JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

ANSWER BOOKLET

GRADE 12

FUNCTIONS AND GRAPHS

JENN MAREMATLOU TRAINING INSTITUTE

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TABLE OF CONTENTS

1.	HYPERBOLIC FUNCTION	3 – 18
2.	QUADRATIC FUNCTION	19 – 22
3.	EXPONETIAL FUNCTION	23 – 26
4.	INVERSE FUNCTIONS	27 – 36
5.	COMBINATIONS	37 – 46
6.	FUNCTIONS – SELECTED QUESTIONS	47 - 56

SECTION 1: HYPERBOLIC FUNCTION

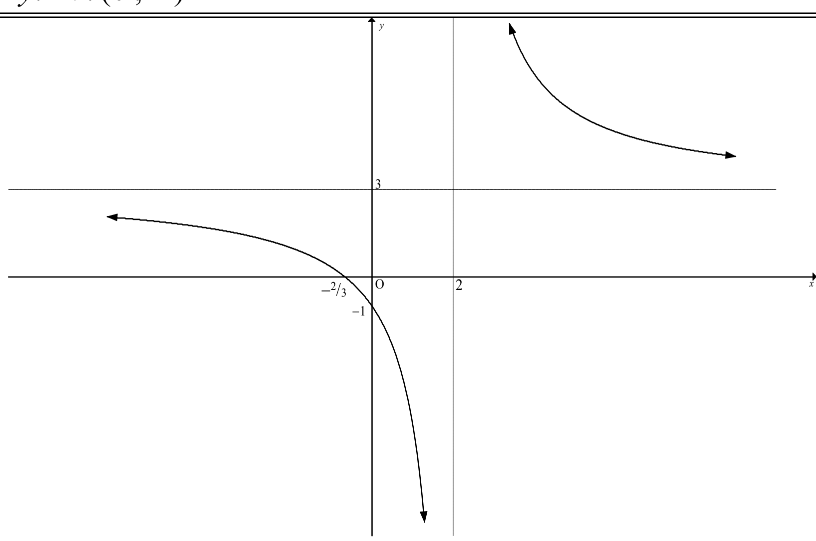
QUESTION 1

1.1.1	$(1\,;-2)$	✓ for/*vir* 1 ✓for/*vir* – 2 (2)
1.1.2	For *x*-intercept/*Vir x-afsnit*: $0=\frac{-9}{x-1}-2$ $2=\frac{-9}{x-1}$ $2(x-1)=-9$ $2x=-7$ $x=-\frac{7}{2}$ $\left(-\frac{7}{2}\,;\,0\right)$ For *y*-intercept/*Vir y-afsnit*: $y=\frac{-9}{0-1}-2$ $=9-2$ $=7$ $(0;7)$	✓$y=0$ ✓ simplification/*vereenv* ✓ answer/*antwoord* ✓$x=0$ ✓ answer/*antwoord* (5)
1.1.3	$y=-x-1$	✓$-x$ ✓ -1 (2)

1.1.4	Closest point is a point of intersection between the axis of symmetry and the hyperbola/*Naaste punt is 'n snypunt tussen die simmetrie-as en die hiperbool*: $-x-1=\frac{-9}{x-1}-2$ $-x+1=\frac{-9}{x-1}$ $x-1=\frac{9}{x-1}$ $(x-1)^2=9$ $x-1=3$ or $x-1=-3$ $x=4$ $\quad x=-2$ in the fourth quadrant, $x>0$, hence $x=4$ *only* $y=-4-1$ $y=-5$ Point/*Punt* is $(4\,;-5)$	✓ equating/*vgl* ✓ $(x-1)^2=9$ ✓ answers for/*antwoord vir x* ✓ selects $x = 4$ only/ *kies slegs x = 4* ✓ answer for/*antwoord vir y* (5)
	OR/*OF* Closest point is a point of intersection between the axis of symmetry and the hyperbola/*Naaste punt is 'n snypunt tussen die simmetrie-as en die hiperbool*: $-x-1=\frac{-9}{x-1}-2$ $(-x-1)(x-1)=-9-2(x-1)$ $-x^2+1=-9-2x+2$ $0=x^2-2x-8$ $0=(x-4)(x+2)$ $x=4$ $\quad x=-2$ in the fourth quadrant, $x>0$, hence $x=4$ *only* $y=-4-1$ $y=-5$ Point is $(4;-5)$ **OR/**	✓ equating/*vgl* ✓ $0=x^2-2x-8$ ✓ answers for/*antwoord vir x* ✓ selects $x = 4$ only/ *kies slegs x = 4* ✓ answer for/*antwoord vir y* (5)

	Onder die translasie 1 regs en 2 na onder, sal punte in die vierde kwadrant steeds in die vierde kwadrant wees. *Die oorsprong word A onder die translasie 1 regs en 2 na onder, en die punt in die vierde kwadrant wat die naaste punt aan* $y=\frac{-9}{x}$ *tot die oorsprong is, is* $(3;-3)$. *Die naaste punt op f aan A is* $(3+1;-3-2)$ *d.i.* $(4;-5)$	✓*punte in 4*^{de} *kwad bly in 4*^{de} *kwad* ✓*oorsprong word A* ✓*naaste punt aan oorsprong op moederfunksie is* $(3;-3)$ ✓✓answer/*antwoord* (5)
1.1.5	$y=\frac{9}{x-1}+2$	✓ $\frac{9}{x-1}$ ✓ +2 (2)

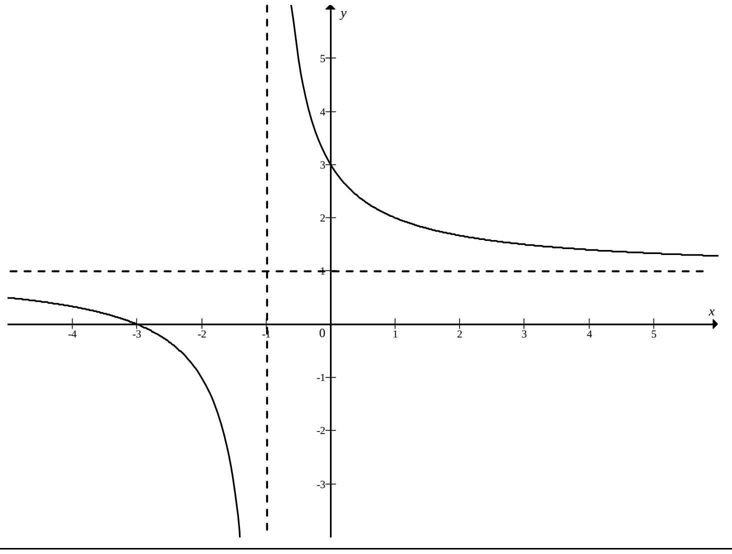
QUESTION 2

2.1	$x=2$ $y=3$	✓ $x=2$ ✓ $y=3$ (2)
2.2	$x.\text{int}: \frac{8}{x-2}+3=0$ $8+3(x-2)=0$ $3x+2=0$ $\therefore x=-\frac{2}{3}$ $\therefore x-\text{int}\left(-\frac{2}{3};0\right)$ $y=\frac{8}{0-2}+3$ $y=-1$ $y.\text{int}:(0;-1)$	✓ $\frac{8}{x-2}+3=0$ ✓ $\left(-\frac{2}{3};0\right)$ ✓ $(0;-1)$ (3)
2.3		✓asymptotes ✓intercepts with axes ✓shape (3)
2.4	$3=2+k$ $k=1$ **OR** $y=(x-2)+3$ $y=x+1$ $\therefore \quad k=1$	✓ substitute ✓ answer (2) ✓ $y=x+1$ ✓answer (2) **[10]**

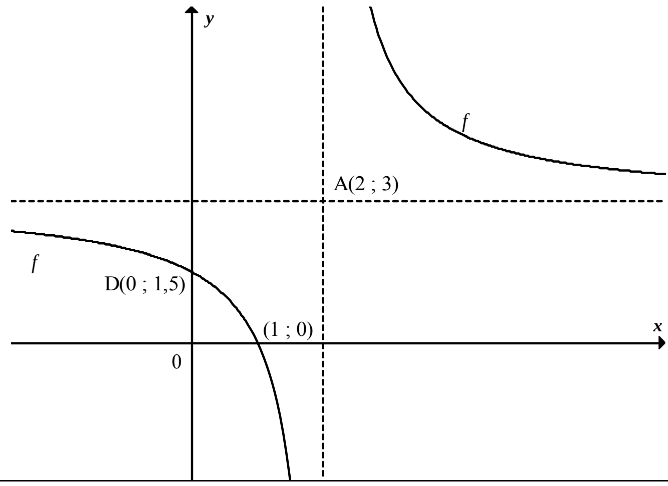
QUESTION 3

3.1	$p = 4$ $q = 2$ $3 = \frac{a}{5-4} + 2$ $1 = \frac{a}{1}$ $a = 1$	✓ answer *p* ✓ answer *q* ✓ substitution of (5; 3) ✓ answer (4) Answer for *p* 1 mark Answer for *q* 1 mark Answer for *a* 2 marks
3.2	$y = -x + c$ substitute (4 ; 2) $2 = -4 + c$ $c = 6$ **OR** Translation of the line $y = -x$ 2 units up and 4 units right $y = -(x-4)+2$ $y = -x+6$	✓ correct point (4 ; 2) ✓ substitution ✓ answer (3) ✓ substitution of $x - 4$ ✓ adding 2 ✓ answer (3) Substitution of T(3 ; 5): 0 / 3 Answer only: 3 / 3 **[7]**

QUESTION 4

4.1	$f(0)=\frac{0+3}{0+1}$ $f(0)=3$ y-intercept (0 ; 3) x-intercepts $0=\frac{x+3}{x+1}$ ...... $(x\neq -1)$ $x=-3$ x-intercept (– 3 ; 0)	✓ substitution $x = 0$ ✓ answer ✓ substitution $y = 0$ ✓ answer (4)
4.2	$\frac{2}{x+1}+1$ $=\frac{2+x+1}{x+1}$ $=\frac{x+3}{x+1}$ **OR** $\frac{x+3}{x+1}$ $=\frac{(x+1)+2}{x+1}$ $=\frac{x+1}{x+1}+\frac{2}{x+1}$ $=\frac{2}{x+1}+1$	✓ LCD ✓ simplification (2) **OR** ✓ split the fraction ✓ simplification (2)
4.3	Vertical asymptote: $x = – 1$ Horizontal asymptote: $y = 1$	✓ answer ✓ answer (2)
4.4		✓✓ asymptotes ✓ shape ✓ intercepts (4) **NOTE:** If the graph does not represent a function, candidates do not get the mark for shape.
4.5	$\frac{2}{x+1}\geq -1$ $\frac{2}{x+1}+1\geq 0$ $x\in(-\infty\ ;-3]\cup(-1\ ;\infty)$ **OR** $x\leq -3$ or $x>-1$	✓ manipulation ✓✓ answer (3) **NOTE:** 0 marks for $-1<x\leq -3$ **[15]**

QUESTION 5

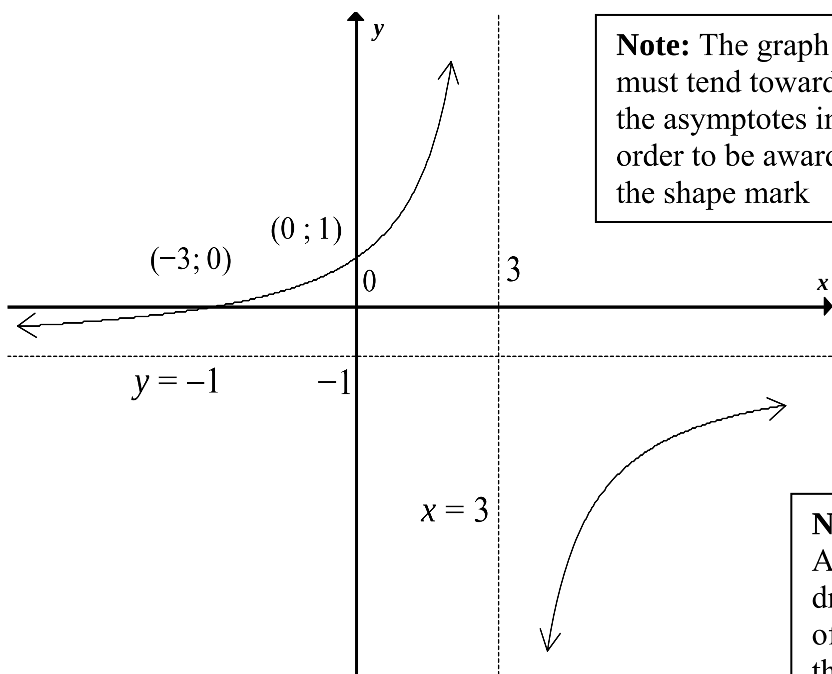
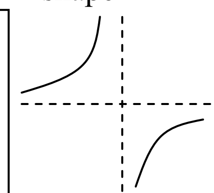


5.1	$x=2$ **OR** x-asymptote $=2$ $y=3$ y-asymptote $=3$ If $x=p$; $y=q$ then 1 mark **Note:** If the candidate just writes down the number 2 or 3 or just coordinates (2 ; 3), then no marks	✓ answer ✓ answer (2)
5.2	$f(x)=\frac{a}{x-2}+3$ $0=\frac{a}{1-2}+3$ $0=-a+3$ $a=3$ $f(x)=\frac{3}{x-2}+3$ If the asymptotes are swopped in 4.1, then $f(x)=\frac{a}{x-3}+2$ $0=\frac{a}{1-3}+2$ $a=4$ $f(x)=\frac{4}{x-3}+2$ **OR** $y=\frac{a}{x-2}+3$ $y-3=\frac{a}{x-2}$ $(x-2)(y-3)=a$ But (1;0) lies on the graph $\therefore(-1)(-3)=a=3$ $\therefore(x-2)(y-3)=3$	✓subs in of asymptotes ✓ subs in (1 ; 0) ✓ answer (3) ✓ equation ✓ subs in (1 ; 0) ✓ answer (3)
5.3	When $x=0,\ y=\frac{3}{0-2}+3$ $=\frac{3}{2}$ $D\left(0\,;\frac{3}{2}\right)$ If asymptotes swopped: $x=0$ $y=\frac{4}{0-3}+2$ $y=\frac{2}{3}$ $D\left(0\,;\frac{2}{3}\right)$	✓ $x=0$ ✓ $y=\frac{3}{2}$ (2)

5.4	$m_{AD} = \frac{3-1,5}{2-0}$ $= \frac{3}{4}$ $y = \frac{3}{4}x + \frac{3}{2}$ **OR** $4y = 3x + 6$ If asymptotes swopped: $m_{AD} = \frac{3-\frac{2}{3}}{2-0}$ $= \frac{7}{3} \times \frac{1}{2}$ $= \frac{7}{6}$ $y = \frac{7}{6}x + \frac{2}{3}$	✓ substitution into gradient ✓ $\frac{3}{4}$ ✓ answer (3)
	OR $y = mx + \frac{3}{2}$ $3 = m(2) + \frac{3}{2}$ $m = \frac{3}{4}$ $y = \frac{3}{4}x + \frac{3}{2}$	✓ substitution of point (2 ; 3) and $c = \frac{3}{2}$ ✓ $\frac{3}{4}$ ✓ answer (3)
5.5	$\frac{p+0}{2} = 2$ $p = 4$ $\frac{q+\frac{3}{2}}{2} = 3$ $q = 4\frac{1}{2}$ Other point of intersection is $\left(4\,;\,4\frac{1}{2}\right)$ **Answer only:** Full Marks	✓ $\frac{p+0}{2} = 2$ ✓ $\frac{q+\frac{3}{2}}{2} = 3$ ✓ $x = 4$ ✓ $y = 4\frac{1}{2}$ (4)
	OR By symmetry the rule to calculate the point of intersection is $(x\,;\,y) \rightarrow \left(x+2\,;\,y+\frac{3}{2}\right)$ Other point of intersection is $\left(2+2\,;\,3+\frac{3}{2}\right)$ $= \left(4\,;\,4\frac{1}{2}\right)$ To help with applying CA the *y*-coordinate will be 3 + (3 – *y*)	✓✓ *x*-answer ✓✓ *y*-answer (4)

	OR $\frac{3}{4}x+\frac{3}{2}=\frac{3}{x-2}+3$ $3x(x-2)+6(x-2)=12+12(x-2)$ $3x^2-6x+6x-12=12+12x-24$ $3x^2-12x=0$ $3x(x-4)=0$ $x=0$ and $x=4$ Other point of intersection is $\left(4\,;4\frac{1}{2}\right)$ **Note:** If the candidate does not select the *x*-value greater than 2 i.e. a realistic answer, max 3 / 4 marks If asymptotes swopped: $\frac{7}{6}x+\frac{2}{3}=\frac{4}{x-3}+2$ $7x(x-3)+4(x-3)=4(6)+2(6)(x-3)$ $7x^2-29x=0$ $x(7x-29)=0$ $x=0$ or $x=\frac{29}{7}$ Other point of intersection is $\left(\frac{29}{7};\frac{11}{2}\right)$	✓ equating ✓ standard form ✓ *x*-values ✓ *y*-value (4) **[14]**

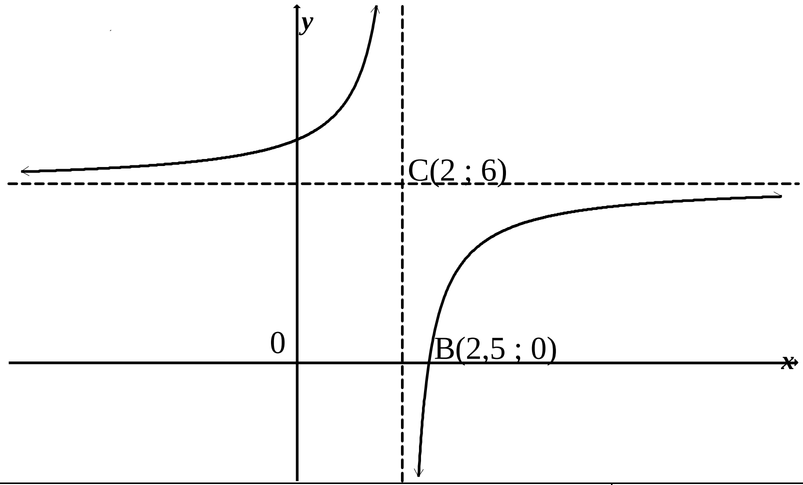
QUESTION 6

6.1.1	$y = f(0)$ $= \frac{-6}{0-3} - 1$ $= 1$ $(0\ ;\ 1)$ **OR** $x = 0$ and $y = 1$ **Note:** Mark 5.1.1 and 5.1.2 as a single question. If the intercepts are interchanged: max 3/5 marks	✓ $y = 1$ ✓ $x = 0$ (2)
6.1.2	$0 = \frac{-6}{x-3} - 1$ $1 = \frac{-6}{x-3}$ $x - 3 = -6$ $x = -3$ $(-3\ ;\ 0)$	✓ $y = 0$ ✓ $x - 3 = -6$ ✓ answer (3)
6.1.3	 **Note:** The graph must tend towards the asymptotes in order to be awarded the shape mark **Note:** A candidate who draws only one 'arm' of the hyperbola loses the 'shape' mark i.e. max 3/4 marks	✓ shape  ✓ both intercepts correct ✓ horizontal asymptote ✓ vertical asymptote (4)
6.1.4	$-3 < x < 3$ **OR** $(-3; 3)$ **OR** $-3 < x$ and $x < 3$ **Note**: if candidate writes $-3 < x$ only: 1/2 marks **Note**: if candidate writes $x < 3$ only: 1/2 marks	✓-3 and 3 ✓inequality OR interval notation (2)
6.1.5	$y = \frac{-6}{-2-3} - 1$ $= \frac{1}{5}$ $m = \frac{1 - \frac{1}{5}}{0-(-2)}$ $= \frac{2}{5}$	✓ $\frac{1}{5}$ ✓ formula ✓ substitution ✓ answer (4)

QUESTION 7

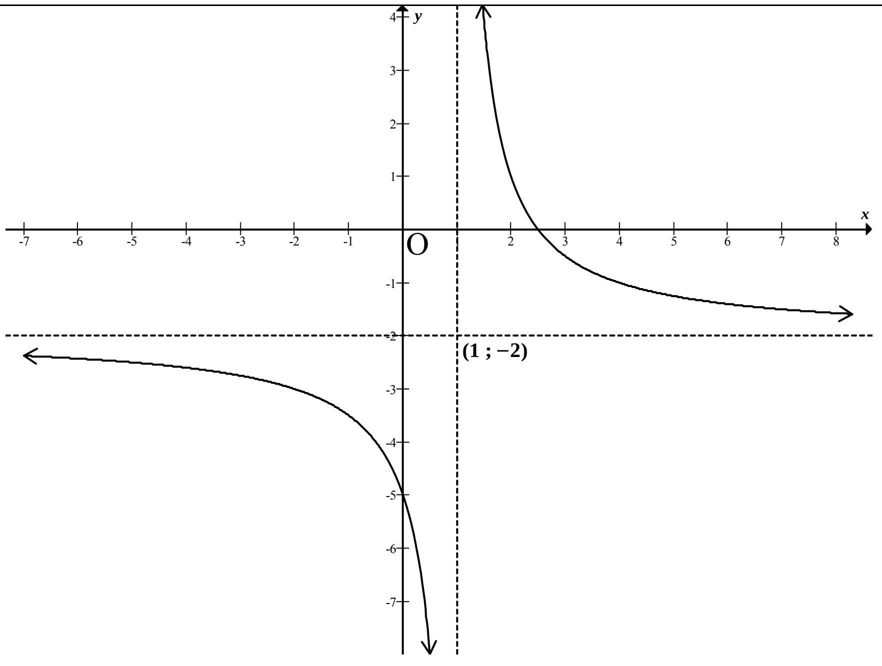
	$f(x)=\dfrac{a}{x-5}+1$ $0=\dfrac{a}{(2)-5}+1$ $-1=\dfrac{a}{-3}$ $a=3$ $f(x)=\dfrac{3}{x-5}+1$ **OR** $(x-5)(y-1)=k$ $(2-5)(0-1)=k$ $k=3$ $(x-5)(y-1)=3$ $y=\dfrac{3}{x-5}+1$ **NOTE:** $f(x)=\dfrac{x-2}{x-5}$ as an alternative simplified form.	✓ $x-5$ ✓ $+1$ ✓ substitution of (2 ; 0) ✓ $a=3$ (4) ✓ $(x-5)$ ✓ $(y-1)$ ✓ substitution of (2 ; 0) ✓ $k=3$ (4) **[4]**

QUESTION 8

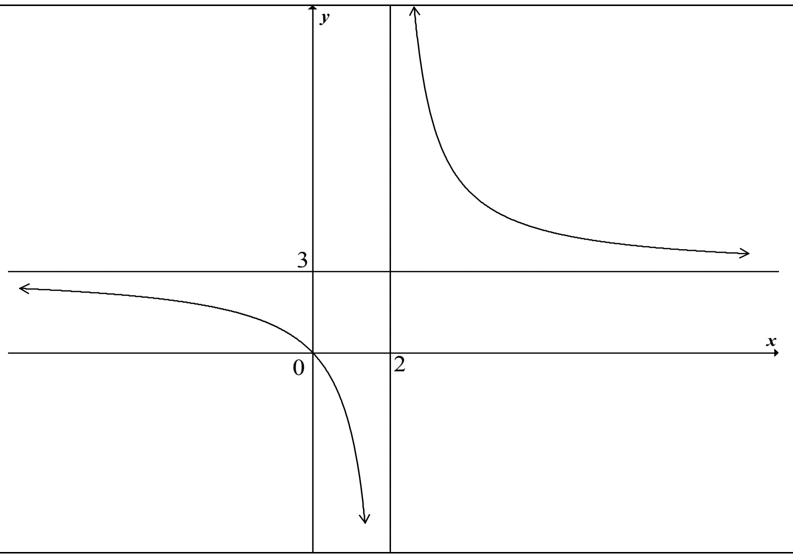


8.1	$g(x)=\dfrac{a}{x-2}+6$ $0=\dfrac{a}{2{,}5-2}+6$ $0=2a+6$ $a=-3$ $g(x)=\dfrac{-3}{x-2}+6$	✓ $p=2$ ✓ $q=6$ ✓ substitute B(2,5 ; 0) ✓ $a=-3$ (4)
8.2	$x_f=2-\dfrac{1}{2}$ $x_f=\dfrac{3}{2}$ $y_f=6+6$ $y_f=12$ $F\left(\dfrac{3}{2};12\right)$	✓*x*-coordinate ✓*y*-coordinate (2) **[6]**

QUESTION 9

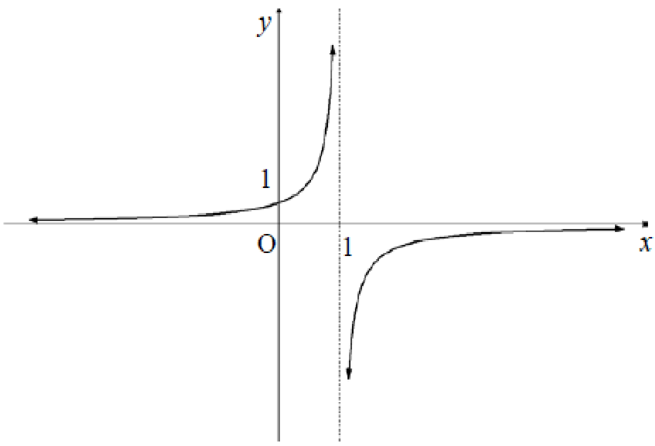
9.1	$x = 1$ $y = -2$	✓✓ answers (2)
9.2	y-intercept: $y = \frac{3}{0-1} - 2 = -5$ x-intercept: $\left(\frac{5}{2}\ ;\ 0\right)$ $0 = \frac{3}{x-1} - 2$ $2 = \frac{3}{x-1}$ $2x - 2 = 3$ $2x = 5$ $x = \frac{5}{2}$	✓ y = -5 ✓ substitute y = 0 ✓ answer (3)
9.3		✓ asymptotes ✓ y–intercept ✓ shape (3)
9.4	$-f(x) = \frac{-3}{x-1} + 2$ $y \in R - \{2\}$ OR $y \in (-\infty\ ;\ 2) \cup (2\ ;\ \infty)$ OR $y \in R; y \neq 2$	✓ answer (1)
9.5	$g(x) = \frac{-3}{x+1} - 2$ $= \frac{3}{-x-1} - 2$ Reflection of f about the y-axis. OR (i) horizontal shift 2 units to the left followed by (ii) reflection in x-axis, followed by (iii) vertical downward shift of 4 units	✓ manipulation ✓ answer (2) **[11]**

QUESTION 10

10.1	$x = 2$ $y = 3$	✓ $x = 2$ ✓ $y = 3$ (2)
10.2	**R** ; $x \neq 2$ **OR** $(-\infty ; 2) \cup (2 ; \infty)$ **OR** **R** $-\{2\}$	✓ answer (1)
10.3		✓ shape ✓ intercept at origin ✓✓asymptotes (4)
10.4	$y = x + 3$ and $y = -x + 1$ $x + 3 = -x + 1$ $2x = -2$ $x = -1$ $y = -1 + 3$ $= 2$ Point of intersection of asymptotes: $(-1 ; 2)$ *Die snypunt van die asimptote:* The transformation is a translation 3 units left and 1 unit down *Die transformasie is 'n translasie van 3 eenhede na links en 1 eenheid na onder* **OR** The transformation is $(x ; y) \rightarrow (x - 3 ; y - 1)$	✓ $x + 3 = -x + 1$ ✓$x = -1$ ✓ $y = 2$ ✓ transformation (4) **[11]**

November 2018

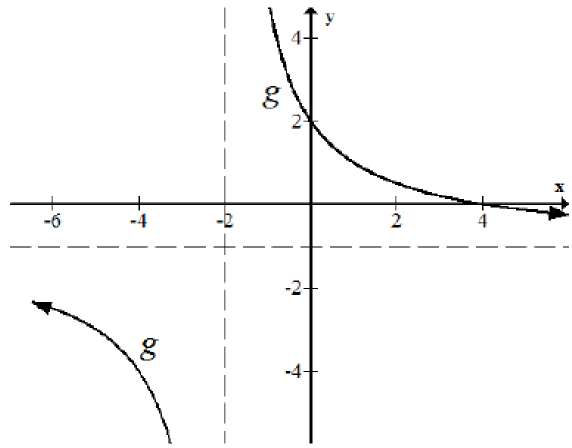
QUESTION/*VRAAG* 5

5.1	Domain: $x \in R$; $x \neq 1$ **OR/OF** $x \in (-\infty;1) \cup (1;\infty)$	✓answer (1)
5.2	$x = 1$ $y = 0$	✓$x = 1$ ✓$y = 0$ (2)
5.3		✓ y intercept ✓vertical asymptote ✓shape (3)
5.4	$x \geq 0$; $x \neq 1$ **OR/OF** $0 \leq x < 1$ or $x > 1$ **OR/OF** $x \in [0;1) \cup (1;\infty)$	✓$x \geq 0$ ✓$x \neq 1$ (2) **OR/OF** ✓ $0 \leq x < 1$ ✓ $x > 1$
		[8]

March 2015

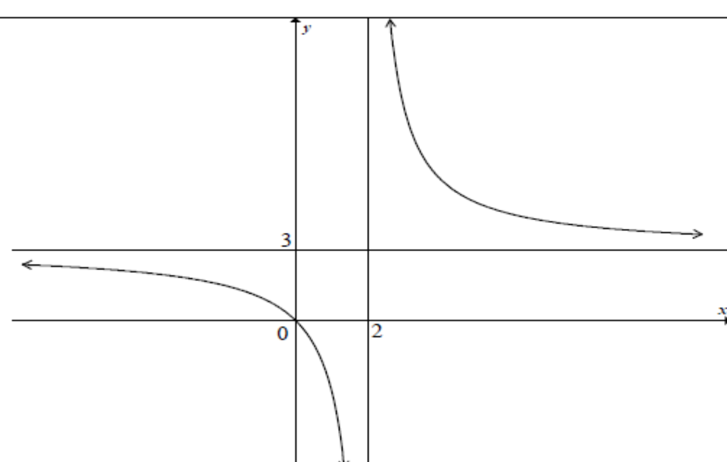
QUESTION/*VRAAG* 4

4.1	$x = -2$ $y = -1$	✓ $x = -2$ ✓ $y = -1$ (2)
4.2.1	$g(0) = \frac{6}{0+2} - 1$ $= 2$ y-intercept/*afsnit* (0 ; 2)	✓ answer/*antwoord* (1)
4.2.2	$0 = \frac{6}{x+2} - 1$ $1 = \frac{6}{x+2}$ $x + 2 = 6$ $x = 4$ x-intercept/*afsnit* (4 ; 0)	✓ equating to/*stel gelyk aan* 0 ✓ answer/*antwoord* (2)

4.3		✓ asymptotes/*asimptote* ✓ intercepts/*afsnitte* ✓ shape/*vorm* (3)
4.4	$y+1=-(x+2)$ $y=-x-3$ **OR/*OF*** Using general formula/*Gebruik algemene formule*: $y=-(x+p)+q$ $y=-(x+2)-1$ $y=-x-3$	✓$m=-1$ ✓substitution of $(-2\ ;-1)$ ✓answer (3) ✓formula/*formule* ✓substitution of p and q values/*substitusie van p- en q-waardes* ✓answer/*antwoord* (3)
4.5	$x>-2$	✓✓ answer (2) **[13]**

March 2014

QUESTION/*VRAAG* 6

6.1	$x=2$ $y=3$	✓ $x=2$ ✓ $y=3$ (2)
6.2	**R** ; $x \neq 2$ **OR** $(-\infty\ ;2)\cup(2\ ;\infty)$ **OR** **R** $-\{2\}$	✓ answer (1)
6.3		✓ shape ✓ intercept at origin ✓✓asymptotes (4)

6.4	$y = x + 3$ and $y = -x + 1$ $x + 3 = -x + 1$ $2x = -2$ $x = -1$ $y = -1 + 3$ $= 2$ Point of intersection of asymptotes: $(-1\ ;\ 2)$ *Die snypunt van die asimptote:* The transformation is a translation 3 units left and 1 unit down *Die transformasie is 'n translasie van 3 eenhede na links en 1 eenheid na onder* **OR** The transformation is $(x\ ;\ y) \to (x - 3\ ;\ y - 1)$	✓ $x + 3 = -x + 1$ ✓ $x = -1$ ✓ $y = 2$ ✓ transformation (4) **[11]**

SECTION 2 : QUADRATIC FUNCTIONS

QUESTION 1

1.1	C(– 2 ; 0)	✓ answer (1)
1.2	$f(x) = ax^2 + q$ $f(x) = a\left(x^2 - 4\right)$ $2{,}5 = a((-3)^2 - 4)$ $2{,}5 = 5a$ $a = \frac{1}{2}$ $f(x) = \frac{1}{2}\left(x^2 - 4\right)$	✓ $f(x) = a\left(x^2 - 16\right)$ ✓ substitution of (– 5 ; 2,25) ✓ answer (3)
1.3	Range of *f*: $[-2 ; \infty)$	✓ answer (1)
1.4	Range of *h*: $(-\infty ; 0]$	✓ notation ✓ critical values (2)
1.5	$g(x) = b^x - 4$ $0 = b^2 - 4$ $4 = b^2$ $b = 2$ $g(x) = 2^x - 4$	✓ $g(x) = b^x - 4$ ✓ substitution ✓ answer (3) **[10]**

QUESTION 2

2.1	$d - 5 + d - 1 = 0$ $2d = 6$ $d = 3$	✓ $d - 5 + d - 1 = 0$ ✓ $d = 3$ (2)
2.2	$y = a(x-2)(x+2)$ $-9 = a(1-2)(1+2)$ $-9 = a(-1)(3)$ $-3a = -9$ $a = 3$ $f(x) = 3\left(x^2 - 4\right)$ $= 3x^2 - 12$ $c = -12$	✓ $y = a(x-2)(x+2)$ ✓ subs (1 ; –9) ✓ $a = 3$ ✓ $c = -12$ (4) **[6]**

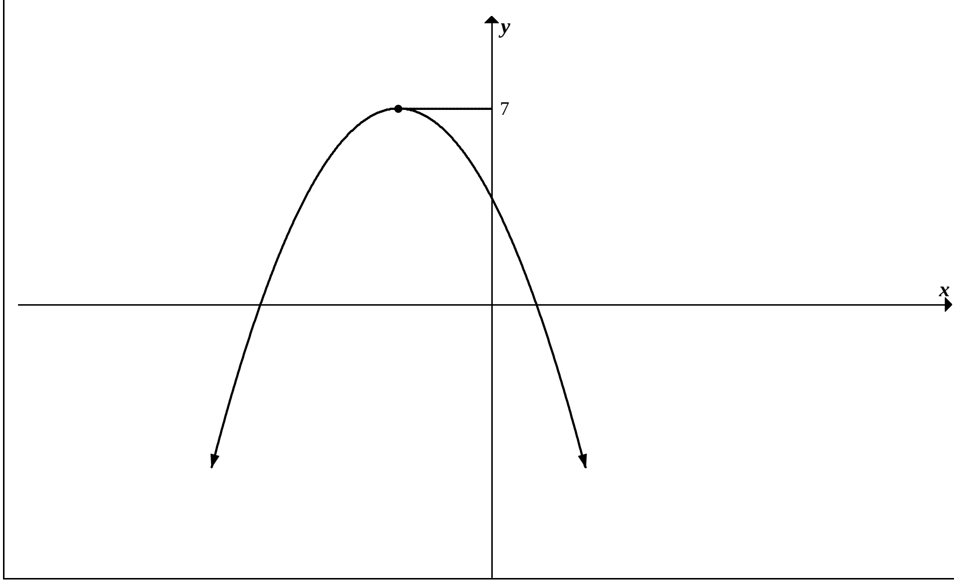
QUESTION 3

3.1	$f(x) = (x - 3)(x + 1) = x^2 - 2x - 3$	(3)	✓ x^2 ✓ $-2x$ ✓ -3

QUESTION 4

4.1	$y = a(x-2)^2 + 9$ Substitution $(0\,;5)$: $5 = a(0-2)^2 + 9$ $5 = 4a + 9$ $a = -1$ $y = -1(x-2)^2 + 9$ $= -(x^2 - 4x + 4) + 9$ $= -x^2 + 4x + 5$	✓ substitution coordinates of TP ✓ substitution of/*van* $(0;5)$ ✓ value of/*waarde van a* ✓ simplification/*vereenv* (4)
4.2	Average Gradient $= \dfrac{9-5}{2-0}$ or $\dfrac{5-9}{0-2}$ $= 2$	✓ $\dfrac{9-5}{2-0}$ or $\dfrac{5-9}{0-2}$ ✓answer/*antwoord* (2)
4.3	x-intercepts of/*x-afsnitte van f*: $\frac{1}{2}x^2 - 8 = 0$ $x^2 = 16$ $x = 4$ or -4 At/*By* B: $x = -4$ x-intercepts of/*x-afsnitte van g*: $-x^2 + 4x + 5 = 0$ $x^2 - 4x - 5 = 0$ $(x-5)(x+1) = 0$ $x = -1$ or 5 At/*By* D: $x = 5$ Length of/*Lengte van* BD: 4 + 5 = 9	✓ $\frac{1}{2}x^2 - 8 = 0$ ✓ -4 ✓ factors/*faktore* ✓ 5 ✓answer/*antwoord* (5)
4.4.1	$x \leq -4$ or $x \geq 4$	✓ $x \leq -4$ ✓ $x \geq 4$ (2)
4.4.2	$0 < x < 2$	✓ endpoints/*eindpunte* ✓ notation/*notasie* (2) **[15]**

QUESTION 5

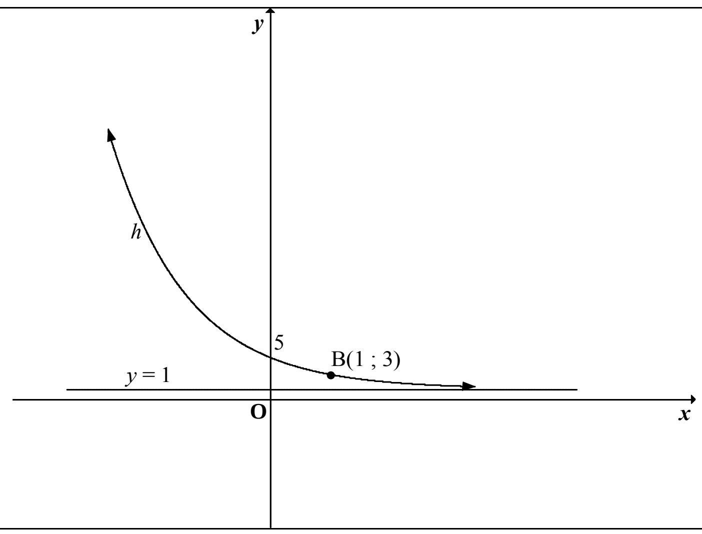
Range of $f\ (-\infty;7]\ \Rightarrow y$-part of turning point [Max value of $f(x)$] is 7 $a < 0$ and shape  $b < 0\ \Rightarrow\ b$ negative $\Rightarrow$ axis of symmetry on left of y-axis roots real, unequal & opposite signs $\Rightarrow$ x-ints on opposite sides of y-axis  	✓ shape ✓turning point at $y = 7$ ✓ axis of symmetry on left of y-axis ✓ roots are on opposite sides **[4]**

SECTION 3: EXPONENTIAL FUNCTIONS

QUESTION 1

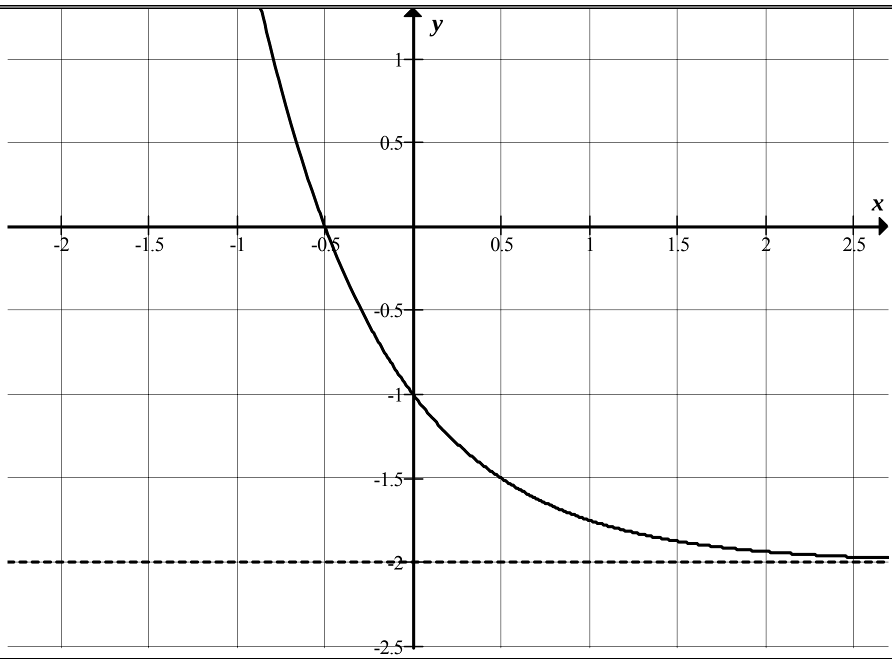
1.1	$q = -6$	✓answer (1)
1.2	$-5\frac{1}{4} = a.2^{-1-1} - 6$ $\frac{3}{4} = \frac{1}{4}a$ $a = 3$	✓substitute x ✓substitute y ✓simplifying ✓answer (4)
1.3	$x\text{int}: 2^{x-1} = 2 \quad \therefore x = 2 \; \therefore (2;0)$ $y\text{int}: y = 3.2^{-1} - 6 = -4\frac{1}{2} \quad \therefore \left(0; -4\frac{1}{2}\right)$ Average Gradient $= \frac{0 + 4\frac{1}{2}}{2 - 0}$ $= \frac{9}{4}$ or $2\frac{1}{4}$	✓ $2^{x-1} = 2$ ✓ $x = 2$ ✓$y = -4\frac{1}{2}$ ✓subst. into gradient formula ✓answer (5)
1.4	$y = 3.2^{x-3} - 6$	✓✓answer (2) **[12]**

QUESTION 2

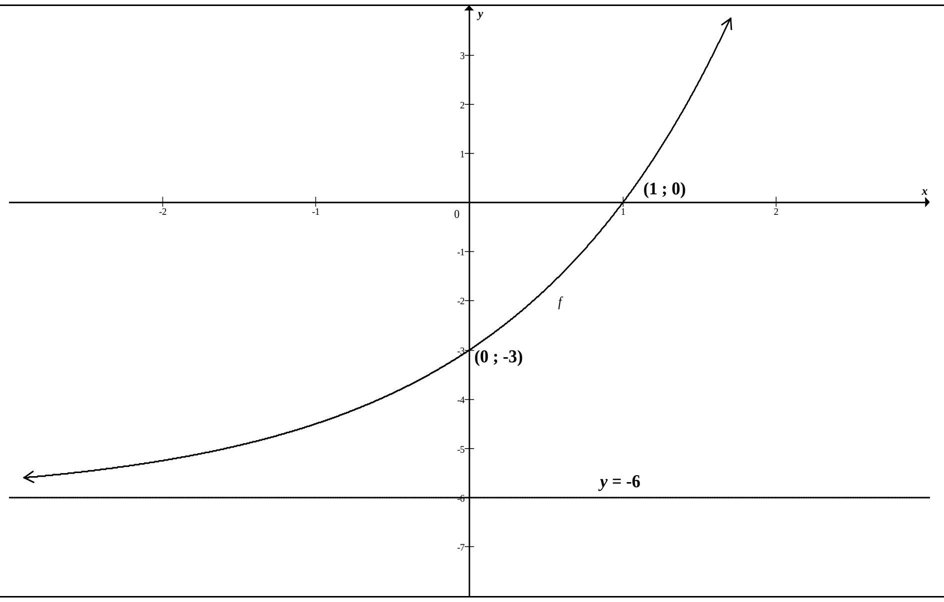
.2.1	For y-intercept/*Vir y-afsnit* substitution $x = 0$: $y = 4.2^0 + 1$ $= 5$ H(0 ; 5)	✓$x = 0$ substitution into the equation/*in die vgl* ✓$y = 5$ (2)
.2.2	For x-intercept/*Vir y-afsnit* $y = 0$ i.e./*d.i.* $4.2^{-x} + 1 = 0$ $4.2^{-x} = -1$ $2^{-x} = -\frac{1}{4}$, which is impossible, since $2^{-x} > 0$ for $x \in R$, *wat onmoontlik is omdat* $2^{-x} > 0$ *vir* $x \in R$ Therefore/*Dus*: no solution/*geen oplossing*, which means there will be no x-intercept/*wat beteken daar sal geen x-afsnit wees nie*. **OR/*OF*** The graph lies above its asymptote $y = 1$ because the coefficient of 2^{-x} is 4/*Die grafiek lê bokant sy asimptoot y = 1 want die koëffisiënt van* 2^{-x} *is* 4. **OR/*OF*** The range is $(1\ ;\infty)$ or $y > 1$ *Die waardeversameling is* $(1\ ;\infty)$ of $y > 1$	✓$4.2^{-x} + 1 = 0$ ✓$2^{-x} = -\frac{1}{4}$ and explanation/*en verduideliking* (2) ✓above/*bokant* ✓ $y = 1$ (2) ✓✓correct range/*korrekte waardeversameling* (2)
.2.3		✓ shape/*vorm* ✓y-intercept and other point/*y-afsnit en ander punt* ✓ asymptote/*asimptoot* (3)

.2.4	$g(x) = 4(2^{-x} + 2)$ $= 4.2^{-x} + 8$ The graph of h is translated 7 units upwards to form g/*Die grafiek van h word 7 eenhede na bo getransleer om g te vorm*.	✓ 7 units/*eenhede* ✓ upwards/*opwaarts* (2) **[25]**

QUESTION 3

3.1	$f(x)=4^{-x}-2$ *y*-intercept: $x=0;\ y=4^{0}-2=-1\ ;\ (0\ ;-1)$ *x*-intercept: $4^{-x}-2=0$ $4^{-x}=2$ $2^{-2x}=2$ $-2x=1$ $x=-\frac{1}{2}$ **OR** $4^{-x}-2=0$ $4^{-x}=2$ $\log 4^{-x}=\log 2$ $-x=\frac{\log 2}{\log 4}$ $-x=\frac{\log 2}{2\log 2}$ $x=-\frac{1}{2}$ **OR** $4^{-x}-2=0$ $4^{-x}=2$ $4^{-x}=4^{\frac{1}{2}}$ $-x=\frac{1}{2}$ $x=-\frac{1}{2}$ *x*-intercept is $\left(-\frac{1}{2}\ ;0\right)$	✓✓ *y*-intercept ✓✓ *x*-intercept (4) **Note:** No penalty if the answer is not left as a coordinate.
3.2	$y=-2$	✓ equation (1)
3.3		✓ asymptote ✓ *y*-intercept or *x*-intercept ✓ shape (decreasing) (3)
3.4	$g(x)=4^{-x}-2+2$ $g(x)=4^{-x}$ **OR** $g(x)=\left(\frac{1}{4}\right)^{x}$ **OR** $g(x)=2^{-2x}$ **OR** $g(x)=\left(\frac{1}{2}\right)^{2x}$	✓ equation (1)
3.5	$4^{-x}-2=3$ $4^{-x}=5$ $-x\log 4=\log 5$ $x=-\frac{\log 5}{\log 4}$ **OR** $x=-\log_4 5$ **OR** $x=\log_{\frac{1}{4}} 5$ **OR** $x=\log_4 \frac{1}{5}$ **OR** $x=-1{,}16$ **OR** $x=\frac{\log 5}{\log \frac{1}{4}}$ **OR** $x=\frac{\log \frac{1}{5}}{\log 4}$	✓ $4^{-x}=5$ ✓ $-x\log 4=\log 5$ ✓ answer (3) **[12]**

QUESTION 4

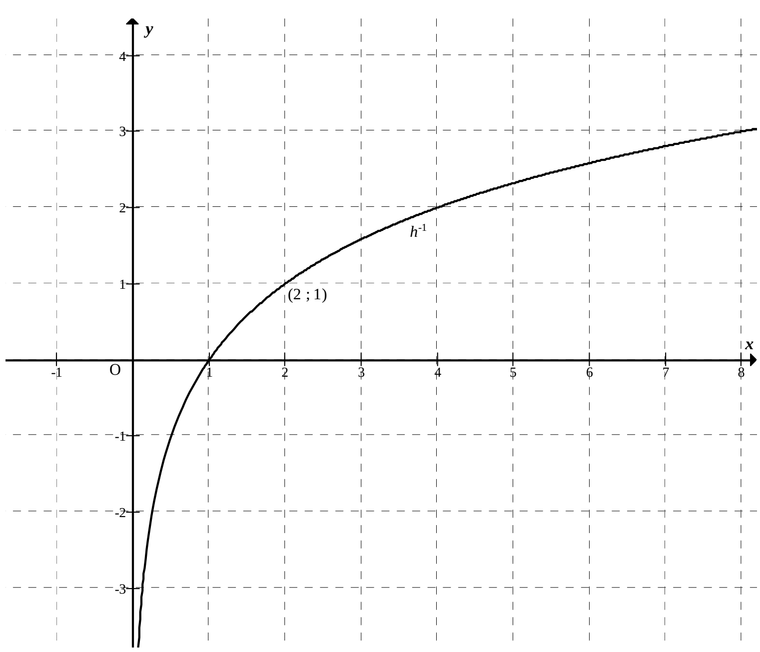
4.1.1	$y = 3.2^0 - 6$ $y = 3 - 6$ $y = -3 \quad (0\,;-3)$	✓ answer (1)
4.1.2	$0 = 3.2^x - 6$ $3.2^x = 6$ $2^x = 2^1$ $x = 1 \quad (1\,;0)$ **Note:** If a candidate interchanges question 4.1.1 and 4.1.2: 0/3 marks **Note:** If a candidate says that $3.2^x = 6^x$ (i.e. wrong mathematics) s/he will arrive at correct answer BUT award max 1/2	✓ $y = 0$ ✓ x-value (2)
4.1.3		✓intercepts ✓ asymptote ✓ shape (3)
4.1.4	$y > -6$ OR $(-6\,;\infty)$	✓ answer (1)

QUESTION 5

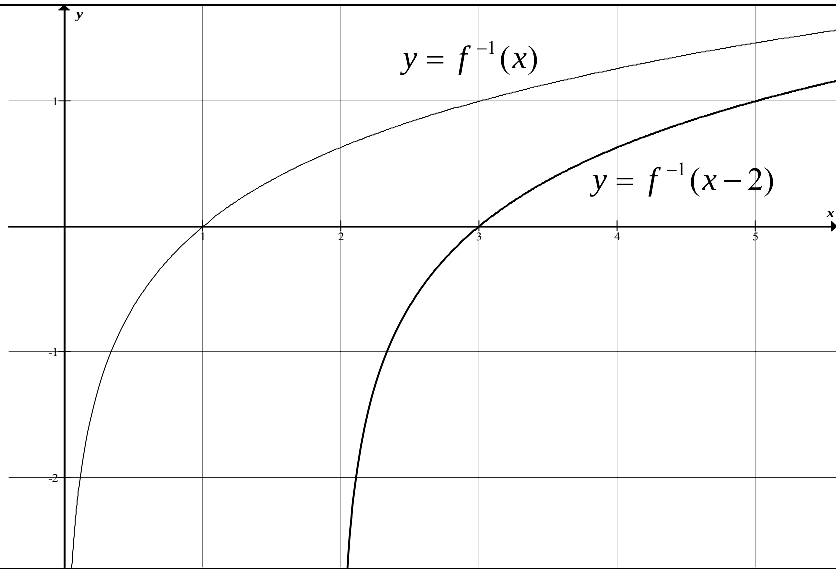
5.1	$f(x) = 2 \times a^x - 1$ $5 = 2.a^1 - 1$ $6 = 2a$ $a = 3$	✓ substitution/substitusie ✓ simplify/vereenvoudig (2)
5.2	$f(x) = 2.3^x - 1$ $y = 2.3^0 - 1$ $y = 2 - 1$ $y = 1$	✓ $x = 0$ ✓ $y = 1$ (2)
5.3	$y > -1$	✓ answer/antwoord (1)
5.4	$f(0{,}23) = 2 \times 3^{0{,}23} - 1$ $= 1{,}575$	✓ substitution/substitusie ✓ answer/antwoord (2)
5.5	$f(x) = -2 \times 3^{x+2} + 1$	✓ $x + 2$ ✓ $-2 \times 3^{x+2} + 1$ (2)
		[9]

SECTION 4: INVERSE FUNCTIONS

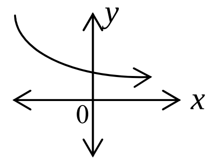
QUESTION 1

1.1	Any base raised to the power 0 is 1 which means the *y*-intercept of the graph $h(x) = a^x$ will be (0 ; 1) therefore Q(0 ; 1) **OR** $h(0) = a^0 = 1$ $\therefore Q(0;1)$	✓ *y*-intercept ✓ any base raised to power 0 is 1 (2)
1.2	$a^{-1} = \frac{1}{2}$ $\frac{1}{a} = \frac{1}{2}$ $a = 2$	✓ substitution ✓ answer (2)
1.3	$2^y = x$ $y = \log_2 x$	✓ interchanging *x* and *y* ✓ answer (2)
1.4		✓ point (0,5 ; – 1) or any other valid point ✓ point (1 ; 0) ✓ shape (3)
1.5	$x > 0,5$	✓ reading off from graph ✓ answer (2)
1.6	$\therefore 2 + x\log 3 = x \log 2$ $\therefore x = \frac{2}{\log \frac{2}{3}} = -11.36$ **OR** $\left(\frac{2}{3}\right)^x = 100$ $\therefore x \log\left(\frac{2}{3}\right) = 2$ $\therefore x = \frac{2}{\log \frac{2}{3}} = -11.36$	✓ equating ✓ logs both sides ✓ answer (3) **[14]**

QUESTION 2

2.1	$y = \log_3 x$	✓ answer (1)
2.2		$y = f^{-1}(x)$ ✓ x-intercept ✓ shape $y = f^{-1}(x-2)$ ✓ x-intercept ✓ shape (4)
2.3	$2 < x < 5$	✓✓ answer (2) **[7]**

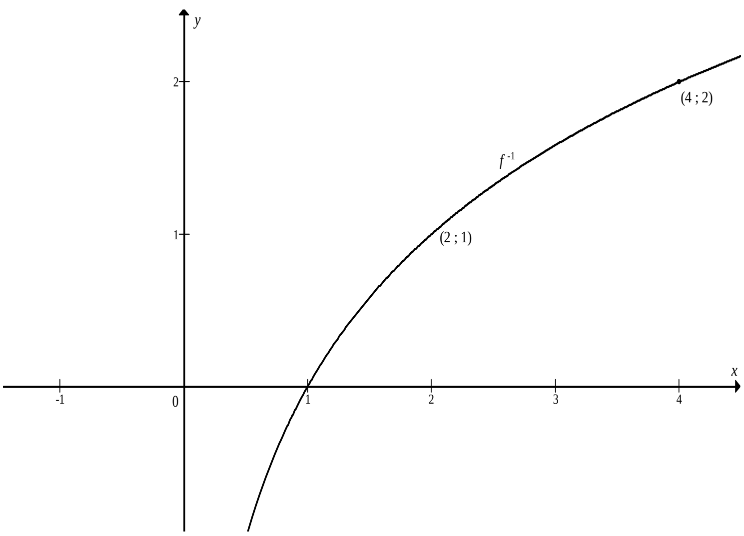
QUESTION 3

3.1	Decreasing function Since $0 < a < 1$ OR As x increases, $f(x)$ decreases	✓ decreasing ✓ $a < 1$ (2)
3.2	$f^{-1}: \quad x = \left(\frac{1}{3}\right)^y$ OR  $y = \log_{\frac{1}{3}} x$ OR $f^{-1}: \quad x = \left(\frac{1}{3}\right)^y$ $y = -\log_3 x$	✓ $x = \left(\frac{1}{3}\right)^y$ ✓ $y = \log_{\frac{1}{3}} x$ or $y = -\log_3 x$ (2)
3.3	$y = -5$	✓ answer (1)
3.4	Reflection about $y = x$. Reflection about the x-axis. **OR** Reflection about the y-axis. Then reflection about the line $y = x$. **OR** Reflection about the line $y = -x$ followed by reflection about the y-axis. **OR** Rotation through $90°$ in a clockwise direction. **OR** Rotation through $90°$ in an anti-clockwise direction. Reflection through the origin.	✓ reflection about $y = x$ ✓ reflection about y-axis (2) ✓ reflection about y-axis ✓ reflection about $y = x$ (2) ✓ rotation through $90°$ ✓ clockwise direction (2) ✓ answer ✓ answer (2) **[7]**

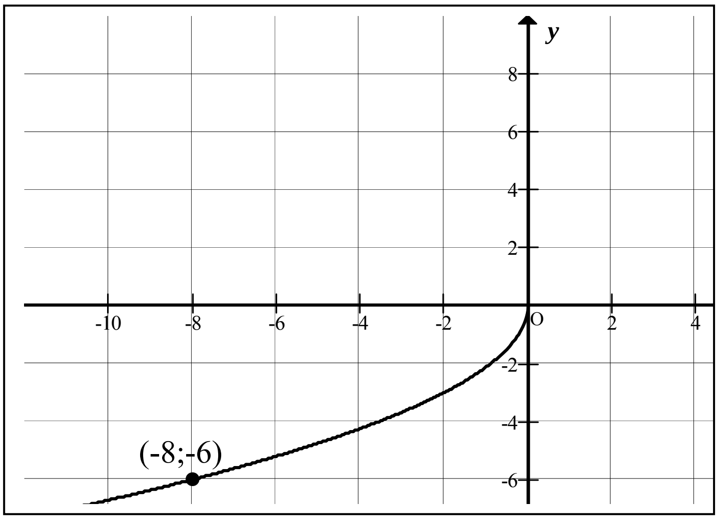
QUESTION 4

4.1	$x > 0$ **OR** $x \in (0; \infty)$	✓ answer (1)
4.2	$y = 2^{-x}$ **OR** $y = \left(\frac{1}{2}\right)^x$	✓ answer (1)
4.3	$y = 0$	✓ answer (1)
4.4.1	Reflect the graph of *f* over the *x*-axis OR For each point the *y*-coordinate changes sign. **NOTE**: Reflect only : 0 / 1	✓ answer (1)
4.4.2	Reflect the graph of *f* over the line $y = x$. Then shift the graph down 5 units	✓✓ answer ✓ answer (3)
4.4.2 contd	**OR** Sketch the graph of the inverse of *f*. Shift the graph of the inverse of *f* down by 5 units. **OR** Shift the graph 5 units LEFT. Reflect the graph over the line $y = x$.	
4.5	$\log_2 x < 3$ $-\log_2 x > -3$ For $-\log_2 x = -3$ $2^3 = x$ $x = 8$ $f(x) > -3$ $0 < x < 8$ or $x \in (0\,;8)$ **NOTE:** Notation incorrect: Answer $x < 8$: 2 / 3 Answer only correct: 3 / 3	✓ multiplication by – 1 ✓ Notation ✓ critical values (3) **[10]**

QUESTION 5

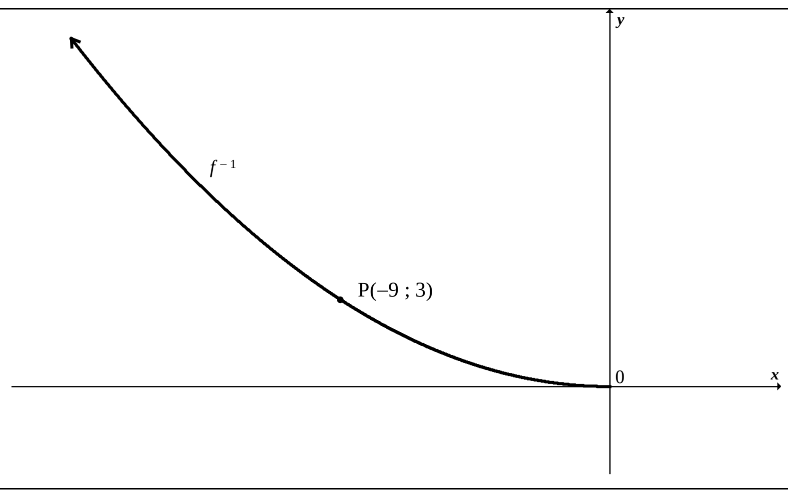
5.1	$g(x) = -(x-1)^2 + 2$ Turning point of *g*: (1 ; 2) $f(1) = 2^1 = 2$ (1 ; 2) lies on *f*. (1 ; 2) lies on both *f* and *g* D(1 ; 2)	✓ (1 ; 2) TP ✓ substitution into *f* ✓ (1 ; 2) lies on both *f* and *g*. (2)
5.2	$y = \log_2 x$	✓ answer (1)
5.3		✓ *y*-intercept ✓ one other point ✓ shape – increasing (3)

QUESTION 6

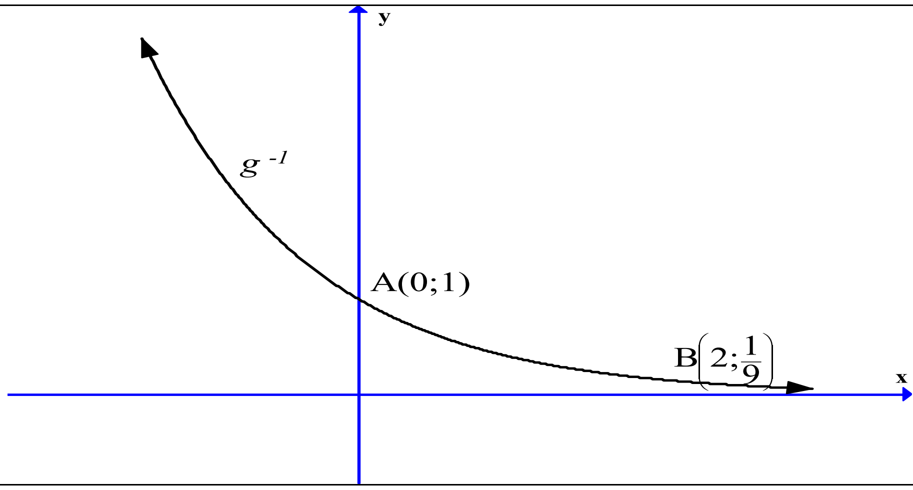
6.1	$f(x)=ax^2$ $-8=a(-6)^2$ $-8=36a$ $a=-\frac{8}{36}$ **OR** $a=-\frac{2}{9}$	✓ substitution ✓ answer (2)
6.2	$f(x):\quad y=-\frac{2}{9}x^2$ $x=-\frac{2}{9}y^2$ $9x=-2y^2$ $-\frac{9x}{2}=y^2$ $y=\pm\sqrt{\frac{-9x}{2}},\quad$ since $\quad y\le 0$ $y=-\sqrt{-\frac{9x}{2}}$ **OR** $\quad y=-3\sqrt{\frac{-x}{2}}$ **Note:** If candidate does not substitute the value of a the answer is $y=-\sqrt{\frac{x}{a}}$ then 2 / 3 marks	✓ swop x and y ✓ $y^2=-\frac{9x}{2}$ or $y=\pm\sqrt{\frac{-9x}{2}}$ ✓ $y=-\sqrt{-\frac{9x}{2}}$ (3)
6.3	$y\le 0$ **OR** $y\in(-\infty\,;0]$	✓ answer (1)
6.4		✓ shape (third quadrant) (concave upward) ✓ Any point other than (0 ; 0) that lies on the graph Point corresponding from original graph will be (– 8 ; – 6) (2)

6.5	$y = -f^{-1}(x)$ $= \sqrt{\dfrac{-9x}{2}}$ **OR** $y = -f^{-1}(x)$ $= 3\sqrt{\dfrac{-x}{2}}$ **OR** $y = -\dfrac{2}{9}x^2$ Reflection in $y = x$: $x = -\dfrac{2}{9}y^2$ $-\dfrac{9}{2}x = y^2$ $y = -\sqrt{-\dfrac{9x}{2}}$ Reflection about y-axis: $y = \sqrt{-\dfrac{9x}{2}}$ **Note:** If candidate has $(x\ ;y) \rightarrow (y\ ; -x)$ then 2 / 3 marks **Note:** If candidate does not substitute the value of a the answer is $y = \sqrt{\dfrac{x}{a}}$ then full marks	✓✓ $-f^{-1}(x)$ ✓ answer (3) ✓ $x = -\dfrac{2}{9}y^2$ ✓ $y = -\sqrt{-\dfrac{9x}{2}}$ ✓ $y = \sqrt{-\dfrac{9x}{2}}$ (3) **[11]**

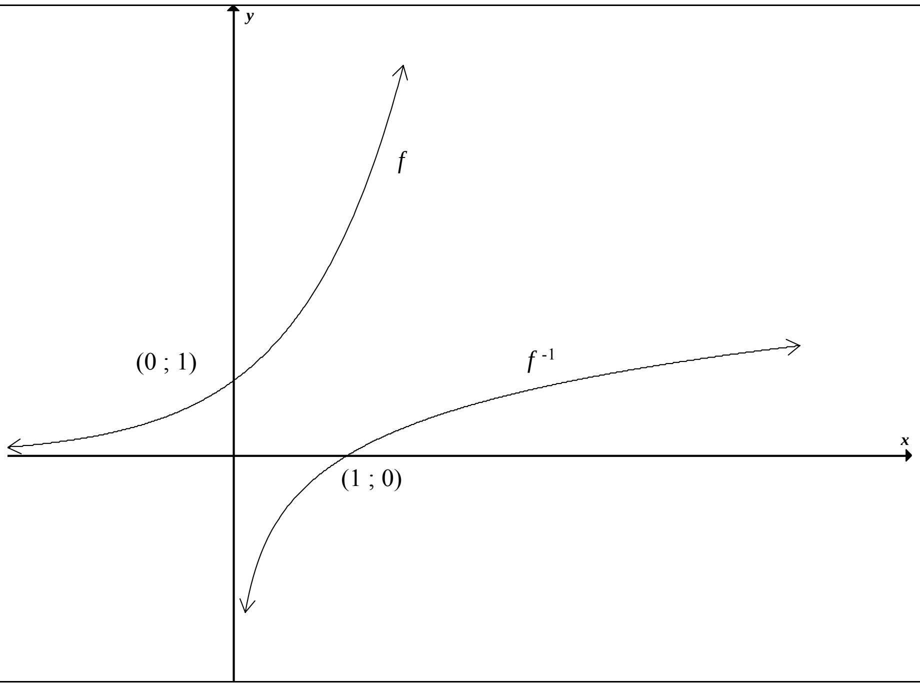
QUESTION 7

7.1	$0 \leq x \leq 3$ **OR** $[0;3]$ **Note:** if the candidate gives $0 < x < 3$, award 1/2 marks	✓ $0 \leq x$ ✓ $x \leq 3$ (2)
7.2	f^{-1} : $x = -\sqrt{27y}$ $x^2 = 27y$ $y = \dfrac{x^2}{27}$ $x \leq 0$ OR $(-\infty;0]$	✓ interchange x- and y- values ✓ $y = \dfrac{x^2}{27}$ ✓ $x \leq 0$ or $(-\infty;0)$ (3)
7.3		✓ shape ✓ end at origin ✓ any other point on the graph (3)
7.4	Reflection about the x-axis **OR** $(x\ ;y) \rightarrow (x\ ; -y)$; $x \geq 0$	✓ answer (1) ✓ answer (1) **[9]**

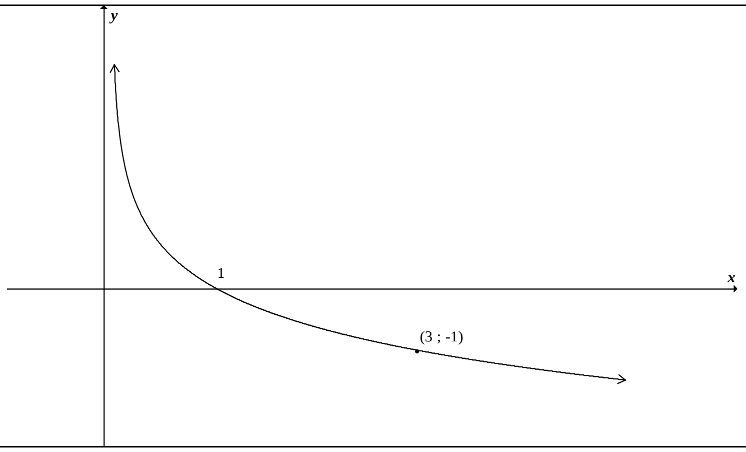
QUESTION 8

8.1	A(1; 0)	✓ answer (1)
8.2		✓shape ✓ A(0 ; 1) ✓ $B\left(2;\frac{1}{9}\right)$ (3)
8.3	**R**	✓ answer (1) **[5]**

QUESTION 9

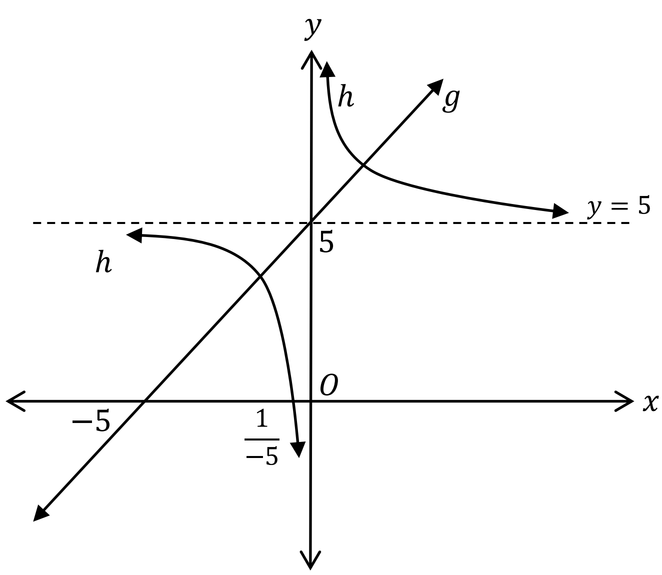
9.1	$f(x)=3^x$ $f^{-1}(x)=\log_3 x$	✓ answer (1)
9.2		$f^{-1}(x)=\log_3 x$ (Log Graph) ✓shape ✓x-intercept $f(x)=3^x$ (Exponential Graph) ✓shape ✓y-intercept (4)
9.3	$x>0$ **OR** $x\in(0\,;\infty)$	✓✓ answer (2)
9.4	$0<x\leq 1$	✓ critical values ✓ notation (2)
9.5	$y>-4$ **OR** $y\in(-4\,;\infty)$	✓✓ answer (2)
9.6	$g(x)=-3^{x-2}$ **OR** $g(x)=-f(x-2)$ **OR** $g(x)=-\frac{1}{9}\left(3^x\right)$ **OR** $g(x)=-\frac{1}{9}f(x)$	✓ – (sign) ✓ $x-2$ (2) ✓ – (sign) ✓ $\frac{1}{9}$ (2) **[13]**

QUESTION 10

10.1	**R** **OR** $(-\infty;\infty)$	✓answer (1)
10.2	$y=0$	✓ $y=0$ (1)
10.3	$x=\left(\frac{1}{3}\right)^y$ $y=\log_{\frac{1}{3}} x$ **OR** $x=\left(\frac{1}{3}\right)^y$ $x=3^{-y}$ $-y=\log_3 x$ $y=-\log_3 x$	✓ $x=\left(\frac{1}{3}\right)^y$ ✓ $y=\log_{\frac{1}{3}} x$ (2) ✓ $x=\left(\frac{1}{3}\right)^y$ ✓ $y=-\log_3 x$ (2)
10.4	 	✓ shape ✓intercept at (1 ; 0) ✓any other correct point (3)
10.5	$x=-2$	✓✓ $x=-2$ (2)
10.6	$\text{LHS}=[f(x)]^2-[f(-x)]^2$ $=\left[\left(\frac{1}{3}\right)^x\right]^2-\left[\left(\frac{1}{3}\right)^{-x}\right]^2$ $=3^{-2x}-3^{2x}$ $\text{RHS}=f(2x)-f(-2x)$ $=\left(\frac{1}{3}\right)^{2x}-\left(\frac{1}{3}\right)^{-2x}$ $=3^{-2x}-3^{2x}$ $\therefore$ LHS = RHS $[f(x)]^2-[f(-x)]^2=f(2x)-f(-2x)$	✓ $\left[\left(\frac{1}{3}\right)^x\right]^2-\left[\left(\frac{1}{3}\right)^{-x}\right]$ ✓ $3^{-2x}-3^{2x}$ ✓ $\left(\frac{1}{3}\right)^{2x}-\left(\frac{1}{3}\right)^{-2x}$ (3) **[12]**

SECTION 5 : COMBINATIONS

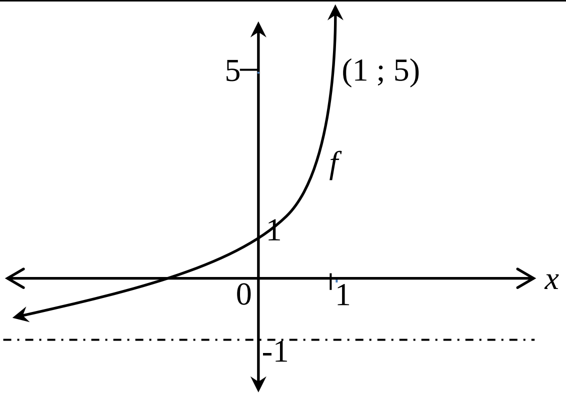
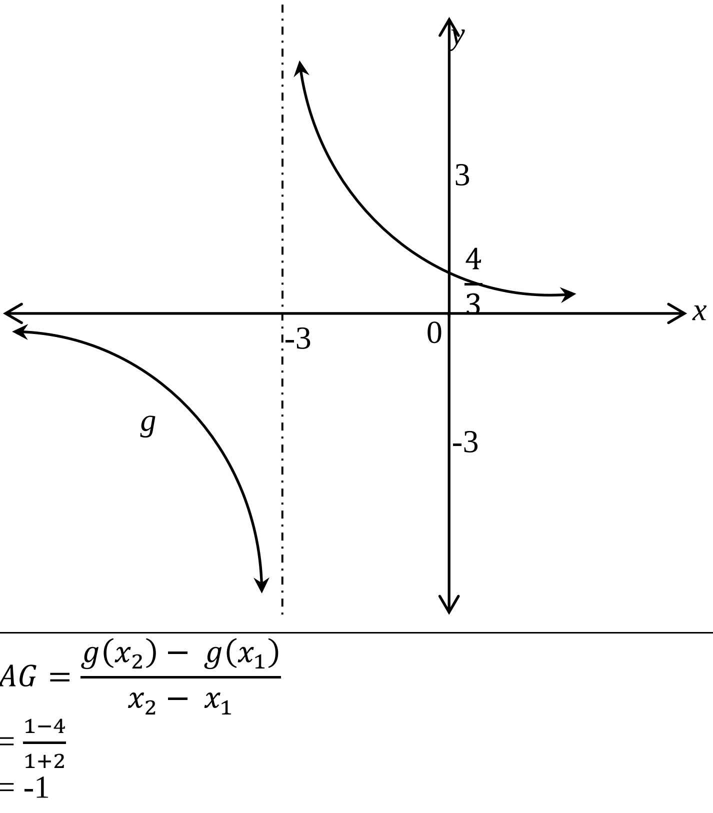
QUESTION 1

1.1	$h(x) = \frac{1}{x} + 5$ Let/stel $y = 0$ $0 = \frac{1}{x} + 5$ $0 = 1 + 5x$ $-5x = 1$ $x = \frac{1}{-5}$ Answer ONLY $x = \frac{1}{-5}$: 2 marks. SLEGS antwoord $x = \frac{1}{-5}$: 2 punte.	✓ $y = 0$ ✓ simplify/vereenvoudig ✓ answer/antwoord (3)
1.2		h ✓ x-intercept/afsnit ✓ asymptote/asimptoot ✓ shape/vorm g ✓ y-intercept/afsnit ✓ x-intercept/afsnit (5)
1.3	$x = 0$	✓ answer/antwoord (1)
1.4	$x + 5 = \frac{1}{x} + 5$ $x^2 + 5x = 1 + 5x$ $x^2 - 1 = 0$ $(x - 1)(x + 1) = 0$ $x = 1$ or/of $x = -1$ $(1; 6)$ or/of $(-1; 4)$	✓ equation/vergelyking ✓ simplify/vereenvoudig ✓ x-values/waardes ✓ $(1; 6)$ ✓ $(-1; 4)$ (5)
1.5	$f(x) = -x + 3$	✓ $-x$ ✓ 3 (2)
1.6	$h(x) = \frac{1}{x+2} + 3$	✓ $x + 2$ ✓ $+3$ (2)
		[18]

QUESTION 2

2.1	$3y = x - 5$ Let/stel $y = 0$ $0 = x - 5$ $x = 5$ $(5; 0)$ Do not penalise if not in coordinate form. Moenie penaliseer indien nie in koördinaatvorm nie.	✓ $y = 0$ ✓ answer/antwoord (2)
2.2	$f(x) = a(x+2)(x-5)$ $(-1; 3)$ $3 = a(-1+2)(-1-5)$ $3 = a(1)(-6)$ $3 = -6a$ $a = \frac{1}{-2}$ NOTE/LET WEL: No reference can be made to 9.3/Geen verwysing kan na 9.3 gemaak word nie. $f(x) = \frac{1}{-2}(x+2)(x-5)$ $f(x) = \frac{1}{-2}(x^2 - 3x - 10)$ $f(x) = \frac{1}{-2}x^2 + \frac{3}{2}x + 5$	✓ setting up equation/ opstel van vergelyking ✓ substitution/substitusie $(-1; 3)$ ✓ a-value/waarde ✓ simplification/vereenvoudiging (4)
2.3	$x = \frac{-2+5}{2} = \frac{3}{2}$ OR/OF $x = \frac{-b}{2a} = \frac{\frac{-3}{2}}{2\left(\frac{-1}{2}\right)} = \frac{3}{2}$ $f\left(\frac{3}{2}\right) = \frac{-1}{2}\left(\frac{3}{2}\right)^2 + \frac{3}{2}\left(\frac{3}{2}\right) + 5$ $= 6\frac{1}{8}$ or/of $\frac{49}{8}$ or/of 6,125 $\left(\frac{3}{2}; 6\frac{1}{8}\right)$ Do not penalise if not in coordinate form. Moenie penaliseer indien nie in koördinaatvorm nie.	✓ $x = \frac{3}{2}$ ✓ $y = 6\frac{1}{8}$ (2)
2.4	E: $3y = x - 5$ Let/stel $x = -1$ $\therefore 3y = -1 - 5$ $3y = -6$ $y = -2$ $E(-1; -2)$ $DE = 5$ units/eenhede	✓ substitute/vervang $x = -1$ ✓ $y = -2$ ✓ answer/antwoord (3)
2.5	$D(-1; 3)$; $B(5; 0)$ $m = \frac{3-0}{-1-5} = \frac{3}{-6} = \frac{1}{-2}$	✓ answer/antwoord (1)
2.6	$x \leq -2$ or/of $0 \leq x \leq 5$	✓ $x \leq -2$ ✓ $0 \leq x \leq 5$ (2)
		[14]

QUESTION 3

3.1	$y = -1$	(1)	✓ y = -1
3.2	y-intercept: $x = 0$ $y = 2.3^0 - 1$ $= 2.1 - 1$ $= 1$ ∴ (0 ; 1)	(2)	✓ value of y waarde van y ✓ coordinate koördinaat
3.3	$x = 1$: $y = 2.3^1 - 1 = 5$ ∴ (1 ; 5)	(2)	✓ x-coordinate x-koördinaat ✓ y-coordinate y-koördinaat
3.4		(3)	✓ shape vorm ✓ y-intercept y-afsnit ✓ y-asymptote y-asimptote
3.5	$y > -1$	(1)	✓ $y > -1$
3.6	x = 3 y = 0	(2)	✓ x-asymtote/ x-asimptote ✓ y-asymtote/ y-asimptote
3.7	$g(0) = \frac{4}{0+3} = \frac{4}{3}$ $\therefore \left(0 ; \frac{4}{3}\right)$	(2)	✓ $g(0)$ ✓ coordinate koördinaat
3.8	$y = x + 3$ $y = -x - 3$	(2)	✓ $y = x + 3$ ✓ $y = -x - 3$
3.9		(4)	✓ asymptote asymptote ✓ y-intercept y-afsnit ✓✓ one mark for each branch een punt vir elke tak
3.10	$AG = \frac{g(x_2) - g(x_1)}{x_2 - x_1}$ $= \frac{1-4}{1+2}$ = -1	(3)	✓ formula formule ✓ substitution vervanging ✓ answer antwoord

[22]

QUESTION 4

4.1	4.1.1	$y = a(x - x_1(x - x_2)$ $y = a(x+2)(x-3) = a(x^2 - x - 6)$ At (0 ; -12): -12 = a(-6) $\therefore a = 2$ $a = 2$ $\therefore y = 2x^2 - 2x - 12$	(4)	✓ factors faktore ✓ simplification vereenvoudiging ✓ value of *a* waarde van *a* ✓ equation vergelyking
	4.1.2	$y = 2(2x^2 - 2x - 12)$ $= 2(x^2 - x - 6)$ $= 2(x^2 - x + \frac{1}{4} - 6 - \frac{1}{4})$ $= 2\left[\left(x - \frac{1}{2}\right)^2 - 6\frac{1}{4}\right]$ $= 2\left(x - \frac{1}{2}\right)^2 - 25/2$	(3)	✓ factorisation faktorisering ✓ completion of the square voltooiing van die kwadraat ✓ simplification vereenvoudiging
4.2	4.2.1	$f(x) = -(x^2 - x - 12)$ $= -(x-4)(x+3)$ C(0 ; 12) and/en D(4 ; 0)	(3)	✓ factorisation faktorisering ✓ C-coordinate C-koördinaat ✓ D-coordinate D-koördinaat
	4.2.2	*m* = -3 and/en *c* = 12 $\therefore g(x) = -3x + 12$	(2)	✓ *m* and/en C ✓ *g(x)=-3x+12*
	4.2.3	OB = $\frac{1}{2}$ or/of $\therefore g\left(\frac{1}{2}\right) = -3\left(\frac{1}{2}\right) + 12 = 10\frac{1}{2}$ $f(x) = -x^2 + x + 12$ $= -(\frac{1}{2})^2 + \frac{1}{2} + 12$ $= -\frac{1}{4} + \frac{2}{4} + 12$ AE = AB – EB $= \frac{1}{4} + 12$ $\therefore$AE = $12\frac{1}{4} - 10\frac{1}{2}$ $f(x) = 12\frac{1}{4}$ $= 1\frac{3}{4}$	(3)	✓ $g\left(\frac{1}{2}\right)$ ✓ $10\frac{1}{2}$ ✓ length of AE lengte van AE
	4.2.4	$x > \frac{1}{2}$	(1)	✓ $x > \frac{1}{2}$
	4.2.5	$y \leq 12\frac{1}{4}$	(1)	✓ $y \leq 12\frac{1}{4}$

QUESTION 5

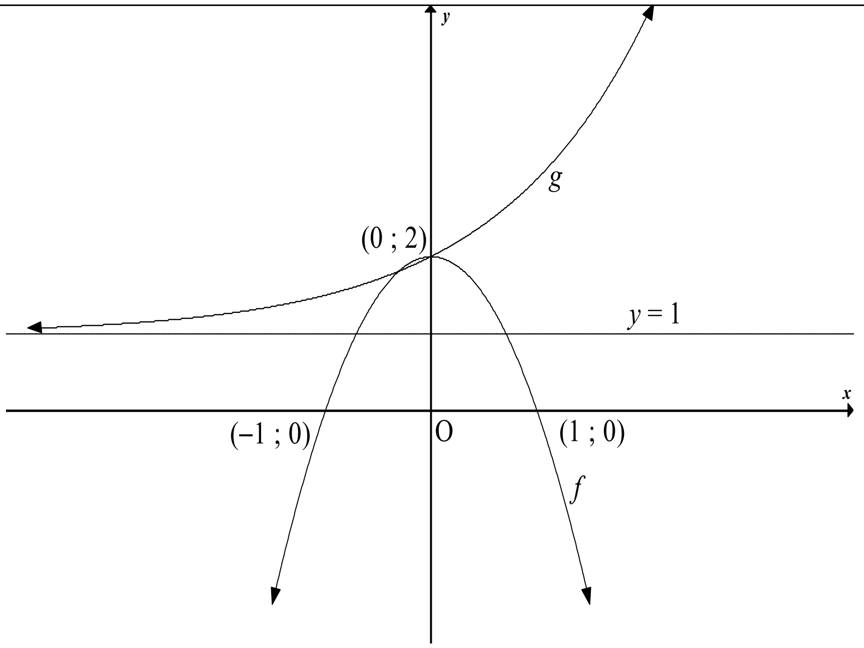
5.1	C(– 1 ; 0)	✓ C(– 1 ; 0) (1)
5.2	$y=(x-3)(x+1)$ $y=x^2-2x-3$	✓ $(x-3)$ ✓ $(x+1)$ ✓ $y=x^2-2x-3$ (3)
5.3	TP : $y=(1)^2-2(1)-3$ $y=-4$ R: $y\in[-4;\infty)$ **OR** $y\geq-4$	✓ $y=-4$ ✓ $[-4;\infty)$ (2) ✓ $y\geq-4$
5.4	$m=\dfrac{0+4}{3-1}=2$ $y-0=2(x-3)$ $y=2x-6$	✓substituting into gradient formula ✓$m=2$ ✓ equation (3)
5.5.1	$x\leq-1$ or $x\geq3$ **OR** $x\in(-\infty;-1]\cup[3;\infty)$	✓ $x\leq-1$ ✓ $x\geq3$ (2) ✓ $(-\infty;-1]$ ✓ $[3;\infty)$ (2)
5.5.2	$-1<x<3$ or $x>3$ **OR** $x>-1$; $x\neq3$ **OR** $(-1;3)\cup(3;\infty)$	✓ critical values ✓ notation (2) ✓ $x>-1$ ✓ $x\neq3$ (2) ✓ $(-1;3)$ ✓ $(3;\infty)$ (2)
5.5.3	$-1<x<0$ or $x>3$ **OR** $(-1;0)\cup(3;\infty)$	✓ critical values ✓ notation (2) ✓ $(-1;0)$ ✓ $(3;\infty)$ (2)

5.6	$x^2 - 2x - p = 0$ $\Delta = (-2)^2 - 4(1)(-p)$ $= 4 + 4p$ $\text{for non - real roots } \Delta < 0$ $4 + 4p < 0$ $4p < -4$ $\therefore p < -1$ **OR** $A(1; -4)$ $x^2 - 2x - 3 = 0$ $x^2 - 2x - p = 0$ $-p > 1$ $\therefore p < -1$	✓ $4 + 4p < 0$ ✓ $p < -1$ (2) ✓ $-p > 1$ ✓ $p < -1$ (2)
5.7	$PM = (2x - 6) - (x^2 - 2x - 3)$ $= -x^2 + 4x - 3$ $x = -\frac{b}{2a}$ $= -\frac{4}{2(-1)} = 2$ $Max.PM = -(2)^2 + 4(2) - 3 = 1 \text{ unit}$ **OR** $PM = (2x - 6) - (x^2 - 2x - 3)$ $= -x^2 + 4x - 3$ $= -(x^2 - 4x + 4 - 4 + 3)$ $= -[(x - 2)^2 - 1]$ $= -(x - 2)^2 + 1$ $Max.PM = 1 \text{ unit}$	✓subtraction ✓quadratic expression ✓ method ✓maximum value (4) ✓subtraction ✓quadratic expression ✓method ✓maximum value (4) **[21]**

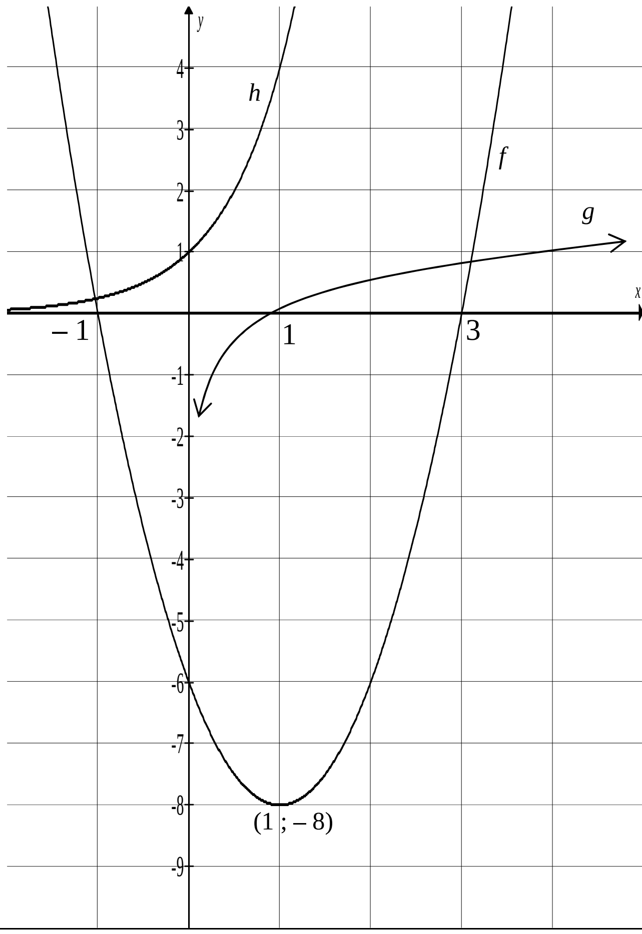
QUESTION 6

6.1	$\text{CD} = 2x+3-\left(-2x^2+14x+k\right)$ $= 2x+3+2x^2-14x-k$ $= 2x^2-12x+3-k$	✓✓ $f(x)-g(x)$ ✓ answer/*antwoord* (3)
6.2	Minimum value occurs at/*Minimum waarde vind plaas by* $x=\frac{-b}{2a}$ $=\frac{12}{2(2)}$ $=3$ Minimum value/*Minimum waarde* $5 = 2(3)^2-12(3)+3-k$ $5 = 18-36+3-k$ $k=-20$ **OR/*OF*** $\text{CD} = 2x^2-12x+3-k$ $= 2\left(x^2-6x\right)+3-k$ $= 2\left[(x-3)^2-9\right]+3-k$ $= 2(x-3)^2-18+3-k$ $= 2(x-3)^2-15-k$ Hence the minimum value of CD is $-15-k$ The minimum value of CD is given to be 5 *Vervolgens is die minimum waarde van CD* $-15-k$ *Die minimum waarde van CD is gegee as 5* $5=-15-k$ $k=-20$	✓ $x=\frac{-b}{2a}$ ✓x-value for minimum *x-waarde vir minimum* ✓ subst 5 ✓ answer/*antwoord* (4) ✓ $2(x-3)^2$ ✓ $\text{CD} = 2(x-3)^2-15-k$ ✓ $5=-15-k$ ✓answer/*antwoord* (4) **[7]**

QUESTION 7

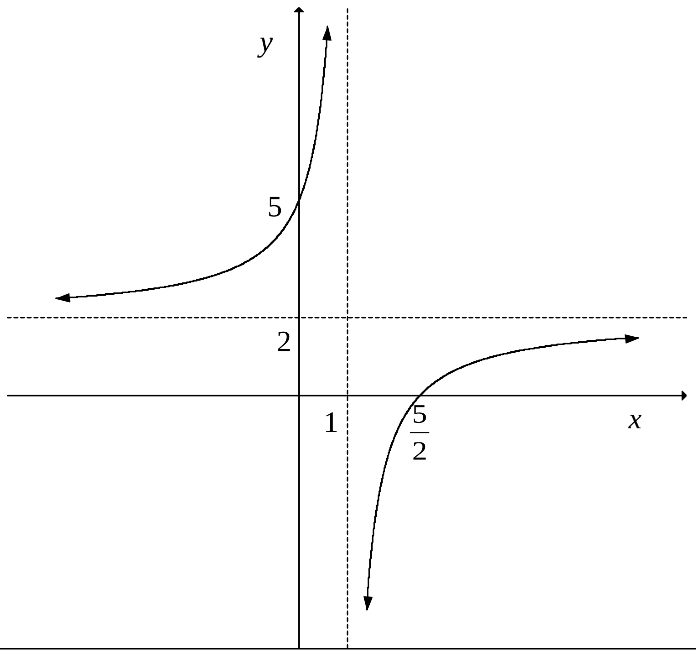
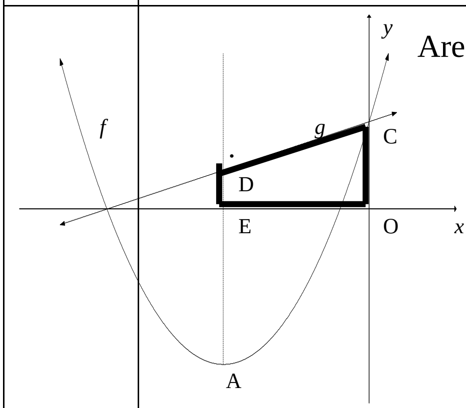
7.1	$y = 1$	✓ answer/*antwoord* (1)
7.2		*f:* ✓ shape of *f/vorm van f* ✓ *x*-intercepts of *f/ x-afsnitte van f* ✓ *y*-intercept (TP) of *f/y-afsnit (DP) van f* *g:* ✓ shape of *g/vorm van g* ✓ asymptote of *g/ asimptoot van g* ✓ *y*-intercept of *g/ y-afsnit van g* (6)
7.3	Range of *f/Waardeversameling van f*: $(-\infty\,;\,2]$ **OR/OF** Range of *f/Waardeversameling van f*: $y \leq 2$	✓ $(-\infty\,;\,2]$ (1) ✓ $y \leq 2$ (1)
7.4	Maximum of $3^{f(x)}$ will be obtained when $f(x)$ is at maximum. Max of $f(x)$ is 2 Max of h will be $3^2 = 9$ *Maksimum van* $3^{f(x)}$ *sal verkry word wanneer f(x) by maksimum is.* *Maks van f(x) is 2* *Maks van h sal* $3^2 = 9$ *wees.*	✓ Max of $f(x)$ is 2/ *Maks van f(x) is 2* ✓ Max of $h = 9$/ *Maks van h = 9* (2)
7.5	*f* would have been reflected in the *x*-axis *f sou in die x-as gereflekteer gewees het*	✓ reflected/*gereflekteer* ✓in the *x*-axis/ *in die x-as* (2) **[12]**

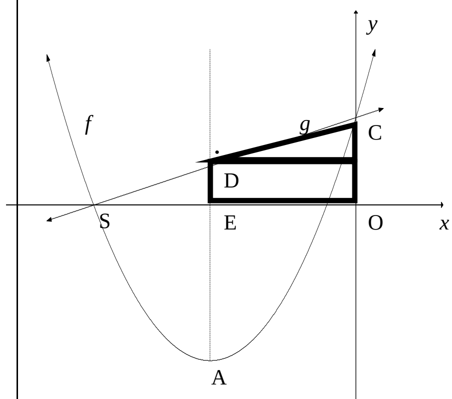
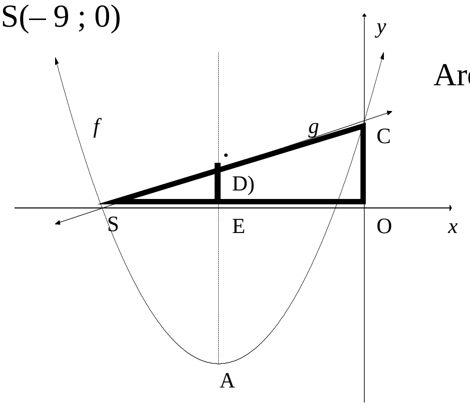
QUESTION 8

8.1 & 8.2		EXPONENTIAL ✓ shape (must be increasing above *x*-axis) ✓ *y*-int PARABOLA ✓ shape ✓✓ turning point ✓ *y*-intercept ✓✓ *x*-intercepts (8) INVERSE/LOG ✓ *x*-int ✓ shape (must be increasing on the right of the *y*-axis) (2) Note: If *x*-intercepts not shown but correct on graph 2/2 for *x*-intercepts.
	Calculation of *x*-intercepts of parabola $0 = 2(x-1)^2 - 8$ $8 = 2(x-1)^2$ $4 = (x-1)^2$ $2 = x-1 \;\; or \;\; -2 = x-1$ $x = 3 \quad or \quad x = -1$ OR $0 = 2(x-1)^2 - 8$ $0 = 2(x^2 - 2x + 1) - 8$ $0 = 2x^2 - 4x - 6$ $0 = x^2 - 2x - 3$ $0 = (x-3)(x+1)$ $x = 3 \quad or \quad x = -1$	
8.3	$y = 2(x+1)^2 - 8$ **OR** $y = 2x^2 + 4x - 6$	✓ -8 ✓ $+1$ (2) ✓ -6 ✓ $+4$ (2)

8.4	$h\left(x+\frac{1}{2}\right)=4^{x+\frac{1}{2}}$ $=4^{x}.4^{\frac{1}{2}}$ $=2\left(4^{x}\right)$ $=2h(x)$	✓ substitution ✓ $4^{x}.4^{\frac{1}{2}}$ ✓ $2\left(4^{x}\right)$ (3)
	OR	
	$h\left(x+\frac{1}{2}\right)=4^{x+\frac{1}{2}}$ $=(2^{2})^{x+\frac{1}{2}}$ $=2^{2x+1}$ $=2^{2x}.2$ $=2.\left(4^{x}\right)$ $=2h(x)$	✓ substitution ✓ $(2^{2})^{x+\frac{1}{2}}$ ✓ $2\left(4^{x}\right)$ (3) Note: If numerical examples are used : 1 / 3 **[15]**

QUESTION/*VRAAG* 4

4.1.1	$x = 1$ $y = 2$	✓ $x = 1$ ✓ $y = 2$ (2)
4.1.2	$y = mx + c$ $2 = -1 + c$ $c = 3$ $y = -x + 3$ **or** $y - y_1 = m(x - x_1)$ $y - 2 = -1(x - 1)$ $y - 2 = -x + 1$ $y = -x + 3$ **or** $y = -(x - p) + q$ $= -(x - 1) + 2$ $y = -x + 3$	✓ substitution of $m = -1$ and $(1 ; 2)$ ✓ answer (2)
4.1.3		✓ vertical asymptote: $x = 1$ and horizontal asymptote: $y = 2$ ✓ x-intercept: $\frac{5}{2}$ ✓ y-intercept: 5 ✓ shape (A) (4)
4.2.1	$(-5 ; -8)$	✓ $x = -5$ ✓ $y = -8$ (2)
4.2.2	$y \geq -8$ **or** $[-8;\infty)$	✓ answer (1)
4.2.3	$m = -5$ $n = g(-5)$ $= \frac{1}{2}(-5) + \frac{9}{2}$ $= 2$	✓ $m = -5$ ✓substitution ✓ $n = 2$ (3)
4.2.4		
	Area trapezium $= \frac{1}{2}(\text{DE} + \text{OC}) \times \text{OE}$ $= \frac{1}{2}(2 + 4{,}5) \times 5$ $= \frac{65}{4}$ or 16,25 **OR**	✓ method ✓correct substitution ✓answer (3) **OR**

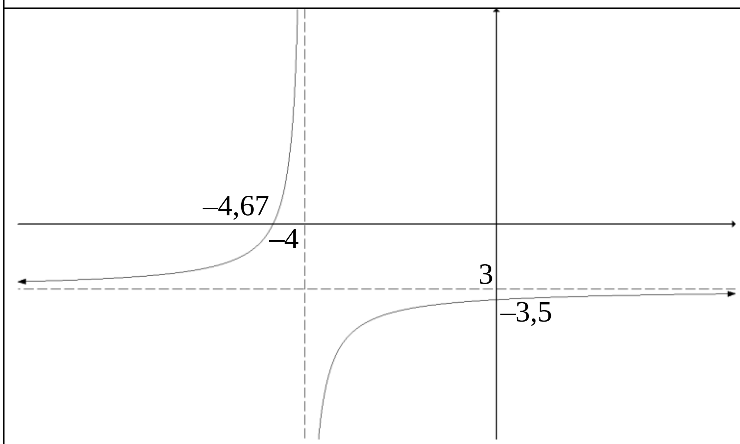
	 Area Δ = $\frac{1}{2}b.h$ $= \frac{1}{2}(5)\left(\frac{5}{2}\right)$ $= \frac{25}{4}$ Area rect = $b.h$ = (5)(2) = 10 Area trapezium = $\frac{25}{4} + 10 = \frac{65}{4}$ or 16,25	✓ method ✓correct substitution ✓answer (3)
	OR S(– 9 ; 0)  Area Δ SOC = $\frac{1}{2}b.h$ $= \frac{1}{2}(9)\left(\frac{9}{2}\right)$ $= \frac{81}{4}$ Area Δ SED = $\frac{1}{2}b.h = \frac{1}{2}(4)(2) = 4$ Area trapezium = area Δ SOC – Area Δ SED $= \frac{81}{4} - 4$ $= \frac{65}{4}$ or 16,25	**OR** ✓ method ✓correct substitution ✓answer (3)
4.2.5	g^{-1}: $x = \frac{1}{2}y + \frac{9}{2}$ g^{-1}: $y = 2x - 9$	✓ changing *x* and *y* ✓answer (2)

4.2.6	$f(x)=\frac{1}{2}(x+5)^2-8$ $f(x)=\frac{1}{2}(x^2+10x+25)-8$ $f(x)=\frac{1}{2}x^2+5x+4{,}5$ $f^{/}(x)=x+5$ $h(x)=2x-9+k$ $x+5=2$ $x=-3 \quad y=-6$ $(-3\,;-6)$ **OR** $f(x)=h(x)$ $\frac{1}{2}(x+5)^2-8=2x-9+k$ $\frac{1}{2}x^2+3x+\frac{27}{2}-k=0$ $x=\frac{-3}{2\left(\frac{1}{2}\right)}=-3 \qquad b^2-4ac=0$ $y=-6$ $(-3\,;-6)$	✓ $f^{/}(x)$ ✓ $x+5=2$ ✓ $x=-3$ ✓ $y=-6$ (4) **OR** ✓ equating ✓ turning point / △ = 0 ✓ $x=-3$ ✓ $y=-6$ (4)
		[23]

QUESTION/*VRAAG* 5

5.1	A(0 ; 1)	✓ answer (1)
5.2	$9=3^{-x}$ $3^2=3^{-x}$ $x=-2$ B(–2 ; 9)	✓ equating ✓ $3^2=3^{-x}$ ✓ $x=-2$ (3)
5.3	$x\in(0;\infty)$ or $x>0$	✓✓answer (2)
5.4	$h(x)=27.3^{-x}$ $h(x)=3^{-(x-3)}$ *f* shifted 3 units to the right	✓ $h(x)=3^{-(x-3)}$ ✓3 units ✓ right (3)
5.5	$\frac{27}{3^x}<1 \qquad 3^{-x+3}<1$ $3^x>27$ **or** $3^{-x+3}<3^0$ $3^x>3^3 \qquad -x+3<0$ $x>3 \qquad x>3$ **OR** The graph shifts 3 units to the right Thus the *y*-intercept shift 3 units to the right (3 ; 1) $\therefore x>3$	✓ $3^x>27$ **or** $3^{-x+3}<3^0$ ✓ $3^x>3^3$ **or** $-x+3<0$ ✓ $x>3$ (3) **OR** ✓ translation ✓ *y*-intercept ✓answer (3)
		[12]

QUESTION/*VRAAG* 4

4.1	$x+1=-x-7$ $2x=-8$ $x=-4$ $\therefore y=-3$ $\therefore f(x)=\frac{-2}{x+}-3$ $\therefore p=4$ and $q=-3$ **OR/OF** $p+q=1$ ……(1) $-p+q=-7$ $q=p-7$ …….(2) subs. (2) into (1) $p+p-7=1$ $2p=8$ $p=4$ $q=-3$	✓ $x+1=-x-7$ ✓ $2x=-8$ ✓ $x=-4$ ✓ $y=-3$ (4) **OR/OF** ✓ $p+q=1$ ✓ $q=p-7$ ✓ substitution ✓ simplification (4)
	$y=\frac{-2}{x+}-3$ $0=\frac{-2}{x+}-3$ $-2-3(x+4)=0$ $-3x-14=0$ $\therefore x=-\frac{}{3}$	✓ $y=0$ ✓ $x=-\frac{}{3}$ (2)
	 	✓ ✓ vertical asymptote ✓ y intercept ✓ shape (4)
		[10]

50

QUESTION/*VRAAG* 5

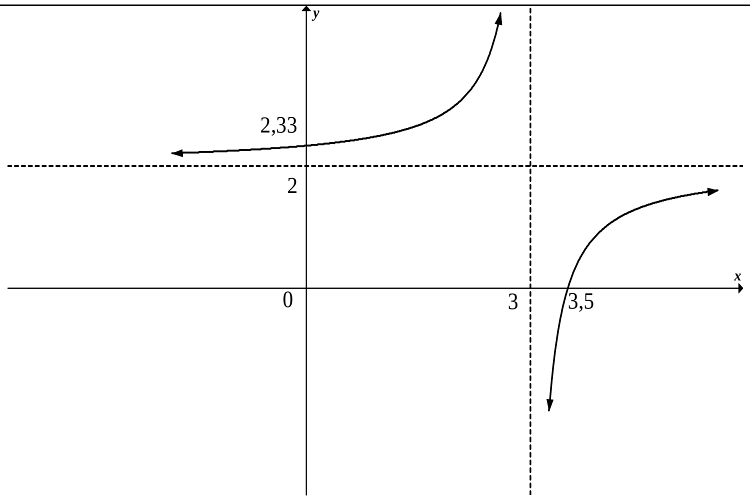
5.1	$-2x^2+4x+16=0$ $x^2-2x-8=0$ $(x-4)(x+2)=0$ $x=4$ or $x=-2$ $\therefore$ A$(-2;0)$ and B$(4;0)$	✓ factors ✓ $x=-2$ ✓ $x=4$ (3)
5.2	$f(x)=-2x^2+4x+16$ $-\frac{b}{2a}=-\frac{-4}{-2(2)}=1$ $f(1)=-2(1)^2+4(1)+16=18$ $\therefore$ C(1;18) **OR/OF** $f(x)=-2x^2+4x+16$ $f^{/}(x)=-4x+4$ $-4x+4=0$ $x=1$ $f(1)=-2(1)^2+4(1)+16=18$ $\therefore$ C(1;18)	✓ 1 ✓ 18 (2) **OR/OF** ✓ 1 ✓ 18 (2)
5.3	$y\leq 18$ **OR/OF** $y\in(-\infty;18]$	✓ $y\leq 18$ (1) **OR/OF** ✓ $y\in(-\infty;18]$ (1)
5.4	TP (1 ; 18) for f TP (2 ; 15) for h $\therefore p=-1$ $q=-3$	✓ TP for h at (2 ; 15) ✓ $p=-1$ ✓ $q=-3$ (3)
5.5	$y=2x+4$ $x=2y+4$ $\therefore y=\frac{1}{2}x-2$	✓swop x and y ✓ $y=\frac{1}{2}x-2$ (2)
5.6	$g(x)=0$ or $g^{-1}(x)=0$ $x=4$ or $x=-2$ (product 0 at x-intercepts)	✓ $x=4$ ✓ $x=-2$ (2)

5.7	$-2x^2+4x+16+k=2x+4$ $-2x^2+2x+12+k=0$ $b^2-4ac<0$ $(2)^2-4(-2)(12+k)<0$ $4+8(12+k)<0$ $100+8k<0$ $k<-12{,}5$ **OR/OF** $g'(x)=2$ $f'(x)=-4x+4=2$ $x=\frac{1}{2}$ $f\left(\frac{1}{2}\right)=17{,}5$ $g\left(\frac{1}{2}\right)=5$ $\therefore k<-12{,}5$	✓equating ✓ standard form ✓ $b^2-4ac<0$ ✓ substitution ✓ answer (5) **OR/OF** ✓ $g'(x)=2$ ✓ $f'(x)=-4x+4$ ✓ $f\left(\frac{1}{2}\right)=17{,}5$ ✓ $g\left(\frac{1}{2}\right)=5$ ✓ answer (5)
		[18]

QUESTION/*VRAAG* 6

6.1.1	$y=3^x$ $x=3^y$ $y=\log_3 x$	✓ swop *x* and *y* ✓ equation (2)
6.1.2	$h(x)=3^{x-4}+2$ Transformation: 4 units left, 2 units down $P'(2;9)$	✓ *x* = 2 (A) ✓ *y* = 9 (A) (2)
6.2	$f(x)=2^{x+p}+q$ $q=-16$ $16=2^{p+3}-16$ $2^{p+3}=32$ $2^{p+3}=2^5$ $\therefore p+3=5$ $p=2$	✓ $q=-16$ ✓ substitute (3 ; 16) ✓ $2^{p+3}=2^5$ or $p+3=\log_2 32$ ✓ $p=2$ (4)
		[8]

QUESTION/VRAAG 5

5.1	$x = 3$ $y = 2$	✓ $x = 3$ ✓ $y = 2$ (2)
5.2	$x \in R,\ x \neq 3$ **OR/*OF*** $x \in (-\infty\ ;3) \cup (3\ ;\infty)$ **OR/OF** $x < 3$ or $x > 3$	✓answer (1) **OR/OF** ✓answer (1) **OR/OF** ✓answer (1)
5.3	$0 = \frac{-1}{x-3} + 2$ $-2x + 6 = -1$ $x = \frac{7}{2}$ *x*-int: $\left(\frac{7}{2}\ ;\ 0\right)$	✓ $y = 0$ ✓ answer (2)
5.4	*y*-int: $\left(0\ ;\ \frac{7}{3}\right)$	✓ $x = 0$ ✓ $\frac{7}{3}$ (2)
5.5		✓ asymptotes ✓ intercepts with the axes ✓ shape (3)
		[10]

QUESTION/VRAAG 6

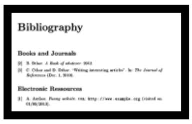
6.1	$f(x)=\log_4 x$ $2=\log_4 k$ $4^2=k$ $\therefore k=16$	✓ substitution of $(k\ ;\ 2)$ ✓ answer (2)
6.2	$-1=\log_4 x\ \ \therefore\ x=\frac{1}{4}$ $\frac{1}{4}\le x\le 16$ or/of $x\in\left[\frac{1}{4}\ ;\ 16\right]$	✓ $x=\frac{1}{4}$ ✓ answer (2)
6.3	$f(x)=\log_4 x$ $y=\log_4 x$ $x=\log_4 y$ $y=4^x$	✓swopping x and y ✓answer (2)
6.4	$x<0$ **OR/*OF*** $x\in(-\infty\ ;\ 0)$	✓✓answer (2) **OR/*OF*** ✓✓answer (2)
		[8]

QUESTION 5

5.1	$g(x)=\frac{a}{x+2}+q$ Subs (1 ; 0): $0=\frac{a}{1+2}+q$ $0=a+3q$ Subs $\left(0\ ;\ -\frac{1}{2}\right)$ $-\frac{1}{2}=\frac{a}{0+2}+q$ $-1=a+2q$ Solving simultaneously: $q=1$ $a=-3$ $\therefore g(x)=\frac{-3}{x+2}+1$	✓ $g(x)=\frac{a}{x+2}+q$ ✓ $0=a+3q$ ✓ $-1=a+2q$ ✓ solving simultaneously ✓ $q=1$ ✓ $a=-3$ (6)
5.2	$y\in R;\ y\neq 1$ **OR/*OF*** $(-\infty\,;1)$ or $(1\,;\infty)$ **OR/*OF*** $y<1$ or $y>1$	✓ answer (1)
5.3	$y-1=1(x+2)$ $y=x+3$ ANSWER ONLY: FULL MARKS **OR/*OF*** $1=1(-2)+c$ $c=3$ $y=x+3$	✓ $m=1$ ✓ subs point (– 2 ; 1) ✓ answer (3)
5.4	$K^{/}$ (–3 ; 4)	✓ x-value ✓ y-value (2)
		[12]

QUESTION 6

6.1	$f(x)=-x^2-6x+7$ $f^{/}(x)=-2x-6$ $-2x-6=0$ **OR/*OF*** $x=-\frac{(-6)}{2(-1)}$ $x=-3$ E(–3 ; 16) ANSWER ONLY: FULL MARKS	✓ method ✓ *x*-value ✓ *y*-value (3)
6.2	$k=f(-5)$ $k=-(-5)^2-6(-5)+7$ $\therefore k=12$	✓ answer (A) (1)
6.3	C(0 ; 7) D(–5 ; 12) $m_{CD}=\frac{12-7}{-5-0}$ $m_{CD}=-1$ Equation of CD: $y=-x+7$	✓ coordinates of C ✓ substitution ✓ *m* ✓ answer (4)
6.4	$-2x-6=-1$ $-2x=5$ $x=-\frac{5}{2}$ $y=f\left(\frac{-5}{2}\right)=-\left(\frac{-5}{2}\right)^2-6\left(\frac{-5}{2}\right)+7=\frac{63}{4}=15,75$ $\therefore \text{P}\left(-\frac{5}{2};\frac{63}{4}\right)$	✓ $f^{/}(x)=-2x-6$ ✓ equating to -1 ✓ *x*-value ✓ *y*-value (A) (4)
6.5	Point by symmetry: (–1 ; 12) $-5<x<-1$ **OR/*OF*** $-x^2-6x+7>12$ $-x^2-6x-5>0$ $x^2+6x+5<0$ $(x+1)(x+5)<0$ $-5<x<-1$ ANSWER ONLY: FULL MARKS	✓ –1 ✓ answer (2) ✓ –1 ✓ answer (2)
		[14]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Books and Journals

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Marking Guidelines

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2. **MATHEMATICS NOVEMBER AND MARCH NATIONAL PAPERS GRADE 12 (2008 – 2022)**