NATIONAL SENIOR CERTIFICATE

PUSH – ONE INTERVENTION PROGRAM

MATHEMATICS

GRADE 12

LAST PUSH

2022

EUCLIDEAN GEOMETRY
SIMILARITY & PROPORTIONALITY

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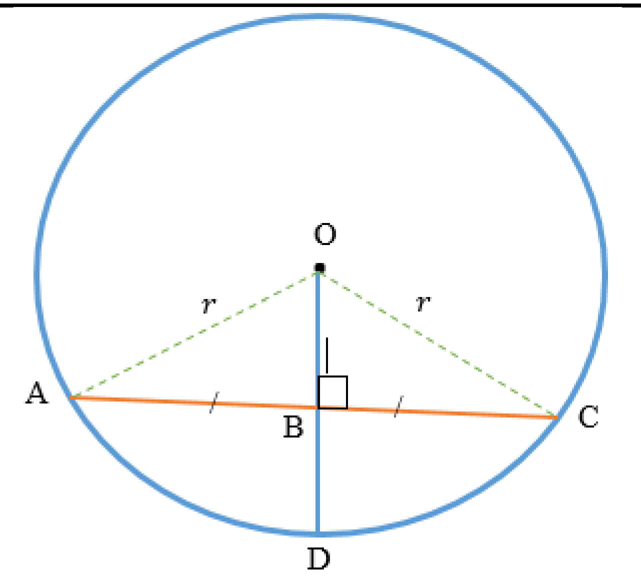
CONCEPT	HOW TO LEARN IT?	RELEVANT FORMULAE AND KEYWORDS
Theorem 1: Line segment through centre, mid-point and converse. **Theorem 2:** Perpendicular bisector of a chord.	Learn and use Congruency; 90°; H; S or A; A; S Lookout for the midpoint, then conclude 90° at the midpoint.	
	This theorem brings a right-angle triangle in the picture, so Pythagoras is important and the midpoint (as in Analytical Geometry).	Look out for the centre or midpoint on a chord as well as mid-points of two sides of a triangle. Look out for the centre
	Congruency to prove the theorem (Theorem 1). The use of Pythagoras as this theorem brings along 90° angle and a right-angled triangle. Ensure that two sides of a triangle are bisected by the same line. First Midpoint of a line, then 90° will follow. You need to ensure that there is a centre and 90°, then the line will pass through the centre.	

<table>
<tr>
<td></td>
<td colspan="2">If $\hat{A} = \hat{B}$ and $\hat{B} = \hat{C}$, then $\hat{A} = \hat{C}$

If $\hat{A} + \hat{B} = \hat{C}$ and $\hat{A} + \hat{D} = \hat{C}$, then $\hat{B} = \hat{C}$.

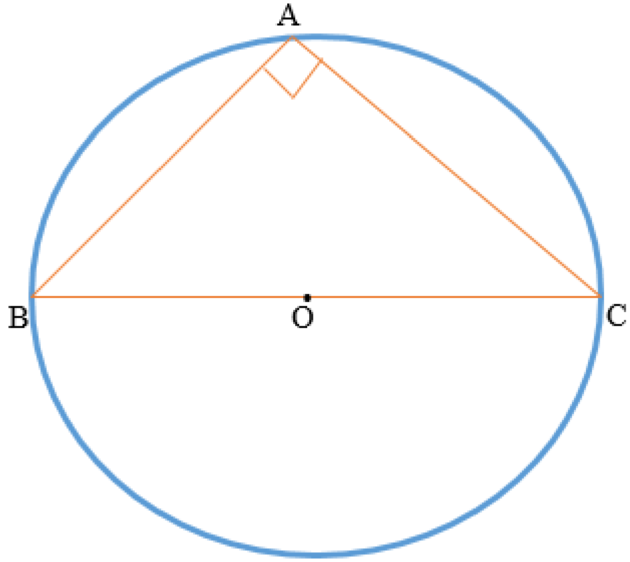
Applications of these rules without numerical values, that is given variables such as x or y.

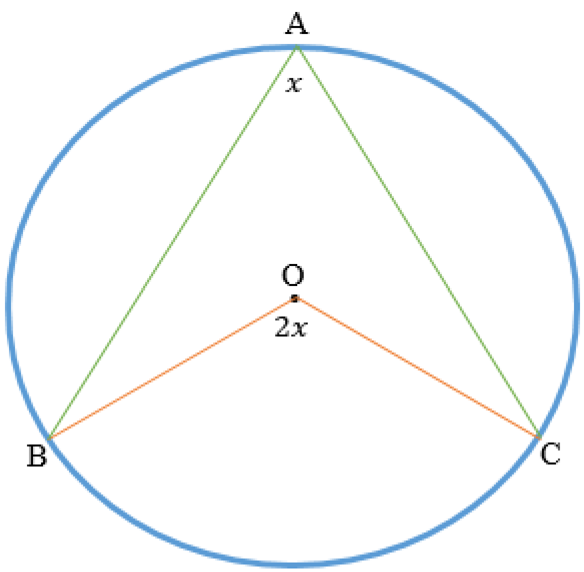
Expressing one variable in terms of the other information, that is changing the subject of the formula of the equation.

Substitution of equal quantities.</td>
</tr>
<tr>
<td>Theorem 3:

Angle at centre is twice angle at circumference.

Diameter subtends right angle at circumference.</td>
<td>Learn exterior angle of a triangle and properties of an Isosceles triangle. The proof of this theorem is for examination purpose. (That is, it is a possible exam question).



Use the previous theorem or lookout for the centre, diameter and the angle subtended by the line that passes through the centre (diameter).</td>
<td>
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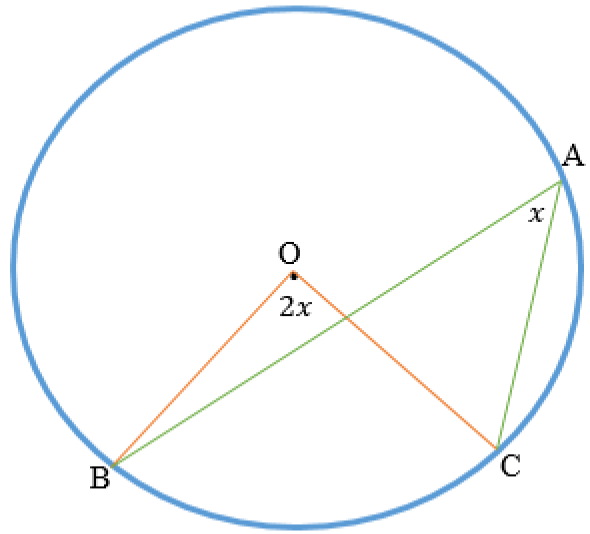
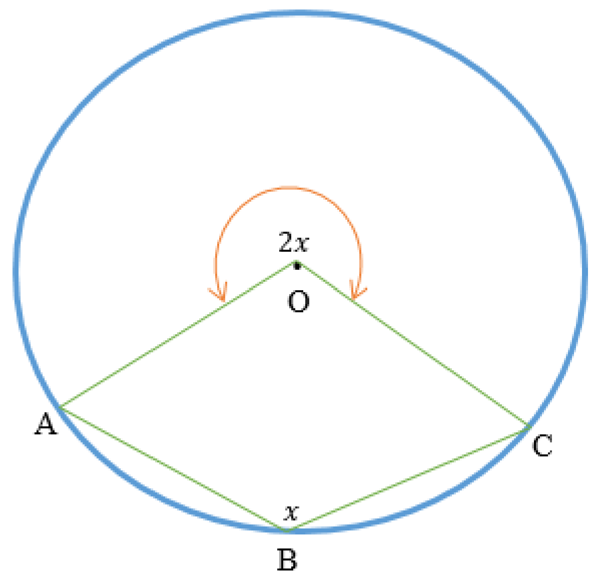
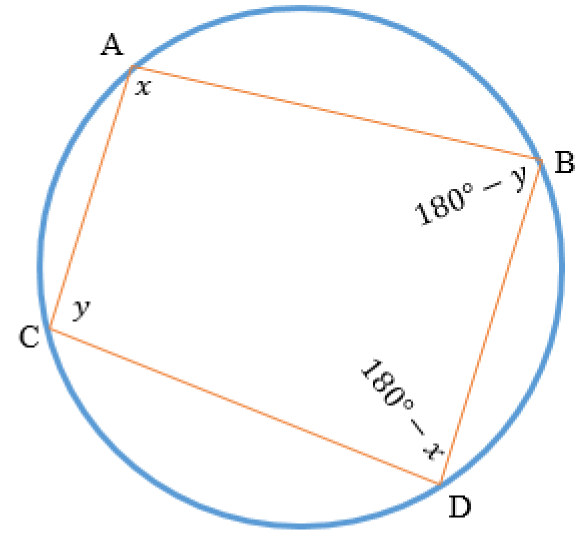
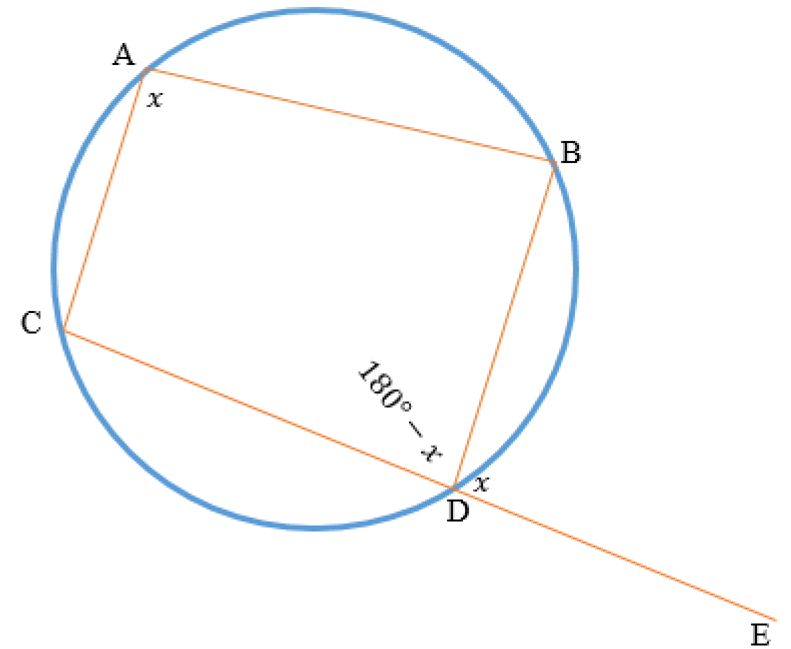
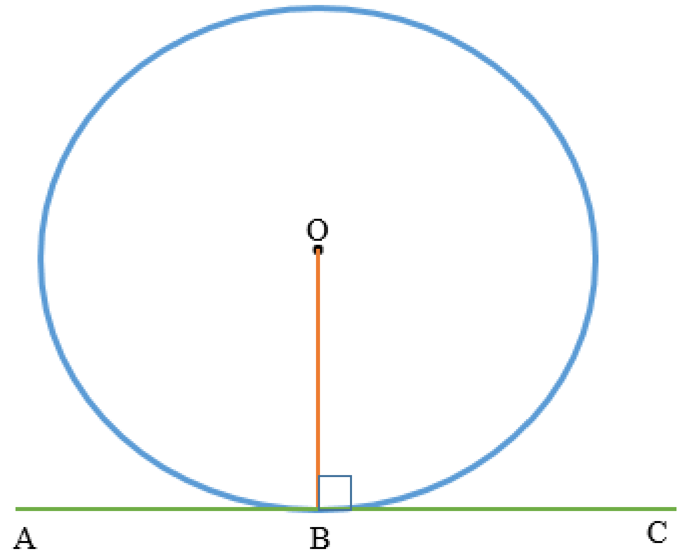
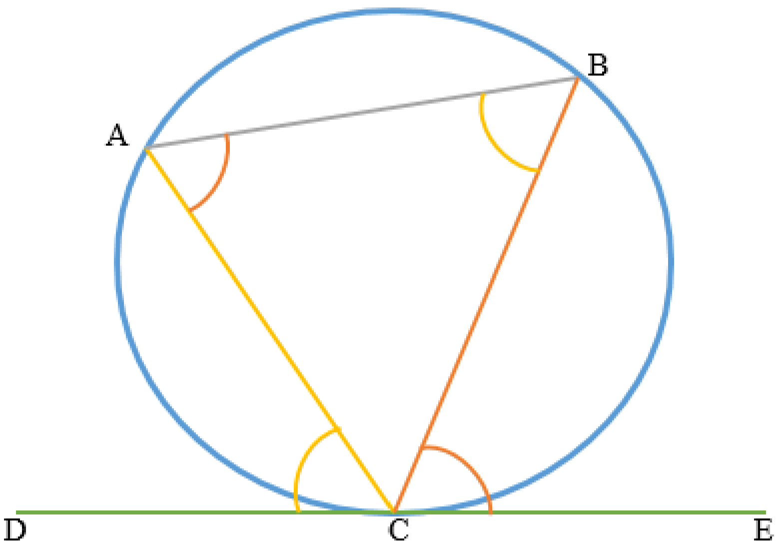
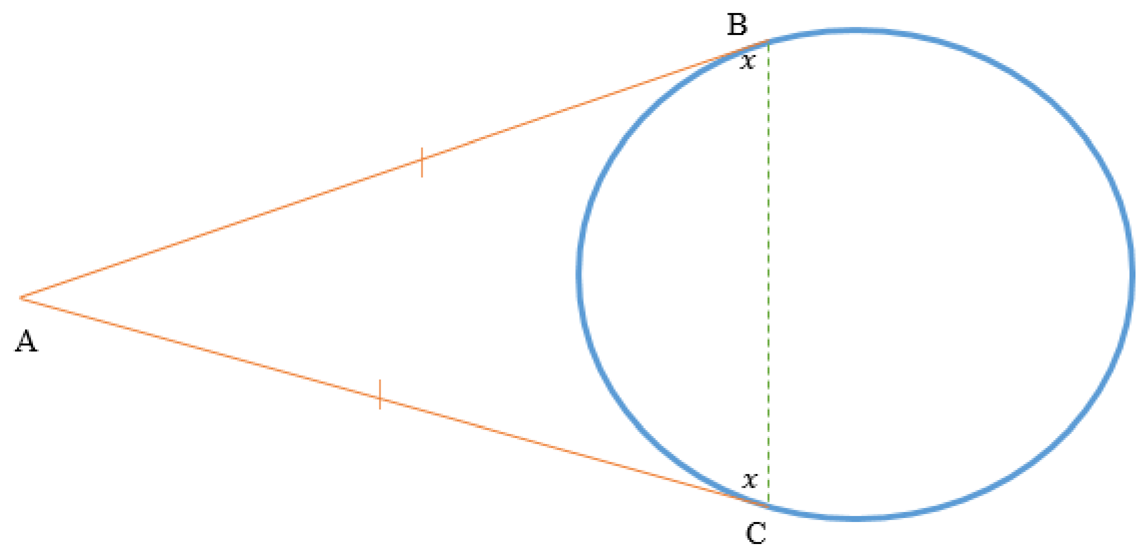
		 
	Arc: The angle at the centre and the one on the	Centre

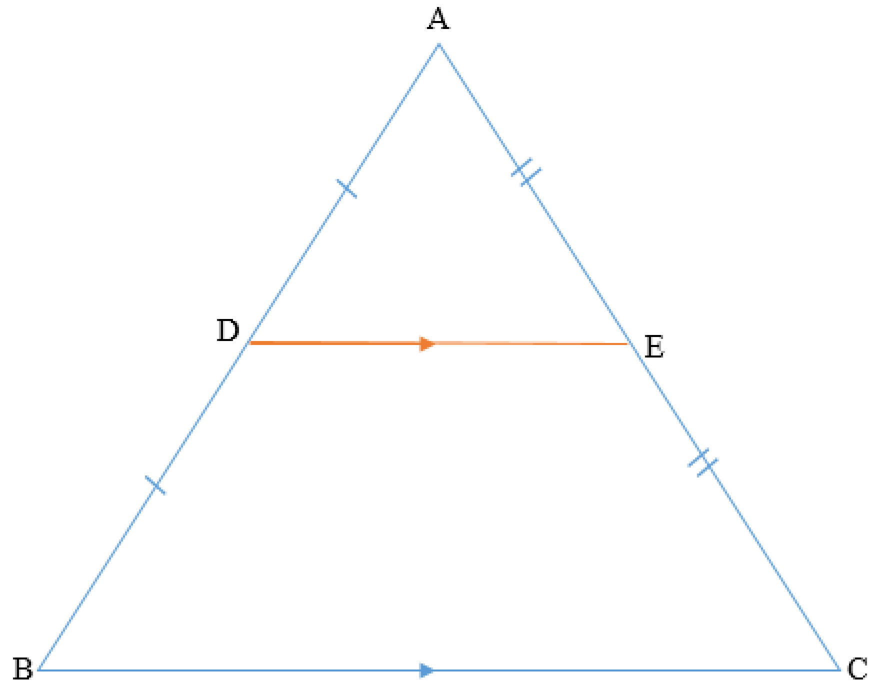
	circle must be on the same arc or chord. This is a special case of the above theorem; it also brings about a 90° angle into the picture a right-angled triangle.	Centre
	Always lookout for an arc that supports the angle at the centre, then look for the angle at the circle that is also supported by the arc. (Think about subtend or support, as a shouting mouth, shouting at the arc or chord).	
Opposite angles of a cyclic Quadrilateral Exterior angle of a cyclic quadrilateral. Proving that a quadrilateral is cyclic.	The proof of this theorem is for examination purpose. For the proof use the theorem about the angle at the centre and a revolution. Look out for the straight line coming from one of the vertices of the cyclic quad.	

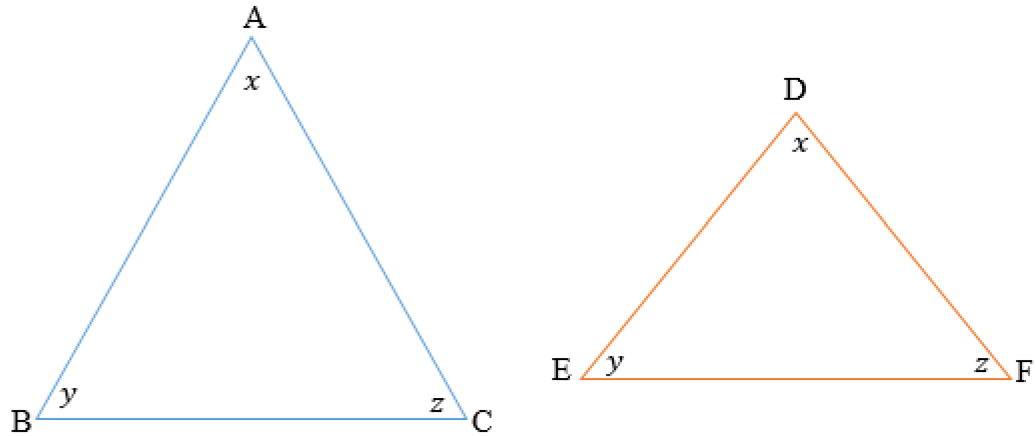
		
	This theorem brings = 180° into the picture. There are three options: Lookout to see if you can have opposite angles of a quad = 180°, then that quad will be cyclic. Lookout to see if you can have ext. angle of a quad = interior opposite angle, then the quad will be cyclic. Lookout to see if you can have two angles that are subtended by a line, then the four vertices will be a cyclic quad.	

	Two properties of the cyclic quad.	
Tan-chord theorem Tangents from same point Tangent perpendicular to radius	Proof for examination purposed. To prove: use the diameter (angle at the centre), Tangent Radius Theorem (Tan – Radius), sum of angles of a triangle and second logical reason of addition.	 


B
x
A
x
C

	Divide the circle at the point of contact (between the chord and the tangent) to be able to identify the alternate segment, then focus on the chord and look for any angle that is supported by the chord in the other segment.	Tangent (see solving riders).
	Three properties of the tangent to a circle.	
Solving riders	Lookout for the key words connected to theorems in the explicitly given information, for instance, centre (connected theorems, include theorem 1, 2, angle at the centre together(diameter). Tangent (connected theorems include, (tan-chord, tan-radius and tangents drawn from the same point) and Cyclic Quad. (Connected theorems, include opposite angles are supplementary and ext. angle of a cyclic quad = to the opposite interior angle). Examine the implicitly given information from the diagram, then make conclusion on what you get from the diagram. Create a short list of statements and reasons, see if you cannot make logical conclusion from the list using important logical reasoning. Lookout for the angles on the same segment in the diagram. Lookout for angles on equal chords (the angles are	

	equal). Then answer the questions.	
	See above Solving riders.	
	The more you practice, the better you become.	
Theorem 1: Line drawn parallel to one side of triangle. The midpoint theorem	For examination purpose. Remember that a triangle can have its height inside the triangle or outside the triangle. The area of triangle between parallel lines and share a base have the equal area. After the constructions rotate the two sides that are not parallel to any of the lines so that it is horizontal, you will see the heights that are being shared by different triangles. Lookout for the midpoint of two sides of the triangle.	
	Properties (the conclusion of the theorem, the part that follows the word **then** in the theorem) of the theorem.	
Theorem 2:	Examination purpose. Use congruency S: A: S to create a line parallel to one side of a triangle, so that you can form proportions using one triangle.	

<table>
<tr><td>Equiangular triangles are similar.</td><td colspan="2">Application: Remember that this theorem requires that you use more than one triangle and the order in naming the triangles is also important as well.
Otherwise, lookout for corresponding sides to prove similarity.


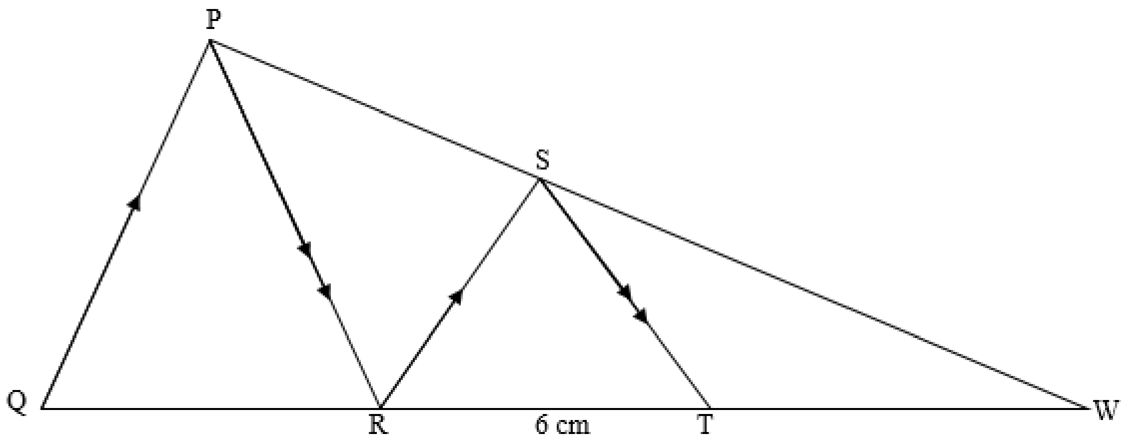
$\Delta ABC|||\Delta DEF$</td></tr>
<tr><td></td><td>Do plenty of examples.</td><td></td></tr>
<tr><td>Theorem of Pythagoras</td><td>Proof is not required for examination purpose. Otherwise, first write down the sum of the squares of the two side = the square of the hypotenuse.

Then look out for either equal sides from the diagram or the sum of two sides making one side.</td><td></td></tr>
<tr><td></td><td>Do plenty of examples.</td><td></td></tr>
<tr><td>Proportion</td><td>Work with one triangle</td><td></td></tr>
<tr><td>Similarity</td><td>Work with two triangles. Remember after similarity (equiangular) you also get proportion, in this case use
two triangles. If asked to prove proportion look for the allocated marks, if the marks are at least 5,</td><td></td></tr>
</table>

	then start by proving similarity using (equiangular) by rearranging the proportion as follows, the first side on the right becomes the denominator on the left and the second side on the left becomes the denominator on the right, in that way you the numerators come from the same triangle and the denominators come from the second triangle. E.g. if asked to prove: $AB \times PQ = PR \times BC$ Then rearranging becomes: $\frac{AB}{PR} = \frac{BC}{PQ}$ so that the triangles are triangle ABC the numerators and triangle PQR from the denominators.	

Activity 1

In ΔPQW, S is a point on PW and R is a point on QW such that SR||PQ. T is a point on QW such that ST||PR. $RT = 6$ cm, $WS:SP = 3:2$.



Calculate:

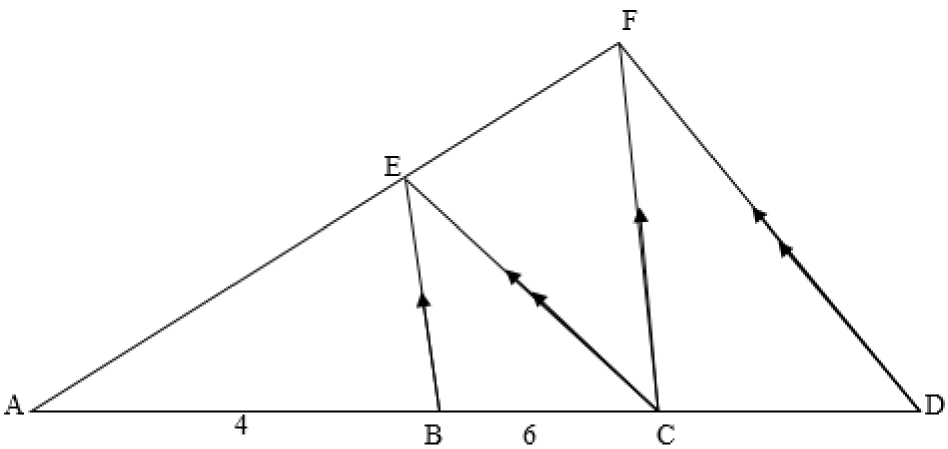
1.1 WT (3)

1.2 WQ (4)

[7]

Activity 2

In ΔADF, DF||CE and CF||BE. If $AB = 4$ units and $BC = 6$ units.



Calculate:

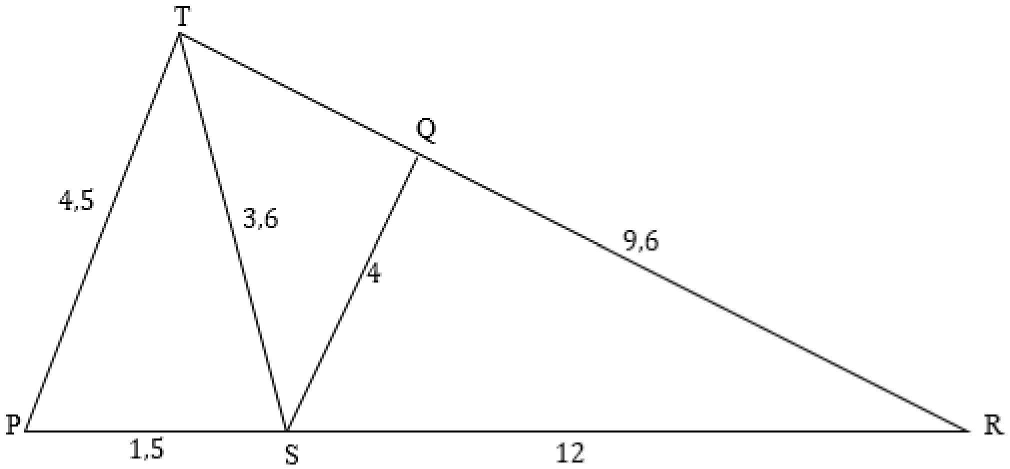
2.1 The length of CD. (3)

2.2 The numerical value of: $\frac{\text{Area of } \Delta FEC}{\text{Area of } \Delta FAD}$ (4)

[7]

Activity 3

In the diagram, TRP is a straight with TP $= 4{,}5$ units, Q and S are points on TR and PR respectively. QR $= 9{,}6$ units, QS $= 4$ units, TS $= 3{,}6$ units, PS $= 1{,}5$ units and SR $= 12$ units.



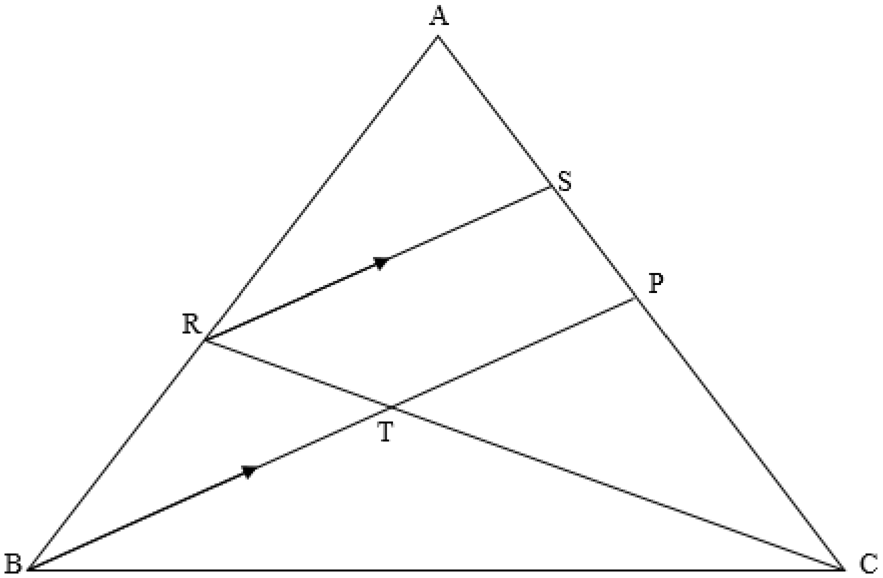
3.1 Prove that PT is a tangent to the circle which passes through the points T, S and R. (7)

3.2 Calculate the length of TQ. (5)

[12]

Activity 4

In the diagram below, P is the midpoint of AC in ΔABC. R is a point on AB such that RS||BP and $\frac{AR}{AB} = \frac{3}{5}$. RC cuts at T.



Determine, giving reasons, the following ratios:

4.1 $\frac{\text{AS}}{\text{SC}}$ (4)

4.2 $\frac{\text{RT}}{\text{TC}}$ (3)

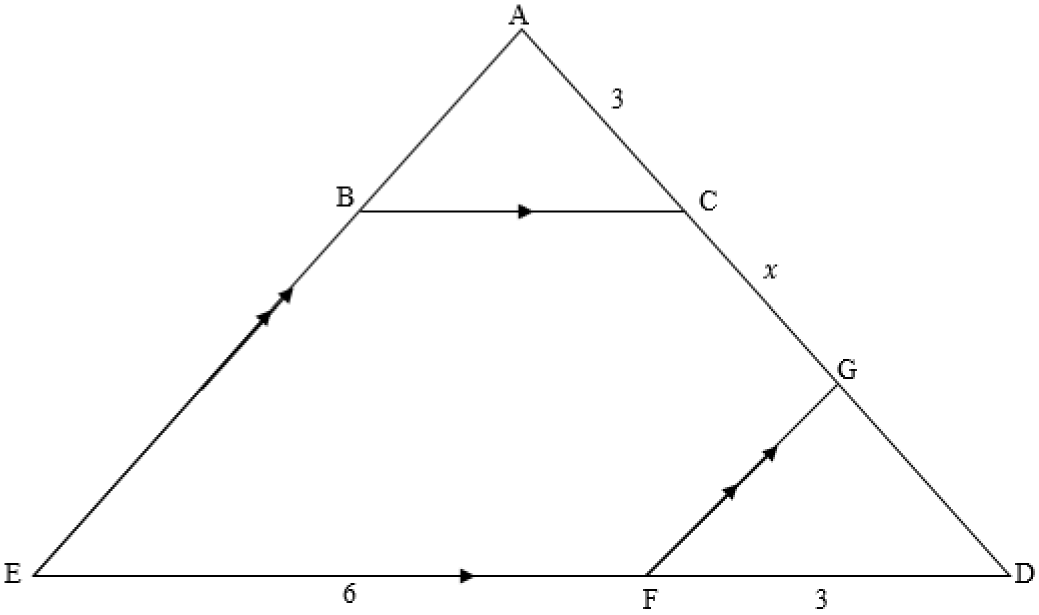
4.3 $\frac{\text{Area of }\Delta\text{RSA}}{\text{Area of }\Delta\text{RSC}}$ (2)

4.4 $\frac{\text{Area of }\Delta\text{TPC}}{\text{Area of }\Delta\text{RSC}}$ (4)

[13]

Activity 5

In the diagram, ADE is a triangle having BC||ED and AE||GF. It is also given $\text{AB}: \text{BE} = 1: 3$, $\text{AC} = 3$ units, $\text{EF} = 6$ units, $\text{FD} = 3$ units and $\text{CG} = x$ units.



Calculate, giving reasons:

5.1 The length of CD. (3)

5.2 The value of x. (4)

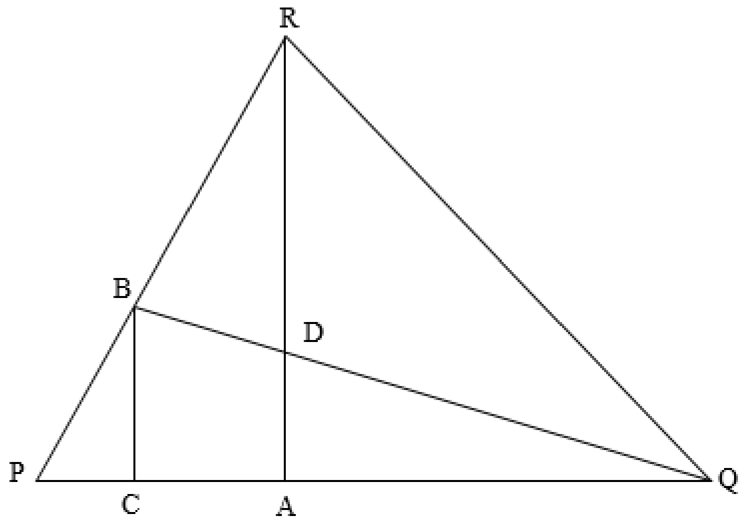
5.3 The length of BC. (5)

5.4 The value: $\frac{\text{Area of }\Delta\text{ABC}}{\text{Area of }\Delta\text{GFD}}$ (5)

[17]

Activity 6

$\frac{PA}{PQ} = \frac{4}{9}$ and $\frac{PB}{BR} = \frac{1}{2}$. BC||RA.



Determine:

6.1 BD: DQ (5)

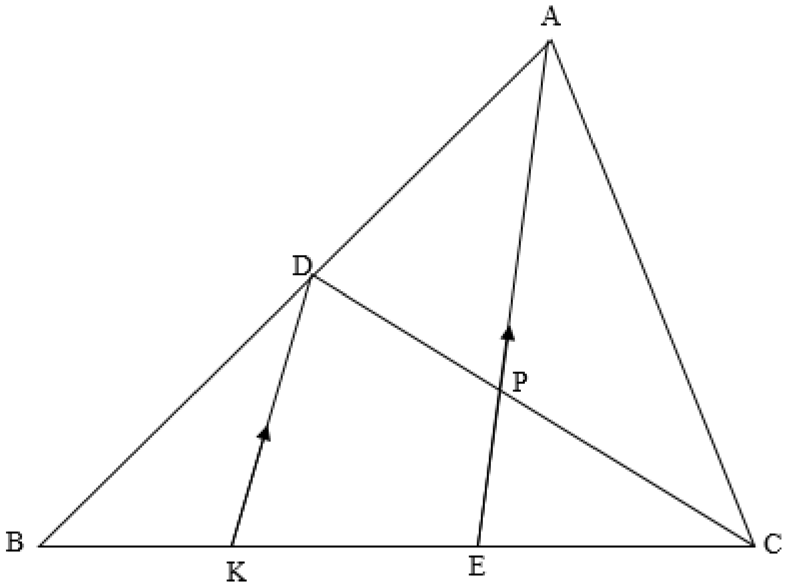
6.2 $\frac{\text{Area of } \Delta PRA}{\text{Area of } \Delta QRA}$ (3)

6.3 $\frac{\text{Area of } \Delta BQC}{\text{Area of } \Delta RPQ}$ (6)

[14]

Activity 7

D and E are points on sides AB and BC respectively of ΔABC such that AD: DB = 2: 3 and BE = $\frac{5}{3}$ EC. If DK||AE and AE and CD intersect at P, find the ratio of CP: PD.

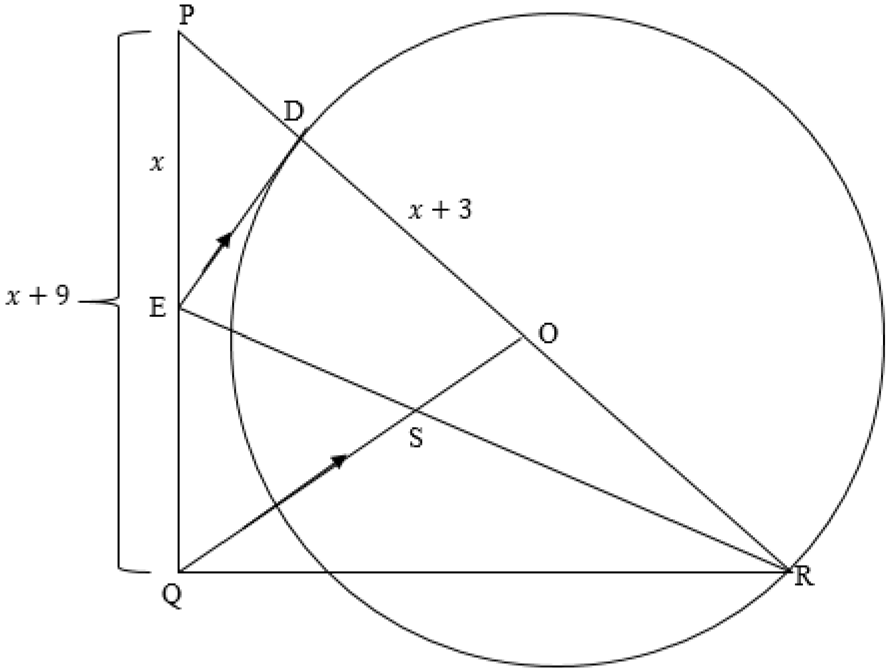


[5]

Activity 8

In the diagram below, the circle with centre O is drawn. OQ is drawn parallel to a tangent to the circle at D. ER is drawn with S on OQ. RD is produced to P and PQ is joined.

PE $= x$ units, PQ $= x + 9$ units, PD $= \frac{2}{3}x$ units and DO $= x + 3$ units.



8.1 Calculate the length of RO. (4)

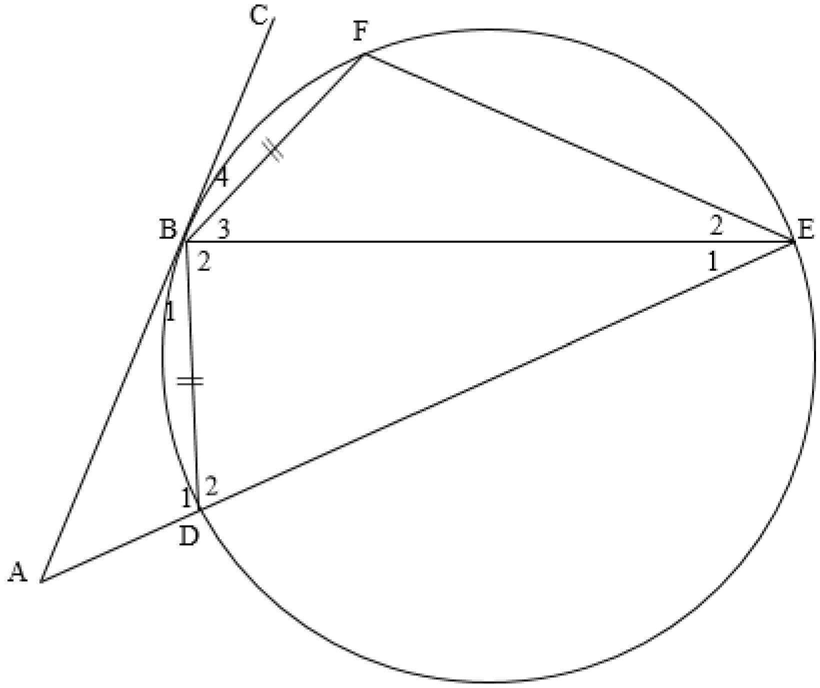
8.2 If OS $= 1{,}4$ units and S is the midpoint of ER, determine the length of DE. (2)

8.3 If the area of ΔPED $= 2{,}7$ units^2, find the area of ΔPER. (4)

[10]

Activity 9

In the diagram, ABC is a tangent to the circle at B. BDEF is a cyclic quadrilateral with DB = BF. BE is drawn and ED produced meets the tangent at A.



Prove that:

9.1 $\hat{B}_1 = \hat{E}_2$ (3)

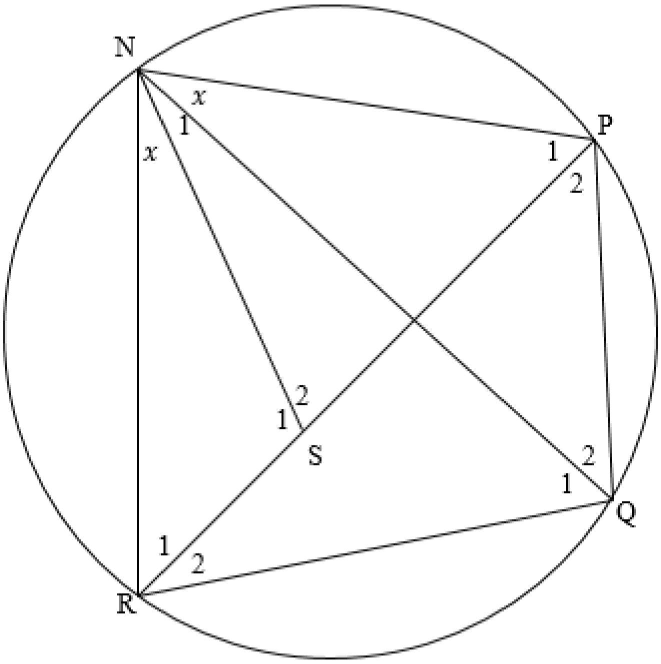
9.2 $\Delta BDA \parallel\!| \Delta EFB$ (4)

9.3 $BD^2 = AD.EF$ (2)

[9]

Activity 10

In the diagram below, NPQR is a cyclic quadrilateral with S, a point on PR. N and S are joined and $R\hat{N}S = P\hat{N}Q = x$.



Prove that:

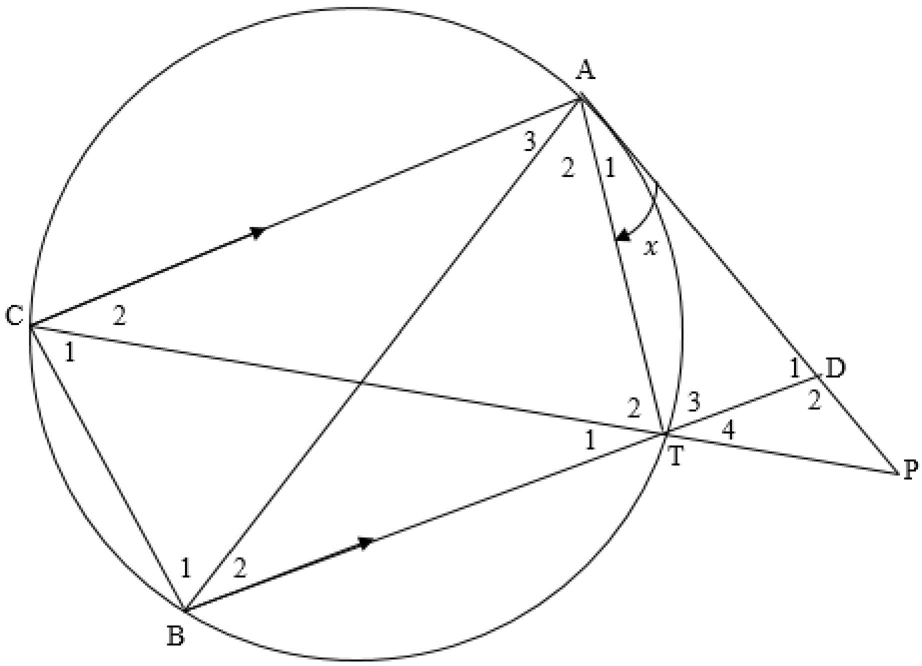
10.1 $\Delta NSR ||| \Delta NPQ$ (3)

10.2 $\Delta NQR ||| \Delta NPS$ (3)

[6]

Activity 11

In the diagram below, DA is a tangent to the circle ACBT at A. CT and AD are produced to meet at P. BT is produced to cut PA at D. AC, CB, AB and AT are joined. AC||BD.
Let $\widehat{A}_1 = x$.



11.1 Prove that ΔABC|||ΔADT. (6)

11.2 Prove that PT is a tangent to the circle ADT at T. (3)

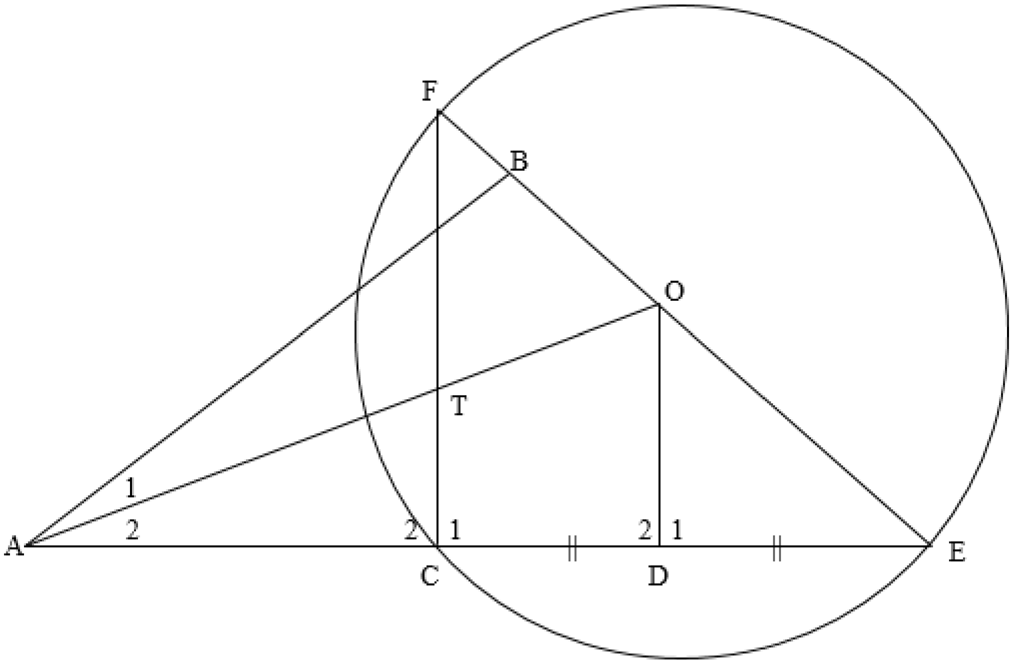
11.3 Prove that ΔATP|||ΔTDP. (3)

11.4 If $AD = \frac{2}{3} AP$, show that $AP^2 = 3PT^2$. (4)

[16]

Activity 12

In the diagram, FBOE is a diameter of a circle with centre O. Chord EC produced meets line BA at A, outside the circle. D is the midpoint of CE. OD and FC are drawn. AFBC is a cyclic quadrilateral.



Prove, with reasons, that:

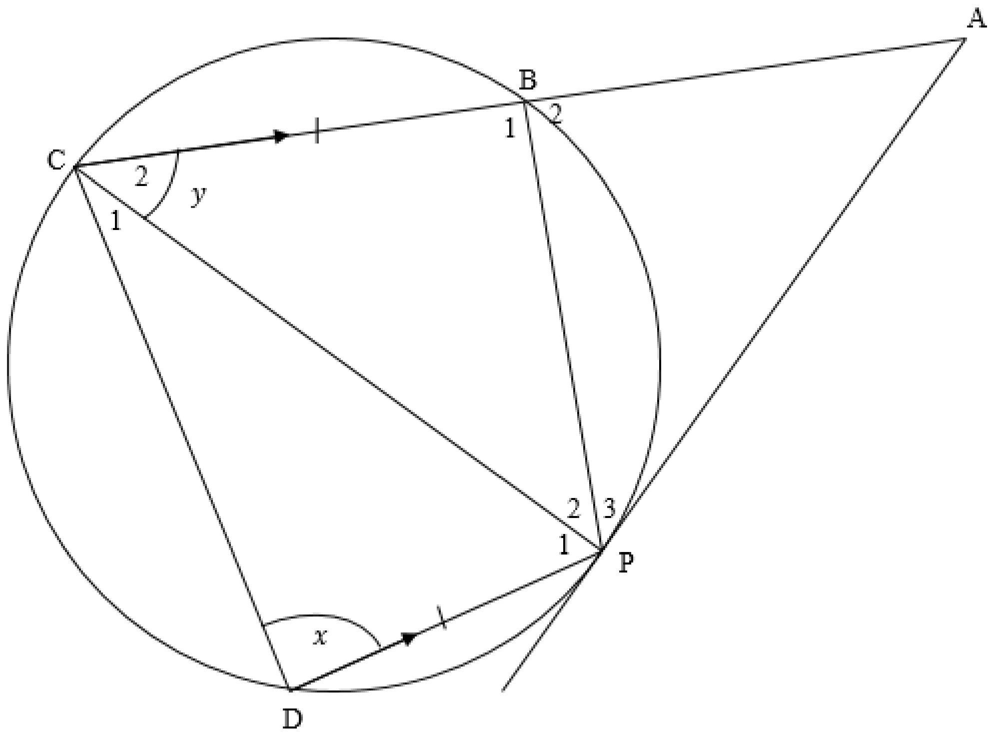
12.1 $FC \parallel OD$ (5)

12.2 $D\hat{O}E = B\hat{A}E$ (4)

[9]

Activity 13

AP is a tangent to the circle at P. CP||DP and $CB = DP$. CBA is a straight line. Let $\hat{D} = x$ and $\hat{C}_2 = y$.



Prove, with reasons that:

13.1 $\Delta APC ||| \Delta ABP$ (4)

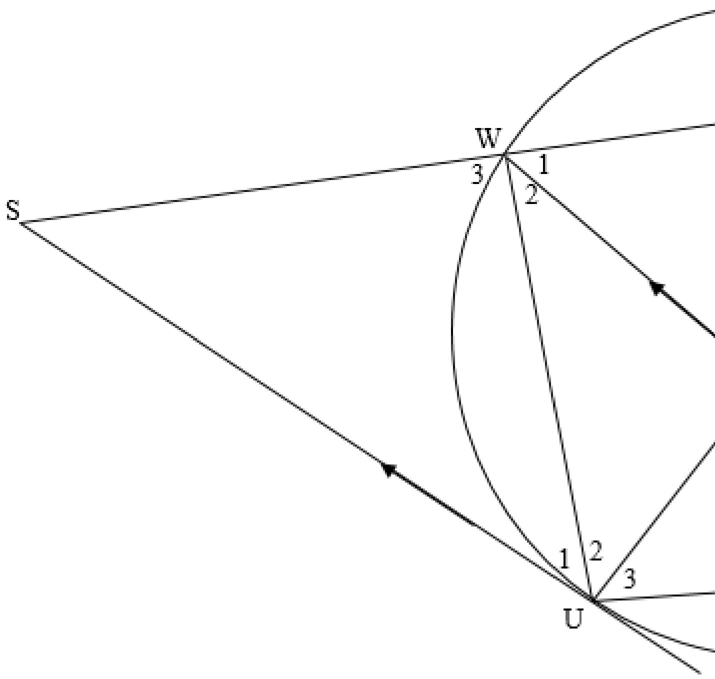
13.2 $AP^2 = AB \times AC$ (1)

13.3 $\Delta APC ||| \Delta CDP$ (4)

[9]

Activity 14

In the diagram below, PWUT is a cyclic quadrilateral with WU = TU. Chord WT and PU intersect at Q. PW is extended to S such that US||TW.



Prove that:

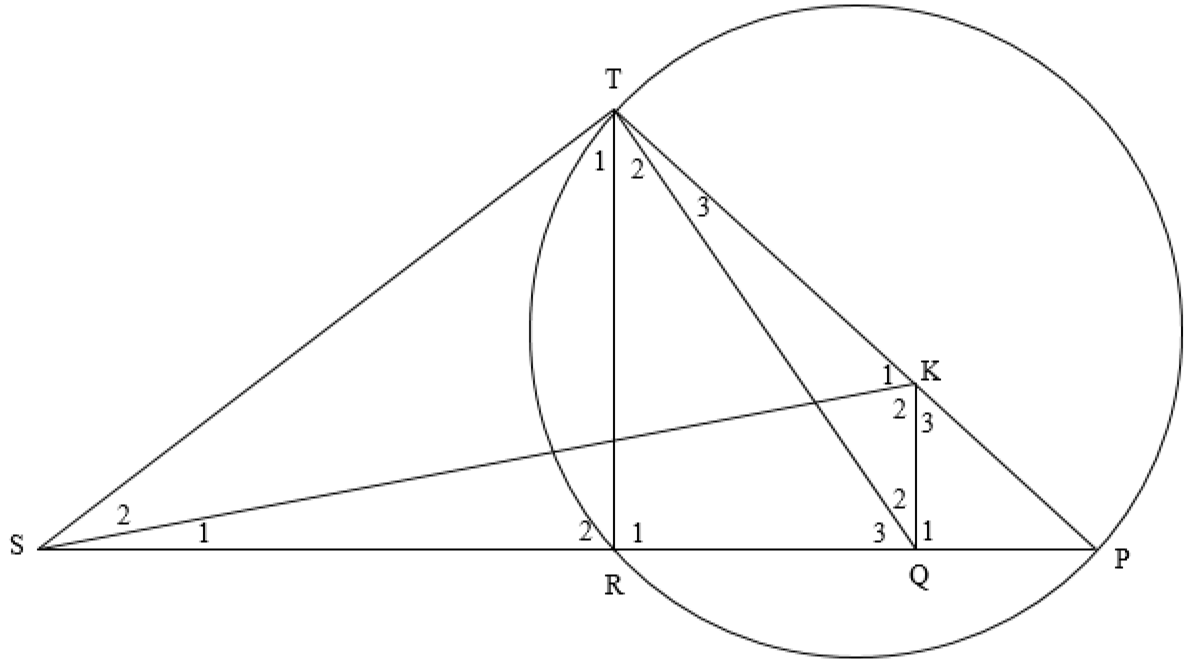
14.1 US is a tangent to the circle PWUT at U. (5)

14.2 ΔSPU|||ΔSUW (4)

[9]

Activity 15

In the diagram below, ST is a tangent to circle TRP. PT is a diameter, SRQP is a secant. K is a point on PT such that $PK:KT = 1:2$ and $PR = \sqrt{18}$ units and $PQ = \sqrt{2}$ units.



15.1 Prove that:

15.1.1 RT||QK (4)

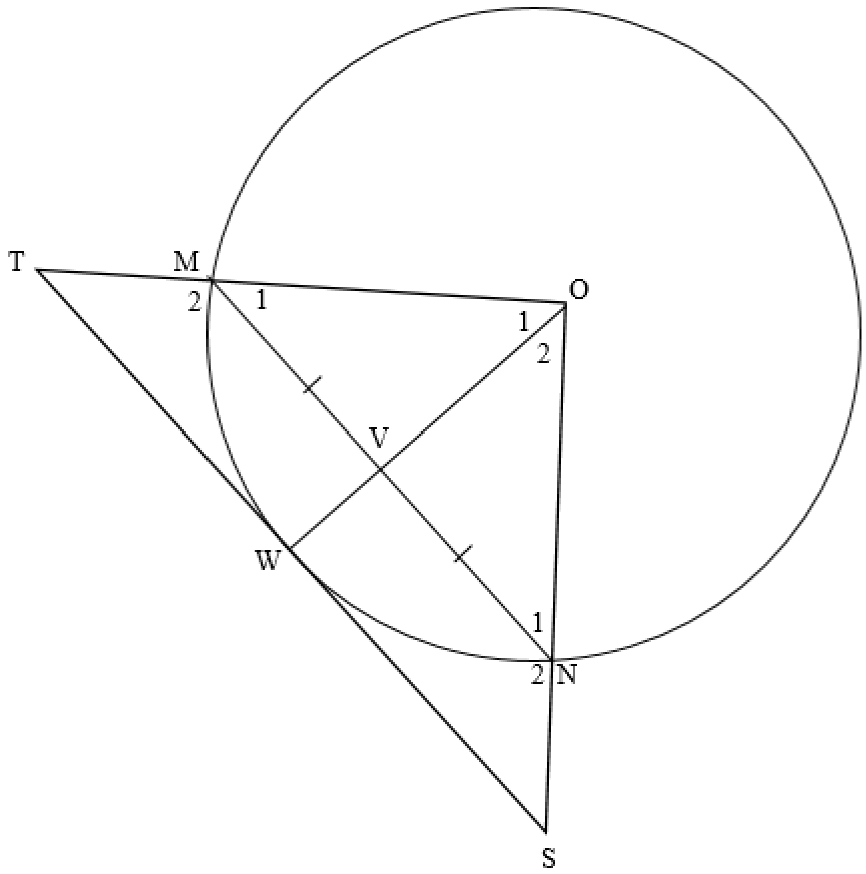
15.1.2 TKQS is a cyclic quadrilateral. (5)

15.1.3 $\Delta QRT|||\Delta KTS$ (4)

[13]

Activity 16

In the diagram, W is a point on circle with centre O. V is a point on OW, Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.



16.1 Give a reason why $OV \perp MN$. (1)

16.2 Prove that:

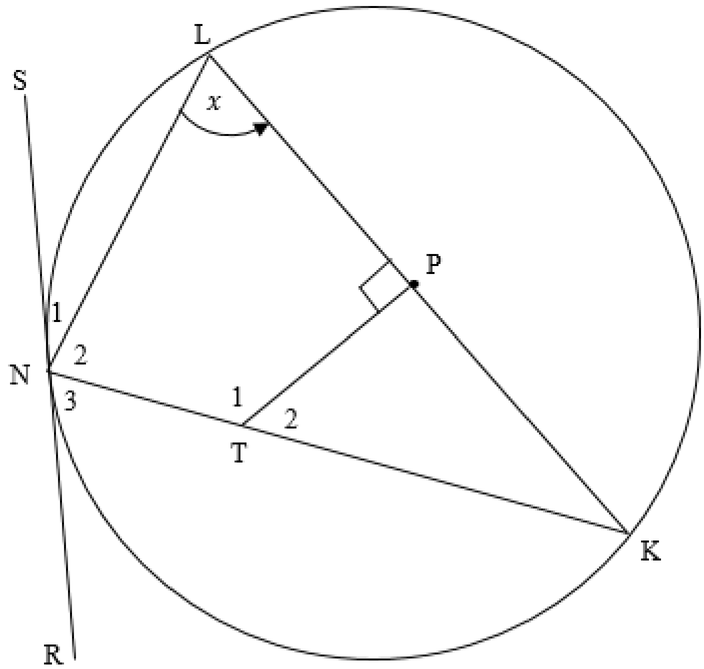
16.2.1 MN||TS (2)

16.2.2 TMNS is a cyclic quadrilateral. (4)

[7]

Activity 17

In the diagram, LK is a diameter of the circle with centre P. RNS is a tangent to the circle at N. T is a point on NK and TP ⊥ KL. $P\hat{L}N = x$.



17.1 Prove that TPLN is a cyclic quadrilateral. (3)

17.2 Determine, giving reasons, the size on $\hat{N}_1$ in terms of x. (3)

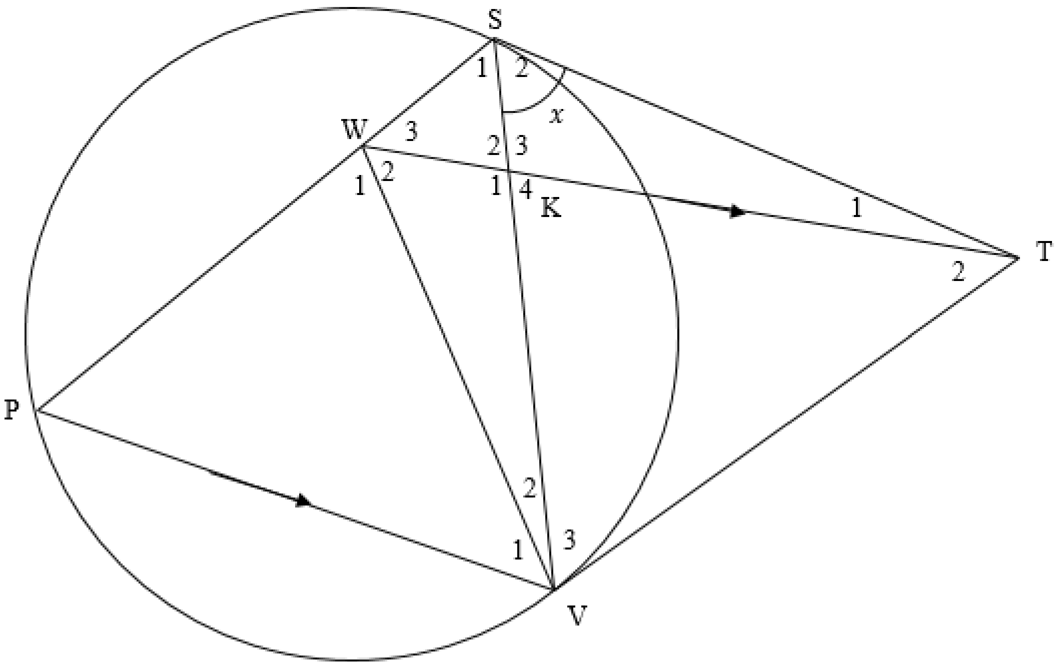
17.3 Prove that:

$\Delta KTP ||| \Delta KLN$ (3)

[9]

Activity 18

In the diagram, ST and VT are tangents to the circle at S and V respectively. P is a point on the circle and W is a point on chord PS such that WT is parallel to PV. SV and WV are drawn. WT intersects SV at K. Let $\hat{S}_2 = x$.



18.1 Write down, with reasons, THREE other angles each equal to x. (6)

18.2 Prove, with reasons, that:

18.2.1 WSTV is a cyclic quadrilateral. (2)

18.2.2 ΔWPV is isosceles (4)

18.2.3 ΔWPV |||ΔTSV (3)

[15]