, with reasons, the size of:

1.1 $\hat{E}_2$ (2)

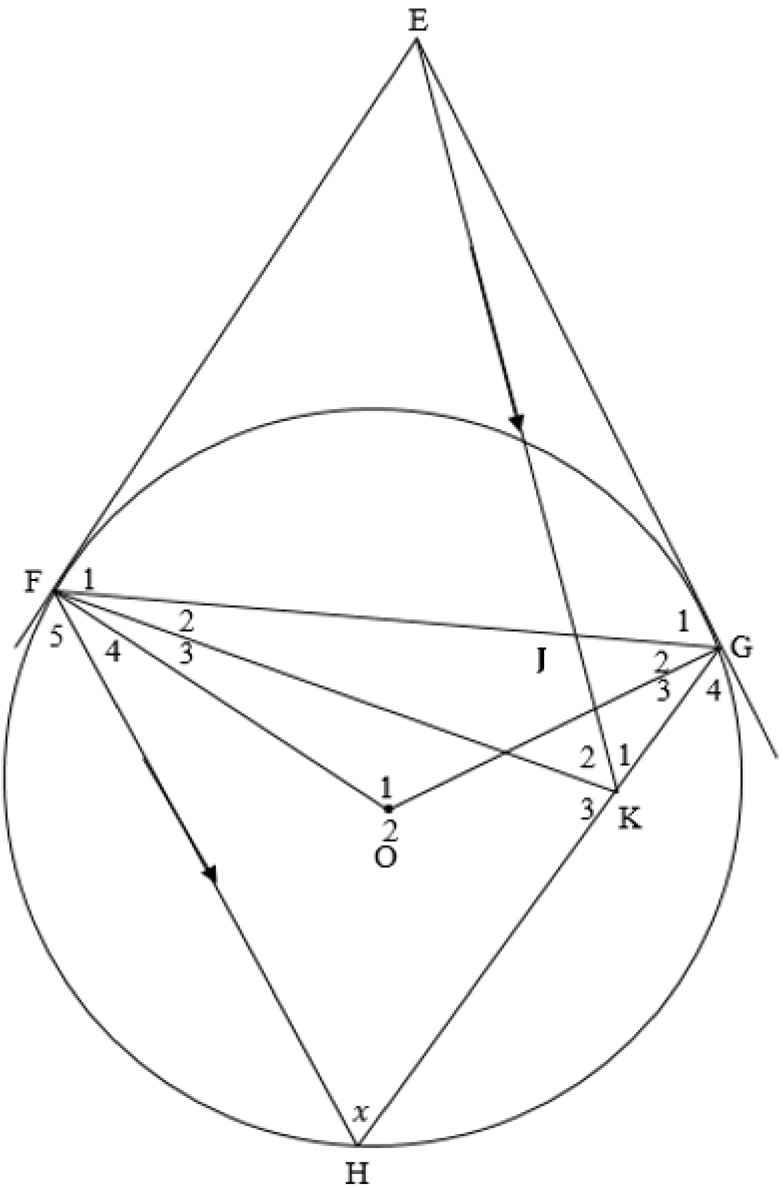
1.2 $E\hat{B}C$ (3)

1.3 $\hat{F}$ (4)

[9]

Activity 2

In the diagram, EF and EG are tangents to circle with centre O. FH||EK, EK intersects FG at J and meets GH at K. $\hat{H} = x$.



Prove that:

2.1 FOGE is a cyclic quadrilateral. (5)

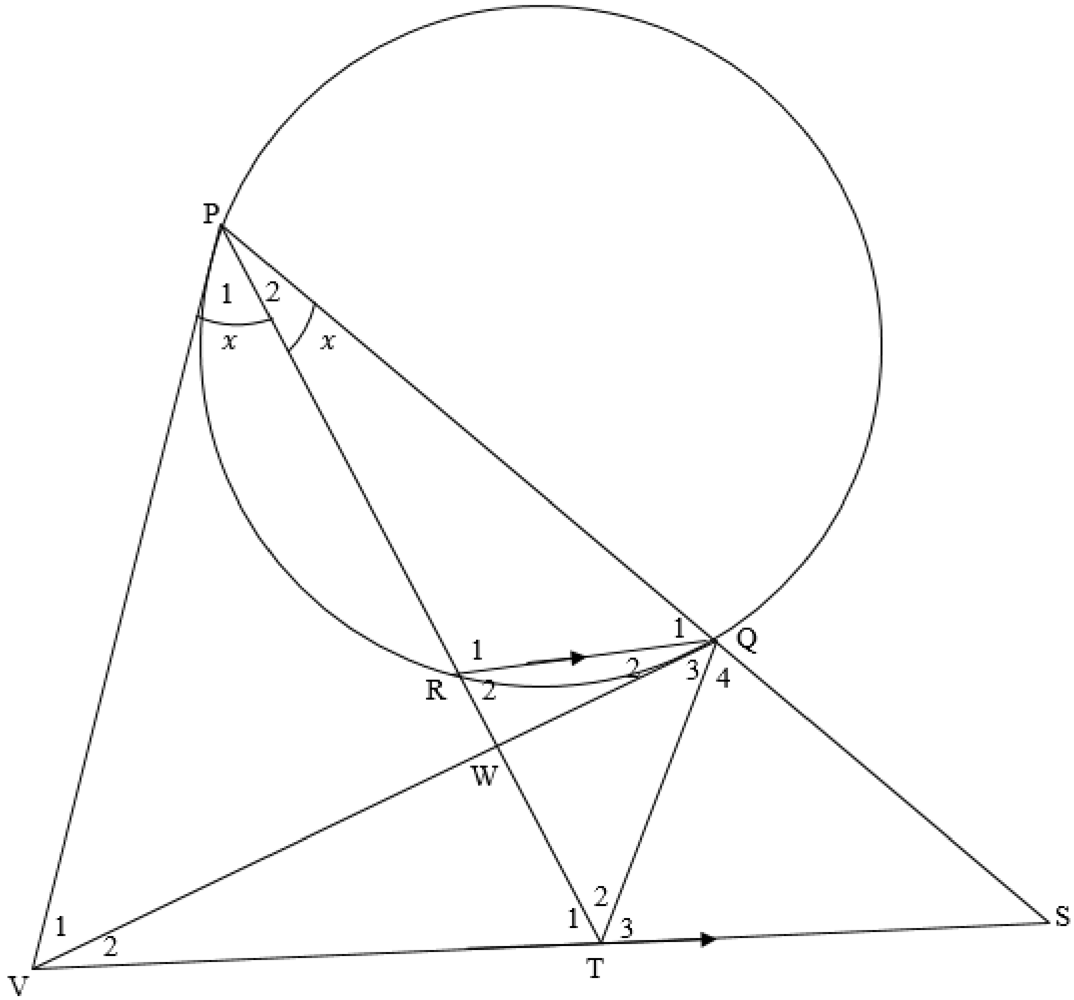
2.2 EG is a tangent to the circle GJK. (5)

2.3 $F\hat{E}G = 180° - 2x$. (3)

[13]

Activity 3

In the diagram, PV and VQ are tangents to the circle at P and Q. PQ is produced to S and chord PR is produced to T such that VTS||RQ. VQ and RT intersect at W. $\hat{P}_1 = \hat{P}_2 = x$.



Prove that:

3.1 $\hat{S} = x$ (4)

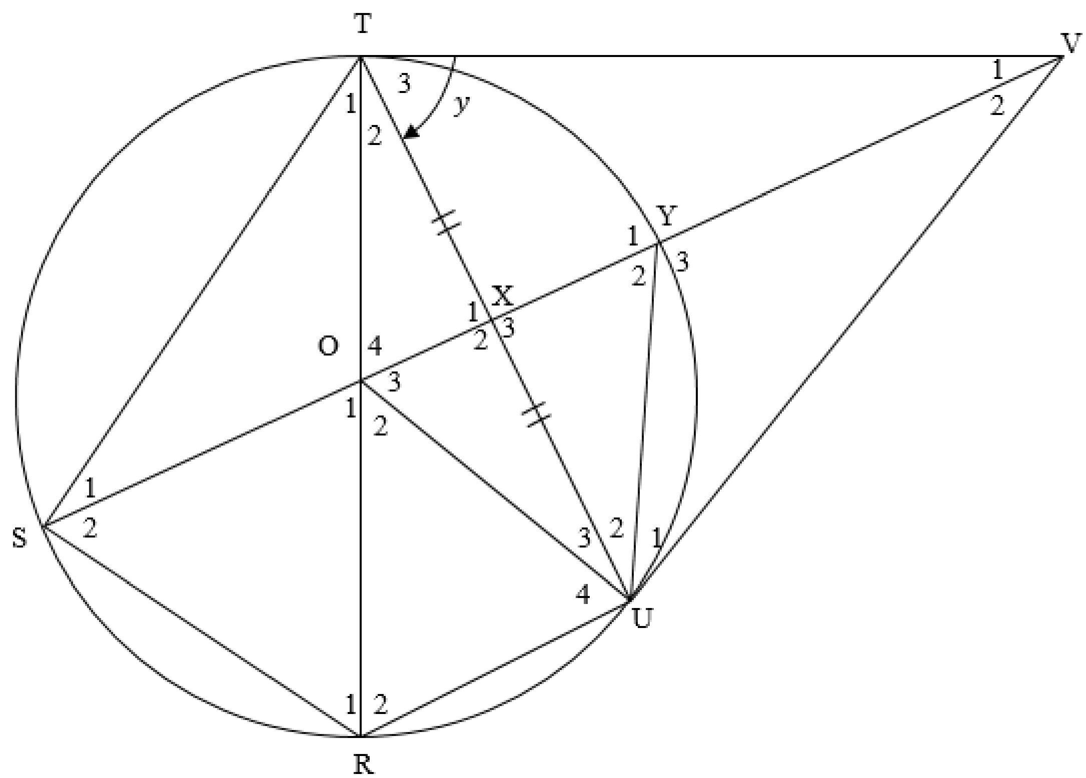
3.2 PQTV is acyclic quadrilateral. (5)

3.3 TQ is a tangent to the circle passing through Q, W and P. (3)

[12]

Activity 4

TV and VU are tangents to the circle with centre O at T and U respectively. TSRUY are points on the circle such chat RT is the diameter. X is the midpoint of chord TU. $\hat{T}_3 = y$.



Prove that:

4.1 RU||SY (5)

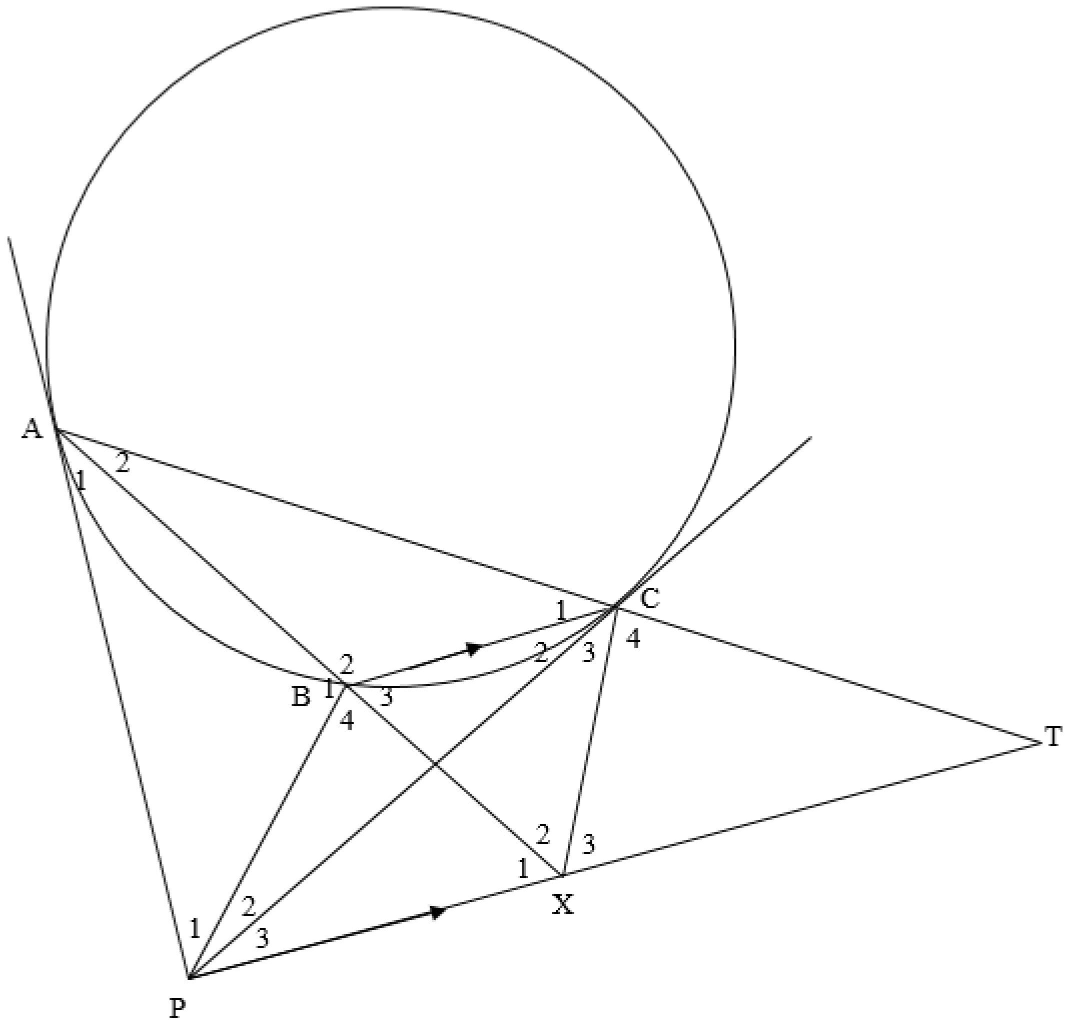
4.2 $\hat{T}_1 = \frac{1}{2}y$ (5)

4.3 TOUV is a cyclic quadrilateral (5)

[15]

Activity 5

In the diagram below, PA and PC are tangents drawn from point P to the circle ABC.
$AB = BC$. ABX is a straight line. AC and PX are produced to meet at T. BC||PT and BP, CP and CX are drawn.



Prove that:

5.1 AB bisects $P\hat{A}C$ (3)

5.2 BC bisects $P\hat{C}A$ (3)

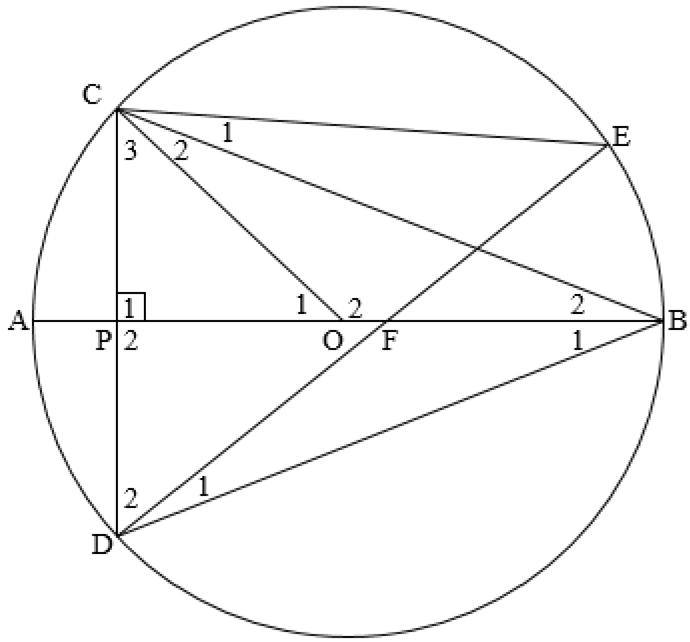
5.3 $P\hat{A}C = \hat{X}_3$ (4)

5.4 BC is a tangent to the circle CPT (4)

[14]

Activity 6

In the diagram below, O is the centre of the circle CADBE with AB as diameter, $CD \perp AB$ and cuts point P. Chord DE cuts AB at F. CE, CB, CO and DB are drawn.



Prove that:

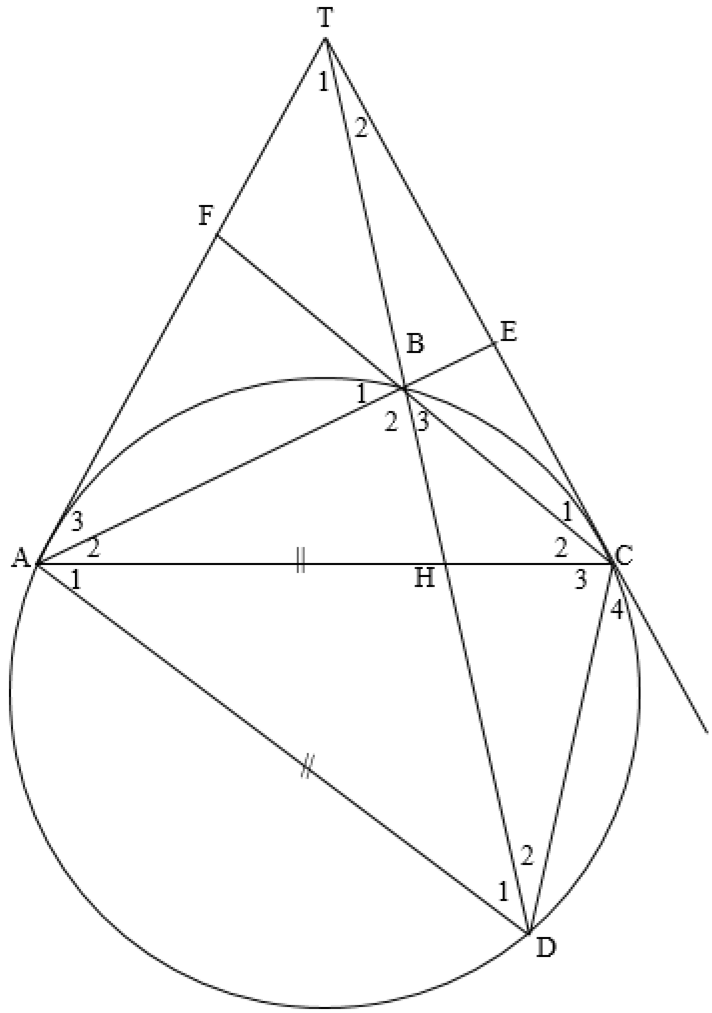
6.1 $\hat{B}_1 = \hat{B}_2$ (4)

6.2 CEFO is a cyclic quadrilateral (5)

[9]

Activity 7

In the diagram below, ABCD is a cyclic quadrilateral with $AC = AD$. Tangents AT and CT touch the circle at A and C respectively. FBC, ABE, AHC and DBT are straight lines.



7.1 $\hat{B}_1 = \hat{B}_2$ (5)

7.2 BECH is a cyclic quadrilateral (4)

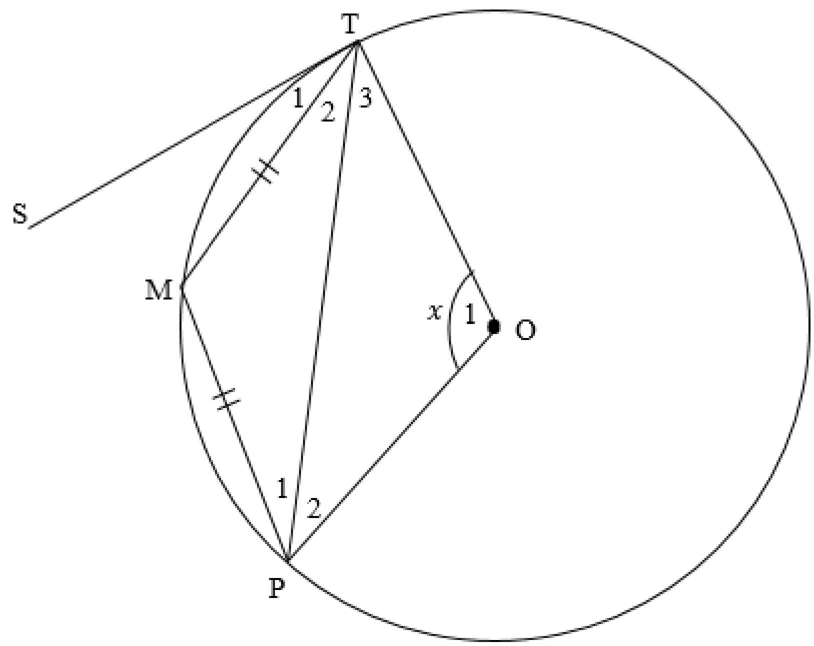
7.3 CA is a tangent to the circle passing through points A, B and T. (5)

[14]

TUTORIAL 4

Activity 1

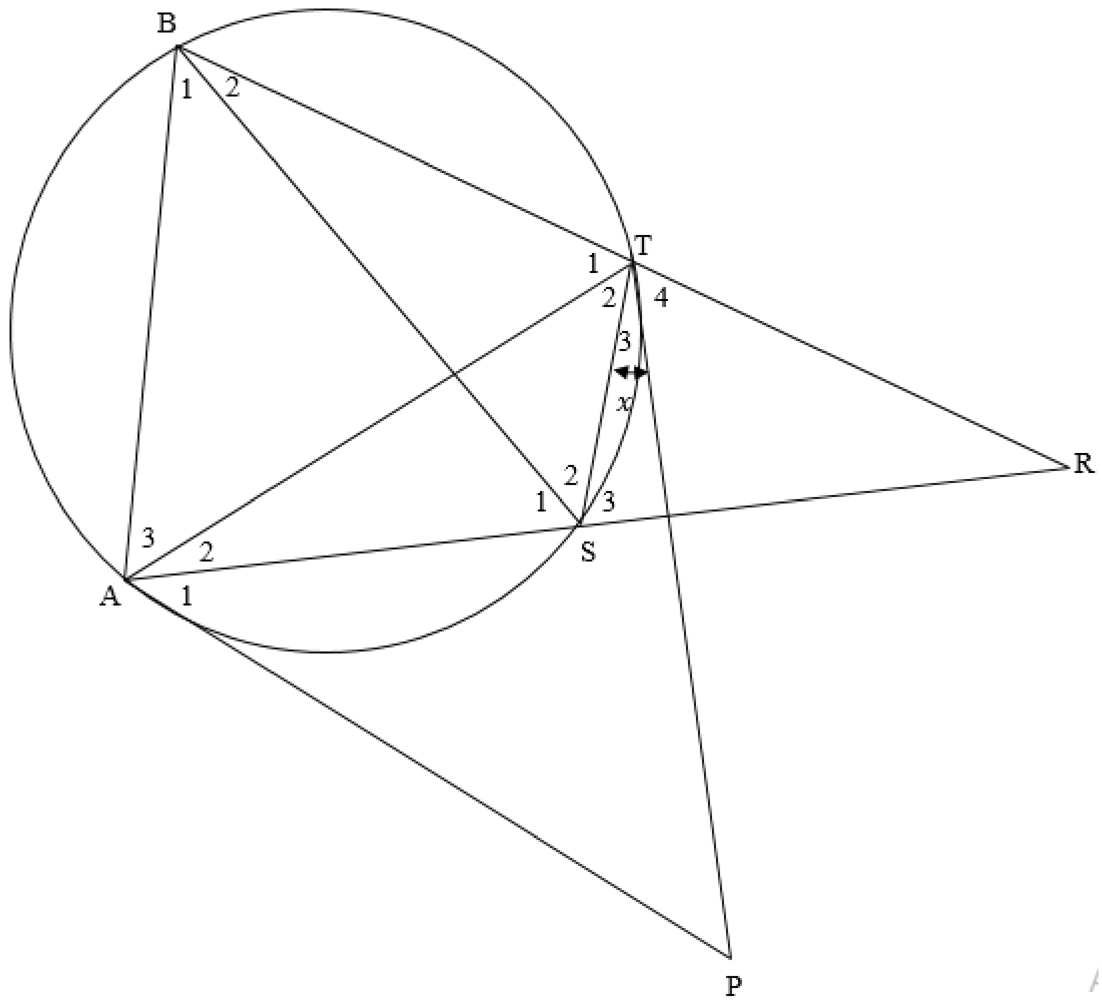
In the diagram, O is the centre of the circle. ST is a tangent to the circle at T. M and P are points on the circle such that TM = MP. OT, OP and TP are drawn. $\hat{O}_1 = x$.



1.1 Prove that $S\hat{T}M = \frac{1}{4}x$. **[7]**

Activity 2

In the diagram, PA and PT are tangents to a circle at A and T respectively. B and S are points on the circle such that BT produced and AS produced meet at R and BR = AR. BS, AT and TS are drawn. $\hat{T}_3 = x$.



2.1 Give a reason why $\hat{T}_3 = \hat{A}_2 = x$. (1)

2.2 Prove that AB||ST. (5)

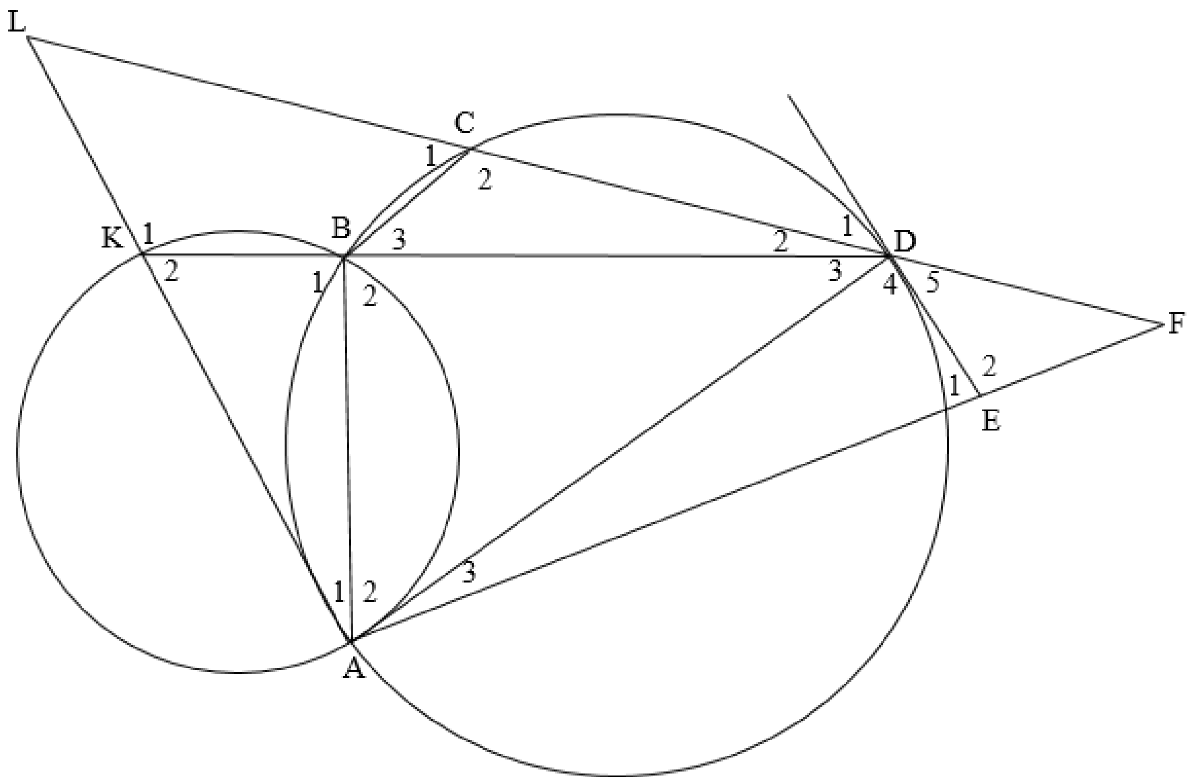
2.3 Prove that $\hat{T}_4 = \hat{A}_1$ (5)

2.4 Prove that RTAP is a cyclic quadrilateral. (2)

[13]

Activity 3

In the diagram below, two circles intersect each other at A and B. ED is a tangent to circle ABCD. DA is a tangent to circle AKB. DBK is a straight line. AK and DC are produced to meet at L. LCD and AE are produced to meet at F. $CD = DF$.



Prove that:

3.1 LKBC is a cyclic quadrilateral. (5)

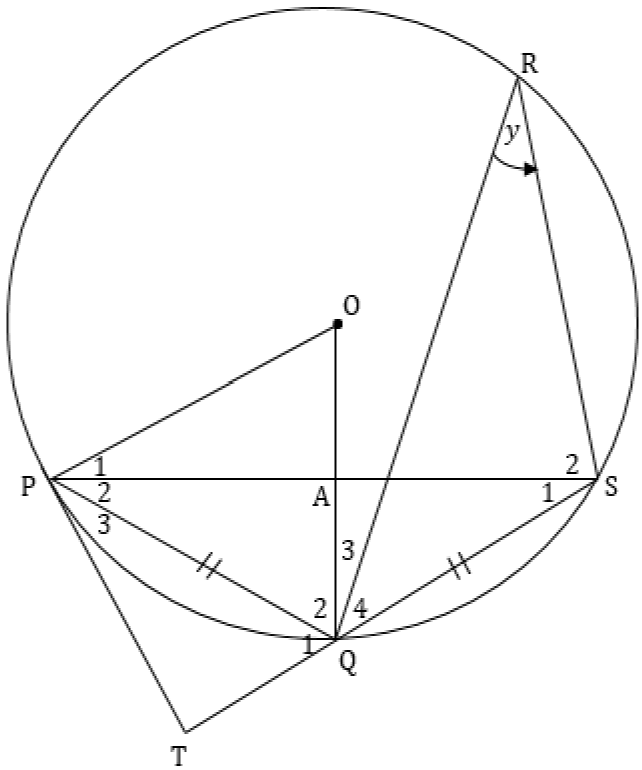
3.2 $\hat{B}_2 = L\hat{A}D$ (3)

3.3 DE||LA (5)

[13]

Activity 4

In the diagram, O is the centre of the circle and P, Q, S and R are points on the circle.
$PQ = QS$ and $Q\hat{R}S = y$. The tangent at P meets SQ produced at T. OQ intersects PS at A.



4.1 Give a reason why $\hat{P}_2 = y$ (1)

4.2 Prove that PQ bisects $T\hat{P}S$. (4)

4.3 Determine $P\hat{O}Q$ in terms of y. (2)

4.4 Prove that PT is a tangent to the circle that passes through P, O and A. (2)

4.5 Prove that $O\hat{A}P = 90°$. (5)

[14]