MATRIC CLASS OF 2020

#SavingTheClassOf2020 MATHS REVISION NOTES

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SESSION 1

Solving Equations Inequalities and Simplifying Expressions Arithmetic Sequences and Series

Nov 2019

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 5x - 6 = 0$ (3)

1.1.2 $4x^2 + 3x - 5 = 0$ (correct to TWO decimal places) (3)

1.1.3 $4x^2 - 1 < 0$ (3)

1.1.4 $\left(\sqrt{\sqrt{32}+x}\right)\left(\sqrt{\sqrt{32}-x}\right) = x$ (4)

1.2 Solve simultaneously for x and y:
$y + x = 12$ and $xy = 14 - 3x$ (5)

1.3 Consider the product $1 \times 2 \times 3 \times 4 \times \ldots \times 30$.
Determine the largest value of k such that 3^k is a factor of this product. (4)

[22]

May-June 2019

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 5x - 6 = 0$ (2)

1.1.2 $(3x-1)(x-4) = 16$ (correct to TWO decimal places) (4)

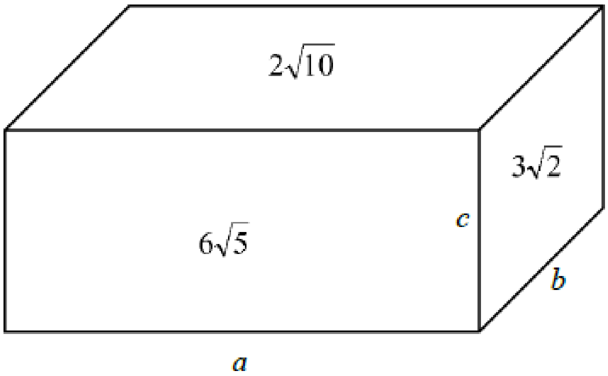
1.1.3 $4x - x^2 \geq 0$ (3)

1.1.4 $\dfrac{5^{2x} - 1}{5^x + 1} = 4$ (3)

1.2 Solve simultaneously for x and y:

$x + 3y = 2$ and $x^2 + 4xy - 5 = 0$ (5)

1.3 A rectangular box has dimensions a, b and c. The area of the surfaces are $2\sqrt{10}$; $3\sqrt{2}$ and $6\sqrt{5}$, as shown in the diagram below.



Calculate, **without using a calculator**, the volume of the rectangular box. (5)

[22]

November 2018

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 4x + 3 = 0$ (3)

1.1.2 $5x^2 - 5x + 1 = 0$ (correct to TWO decimal places) (3)

1.1.3 $x^2 - 3x - 10 > 0$ (3)

1.1.4 $3\sqrt{x} = x - 4$ (4)

1.2 Solve simultaneously for x and y:

$3x - y = 2$ and $2y + 9x^2 = -1$ (6)

1.3 If $3^{9x} = 64$ and $5^{\sqrt{p}} = 64$, calculate, WITHOUT the use of a calculator, the value of: $\dfrac{\left[3^{x-1}\right]^3}{\sqrt{5}^{\sqrt{p}}}$

(4)

[23]

June 2018

QUESTION 1

1.1 Solve for x:

1.1.1 $(3x-1)(x+4)=0$ (2)

1.1.2 $2x^2+9x-14=0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{3-26x}=3x$ (4)

1.1.4 $(x-1)(x-4)>x+11$ (5)

1.2 Simplify fully:

$$\frac{\sqrt{16x^7}-\sqrt{25x^7}}{\sqrt{x}}$$ (3)

1.3 Solve simultaneously for x and y:

$xy=9$ and $x-2y-3=0$ (5)

1.4 Prove that $x^2+2xy+2y^2$ cannot be negative for $x, y \in \mathbb{R}$. (4)

[27]

March 2018

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2-6x-16=0$ (3)

1.1.2 $2x^2+7x-1=0$ (correct to TWO decimal places) (4)

1.2 List all the integers that are solutions to $x^2-25<0$. (4)

1.3 Solve for x and y:

$-2y+x=-1$ and $x^2-7-y^2=-y$ (6)

1.4 Evaluate: $\frac{3^{2018}+3^{2016}}{3^{2017}}$ (2)

1.5 Given: $t(x)=\frac{\sqrt{3x-5}}{x-3}$

1.5.1 For which values of x will $\frac{\sqrt{3x-5}}{x-3}$ be real? (3)

1.5.2 Solve for x if $t(x)=1$. (4)

[26]

November 2017:

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 9x + 14 = 0$ (3)

1.1.2 $4x^2 + 9x - 3 = 0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{x^2 - 5} = 2\sqrt{x}$ (4)

1.2 Solve for x and y if:

$3x - y = 4$ and $x^2 + 2xy - y^2 = -2$ (6)

1.3 Given: $f(x) = x^2 + 8x + 16$

1.3.1 Solve for x if $f(x) > 0$. (3)

1.3.2 For which values of p will $f(x) = p$ have TWO unequal negative roots? (4)

[24]

June 2017:

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 + 10x + 6 = 0$ (correct to TWO decimal places) (3)

1.1.2 $\sqrt{6x^2 - 15} = x + 1$ (5)

1.1.3 $x^2 + 2x - 24 \geq 0$ (3)

1.2 Solve simultaneously for x and y:

$5x + y = 3$ and $3x^2 - 2xy = y^2 - 105$ (6)

1.3 1.3.1 Solve for p if $p^2 - 48p - 49 = 0$ (3)

1.3.2 Hence, or otherwise, solve for x if $7^{2x} - 48(7^x) - 49 = 0$ (3)

[23]

March 2017:

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-3)(x+1)=0$ (2)

1.1.2 $\sqrt{x^3}=512$ (3)

1.1.3 $x(x-4)<0$ (2)

1.2 Given: $f(x)=x^2-5x+2$

1.2.1 Solve for x if $f(x)=0$ (3)

1.2.2 For which values of c will $f(x)=c$ have no real roots? (4)

1.3 Solve for x and y:

$x=2y+2$

$x^2-2xy+3y^2=4$ (6)

1.4 Calculate the maximum value of S if $S=\frac{6}{x^2+2}$. (2)

[22]

ARITHMETIC SEQUENCES AND SERIES

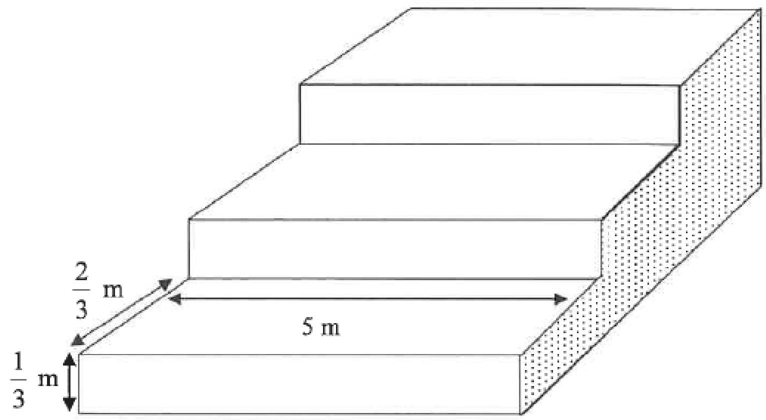
Nov 2019

QUESTION 3

3.1 Without using a calculator, determine the value of: $\sum_{y=3}^{10}\frac{1}{y-2}-\sum_{y=3}^{10}\frac{1}{y-1}$ (3)

3.2 A steel pavilion at a sports ground comprises of a series of 12 steps, of which the first 3 are shown in the diagram below.

Each step is 5 m wide. Each step has a rise of $\frac{1}{3}$ m and has a tread of $\frac{2}{3}$ m, as shown in the diagram below.



The open side (shaded on sketch) on each side of the pavilion must be covered with metal sheeting. Calculate the area (in m^2) of metal sheeting needed to cover both open sides. (6)

[9]

May-June 2019

QUESTION 3

3.1 The first three terms of an arithmetic sequence are: $2p+3$; $p+6$ and $p-2$.

3.1.1 Show that $p = 11$. (2)

3.1.2 Calculate the smallest value of n for which $T_n < -55$. (3)

3.2 Given that $\sum_{k=1}^{6}(x-3k) = \sum_{k=1}^{9}(x-3k)$, prove that $\sum_{k=1}^{15}(x-3k) = 0$. (5)

[10]

Nov 2018

QUESTION 2

2.1 Given the quadratic sequence: 2 ; 3 ; 10 ; 23 ; …

2.1.1 Write down the next term of the sequence. (1)

2.1.2 Determine the n^{th} term of the sequence. (4)

2.1.3 Calculate the 20^{th} term of the sequence. (2)

2.2 Given the arithmetic sequence: 35 ; 28 ; 21 ; …

Calculate which term of the sequence will have a value of –140. (3)

2.3 For which value of n will the sum of the first n terms of the arithmetic sequence in QUESTION 2.2 be equal to the n^{th} term of the quadratic sequence in QUESTION 2.1? (6)

[16]

June 2018

QUESTION 2

2.1 Given the quadratic pattern: 5 ; 10 ; 17 ; 26 ; …

2.1.1 Write down the next TWO terms of the pattern. (2)

2.1.2 Determine the formula for the n^{th} term of the pattern. (4)

2.1.3 Which term of the pattern will have a value of 1 765? (4)

2.2 The first 24 terms of an arithmetic series are: $35 + 42 + 49 + \ldots + 196$.

Calculate the sum of ALL natural numbers from 35 to 196 that are NOT divisible by 7. (5)

[15]

March 2018

QUESTION 3

The first three terms of an arithmetic sequence are -1 ; 2 and 5.

3.1 Determine the n^{th} term, T_n, of the sequence. (2)

3.2 Calculate T_{43}. (2)

3.3 Evaluate $\sum_{k=1}^{n} T_k$ in terms of n. (3)

3.4 A quadratic sequence, with general term T_n, has the following properties:

- $T_{11} = 125$
- $T_n - T_{n-1} = 3n - 4$

Determine the first term of the sequence. (6)

[13]

November 2017:

2.2 The first three terms of an arithmetic sequence are $2k-7$; $k+8$ and $2k-1$.

2.2.1 Calculate the value of the 15^{th} term of the sequence. (5)

2.2.2 Calculate the sum of the first 30 even terms of the sequence. (4)

June 2017:

2.2 In a Mathematics competition, the total prize money for the finalists is R30 500. Each finalist will receive a part of the prize money according to his/her position at the end of the competition. The table below shows the position of the finalists at the end of the competition and the prize money received.

POSITION OF THE FINALIST AT THE END OF THE COMPETITION	PRIZE MONEY
Last	R100
Second from last	R250
Third from last	R400
Fourth from last	R550
. . . .	. . . .
First	Rx

2.2.1 Calculate the prize money of the finalist finishing 18th from last. (2)

2.2.2 Calculate x. (6)

SESSION 2

Geometric Sequences and Series Quadratic Sequences Financial Mathematics

Nov 2019

2.2 Given the geometric series: $\frac{5}{8}+\frac{5}{16}+\frac{5}{32}+\ldots=K$

2.2.1 Determine the value of K if the series has 21 terms. (3)

2.2.2 Determine the largest value of n for which $T_n > \frac{5}{8192}$ (4)

[19]

May-June 2019

2.2 Given a geometric sequence: 36 ; −18 ; 9 ; …

2.2.1 Determine the value of r, the common ratio. (1)

2.2.2 Calculate n if $T_n = \frac{9}{4\,096}$ (3)

2.2.3 Calculate S_∞ (2)

2.2.4 Calculate the value of $\dfrac{T_1 + T_3 + T_5 + T_7 + \ldots + T_{499}}{T_2 + T_4 + T_6 + T_8 + \ldots + T_{500}}$ (4)

[17]

<u>Nov 2018</u>

QUESTION 3

A geometric series has a constant ratio of $\frac{1}{2}$ and a sum to infinity of 6.

3.1 Calculate the first term of the series. (2)

3.2 Calculate the 8^{th} term of the series. (2)

3.3 Given: $\sum_{k=1}^{n} 3(2)^{1-k} = 5{,}8125$ Calculate the value of n. (4)

3.4 If $\sum_{k=1}^{20} 3(2)^{1-k} = p$, write down $\sum_{k=1}^{20} 24(2)^{-k}$ in terms of p. (3)

[11]

<u>June 2018</u>

QUESTION 3

Themba is planning a bicycle trip from Cape Town to Pretoria. The total distance covered during the trip will be 1 500 km. He plans to travel 100 km on the first day. For every following day he plans to cover 94% of the distance he covered the previous day.

3.1 What distance will he cover on day 3 of the trip? (2)

3.2 On what day of the trip will Themba pass the halfway point? (4)

3.3 Themba must cover a certain percentage of the previous day's distance to ensure that he will eventually reach Pretoria. Calculate ALL possible value(s) of this percentage. (3)

[9]

<u>March 2018</u>

QUESTION 2

2.1 Given the following geometric sequence: $30\ ;\ 10\ ;\ \frac{10}{3}\ ;\ldots$

2.1.1 Determine n if the n^{th} term of the sequence is equal to $\frac{10}{729}$. (4)

2.1.2 Calculate: $30+10+\frac{10}{3}+\ldots$ (2)

2.2 Derive a formula for the sum of the first n terms of an arithmetic sequence if the first term of the sequence is a and the common difference is d. (4)

[10]

November 2017:

QUESTION 3

A convergent geometric series consisting of only positive terms has first term a, constant ratio r and n^{th} term, T_n, such that $\sum_{n=3}^{\infty} \mathrm{T}_n = \frac{1}{4}$.

3.1 If $\mathrm{T}_1 + \mathrm{T}_2 = 2$, write down an expression for a in terms of r. (2)

3.2 Calculate the values of a and r. (6)

[8]

June 2017:

2.1 Given the geometric sequence: $3\ ;\ 2\ ;\ k\ ;\ldots$

2.1.1 Write down the value of the common ratio. (1)

2.1.2 Calculate the value of k. (2)

2.1.3 Calculate the value of n if $T_n = \frac{128}{729}$. (4)

March 2017:

QUESTION 2

Given the geometric sequence: $-\frac{1}{4}\ ;\ b\ ;\ -1\ ;\ ......$

2.1 Calculate the possible values of b. (3)

2.2 If $b=\frac{1}{2}$, calculate the 19^{th} term (T_{19}) of the sequence. (3)

2.3 If $b=\frac{1}{2}$, write the sum of the first 20 positive terms of the sequence in sigma notation. (4)

2.4 Is the geometric series formed in QUESTION 2.3 convergent? Give reasons for your answer. (2)

[12]

Quadratic Sequences:

November 2019:

QUESTION 2

2.1 Given the quadratic sequence: 321 ; 290 ; 261 ; 234 ; ....

2.1.1 Write down the values of the next TWO terms of the sequence. (2)

2.1.2 Determine the general term of the sequence in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Which term(s) of the sequence will have a value of 74? (4)

2.1.4 Which term in the sequence has the least value? (2)

May-June 2019

QUESTION 2

2.1 The first FOUR terms of a quadratic pattern are: 15 ; 29 ; 41 ; 51

2.1.1 Write down the value of the 5^{th} term. (1)

2.1.2 Determine an expression for the n^{th} term of the pattern in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Determine the value of T_{27} (2)

November 2017:

2.1 Given the following quadratic number pattern: 5 ; –4 ; –19 ; –40 ; …

2.1.1 Determine the constant second difference of the sequence. (2)

2.1.2 Determine the n^{th} term (T_n) of the pattern. (4)

2.1.3 Which term of the pattern will be equal to $-25\,939$? (3)

June 2017:

QUESTION 3

Given the quadratic sequence: 0; 17; 32; ...

3.1 Determine an expression for the general term, T_n, of the quadratic sequence. (4)

3.2 Which terms in the quadratic sequence have a value of 56? (3)

3.3 Hence, or otherwise, calculate the value of $\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$. (4)
[11]

March 2017:

QUESTION 3

3.1 6 ; 6 ; 9 ; 15 ; … are the first four terms of a quadratic number pattern.

3.1.1 Write down the value of the fifth term (T_5) of the pattern. (1)

3.1.2 Determine a formula to represent the general term of the pattern. (4)

3.1.3 Which term of the pattern has a value of 3 249? (4)

3.2 Determine the value(s) of x in the interval $x \in [0^\circ ; 90^\circ]$ for which the sequence -1 ; 2sin3x ; 5 ; ..... will be arithmetic. (4)
[13]

FINANCIAL MATHEMATICS

November 2019

QUESTION 6

6.1 Two friends, Kuda and Thabo, each want to invest R5 000 for four years. Kuda invests his money in an account that pays simple interest at 8,3% per annum. At the end of four years, he will receive a bonus of exactly 4% of the accumulated amount. Thabo invests his money in an account that pays interest at 8,1% p.a., compounded monthly.

Whose investment will yield a better return at the end of four years? Justify your answer with appropriate calculations. (5)

6.2 Nine years ago, a bank granted Mandy a home loan of R525 000. This loan was to be repaid over 20 years at an interest rate of 10% p.a., compounded monthly. Mandy's monthly repayments commenced exactly one month after the loan was granted.

6.2.1 Mandy decided to make monthly repayments of R6 000 instead of the required R5 066,36. How many payments will she make to settle the loan? (5)

6.2.2 After making monthly repayments of R6 000 for nine years, Mandy required money to fund her daughter's university fees. She approached the bank for another loan. Instead, the bank advised Mandy that the extra amount repaid every month could be regarded as an investment and that she could withdraw this full amount to fund her daughter's studies. Calculate the maximum amount that Mandy may withdraw from the loan account. (4)

[14]

May-June 2019

QUESTION 6

6.1 Sandile bought a car for R180 000. The value of the car depreciated at 15% per annum according to the reducing-balance method. The book value of Sandile's car is currently R79 866,96.

6.1.1 How many years ago did Sandile buy the car? (3)

6.1.2 At exactly the same time that Sandile bought the car, Anil deposited R49 000 into a savings account at an interest rate of 10% p.a., compounded quarterly. Has Anil accumulated enough money in his savings account to buy Sandile's car now? (3)

6.2 Exactly 10 months ago, a bank granted Jane a loan of R800 000 at an interest rate of 10,25% p.a., compounded monthly.
The bank stipulated that the loan:

- Must be repaid over 20 years
- Must be repaid by means of monthly repayments of R7 853,15, starting one month after the loan was granted

6.2.1 How much did Jane owe immediately after making her 6^{th} repayment? (4)

6.2.2 Due to financial difficulties, Jane missed the 7^{th}, 8^{th} and 9^{th} payments. She was able to make payments from the end of the 10^{th} month onwards. Calculate Jane's increased monthly payment in order to settle the loan in the original 20 years. (5)

[15]

November 2018

QUESTION 7

7.1 Selby decided today that he will save R15 000 per quarter over the next four years. He will make the first deposit into a savings account in three months' time and he will make his last deposit at the end of four years from now.

7.1.1 How much will Selby have at the end of four years if interest is earned at 8,8% per annum, compounded quarterly? (3)

7.1.2 If Selby decides to withdraw R100 000 from the account at the end of three years from now, how much will he have in the account at the end of four years from now? (3)

7.2 Tshepo takes out a home loan over 20 years to buy a house that costs R1 500 000.

7.2.1 Calculate the monthly instalment if interest is charged at 10,5% p.a., compounded monthly. (4)

7.2.2 Calculate the outstanding balance immediately after the 144^{th} payment was made. (5)

[15]

June 2018

QUESTION 6

6.1 Calculate how many years it will take for the value of a truck to decrease to 50% of its original value if depreciation is calculated at 15% per annum using the reducing-balance method. (4)

6.2 Every month Tshepo deposited R1 500 for his retirement into an account that paid interest at a rate of 9,2% per annum, compounded monthly. Tshepo made his first instalment on his 23^{rd} birthday and the last instalment one month before his 55^{th} birthday. Calculate how much money he had in the account on his 55^{th} birthday. (5)

6.3 Abram has R150 000 to invest in two separate accounts. One account pays interest at a rate of 8,4% per annum, compounded quarterly, and the other account at a rate of 9,6% per annum, compounded monthly. How much money should he invest in each account so that he will collect the same amount from each account at the end of 12 years? (6)

[15]

March 2018

QUESTION 7

7.1 On 30 June 2013 and at the end of each month thereafter, Asif deposited R2 500 into a bank account that pays interest at 6% per annum, compounded monthly. He wants to continue to deposit this amount until 31 May 2018.

Calculate how much money Asif will have in this account immediately after depositing R2 500 on 31 May 2018. (3)

7.2 On 1 February 2018, Genevieve took a loan of R82 000 from the bank to pay for her studies. She will make her first repayment of R3 200 on 1 February 2019 and continue to make payments of R3 200 on the first of each month thereafter until she settles the loan. The bank charges interest at 15% per annum, compounded monthly.

7.2.1 Calculate how much Genevieve will owe the bank on 1 January 2019. (3)

7.2.2 How many instalments of R3 200 must she pay? (5)

7.2.3 Calculate the final payment, to the nearest rand, Genevieve has to pay to settle the loan. (5)

[16]

November 2017:

QUESTION 6

6.1 Mbali invested R10 000 for 3 years at an interest rate of r % p.a., compounded monthly. At the end of this period, she received R12 146,72. Calculate r, correct to ONE decimal place. (5)

6.2 Piet takes a loan from a bank to buy a car for R235 000. He agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a., compounded monthly.

6.2.1 Calculate Piet's monthly instalment. (4)

6.2.2 Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan. (6)

[15]

June 2017:

QUESTION 7

7.1 A company bought a new machine for R500 000. After 3 years, the machine has a book value of R331 527. Calculate the yearly rate of depreciation if the machine was depreciated according to the reducing-balance method. (3)

7.2 Musa takes a personal loan from a bank to buy a motorcycle that costs R46 000. The bank charges interest at 24% per annum, compounded monthly.

How many months will it take Musa to repay the loan, if the monthly instalment is R1 900? (4)

7.3 Neil set up an investment fund. Exactly 3 months later and every 3 months thereafter he deposited R3 500 into the fund. The fund pays interest at 7,5% p.a., compounded quarterly. He continued to make quarterly deposits into the fund for 6½ years from the time that he originally set up the fund.

Neil made no further deposits into the fund, but left the money in the same fund at the same rate of interest. Calculate how much he will have in the fund 10 years after he originally set it up. (6)

[13]

March 2017:

QUESTION 6

6.1 On the 2nd day of January 2015 a company bought a new printer for R150 000.

- The value of the printer decreases by 20% annually on the reducing-balance method.
- When the book value of the printer is R49 152, the company will replace the printer.

6.1.1 Calculate the book value of the printer on the 2nd day of January 2017. (3)

6.1.2 At the beginning of which year will the company have to replace the printer? Show ALL calculations. (4)

6.1.3 The cost of a similar printer will be R280 000 at the beginning of 2020. The company will use the R49 152 that it will receive from the sale of the old printer to cover some of the costs of replacing the printer. The company set up a sinking fund to cover the balance. The fund pays interest at 8,5% per annum, compounded quarterly. The first deposit was made on 2 April 2015 and every three months thereafter until 2 January 2020. Calculate the amount that should be deposited every three months to have enough money to replace the printer on 2 January 2020. (4)

6.2 Lerato wishes to apply for a home loan. The bank charges interest at 11% per annum, compounded monthly. She can afford a monthly instalment of R9 000 and wants to repay the loan over a period of 15 years. She will make the first monthly repayment one month after the loan is granted. Calculate, to the nearest thousand rand, the maximum amount that Lerato can borrow from the bank. (5)

[16]

SESSION 3

Functions Differential Calculus Probability

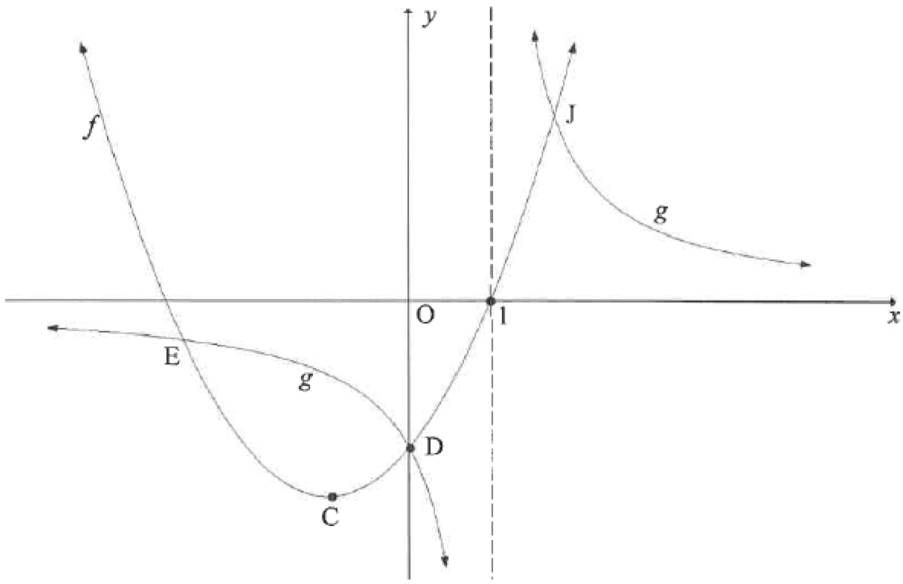
Functions

November 2019

QUESTION 4

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \dfrac{a}{x+p}$.

- f has a turning point at C and passes through the x-axis at $(1\ ;0)$.
- D is the y-intercept of both f and g. The graphs f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



4.1 Write down the value of p. (1)

4.2 Show that $a = 3$ and $b = 2$. (3)

4.3 Calculate the coordinates of C. (4)

4.4 Write down the range of f. (2)

4.5 Determine the equation of the line through C that makes an angle of $45°$ with the positive x-axis. Write your answer in the form $y = \ldots$ (3)

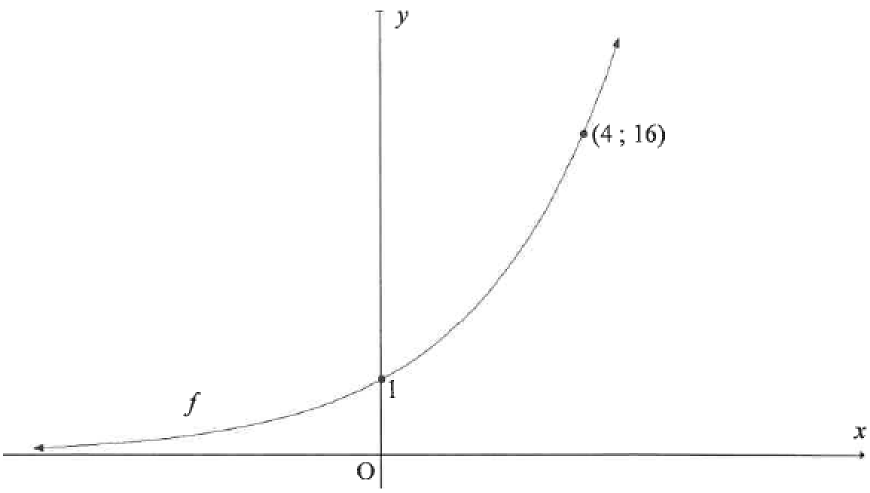
4.6 Is the straight line, determined in QUESTION 4.5, a tangent to f? Explain your answer. (2)

4.7 The function $h(x) = f(m - x) + q$ has only one x-intercept at $x = 0$. Determine the values of m and q. (4)

[19]

QUESTION 5

Sketched below is the graph of $f(x) = k^x$; $k > 0$. The point (4 ; 16) lies on f.



5.1 Determine the value of k. (2)

5.2 Graph g is obtained by reflecting graph f about the line $y = x$. Determine the equation of g in the form $y = \ldots$ (2)

5.3 Sketch the graph g. Indicate on your graph the coordinates of two points on g. (4)

5.4 Use your graph to determine the value(s) of x for which:

5.4.1 $f(x) \times g(x) > 0$ (2)

5.4.2 $g(x) \leq -1$ (2)

5.5 If $h(x) = f(-x)$, calculate the value of x for which $f(x) - h(x) = \frac{15}{4}$ (4)

[16]

May-June 2019

QUESTION 4

Given the exponential function: $g(x)=\left(\frac{1}{2}\right)^x$

4.1 Write down the range of g. (1)

4.2 Determine the equation of g^{-1} in the form $y = \ldots$ (2)

4.3 Is g^{-1} a function? Justify your answer. (2)

4.4 The point $M(a\ ;\ 2)$ lies on g^{-1}.

4.4.1 Calculate the value of a. (2)

4.4.2 $M^{/}$, the image of M, lies on g. Write down the coordinates of $M^{/}$. (1)

4.5 If $h(x) = g(x + 3) + 2$, write down the coordinates of the image of $M^{/}$ on h. (3)

[11]

QUESTION 5

5.1 Given: $f(x)=\frac{1}{x+2}+3$

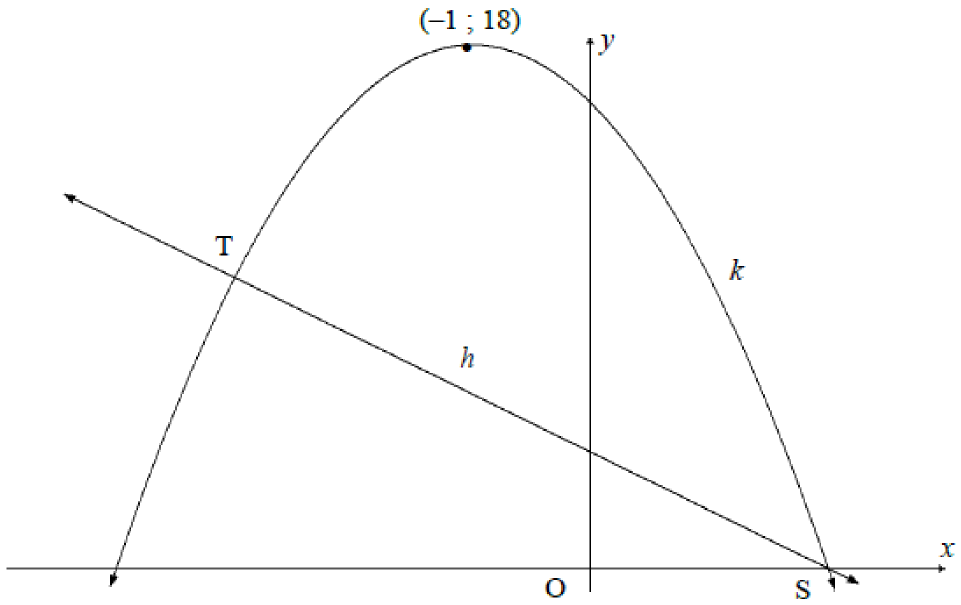
5.1.1 Determine the equations of the asymptotes of f. (2)

5.1.2 Write down the y-intercept of f. (1)

5.1.3 Calculate the x-intercept of f. (2)

5.1.4 Sketch the graph of f. Clearly label ALL intercepts with the axes and any asymptotes. (3)

5.2 Sketched below are the graphs of $k(x)=ax^2+bx+c$ and $h(x)=-2x+4$. Graph k has a turning point at $(-1\ ;\ 18)$. S is the x-intercept of h and k. Graphs h and k also intersect at T.



5.2.1 Calculate the coordinates of S. (2)

5.2.2 Determine the equation of k in the form $y = a(x+p)^2 + q$ (3)

5.2.3 If $k(x) = -2x^2 - 4x + 16$, determine the coordinates of T. (5)

5.2.4 Determine the value(s) of x for which $k(x) < h(x)$. (2)

5.2.5 It is further given that k is the graph of $g'(x)$.

(a) For which values of x will the graph of g be concave up? (2)

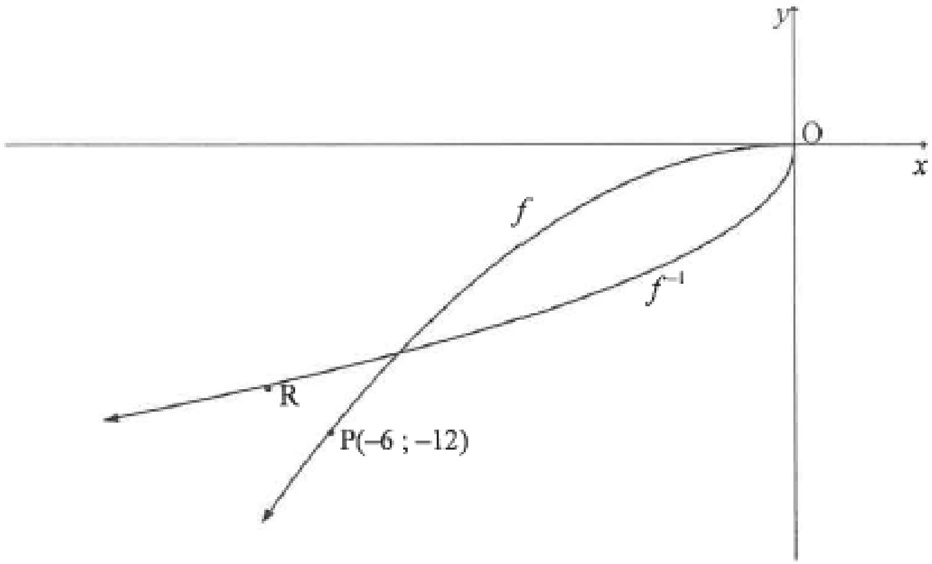
(b) Sketch the graph of g, showing clearly the x-values of the turning points and the point of inflection. (3)

[25]

November 2018

QUESTION 4

In the diagram below, the graph of $f(x) = ax^2$ is drawn in the interval $x \leq 0$.
The graph of f^{-1} is also drawn. P(–6 ; –12) is a point on f and R is a point on f^{-1}.



4.1 Is f^{-1} a function? Motivate your answer. (2)

4.2 If R is the reflection of P in the line $y = x$, write down the coordinates of R. (1)

4.3 Calculate the value of a. (2)

4.4 Write down the equation of f^{-1} in the form $y = ...$ (3)

[8]

QUESTION 5

Given: $f(x) = \dfrac{-1}{x-1}$

5.1 Write down the domain of f. (1)

5.2 Write down the asymptotes of f. (2)

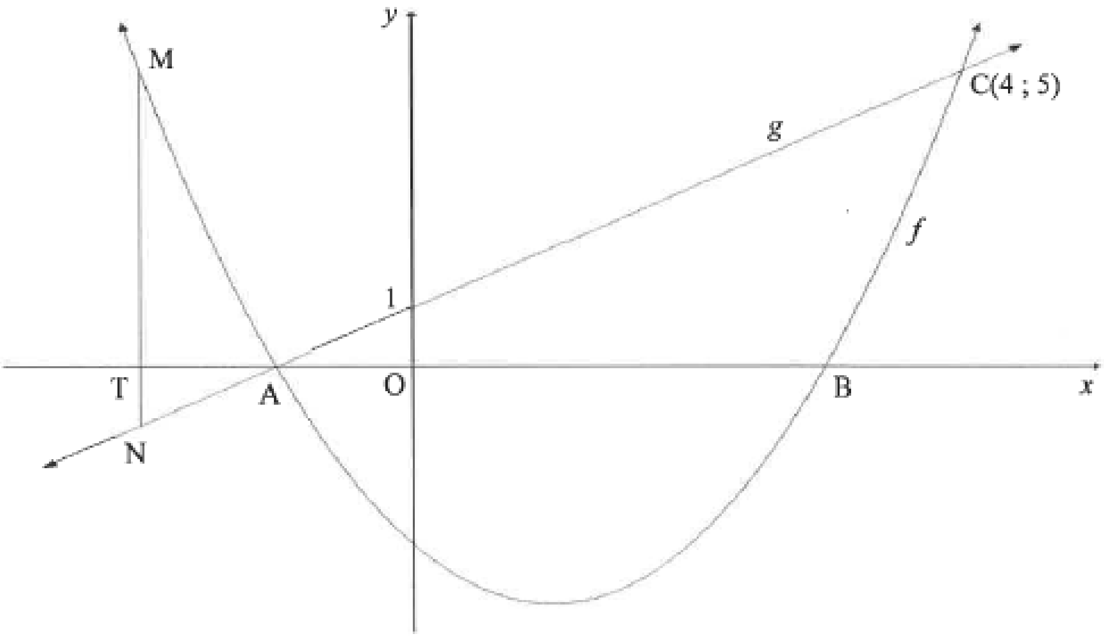
5.3 Sketch the graph of f, clearly showing all intercepts with the axes and any asymptotes. (3)

5.4 For which values of x will $x.f'(x) \geq 0$? (2)

[8]

QUESTION 6

In the diagram below, A and B are the x-intercepts of the graph of $f(x) = x^2 - 2x - 3$.
A straight line, g, through A cuts f at C(4 ; 5) and the y-axis at (0 ; 1).
M is a point on f and N is a point on g such that MN is parallel to the y-axis.
MN cuts the x-axis at T.

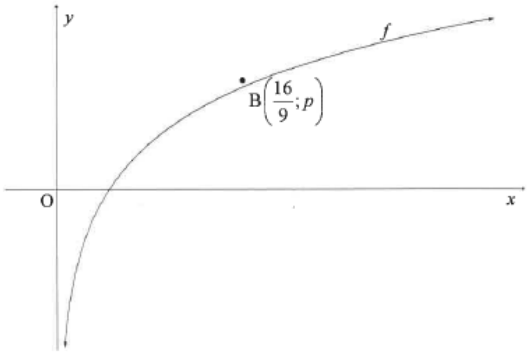


6.1	Show that $g(x) = x + 1$.	(2)
6.2	Calculate the coordinates of A and B.	(3)
6.3	Determine the range of f.	(3)
6.4	If MN = 6:	
	6.4.1 Determine the length of OT if T lies on the negative x-axis. Show ALL your working.	(4)
	6.4.2 Hence, write down the coordinates of N.	(2)
6.5	Determine the equation of the tangent to f drawn parallel to g.	(5)
6.6	For which value(s) of k will $f(x) = x^2 - 2x - 3$ and $h(x) = x + k$ NOT intersect?	(1)
		[20]

June 2018

QUESTION 4

The graph of $f(x) = \log_{\frac{4}{3}} x$ is drawn below. $B\left(\frac{16}{9}; p\right)$ is a point on f.



4.1 For which value(s) of x is $\log_{\frac{4}{3}} x \leq 0$? (2)

4.2 Determine the value of p, without the use of a calculator. (3)

4.3 Write down the equation of the inverse of f in the form $y = \ldots$ (2)

4.4 Write down the range of $y = f^{-1}(x)$. (2)

4.5 The function $h(x) = \left(\frac{3}{4}\right)^x$ is obtained after applying two reflections on f.

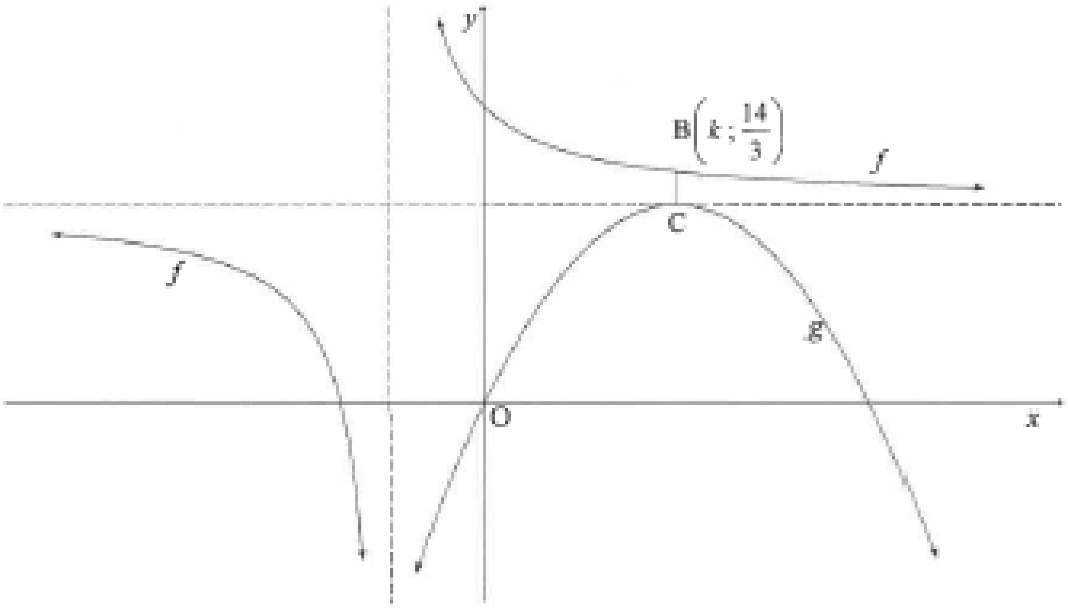
Write down the coordinates of B'', the image of B on h. (2)

[11]

QUESTION 5

The graphs of $f(x)=\frac{2}{x+1}+4$ and parabola g are drawn below.

- C, the turning point of g, lies on the horizontal asymptote of f.
- The graph of g passes through the origin.
- B $\left(k;\frac{14}{3}\right)$ is a point on f such that BC is parallel to the y-axis.



5.1 Write down the domain of f. (2)

5.2 Determine the x-intercept of f. (2)

5.3 Calculate the value of k. (3)

5.4 Write down the coordinates of C. (2)

5.5 Determine the equation of g in the form $y=a(x+p)^2+q$. (3)

5.6 For which value(s) of x will $\frac{f(x)}{g(x)}\leq 0$? (4)

5.7 Use the graphs of f and g to determine the number of real roots of $\frac{2}{x}-5=-(x-3)^2-5$. Give reasons for your answer. (4)

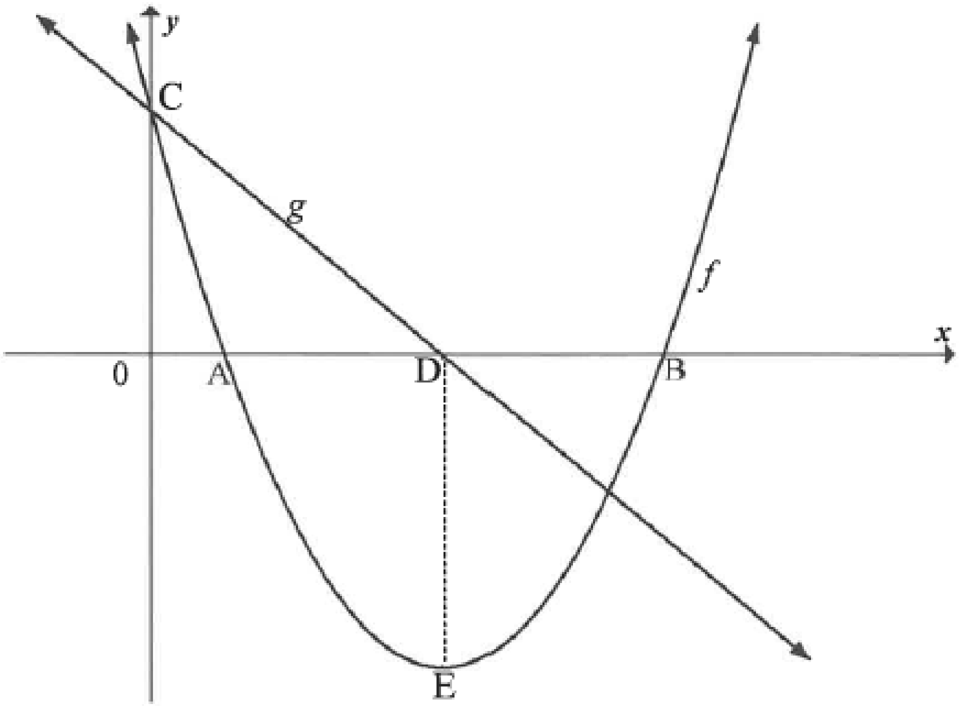
[20]

March 2018

QUESTION 4

Below are the graphs of $f(x)=(x-4)^2-9$ and a straight line g.

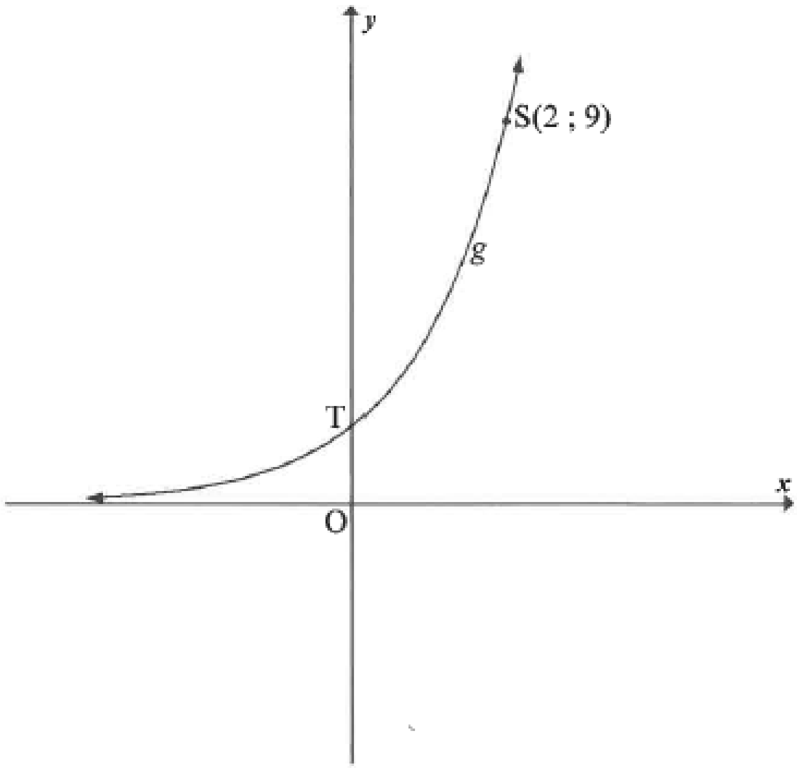
- A and B are the x-intercepts of f and E is the turning point of f.
- C is the y-intercept of both f and g.
- The x-intercept of g is D. DE is parallel to the y-axis.



4.1	Write down the coordinates of E.	(2)
4.2	Calculate the coordinates of A.	(3)
4.3	M is the reflection of C in the axis of symmetry of f. Write down the coordinates of M.	(3)
4.4	Determine the equation of g in the form $y=mx+c$.	(3)
4.5	Write down the equation of g^{-1} in the form $y=\ldots$	(3)
4.6	For which values of x will $x(f(x))\le 0$?	(4)
		[18]

QUESTION 5

The graph of $g(x) = a^x$ is drawn in the sketch below. The point S(2 ; 9) lies on g. T is the y-intercept of g.



5.1	Write down the coordinates of T.	(2)
5.2	Calculate the value of a.	(2)
5.3	The graph h is obtained by reflecting g in the y-axis. Write down the equation of h.	(2)
5.4	Write down the values of x for which $0 < \log_3 x < 1$.	(2) **[8]**

QUESTION 6

The function f, defined by $f(x) = \dfrac{a}{x+p} + q$, has the following properties:

- The range of f is $y \in R,\ y \neq 1$.
- The graph f passes through the origin.
- $P\left(\sqrt{2}+2;\sqrt{2}+1\right)$ lies on the graph f.

6.1	Write down the value of q.	(1)
6.2	Calculate the values of a and p.	(5)
6.3	Sketch a neat graph of this function. Your graph must include the asymptotes, if any.	(4) **[10]**

November 2017:

QUESTION 4

Given: $f(x) = -ax^2 + bx + 6$

4.1 The gradient of the tangent to the graph of f at the point $\left(-1\ ;\ \frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

4.2 Calculate the x-intercepts of f. (3)

4.3 Calculate the coordinates of the turning point of f. (3)

4.4 Sketch the graph of f. Clearly indicate ALL intercepts with the axes and the turning point. (4)

4.5 Use the graph to determine the values of x for which $f(x) > 6$. (3)

4.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as f. Clearly indicate ALL intercepts with the axes. (2)

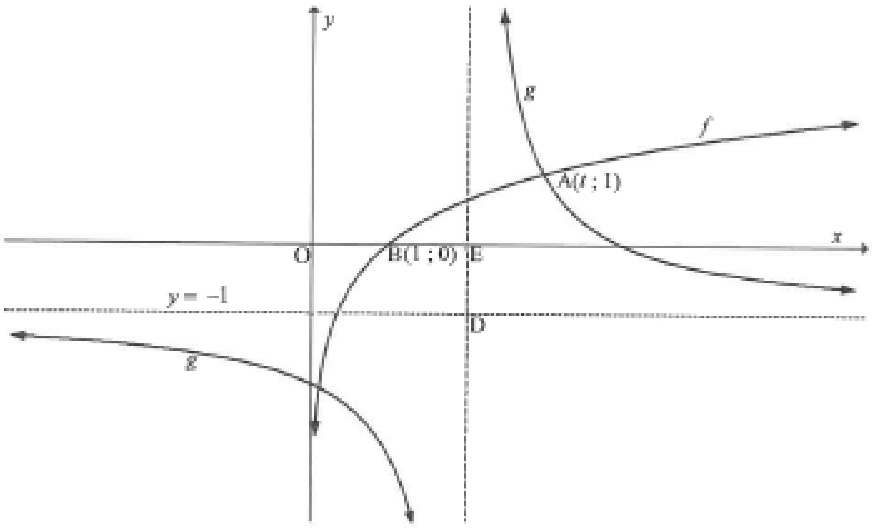
4.7 Write down the values of x for which $f(x).g(x) \leq 0$. (3)

[23]

QUESTION 5

The diagram below shows the graphs of $g(x) = \frac{2}{x+p} + q$ and $f(x) = \log_3 x$.

- $y = -1$ is the horizontal asymptote of g.
- B(1 ; 0) is the x-intercept of f.
- A(t ; 1) is a point of intersection between f and g.
- The vertical asymptote of g intersects the x-axis at E and the horizontal asymptote at D.
- OB = BE.



5.1 Write down the range of g. (2)

5.2 Determine the equation of g. (2)

5.3 Calculate the value of t. (3)

5.4 Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

5.5 For which values of x will $f^{-1}(x) < 3$? (2)

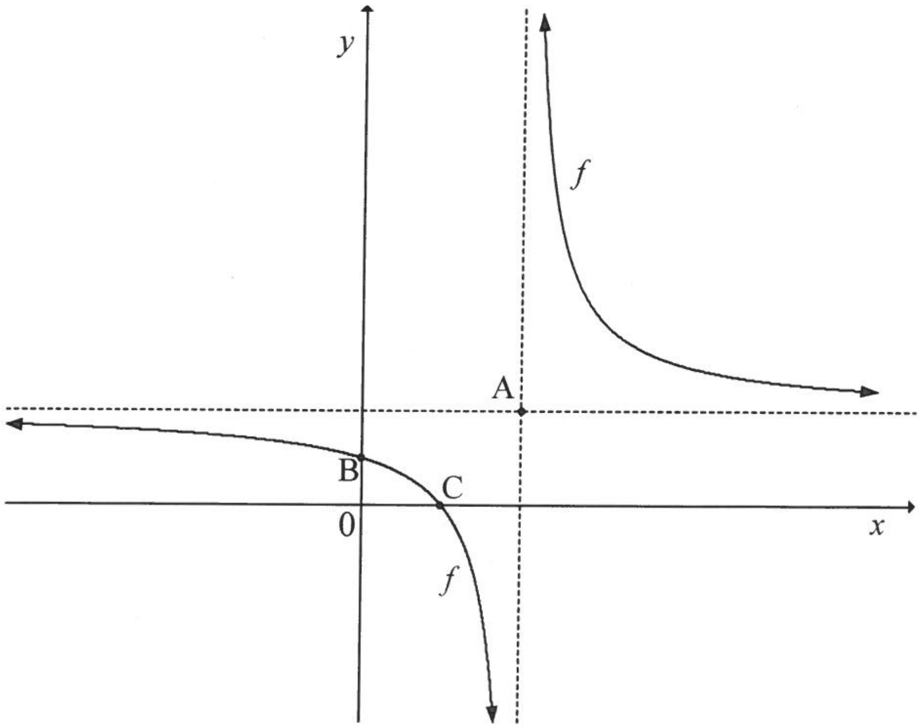
5.6 Determine the point of intersection of the graphs of f and the axis of symmetry of g that has a negative gradient. (3)

[14]

June 2017:

QUESTION 4

The sketch below shows the graph of $f(x)=\dfrac{6}{x-4}+3$. The asymptotes of f intersect at A. The graph f intersects the x-axis and y-axis at C and B respectively.



4.1	Write down the coordinates of A.	(1)
4.2	Calculate the coordinates of B.	(2)
4.3	Calculate the coordinates of C.	(2)
4.4	Calculate the average gradient of f between B and C.	(2)
4.5	Determine the equation of a line of symmetry of f which has a positive y-intercept.	(2) **[9]**

QUESTION 5

Given: $f(x) = x^2 - 5x - 14$ and $g(x) = 2x - 14$

5.1 On the same set of axes, sketch the graphs of f and g. Clearly indicate all intercepts with the axes and turning points. (6)

5.2 Determine the equation of the tangent to f at $x = 2\frac{1}{2}$. (2)

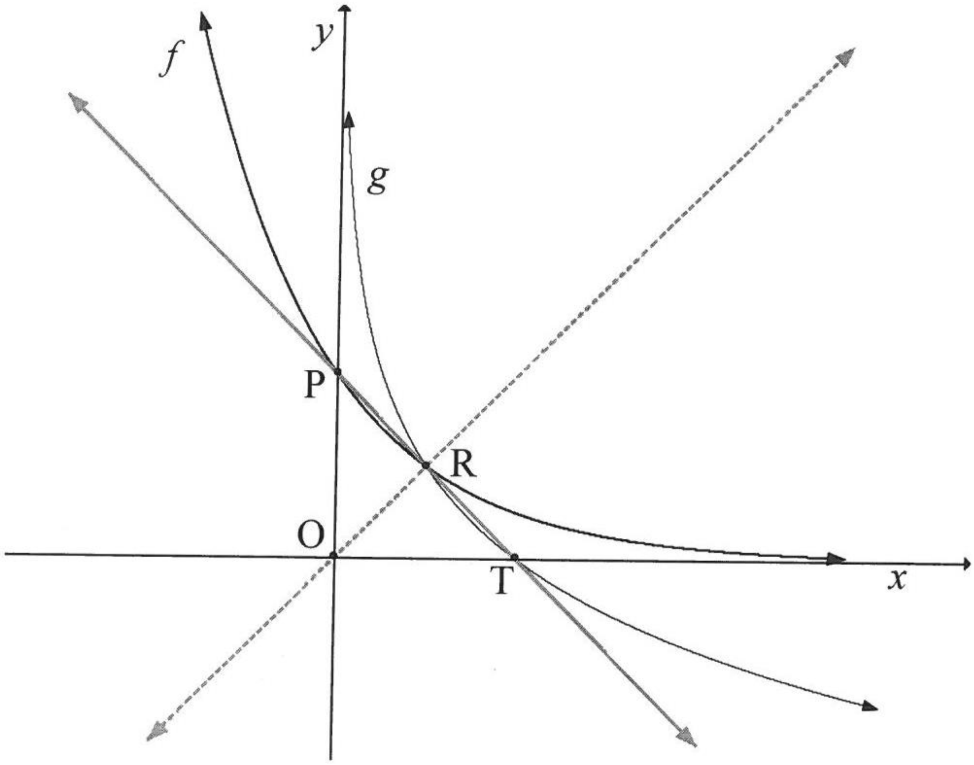
5.3 Determine the value(s) of k for which $f(x) = k$ will have two unequal positive real roots. (2)

5.4 A new graph h is obtained by first reflecting g in the x-axis and then translating it 7 units to the left. Write down the equation of h in the form $h(x) = mx + c$. (2)

[12]

QUESTION 6

In the sketch below, P is the y-intercept of the graph of $f(x) = b^x$. T is the x-intercept of graph g, the inverse of f. R is the point of intersection of f and g. Straight lines are drawn through O and R and through P and T.



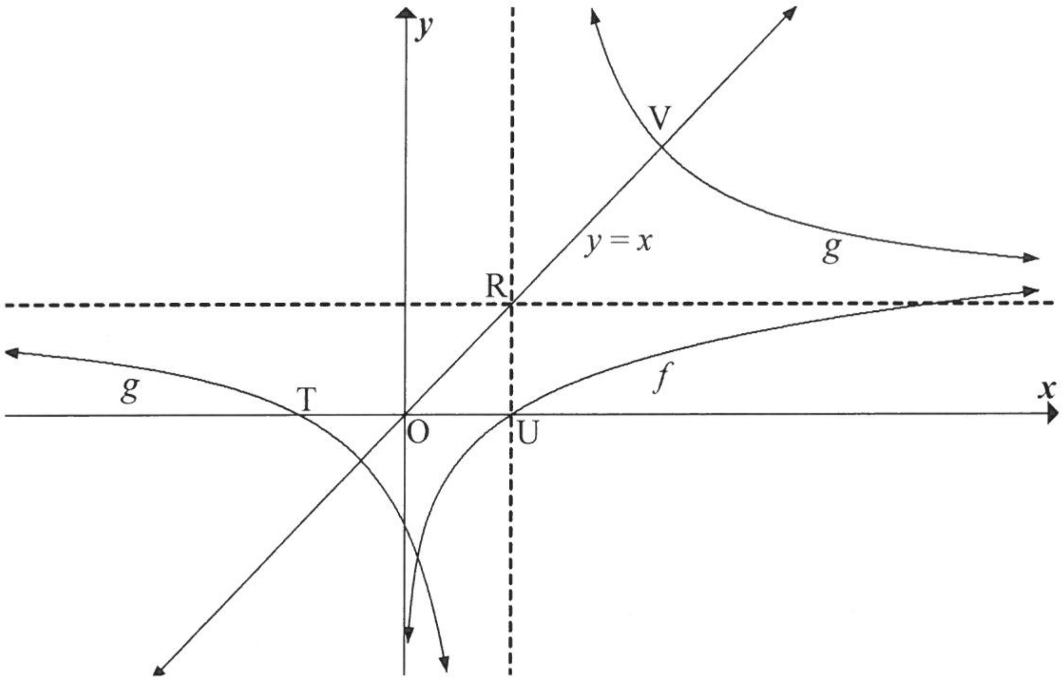
6.1	Determine the equation of g (in terms of b) in the form $y = \ldots$	(2)
6.2	Write down the equation of the line passing through O and R.	(1)
6.3	Write down the coordinates of point P.	(1)
6.4	Determine the equation of the line passing through P and T.	(2)
6.5	Calculate the value of b.	(5)
		[11]

<u>March 2017:</u>

QUESTION 4

The sketch below shows the graphs of $f(x) = \log_5 x$ and $g(x) = \frac{2}{x-1} + 1$.

- T and U are the x-intercepts of g and f respectively.
- The line $y = x$ intersects the asymptotes of g at R, and the graph of g at V.



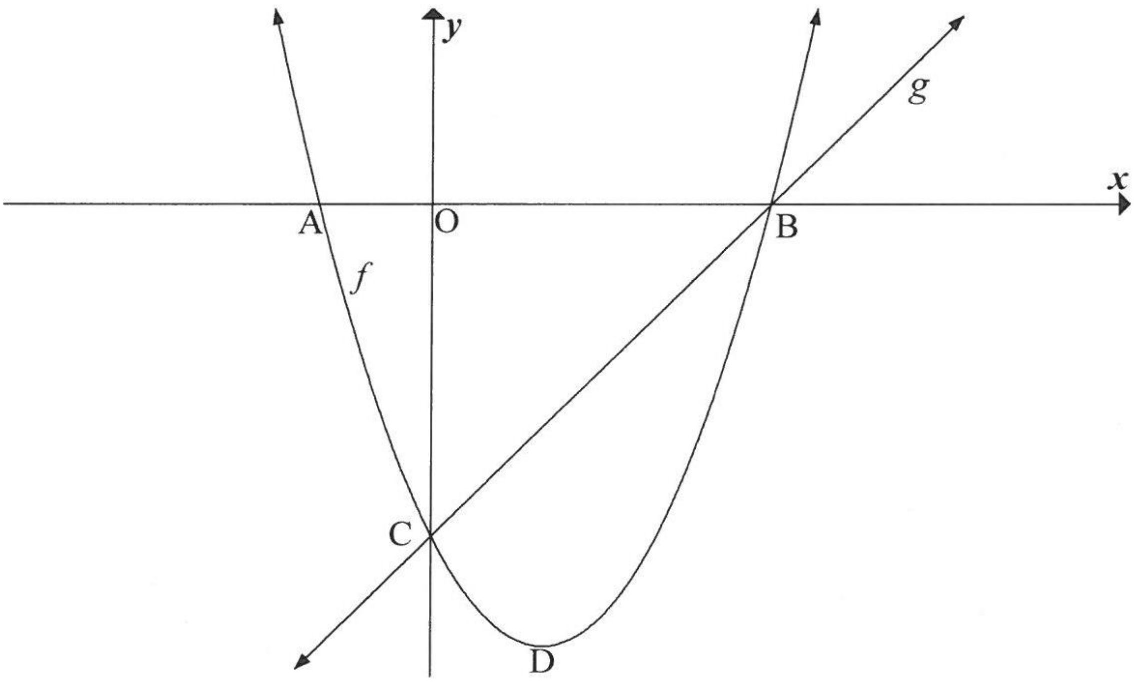
4.1	Write down the coordinates of U.	(1)
4.2	Write down the equations of the asymptotes of g.	(2)
4.3	Determine the coordinates of T.	(2)
4.4	Write down the equation of h, the reflection of f in the line $y = x$, in the form $y = \ldots$	(2)
4.5	Write down the equation of the asymptote of $h(x-3)$.	(1)
4.6	Calculate the coordinates of V.	(4)
4.7	Determine the coordinates of $T^{/}$ the point which is symmetrical to T about the point R.	(2)
		[14]

QUESTION 5

5.1 The sketch below shows the graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x - 3$.

- A and B are the x-intercepts of f.
- The graphs of f and g intersect at C and B.

D is the turning point of f.



5.1.1	Determine the coordinates of C.	(1)
5.1.2	Calculate the length of AB.	(4)
5.1.3	Determine the coordinates of D.	(2)
5.1.4	Calculate the average gradient of f between C and D.	(2)
5.1.5	Calculate the size of $O\hat{C}B$	(2)
5.1.6	Determine the values of k for which $f(x) = k$ will have two unequal positive real roots.	(3)
5.1.7	For which values of x will $f'(x) \cdot f''(x) > 0$?	(3)

5.2 The graph of a parabola f has x-intercepts at $x = 1$ and $x = 5$. $g(x) = 4$ is a tangent to f at P, the turning point of f. Sketch the graph of f, clearly showing the intercepts with the axes and the coordinates of the turning point. (5)

[22]

Differential Calculus:

November 2019

QUESTION 7

7.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 4 - 7x$. (4)

7.2 Determine $\frac{dy}{dx}$ if $y = 4x^8 + \sqrt{x^3}$ (3)

7.3 Given: $y = ax^2 + a$

Determine:

7.3.1 $\frac{dy}{dx}$ (1)

7.3.2 $\frac{dy}{da}$ (2)

7.4 The curve with equation $y = x + \frac{12}{x}$ passes through the point A(2 ; b). Determine the equation of the line perpendicular to the tangent to the curve at A. (4)

[14]

QUESTION 8

After flying a short distance, an insect came to rest on a wall. Thereafter the insect started crawling on the wall. The path that the insect crawled can be described by $h(t) = (t-6)(-2t^2 + 3t - 6)$, where h is the height (in cm) above the floor and t is the time (in minutes) since the insect started crawling.

8.1 At what height above the floor did the insect start to crawl? (1)

8.2 How many times did the insect reach the floor? (3)

8.3 Determine the maximum height that the insect reached above the floor. (4)

[8]

QUESTION 9

Given: $f(x) = 3x^3$

9.1 Solve $f(x) = f'(x)$ (3)

9.2 The graphs f, f' and f'' all pass through the point $(0\,;0)$.

9.2.1 For which of the graphs will $(0\,;0)$ be a stationary point? (1)

9.2.2 Explain the difference, if any, in the stationary points referred to in QUESTION 9.2.1. (2)

9.3 Determine the vertical distance between the graphs of f' and f'' at $x = 1$. (3)

9.4 For which value(s) of x is $f(x) - f'(x) < 0$? (4)

[13]

May-June 2019

QUESTION 7

7.1 Given $f(x) = x^2 + 2$.

Determine $f'(x)$ from first principles. (4)

7.2 Determine $\frac{dy}{dx}$ if:

7.2.1 $y = 4x^3 + \frac{2}{x}$ (3)

7.2.2 $y = 4.\sqrt[3]{x} + (3x^3)^2$ (4)

7.3 If g is a linear function with $g(1) = 5$ and $g'(3) = 2$, determine the equation of g in the form $y = \ldots..$ (3)

[14]

QUESTION 8

A cubic function $h(x) = -2x^3 + bx^2 + cx + d$ cuts the x-axis at $(-3\,;0)$; $\left(-\frac{3}{2}\,;0\right)$ and $(1\,;0)$.

8.1 Show that $h(x) = -2x^3 - 7x^2 + 9$. (3)

8.2 Calculate the x-coordinates of the turning points of h. (3)

8.3 Determine the value(s) of x for which h will be decreasing. (2)

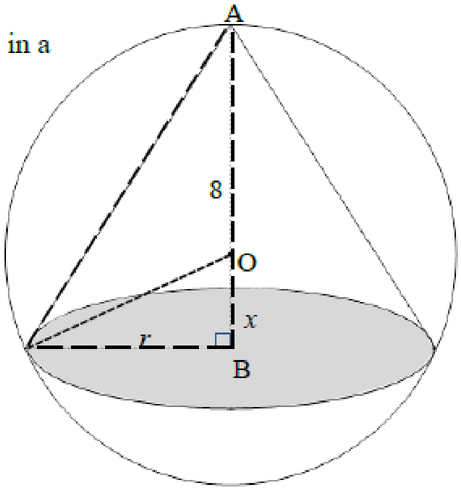
8.4 For which value(s) of x will there be a tangent to the curve of h that is parallel to the line $y - 4x = 7$. (4)

[12]

QUESTION 9

A cone with radius r cm and height AB is inscribed in a sphere with centre O and a radius of 8 cm. OB = x.

Volume of sphere = $\frac{4}{3}\pi r^3$
Volume of cone = $\frac{1}{3}\pi r^2 h$



9.1 Calculate the volume of the sphere. (1)

9.2 Show that $r^2 = 64 - x^2$. (1)

9.3 Determine the ratio between the largest volume of this cone and the volume of the sphere. (7)

[9]

November 2018

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given $f(x) = x^2 - 5$. (5)

8.2 Determine $\frac{dy}{dx}$ if:

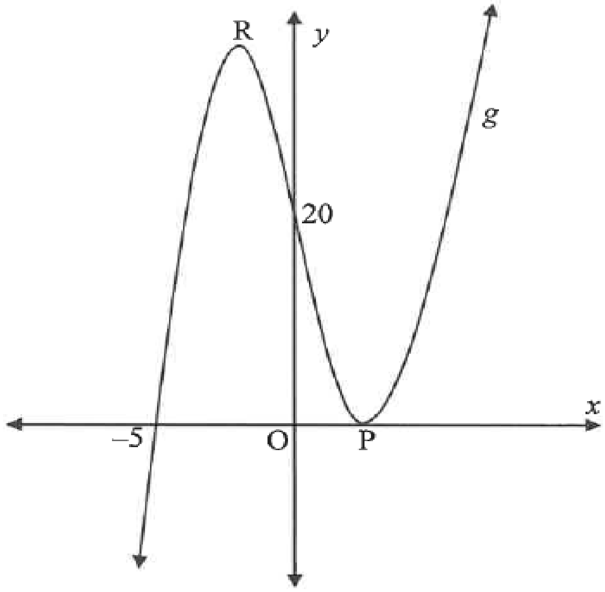
8.2.1 $y = 3x^3 + 6x^2 + x - 4$ (3)

8.2.2 $yx - y = 2x^2 - 2x \quad ; \quad x \neq 1$ (4)

[12]

QUESTION 9

9.1 The graph of $g(x) = x^3 + bx^2 + cx + d$ is sketched below.
The graph of g intersects the x-axis at $(-5 ; 0)$ and at P, and the y-axis at $(0 ; 20)$.
P and R are turning points of g.



9.1.1 Show that $b = 1$, $c = -16$ and $d = 20$. (4)

9.1.2 Calculate the coordinates of P and R. (5)

9.1.3 Is the graph concave up or concave down at $(0 ; 20)$? Show ALL your calculations. (3)

9.2 If g is a cubic function with:

- $g(3) = g'(3) = 0$
- $g(0) = 27$
- $g''(x) > 0$ when $x < 3$ and $g''(x) < 0$ when $x > 3$,

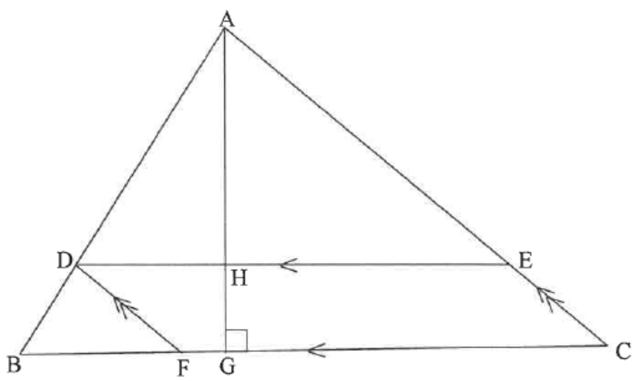
draw a sketch graph of g indicating ALL relevant points. (3)

[15]

QUESTION 10

In ΔABC:

- D is a point on AB, E is a point on AC and F is a point on BC such that DECF is a parallelogram.
- BF : FC = 2 : 3.
- The perpendicular height AG is drawn intersecting DE at H.
- $AG = t$ units.
- $BC = (5 - t)$ units.



10.1 Write down AH : HG. (1)

10.2 Calculate t if the area of the parallelogram is a maximum.
(NOTE: Area of a parallelogram = base × $\perp$ height) (5)
[6]

June 2018

QUESTION 7

7.1 Given: $f(x) = 2 - 3x^2$
Determine $f'(x)$ from first principles. (5)

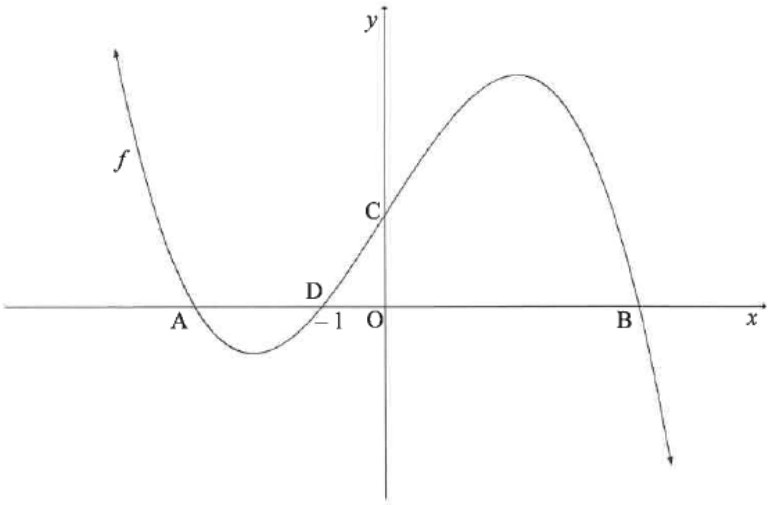
7.2 Determine:

7.2.1 $D_x\left[(4x+5)^2\right]$ (3)

7.2.2 $\frac{dy}{dx}$ if $y = \sqrt[4]{x} + \frac{x^2 - 8}{x^2}$ (4)
[12]

QUESTION 8

The graph of $f(x) = -x^3 + 13x + 12$ is sketched below.
A, B and D(–1 ; 0) are the x-intercepts of f.
C is the y-intercept of f.



8.1 Write down the coordinates of C. (1)

8.2 Calculate the coordinates of A and B. (5)

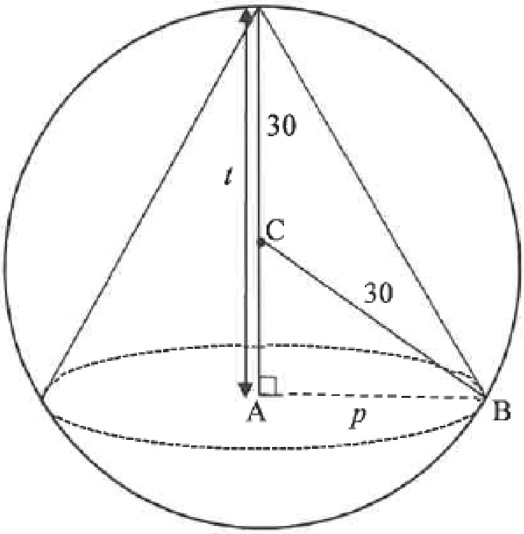
8.3 Determine the point of inflection of g if it is given that $g(x) = -f(x)$. (4)

8.4 Calculate the value(s) of x for which the tangent to f is parallel to the line $y = -14x + c$. (4)

[14]

QUESTION 9

A right circular cone with radius p and height t is machined (cut out) from a solid sphere (with centre C) with a radius of 30 cm, as shown in the sketch.



Sphere: $V = \frac{4}{3}\pi r^3$

Cone: $V = \frac{1}{3}\pi r^2 h$

9.1 From the given information, express the following:

9.1.1 AC in terms of t. (1)

9.1.2 p^2, in its simplest form, in terms of t. (3)

9.2 Show that the volume of the cone can be written as $V(t) = 20\pi t^2 - \frac{1}{3}\pi t^3$. (1)

9.3 Calculate the value of t for which the volume of the cone will be a maximum. (3)

9.4 What percentage of the sphere was used to obtain this cone having maximum volume? (4)

[12]

March 2018

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=4x^2$. (5)

8.2 Determine:

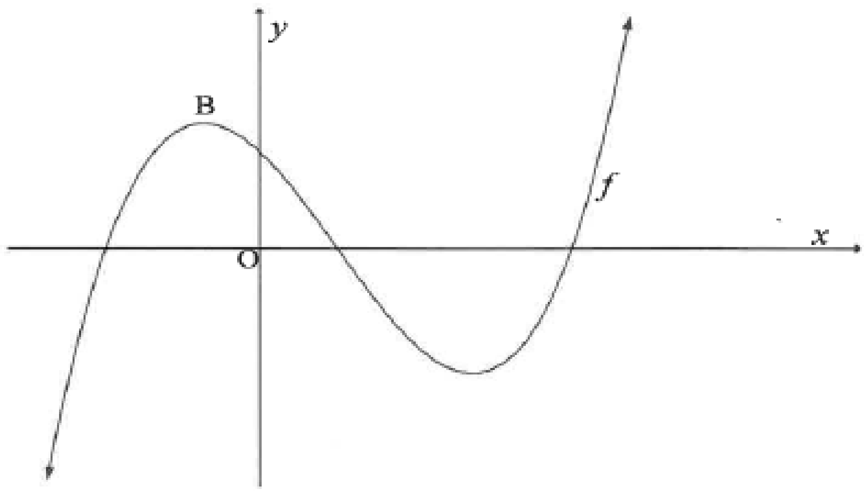
8.2.1 $D_x\left[\dfrac{x^2-2x-3}{x+1}\right]$ (3)

8.2.2 $f''(x)$ if $f(x)=\sqrt{x}$ (3)

[11]

QUESTION 9

The sketch below represents the curve of $f(x)=x^3+bx^2+cx+d$. The solutions of the equation $f(x)=0$ are -2 ; 1 and 4.



9.1 Calculate the values of b, c and d. (4)

9.2 Calculate the x-coordinate of B, the maximum turning point of f. (4)

9.3 Determine an equation for the tangent to the graph of f at $x=-1$. (4)

9.4 In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x- and y-intercepts on your sketch. (3)

9.5 For which value(s) of x is $f(x)$ concave upwards? (2)

[17]

QUESTION 10

Given: $f(x)=-3x^3+x$.

Calculate the value of q for which $f(x)+q$ will have a maximum value of $\frac{8}{9}$. **[6]**

November 2017:

QUESTION 7

7.1 Given: $f(x)=2x^2-x$

Determine $f'(x)$ from first principles. (6)

7.2 Determine:

7.2.1 $D_x[(x+1)(3x-7)]$ (2)

7.2.2 $\frac{dy}{dx}$ if $y=\sqrt{x^3}-\frac{5}{x}+\frac{1}{2}\pi$ (4)

[12]

QUESTION 8

Given: $f(x)=x(x-3)^2$ with $f'(1)=f'(3)=0$ and $f(1)=4$

8.1 Show that f has a point of inflection at $x=2$. (5)

8.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

8.3 For which values of x will $y=-f(x)$ be concave down? (2)

8.4 Use your graph to answer the following questions:

8.4.1 Determine the coordinates of the local maximum of h if $h(x)=f(x-2)+3$. (2)

8.4.2 Claire claims that $f'(2)=1$.

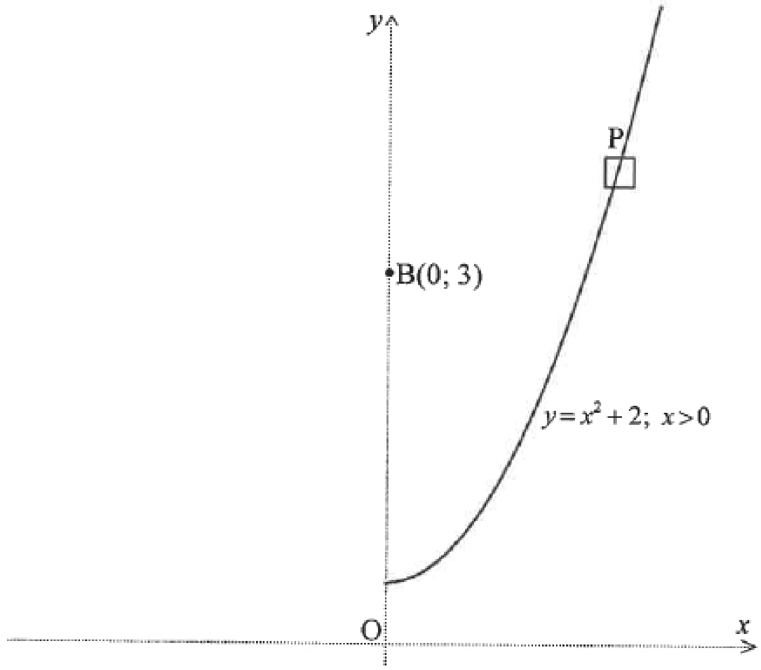
Do you agree with Claire? Justify your answer. (2)

[15]

QUESTION 9

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point B(0 ; 3) and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny. **[7]**

June 2017:

QUESTION 8

8.1 Given $f(x) = 3 - 2x^2$. Determine $f'(x)$, using first principles. (5)

8.2 Determine $\frac{dy}{dx}$ if $y = \frac{12x^2 + 2x + 1}{6x}$. (4)

8.3 The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at (2 ; 4). Calculate the values of b and c. (7)

[16]

QUESTION 9

Given: $f(x) = x^3 - x^2 - x + 1$

9.1 Write down the coordinates of the y-intercept of f. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (5)

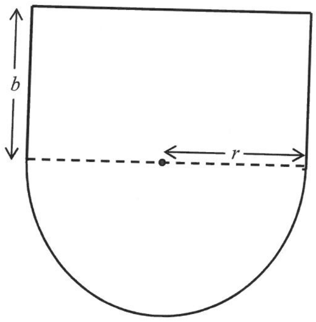
9.3 Calculate the coordinates of the turning points of f. (6)

9.4 Sketch the graph of f in your ANSWER BOOK. Clearly indicate all intercepts with the axes and the turning points. (3)

9.5 Write down the values of x for which $f'(x) < 0$. (2)

[17]

QUESTION 10



The figure above shows the design of a theatre stage which is in the shape of a semicircle attached to a rectangle. The semicircle has a radius r and the rectangle has a breadth b. The perimeter of the stage is 60 m.

10.1 Determine an expression for b in terms of r. (2)

10.2 For which value of r will the area of the stage be a maximum? (6)

[8]

March 2017:

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 - 5$. (5)

7.2 Determine the derivative of: $g(x) = 5x^2 - \frac{2x}{x^3}$ (3)

7.3 Given: $h(x) = ax^2$, $x > 0$.

Determine the value of a if it is given that $h^{-1}(8) = h'(4)$. (6)

[14]

QUESTION 8

Given: $f(x) = 2x^3 - 5x^2 + 4x$

8.1 Calculate the coordinates of the turning points of the graph of f. (5)

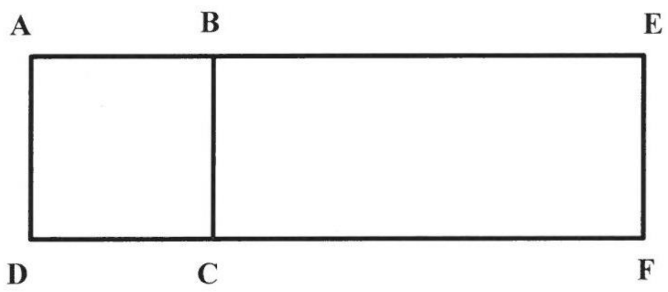
8.2 Prove that the equation $2x^3 - 5x^2 + 4x = 0$ has only one real root. (3)

8.3 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (3)

8.4 For which values of x will the graph of f be concave up? (3)
[14]

QUESTION 9

A piece of wire 6 metres long is cut into two pieces. One piece, x metres long, is bent to form a square ABCD. The other piece is bent into a U-shape so that it forms a rectangle BEFC when placed next to the square, as shown in the diagram below.



Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum. **[7]**

Probability

November 2019

QUESTION 10

The school library is open from Monday to Thursday. Anna and Ben both studied in the school library one day this week. If the chance of studying any day in the week is equally likely, calculate the probability that Anna and Ben studied on:

10.1 The same day (2)

10.2 Consecutive days (3)
[5]

QUESTION 11

11.1 Events **A** and **B** are independent. P(A) = 0,4 and P(B) = 0,25.

11.1.1 Represent the given information on a Venn diagram. Indicate on the Venn diagram the probabilities associated with each region. (3)

11.1.2 Determine P(**A** or NOT **B**). (2)

11.2 Motors Incorporated manufacture cars with 5 different body styles, 4 different interior colours and 6 different exterior colours, as indicated in the table below.

BODY STYLES	INTERIOR COLOURS	EXTERIOR COLOURS
Five body styles	Blue	Silver
		Blue
	Grey	White
	Black	Green
	Red	Red
		Gold

The interior colour of the car must NOT be the same as the exterior colour.

Motors Incorporated wants to display one of each possible variation of its car in their showroom. The showroom has a floor space of $500\ m^2$ and each car requires a floor space of $5\,m^2$.

Is this display possible? Justify your answer with the necessary calculations. (6)

[11]

May-June 2019

QUESTION 10

10.1 A bag contains 7 yellow balls, 3 red balls and 2 blue balls. A ball is chosen at random from the bag and not replaced. A second ball is then chosen. Determine the probability that of the two balls chosen, one is red and the other is blue. (4)

10.2 Learners at a hostel may choose a meal and a drink for lunch. Their selections on a certain day were recorded and shown in the partially completed table below.

		MEAL		TOTAL
		SANDWICH (S)	PASTA (P)	
DRINK	Fruit Juice (F)	a	30	b
	Bottled Water (W)			
TOTAL		200		250

The probability of a learner choosing fruit juice and a sandwich on that day was 0,48.

10.2.1 Calculate the number of learners who chose fruit juice and a sandwich for lunch on that day. (1)

10.2.2 Is the choice of fruit juice independent of the choice of a sandwich for lunch on that day? Show ALL calculations to motivate your answer. (4)

[9]

QUESTION 11

Two learners from each grade at a high school (Grades 8, 9, 10, 11 and 12) are elected to form a sports committee.

11.1 In how many different ways can the chairperson and the deputy chairperson of the sports committee be elected if there is no restriction on who may be elected? (2)

11.2 A photographer wants to take a photograph of the sports committee. In how many different ways can the members be arranged in a straight line if:

11.2.1 Any member may stand in any position? (1)

11.2.2 Members from the same grade must stand next to each other and the Grade 12 members must be in the centre? (3)

[6]

November 2018

QUESTION 11

Given the digits: 3 ; 4 ; 5 ; 6 ; 7 ; 8 and 9

11.1 Calculate how many unique 5-digit codes can be formed using the digits above, if:

11.1.1 The digits may be repeated (2)

11.1.2 The digits may not be repeated (2)

11.2 How many unique 3-digit codes can be formed using the above digits, if:

- Digits may be repeated
- The code is greater than 400 but less than 600
- The code is divisible by 5 (3)

[7]

QUESTION 12

12.1 Given: $P(A) = 0{,}45$; $P(B) = y$ and $P(A \text{ or } B) = 0{,}74$

Determine the value(s) of y if A and B are mutually exclusive. (3)

12.2 An organisation decided to distribute gift bags of sweets to a Grade R class at a certain school. There is a mystery gift in exactly $\frac{1}{4}$ of the total number of bags.

Each learner in the class may randomly select two gift bags of sweets, one after the other. The probability that a learner selects two bags of sweets with a mystery gift is $\frac{7}{118}$. Calculate the number of gift bags of sweets with a mystery gift inside. (6)

[9]

June 2018

QUESTION 10

Ben, Nhlanhla, Owen, Derick and 6 other athletes take part in a 100 m race. Each athlete will be allocated a lane in which to run. The athletic track has 10 lanes.

10.1 In how many different ways can all the athletes be allocated a lane? (2)

10.2 Four athletes taking part in the event insist on being placed in lanes next to each other. In how many different ways can the lanes be allocated to the athletes now? (3)

10.3 If lanes are randomly allocated to athletes, determine the probability that Ben will be placed in lane 1, Nhlanhla in lane 3, Owen in lane 5 and Derick in lane 7. (2)

[7]

QUESTION 11

A survey on their preference of exercise was conducted among 140 people in two age groups. The information is summarised below.

AGE	TENNIS	RUNNING	GYM	TOTAL
35 years and younger	a	28	c	80
Older than 35 years	b	21	d	60
	21	49	70	140

11.1 If it is given that preferring to play tennis and age are independent of each other, determine the value of a. (3)

11.2 If it is given that $a = 12$, determine the probability that a randomly selected person prefers going to the gym or is in the age group 35 years and younger. (5)
[8]

March 2018

QUESTION 11

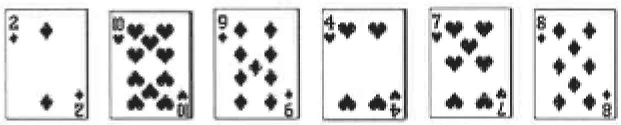
11.1 Veli and Bongi are learners at the same school. Some days they arrive late at school. The probability that neither Veli nor Bongi will arrive late on any day is 0,7.

11.1.1 Calculate the probability that at least one of the two learners will arrive late on a randomly selected day. (1)

11.1.2 The probability that Veli arrives late for school on a randomly selected day is 0,25, while the probability that both of them arrive late for school on that day is 0,15. Calculate the probability that Bongi will arrive late for school on that day. (3)

11.1.3 The principal suspects that the latecoming of the two learners is linked. The principal asks you to determine whether the events of Veli arriving late for school and Bongi arriving late for school are statistically independent or not. What will be your response to him? Show ALL calculations. (3)

11.2 The cards below are placed from left to right in a row.



11.2.1 In how many different ways can these 6 cards be randomly arranged in a row? (2)

11.2.2 In how many different ways can these cards be arranged in a row if the diamonds and hearts are placed in alternating positions? (3)

11.2.3 If these cards are randomly arranged in a row, calculate the probability that ALL the hearts will be next to one another. (3)
[15]

November 2017:

QUESTION 10

A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 use all three.
- 12 use Instagram and Twitter.
- 5 use Twitter and WhatsApp, but not Instagram.
- x use Instagram and WhatsApp, but not Twitter.
- 61 use Instagram.
- 19 use Twitter.
- 73 use WhatsApp.
- 14 use none of these applications.

10.1 Draw a Venn diagram to illustrate the information above. (4)

10.2 Calculate the value of x. (2)

10.3 Calculate the probability that a learner, chosen randomly, uses only ONE of these applications. (2)

[8]

QUESTION 11

A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR 45789.

The letters are chosen from A; D; R; S and U. Letters may be repeated in the code.

The digits 0 to 9 are used, but NO digit may be repeated in the code.

11.1 How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits? (3)

11.2 Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system. (3)

[6]

June 2017:

QUESTION 11

11.1 The letters of the word EQUATION are randomly used to form a new word consisting of five letters. How many of these words are possible if letters may not be repeated? (2)

11.2 It is given that two events, A and B, are independent. $P(A)=\frac{2}{5}$ and $P(B)=0{,}35$. Calculate P(A or B). (4)

11.3 Grade 12 learners in a certain town may choose to attend any one of three high schools. The table below shows the number of Grade 12 learners (as a percentage) attending the different schools in 2016 and the matric pass rate in that school (as a percentage) in 2016.

SCHOOLS	NUMBER OF LEARNERS ATTENDING (%)	MATRIC PASS RATE (%)
A	20	35
B	30	65
C	50	90

If a learner from this town, who was in Grade 12 in 2016, is selected at random, determine the probability that the learner:

11.3.1 Did not attend School A (2)

11.3.2 Attended School B and failed Grade 12 in 2016 (3)

11.3.3 Passed Grade 12 in 2016 (4)

[15]

March 2017:

QUESTION 10

10.1 The events S and T are independent.

- $P(S \text{ and } T)=\frac{1}{6}$
- $P(S)=\frac{1}{4}$

10.1.1 Calculate $P(T)$. (2)

10.1.2 Hence, calculate $P(S \text{ or } T)$. (2)

10.2 A FIVE-digit code is created from the digits 2 ; 3 ; 5 ; 7 ; 9.

How many different codes can be created if:

10.2.1 Repetition of digits is NOT allowed in the code (2)

10.2.2 Repetition of digits IS allowed in the code (1)

10.3 A group of 3 South Africans, 2 Australians and 2 Englishmen are staying at the same hotel while on holiday. Each person has his/her own room and the rooms are next to each other in a straight corridor.

If the rooms are allocated at random, determine the probability that the 2 Australians will have adjacent rooms and the 2 Englishmen will also have adjacent rooms. (4)

[11]

QUESTION 11

The success rate of the Fana soccer team depends on a number of factors. The fitness of the players is one of the factors that influence the outcome of a match.

- The probability that all the players are fit for the next match is 70%
- If all the players are fit to play the next match, the probability of winning the next match is 85%
- If there are players that are not fit to play the next match, the probability of winning the match is 55%

Based on fitness alone, calculate the probability that the Fana soccer team will win the next match. **[5]**

SESSION 4

Trigonometry Statistical Reasoning

<u>TRIGONOMETRY:</u>

<u>November 2019</u>

QUESTION 5

5.1 Simplify the following expression to ONE trigonometric term:

$$\frac{\sin x}{\cos x.\tan x}+\sin(180°+x)\cos(90°-x)$$ (5)

5.2 **Without using a calculator**, determine the value of: $\frac{\sin^2 35°-\cos^2 35°}{4\sin 10°\cos 10°}$ (4)

5.3 Given: $\cos 26° = m$

Without using a calculator, determine $2\sin^2 77°$ in terms of m. (4)

5.4 Consider: $f(x)=\sin(x+25°)\cos 15°-\cos(x+25°)\sin 15°$

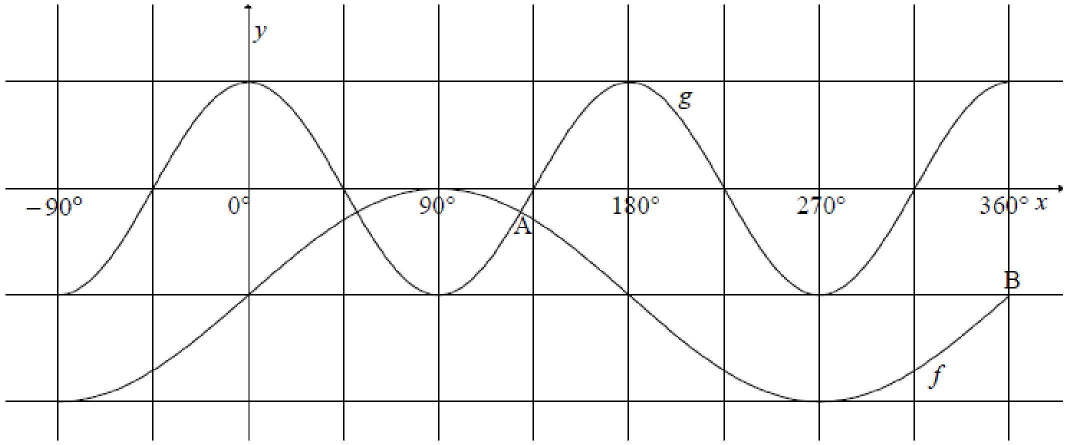
5.4.1 Determine the general solution of $f(x)=\tan 165°$ (6)

5.4.2 Determine the value(s) of x in the interval $x\in[0°;360°]$ for which $f(x)$ will have a minimum value. (3)

[22]

QUESTION 6

In the diagram, the graphs of $f(x) = \sin x - 1$ and $g(x) = \cos 2x$ are drawn for the interval $x \in [-90°; 360°]$. Graphs f and g intersect at A. B(360° ; −1) is a point on f.



6.1 Write down the range of f. (2)

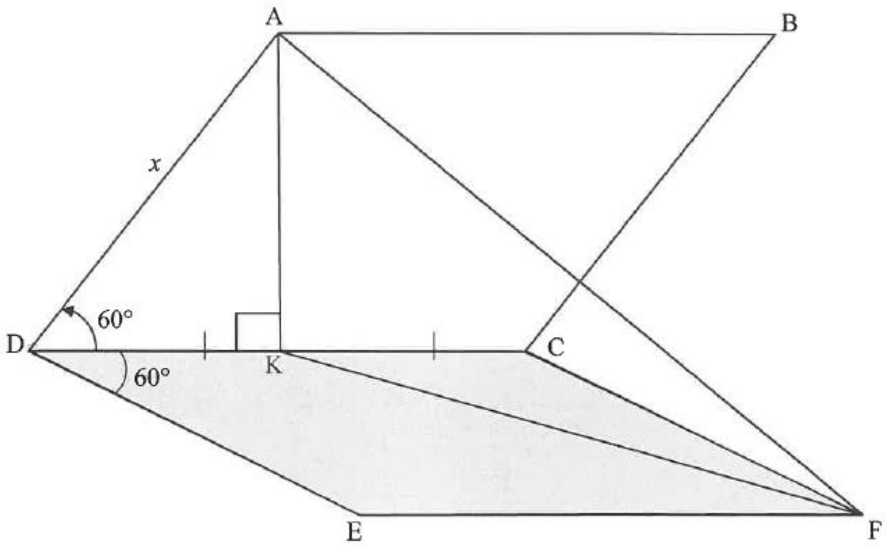
6.2 Write down the values of x in the interval $x \in [-90°; 360°]$ for which graph f is decreasing. (2)

6.3 P and Q are points on graphs g and f respectively such that PQ is parallel to the y-axis. If PQ lies between A and B, determine the value(s) of x for which PQ will be a maximum. (6)

[10]

QUESTION 7

The diagram below shows a solar panel, ABCD, which is fixed to a flat piece of concrete slab EFCD. ABCD and EFCD are two identical rhombuses. K is a point on DC such that DK = KC and AK ⊥ DC. AF and KF are drawn. $A\hat{D}C = C\hat{D}E = 60°$ and AD = x units.



7.1	Determine AK in terms of x.	(2)
7.2	Write down the size of $K\hat{C}F$.	(1)
7.3	It is further given that $A\hat{K}F$, the angle between the solar panel and the concrete slab, is y. Determine the area of ΔAKF in terms of x and y.	(7)
		[10]

May-June 2019

QUESTION 5

5.1 **Without using a calculator,** write the following expressions in terms of $\sin 11°$:

5.1.1	$\sin 191°$	(1)
5.1.2	$\cos 22°$	(1)

5.2 Simplify $\cos(x-180°) + \sqrt{2}\sin(x+45°)$ to a single trigonometric ratio. (5)

5.3 Given: $\sin P + \sin Q = \frac{7}{5}$ and $\hat{P} + \hat{Q} = 90°$

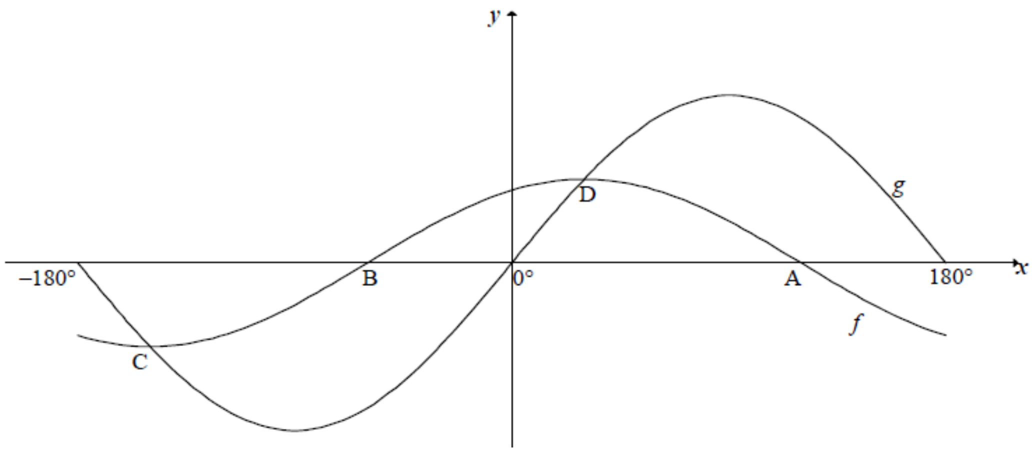
Without using a calculator, determine the value of $\sin 2P$. (5)

[12]

QUESTION 6

6.1 Determine the general solution of $\cos(x-30°) = 2\sin x$. (6)

6.2 In the diagram, the graphs of $f(x) = \cos(x-30°)$ and $g(x) = 2\sin x$ are drawn for the interval $x \in [-180°;180°]$. A and B are the x–intercepts of f. The two graphs intersect at C and D, the minimum and maximum turning points respectively of f.



6.2.1 Write down the coordinates of:

(a) A (1)

(b) C (2)

6.2.2 Determine the values of x in the interval $x \in [-180°; 180°]$, for which:

(a) Both graphs are increasing (2)

(b) $f(x+10°) > g(x+10°)$ (2)

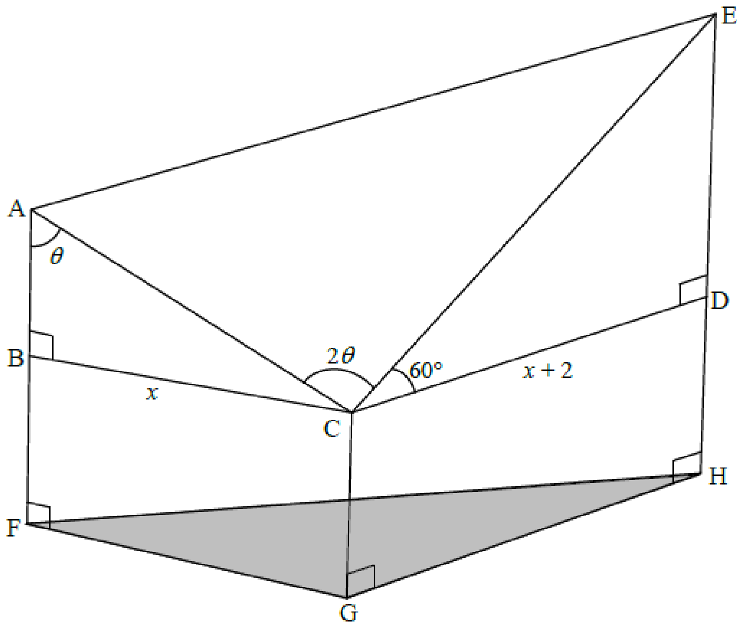
6.2.3 Determine the range of $y = 2^{2\sin x + 3}$ (5)

[18]

QUESTION 7

In the diagram below, CGFB and CGHD are fixed walls that are rectangular in shape and vertical to the horizontal plane FGH. Steel poles erected along FB and HD extend to A and E respectively. ΔACE forms the roof of an entertainment centre.

$BC = x$, $CD = x + 2$, $B\hat{A}C = \theta$, $A\hat{C}E = 2\theta$ and $E\hat{C}D = 60°$



7.1 Calculate the length of:

7.1.1 AC in terms of x and θ (2)

7.1.2 CE in terms of x (2)

7.2 Show that the area of the roof ΔACE is given by $2x(x+2)\cos\theta$. (3)

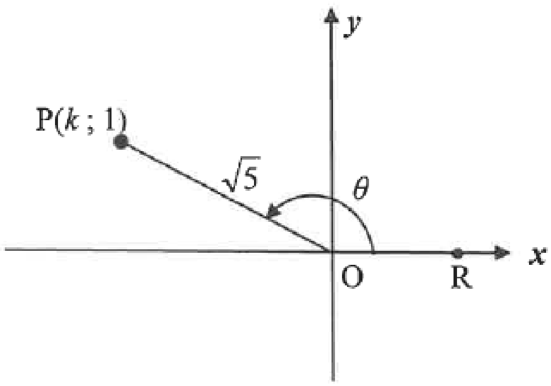
7.3 If $\theta = 55°$ and BC = 12 metres, calculate the length of AE. (4)

[11]

November 2018

QUESTION 5

5.1 In the diagram, $P(k\ ;\ 1)$ is a point in the 2^{nd} quadrant and is $\sqrt{5}$ units from the origin. R is a point on the positive x-axis and obtuse $R\hat{O}P = \theta$.



5.1.1 Calculate the value of k. (2)

5.1.2 **Without using a calculator**, calculate the value of:

(a) $\tan\theta$ (1)

(b) $\cos(180° + \theta)$ (2)

(c) $\sin(\theta + 60°)$ in the form $\dfrac{a+b}{\sqrt{20}}$ (5)

5.1.3 **Use a calculator** to calculate the value of $\tan(2\theta - 40°)$ correct to ONE decimal place. (3)

5.2 Prove the following identity: $\dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$ (5)

5.3 Evaluate, **without using a calculator**: $\sum_{A=38°}^{52°} \cos^2 A$ (5)

[23]

QUESTION 6

Consider: $f(x) = -2\tan\frac{3}{2}x$

6.1 Write down the period of f. (1)

6.2 The point A(t ; 2) lies on the graph. Determine the general solution of t. (3)

6.3 On the grid provided in the ANSWER BOOK, draw the graph of f for the interval $x \in [-120°\ ;\ 180°]$. Clearly show ALL asymptotes, intercepts with the axes and endpoint(s) of the graph. (4)

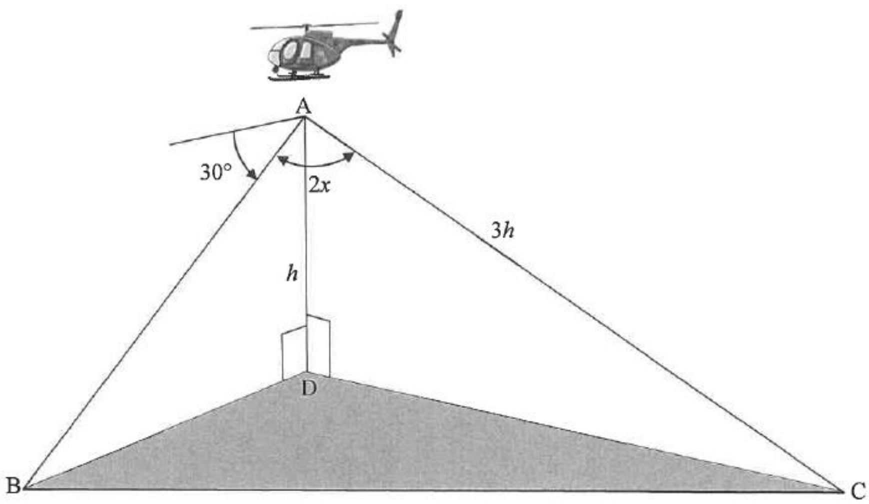
6.4 Use the graph to determine for which value(s) of x will $f(x) \geq 2$ for $x \in [-120°\ ;\ 180°]$. (3)

6.5 Describe the transformation of graph f to form the graph of $g(x) = -2\tan\left(\frac{3}{2}x + 60°\right)$. (2)

[13]

QUESTION 7

A pilot is flying in a helicopter. At point A, which is h metres directly above point D on the ground, he notices a strange object at point B. The pilot determines that the angle of depression from A to B is 30°. He also determines that the control room at point C is $3h$ metres from A and $B\hat{A}C = 2x$. Points B, C and D are in the same horizontal plane. This scenario is shown in the diagram below.



7.1 Determine the distance AB in terms of h. (2)

7.2 Show that the distance between the strange object at point B and the control room at point C is given by $BC = h\sqrt{25 - 24\cos^2 x}$. (4)

[6]

June 2018

QUESTION 5

5.1 In ΔMNP, $\hat{N} = 90°$ and $\sin M = \frac{15}{17}$.

Determine, **without using a calculator**:

5.1.1 $\tan M$ (3)

5.1.2 The length of NP if $MP = 51$ (2)

5.2 Simplify to a single term: $\cos(x-360°).\sin(90°+x)+\cos^2(-x)-1$ (4)

5.3 Consider: $\sin(2x+40°)\cos(x+30°)-\cos(2x+40°)\sin(x+30°)$

5.3.1 Write as a single trigonometric term in its simplest form. (2)

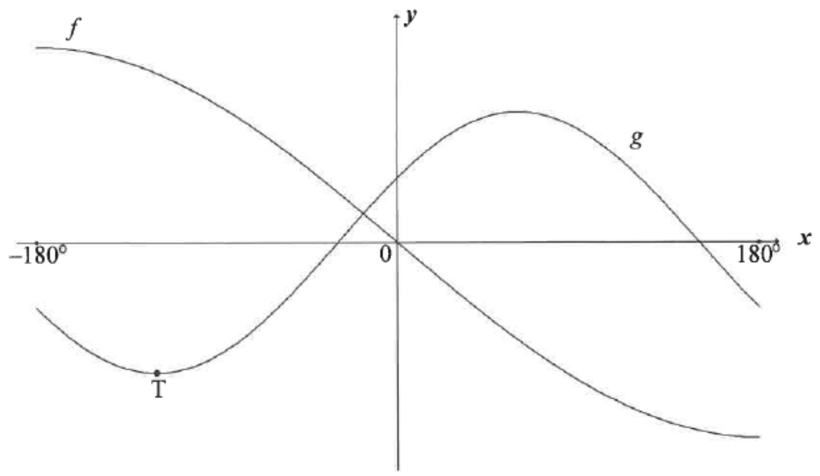
5.3.2 Determine the general solution of the following equation:

$$\sin(2x+40°)\cos(x+30°)-\cos(2x+40°)\sin(x+30°)=\cos(2x-20°)$$ (7)

[18]

QUESTION 6

In the diagram, the graphs of $f(x) = -3\sin\frac{x}{2}$ and $g(x) = 2\cos(x-60°)$ are drawn in the interval $x \in [-180°;180°]$. $T(p\,;q)$ is a turning point of g with $p < 0$.



6.1 Write down the period of f. (1)

6.2 Write down the range of g. (2)

6.3 Calculate $f(p) - g(p)$. (3)

6.4 Use the graphs to determine the value(s) of x in the interval $x \in [-180°; 180°]$ for which:

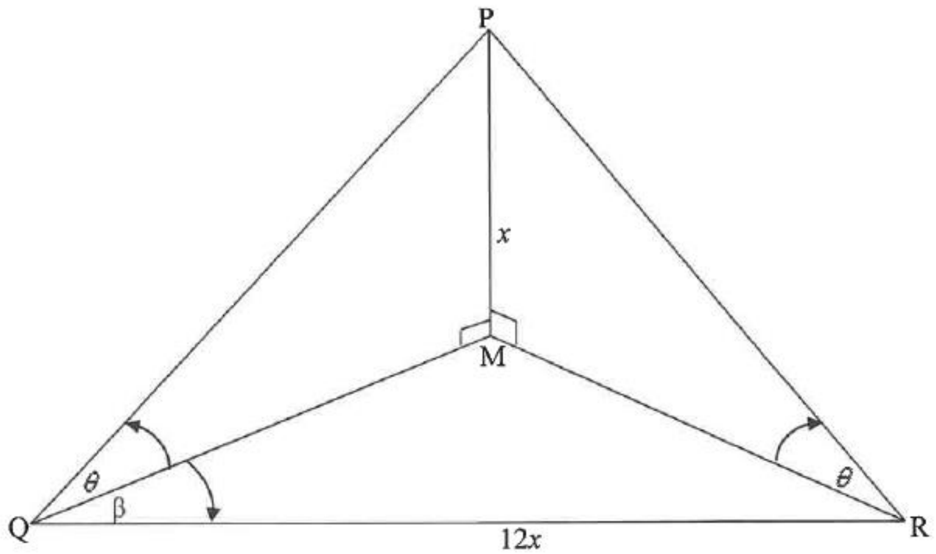
6.4.1 $g(x) > 0$ (3)

6.4.2 $g(x).g'(x) > 0$ (4)

[13]

QUESTION 7

The captain of a boat at sea, at point Q, notices a lighthouse PM directly north of his position. He determines that the angle of elevation of P, the top of the lighthouse, from Q is θ and the height of the lighthouse is x metres. From point Q the captain sails $12x$ metres in a direction β degrees east of north to point R. From point R, he notices that the angle of elevation of P is also θ. Q, M and R lie in the same horizontal plane.



7.1 Write QM in terms of x and θ. (2)

7.2 Prove that $\tan\theta = \dfrac{\cos\beta}{6}$. (4)

7.3 If $\beta = 40°$ and QM = 60 metres, calculate the height of the lighthouse **to the nearest metre.** (3)

[9]

March 2018

QUESTION 5

5.1 If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°;270°]$, calculate, **without using a calculator**, the values in simplest form of:

5.1.1 $\sin 2\theta$ (4)

5.1.2 $\sin^2\theta$ (3)

5.2 Simplify $\sin(180° - x).\cos(-x) + \cos(90° + x).\cos(x - 180°)$ to a single trigonometric ratio. (6)

5.3 Determine the value of $\sin 3x.\cos y + \cos 3x.\sin y$ if $3x + y = 270°$. (2)

5.4 Given: $2\cos x = 3\tan x$

5.4.1 Show that the equation can be rewritten as $2\sin^2 x + 3\sin x - 2 = 0$. (3)

5.4.2 Determine the general solution of x if $2\cos x = 3\tan x$. (5)

5.4.3 Hence, determine two values of y, $144° \leq y \leq 216°$, that are solutions of $2\cos 5y = 3\tan 5y$. (4)

5.5 Consider: $g(x) = -4\cos(x + 30°)$

5.5.1 Write down the maximum value of $g(x)$. (1)

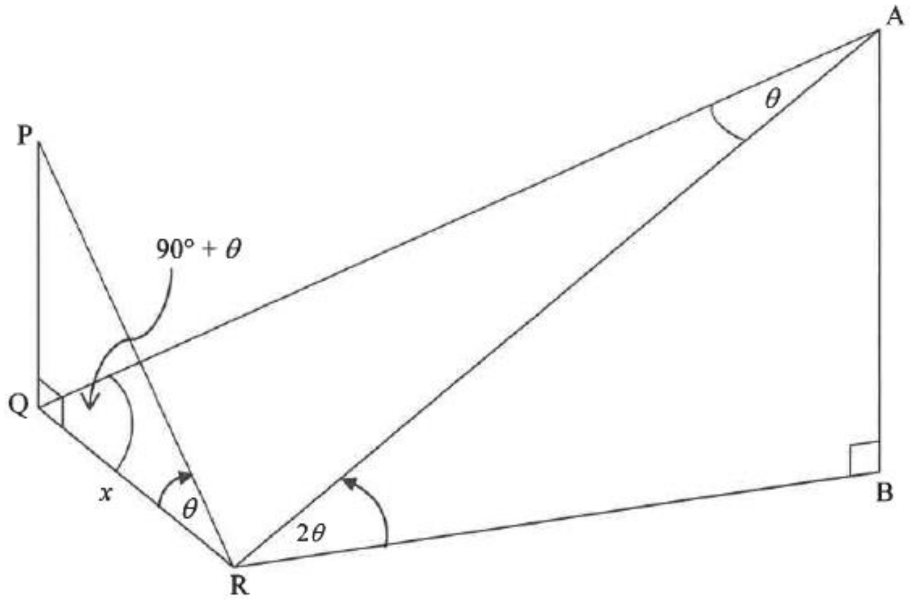
5.5.2 Determine the range of $g(x) + 1$. (2)

5.5.3 The graph of g is shifted $60°$ to the left and then reflected about the x-axis to form a new graph h. Determine the equation of h in its simplest form. (3)

[33]

QUESTION 6

PQ and AB are two vertical towers.
From a point R in the same horizontal plane as Q and B, the angles of elevation to P and A are θ and 2θ respectively.
$A\hat{Q}R = 90° + \theta$, $Q\hat{A}R = \theta$ and $QR = x$.



6.1 Determine in terms of x and θ:

6.1.1 QP (2)

6.1.2 AR (2)

6.2 Show that $AB = 2x\cos^2\theta$ (4)

6.3 Determine $\frac{AB}{QP}$ if $\theta = 12°$. (2)

[10]

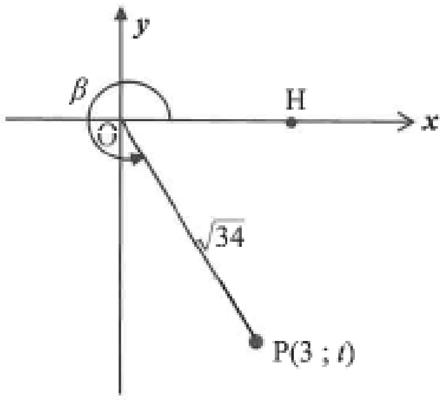
<u>November 2017:</u>

QUESTION 5

5.1 Given: $\dfrac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$

Simplify the expression to a single trigonometric ratio. (6)

5.2 In the diagram, P(3 ; t) is a point in the Cartesian plane. $OP=\sqrt{34}$ and $H\hat{O}P=\beta$ is a reflex angle.



Without using a calculator, determine the value of:

5.2.1 t (2)

5.2.2 $\tan\beta$ (1)

5.2.3 $\cos 2\beta$ (4)

5.3 Prove:

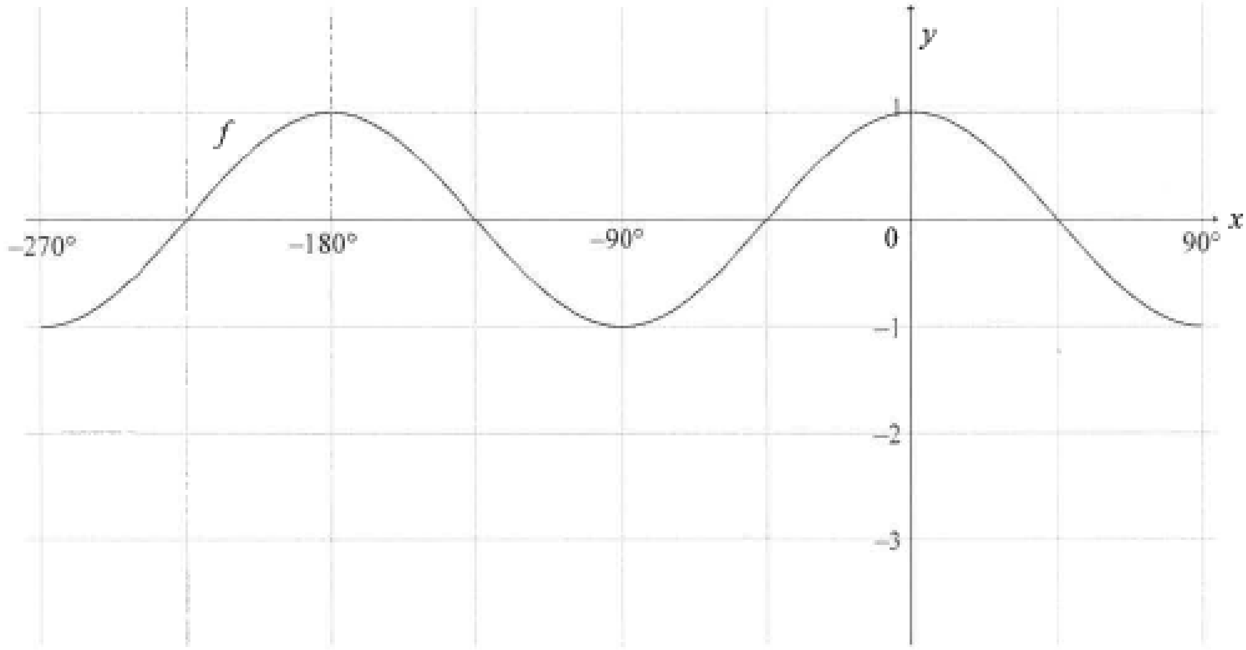
5.3.1 $\sin(A+B)-\sin(A-B)=2\cos A.\sin B$ (2)

5.3.2 **Without using a calculator,** that $\sin 77°-\sin 43°=\sin 17°$ (4)

[19]

QUESTION 6

In the diagram, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [-270°; 90°]$.



6.1 Draw the graph of $g(x) = 2\sin x - 1$ for the interval $x \in [-270°; 90°]$ on the grid given in your ANSWER BOOK. Show ALL the intercepts with the axes, as well as the turning points. (4)

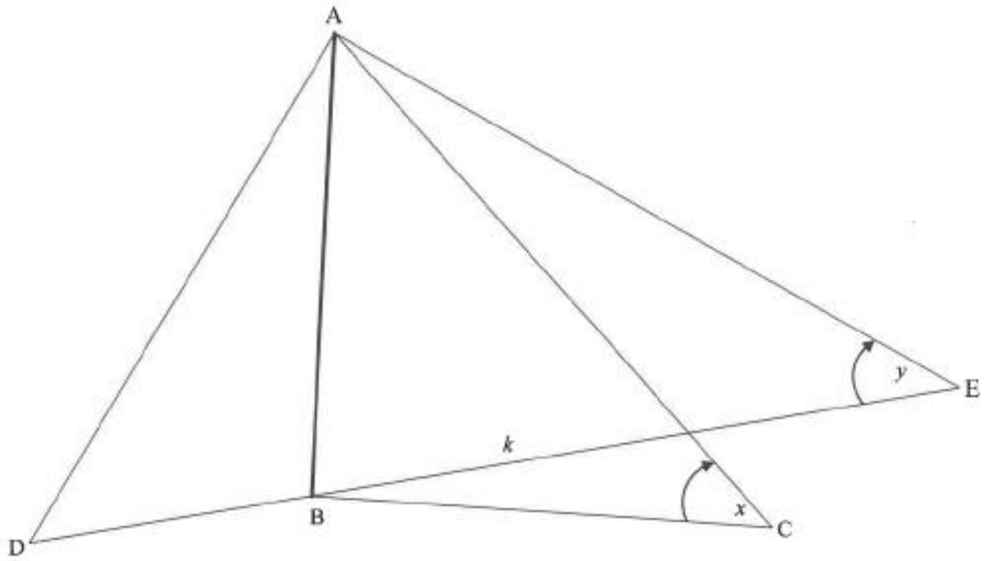
6.2 Let A be a point of intersection of the graphs of f and g. Show that the x-coordinate of A satisfies the equation $\sin x = \frac{-1+\sqrt{5}}{2}$. (4)

6.3 Hence, calculate the coordinates of the points of intersection of graphs of f and g for the interval $x \in [-270°; 90°]$. (4)

[12]

QUESTION 7

AB represents a vertical netball pole. Two players are positioned on either side of the netball pole at points D and E such that D, B and E are on the same straight line. A third player is positioned at C. The points B, C, D and E are in the same horizontal plane. The angles of elevation from C to A and from E to A are x and y respectively. The distance from B to E is k.



7.1 Write down the size of $A\hat{B}C$. (1)

7.2 Show that $AC = \dfrac{k.\tan y}{\sin x}$ (4)

7.3 If it is further given that $D\hat{A}C = 2x$ and AD = AC, show that the distance DC between the players at D and C is $2k\tan y$. (5)
[10]

June 2017:

QUESTION 5

5.1 Given: $\sin A = 2p$ and $\cos A = p$

5.1.1 Determine the value of $\tan A$. (2)

5.1.2 **Without using a calculator,** determine the value of p, if $A \in [180° ; 270°]$. (3)

5.2 Determine the general solution of $2\sin^2 x - 5\sin x + 2 = 0$ (6)

5.3 5.3.1 Expand $\sin(x + 300°)$ using an appropriate compound angle formula. (1)

5.3.2 **Without using a calculator,** determine the value of $\sin(x + 300°) - \cos(x - 150°)$. (5)

QUESTION 5

5.1 Given: $\sin A = 2p$ and $\cos A = p$

5.1.1 Determine the value of $\tan A$. (2)

5.1.2 **Without using a calculator,** determine the value of p, if $A \in [180° ; 270°]$. (3)

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5.3 5.3.1 Expand $\sin(x + 300°)$ using an appropriate compound angle formula. (1)

5.3.2 **Without using a calculator,** determine the value of $\sin(x + 300°) - \cos(x - 150°)$. (5)

5.4 Prove the identity: $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} = \sin x + \cos x.$ (5)

5.5 Consider: $\sin x + \cos x = \sqrt{1+k}$

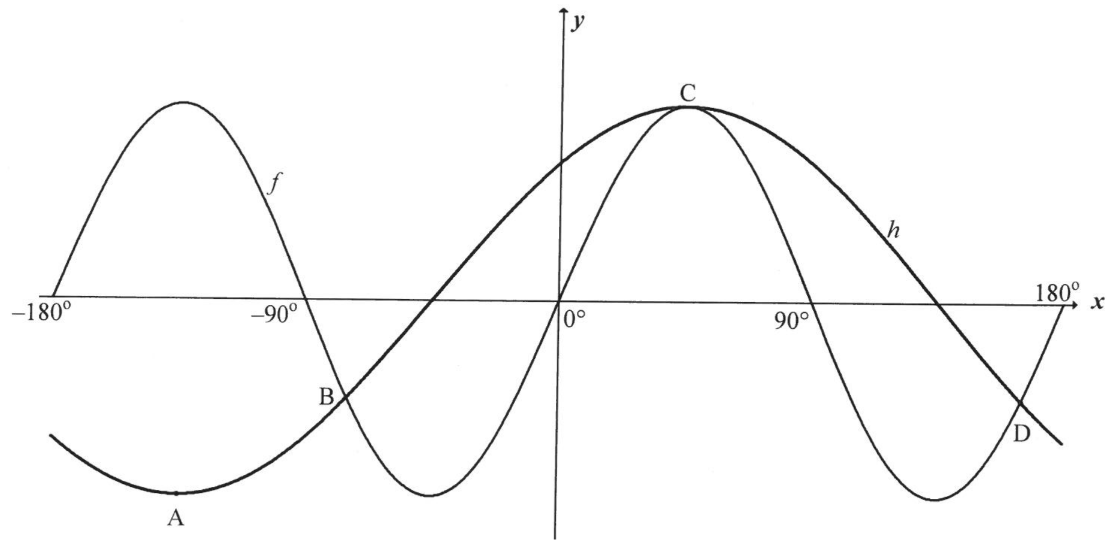
5.5.1 Determine k as a single trigonometric ratio. (3)

5.5.2 Hence, determine the maximum value of $\sin x + \cos x$. (2)

[27]

QUESTION 6

In the diagram are the graphs of $f(x) = \sin 2x$ and $h(x) = \cos(x - 45°)$ for the interval $x \in [-180° ; 180°]$. $A(-135° ; -1)$ is a minimum point on graph h and $C(45° ; 1)$ is a maximum point on both graphs. The two graphs intersect at B, C and $D\left(165°; -\frac{1}{2}\right)$.



6.1 Write down the period of f. (1)

6.2 Determine the x-coordinate of B. (1)

6.3 Use the graphs to solve $2\sin x.\cos x \leq \frac{1}{\sqrt{2}}(\cos x + \sin x)$ for the interval $x \in [-180°\ ;\ 180°]$. Show ALL working. (4)

[6]

QUESTION 7

A rectangular box with lid ABCD is given in FIGURE (i) below. The lid is opened through $60°$ to position HKCD, as shown in the FIGURE (ii) below. $EF = 12$ cm, $FG = 6$ cm and $BG = 8$ cm.

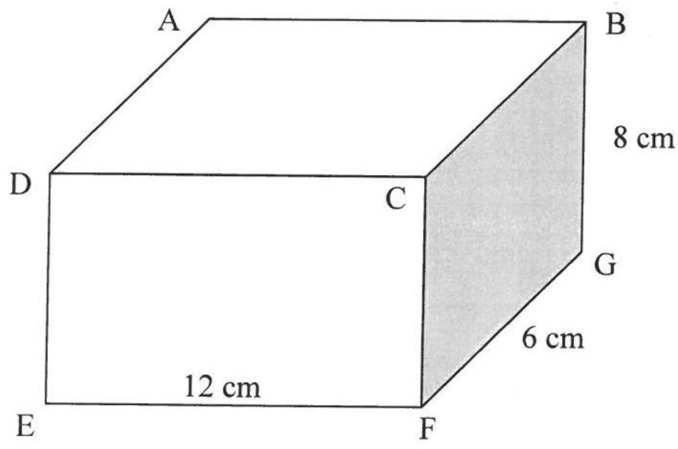


FIGURE (I)

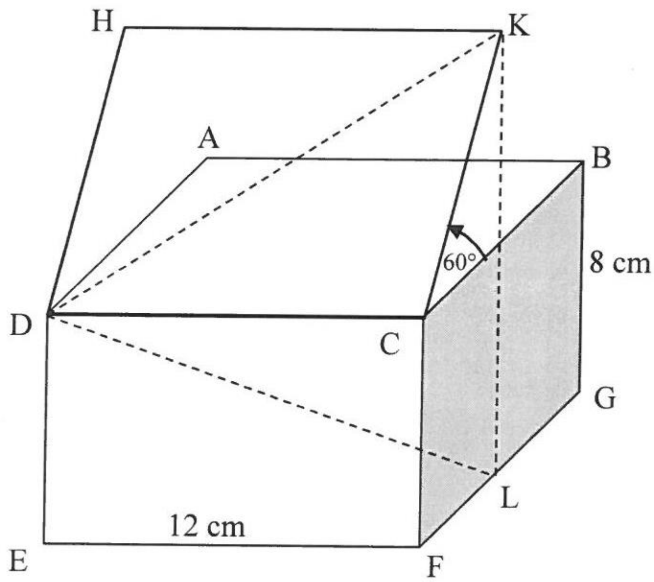


FIGURE (II)

7.1 Write down the length of KC. (1)

7.2 Determine KL, the perpendicular height of K, above the base of the box. (3)

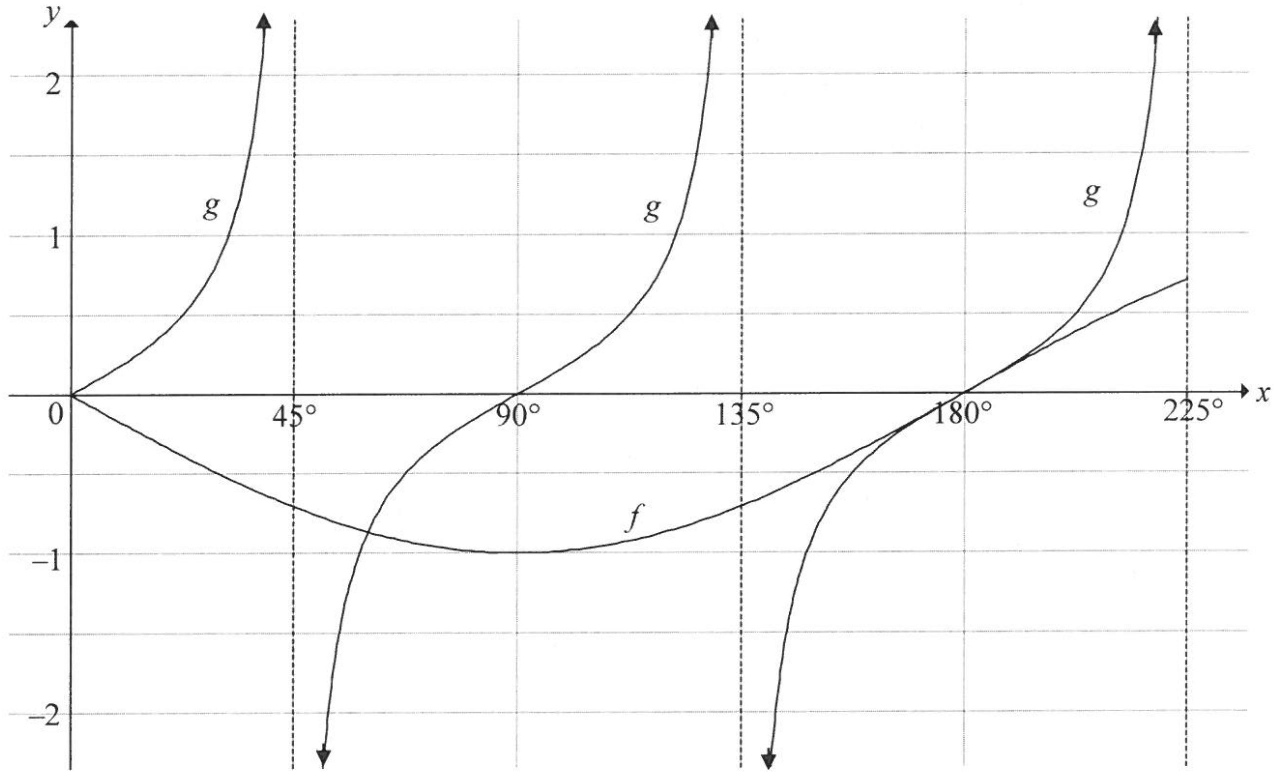
7.3 Hence, determine the value of $\frac{\sin K\hat{D}L}{\sin D\hat{L}K}$. (4)

[8]

March 2017:

QUESTION 5

In the diagram, the graphs of the functions $f(x) = a\sin x$ and $g(x) = \tan bx$ are drawn on the same system of axes for the interval $0° \leq x \leq 225°$.



5.1 Write down the values of a and b. (2)

5.2 Write down the period of $f(3x)$. (2)

5.3 Determine the values of x in the interval $90° \leq x \leq 225°$ for which $f(x).g(x) \leq 0$. (3)

[7]

QUESTION 6

6.1 **Without using a calculator**, determine the following in terms of $\sin 36°$:

6.1.1 $\sin 324°$ (1)

6.1.2 $\cos 72°$ (2)

6.2 Prove the identity: $1-\dfrac{\tan^2\theta}{1+\tan^2\theta}=\cos^2\theta$ (4)

6.3 Use QUESTION 6.2 to determine the general solution of:

$$1-\frac{\tan^2\frac{1}{2}x}{1+\tan^2\frac{1}{2}x}=\frac{1}{4}$$

(6)

6.4 Given: $\cos(A-B)=\cos A\cos B+\sin A\sin B$

6.4.1 Use the formula for $\cos(A-B)$ to derive a formula for $\sin(A-B)$. (4)

6.4.2 **Without using a calculator**, show that

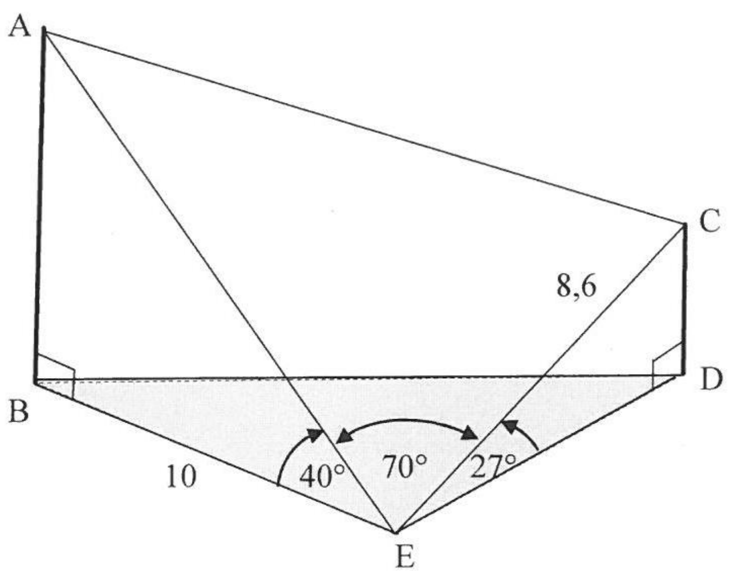
$$\sin(x+64°)\cos(x+379°)+\sin(x+19°)\cos(x+244°)=\frac{1}{\sqrt{2}}$$

for all values of x. (6)

[23]

QUESTION 7

In the diagram, B, E and D are points in the same horizontal plane. AB and CD are vertical poles. Steel cables AE and CE anchor the poles at E. Another steel cable connects A and C. CE = 8,6 m, BE = 10 m, $A\hat{E}B=40°$, $A\hat{E}C=70°$ and $C\hat{E}D=27°$.



Calculate the:

7.1 Height of pole CD (2)

7.2 Length of cable AE (2)

7.3 Length of cable AC (4)
[8]

STATISTICAL REASONING:

November 2019

QUESTION 1

The table below shows the monthly income (in rands) of 6 different people and the amount (in rands) that each person spends on the monthly repayment of a motor vehicle.

MONTHLY INCOME (IN RANDS)	9 000	13 500	15 000	16 500	17 000	20 000
MONTHLY REPAYMENT (IN RANDS)	2 000	3 000	3 500	5 200	5 500	6 000

1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 If a person earns R14 000 per month, predict the monthly repayment that the person could make towards a motor vehicle. (2)

1.3 Determine the correlation coefficient between the monthly income and the monthly repayment of a motor vehicle. (1)

1.4 A person who earns R18 000 per month has to decide whether to spend R9 000 as a monthly repayment of a motor vehicle, or not. If the above information is a true representation of the population data, which of the following would the person most likely decide on:

A Spend R9 000 per month because there is a very strong positive correlation between the amount earned and the monthly repayment.

B NOT to spend R9 000 per month because there is a very weak positive correlation between the amount earned and the monthly repayment.

C Spend R9 000 per month because the point (18 000 ; 9 000) lies very near to the least squares regression line.

D NOT to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. (2)
[8]

QUESTION 2

A survey was conducted among 100 people about the amount that they paid on a monthly basis for their cellphone contracts. The person carrying out the survey calculated the estimated mean to be R309 per month. Unfortunately, he lost some of the data thereafter. The partial results of the survey are shown in the frequency table below:

AMOUNT PAID (IN RANDS)	FREQUENCY
$0 < x \leq 100$	7
$100 < x \leq 200$	12
$200 < x \leq 300$	a
$300 < x \leq 400$	35
$400 < x \leq 500$	b
$500 < x \leq 600$	6

2.1 How many people paid R200 or less on their monthly cellphone contracts? (1)

2.2 Use the information above to show that $a = 24$ and $b = 16$. (5)

2.3 Write down the modal class for the data. (1)

2.4 On the grid provided in the ANSWER BOOK, draw an ogive (cumulative frequency graph) to represent the data. (4)

2.5 Determine how many people paid more than R420 per month for their cellphone contracts. (2)

[13]

May-June 2019

QUESTION 1

Each child in a group of four-year-old children was given the same puzzle to complete. The time taken (in minutes) by each child to complete the puzzle is shown in the table below.

TIME TAKEN (t) (IN MINUTES)	NUMBER OF CHILDREN
$2 < t \leq 6$	2
$6 < t \leq 10$	10
$10 < t \leq 14$	9
$14 < t \leq 18$	7
$18 < t \leq 22$	8
$22 < t \leq 26$	7
$26 < t \leq 30$	2

1.1 How many children completed the puzzle? (1)

1.2 Calculate the estimated mean time taken to complete the puzzle. (2)

1.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

1.4 Draw a cumulative frequency graph (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)

1.5 Use the graph to determine the median time taken to complete the puzzle. (2)
[10]

QUESTION 2

Learners who scored a mark below 50% in a Mathematics test were selected to use a computer-based programme as part of an intervention strategy. On completing the programme, these learners wrote a second test to determine the effectiveness of the intervention strategy. The mark (as a percentage) scored by 15 of these learners in both tests is given in the table below.

LEARNER	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15
TEST 1 (%)	10	18	23	24	27	34	34	36	37	39	40	44	45	48	49
TEST 2 (%)	33	21	32	20	58	43	49	48	41	55	50	45	62	68	60

2.1 Determine the equation of the least squares regression line. (3)

2.2 A learner's mark in the first test was 15 out of a maximum of 50 marks.

2.2.1 Write down the learner's mark for this test as a percentage. (1)

2.2.2 Predict the learner's mark for the second test. Give your answer to the nearest integer. (2)

2.3 For the 15 learners above, the mean mark of the second test is 45,67% and the standard deviation is 13,88%. The teacher discovered that he forgot to add the marks of the last question to the total mark of each of these learners. All the learners scored full marks in the last question. When the marks of the last question are added, the new mean mark is 50,67%.

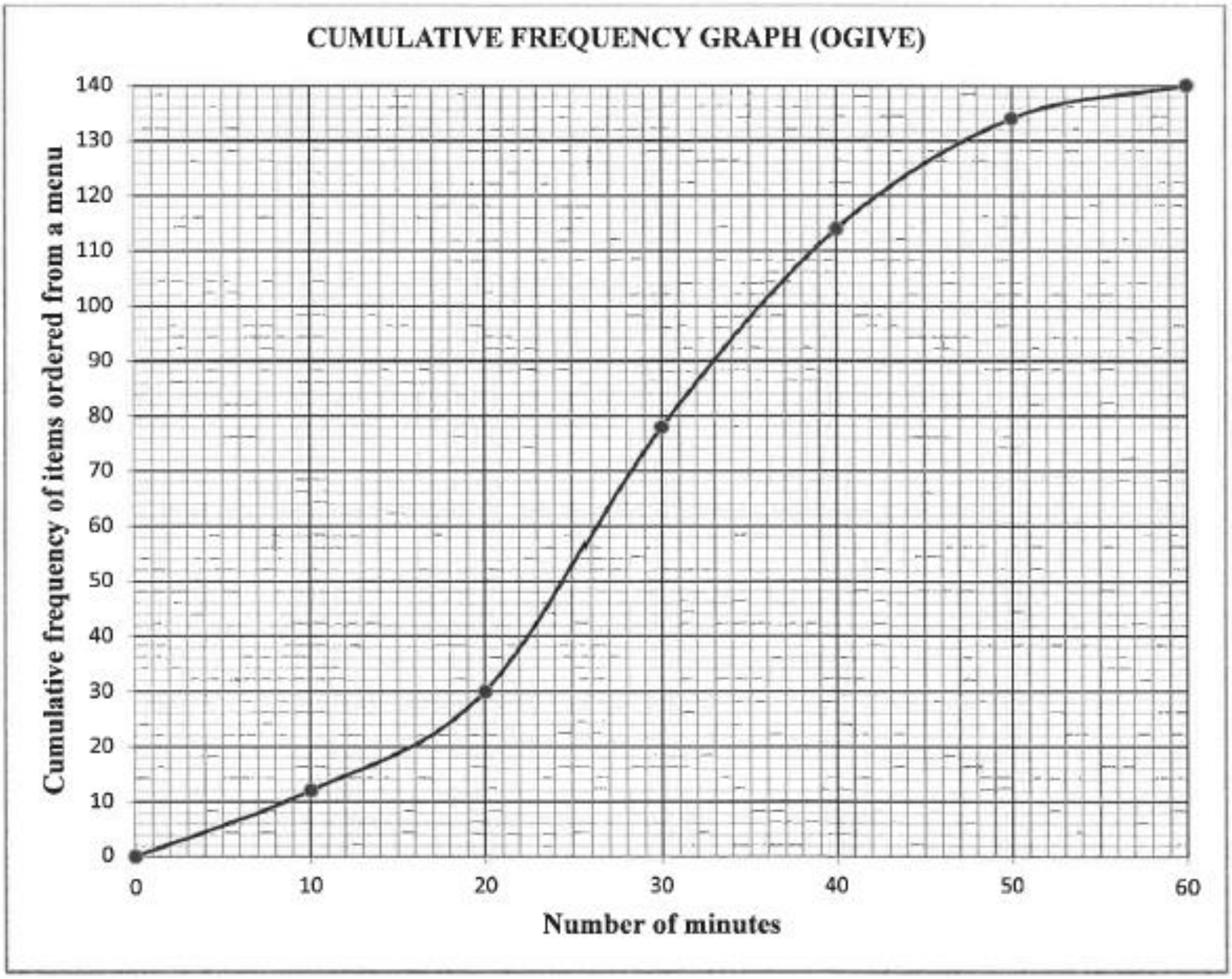
2.3.1 What is the standard deviation after the marks for the last question are added to each learner's total? (2)

2.3.2 What is the total mark of the last question? (2)

[10]

<u>November 2018</u>

1.1 The cumulative frequency graph (ogive) drawn below shows the total number of food items ordered from a menu over a period of 1 hour.



1.1.1	Write down the total number of food items ordered from the menu during this hour.	(1)
1.1.2	Write down the modal class of the data.	(1)
1.1.3	How long did it take to order the first 30 food items?	(1)
1.1.4	How many food items were ordered in the last 15 minutes?	(2)
1.1.5	Determine the 75^{th} percentile for the data.	(2)
1.1.6	Calculate the interquartile range of the data.	(2)

1.2 Reggie works part-time as a waiter at a local restaurant. The amount of money (in rands) he made in tips over a 15-day period is given below.

35	70	75	80	80
90	100	100	105	105
110	110	115	120	125

1.2.1 Calculate:

(a) The mean of the data (2)

(b) The standard deviation of the data (2)

1.2.2 Mary also works part-time as a waitress at the same restaurant. Over the same 15-day period Mary collected the same mean amount in tips as Reggie, but her standard deviation was R14.

Using the available information, comment on the:

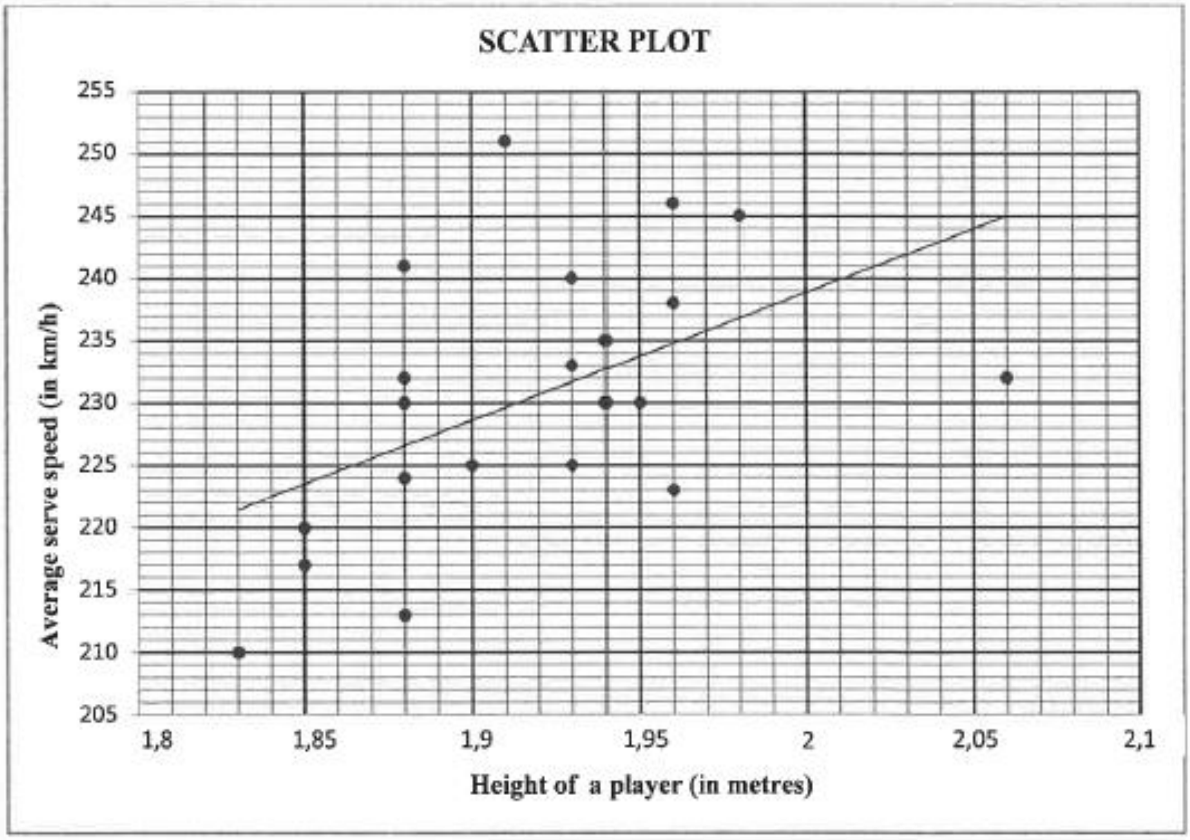
(a) Total amount in tips that they EACH collected over the 15-day period (1)

(b) Variation that EACH of them received in daily tips over this period (1)

[15]

QUESTION 2

A familiar question among professional tennis players is whether the speed of a tennis serve (in km/h) depends on the height of a player (in metres). The heights of 21 tennis players and the average speed of their serves were recorded during a tournament. The data is represented in the scatter plot below. The least squares regression line is also drawn.



2.1 Write down the fastest average serve speed (in km/h) achieved in this tournament. (1)

2.2 Consider the following correlation coefficients:

A. $r = 0{,}93$ B. $r = -0{,}42$ C. $r = 0{,}52$

2.2.1 Which ONE of the given correlation coefficients best fits the plotted data? (1)

2.2.2 Use the scatter plot and least squares regression line to motivate your answer to QUESTION 2.2.1. (1)

2.3 What does the data suggest about the speed of a tennis serve (in km/h) and the height of a player (in metres)? (1)

2.4 The equation of the regression line is given as $\hat{y} = 27{,}07 + bx$.
Explain why, in this context, the least squares regression line CANNOT intersect the y-axis at $(0\,;27{,}07)$. (1)
[5]

June 2018

QUESTION 1

The monthly profit (in thousands of rands) made by a company in a year is given in the table below.

110	112	156	164	167	169
171	176	192	228	278	360

1.1 Calculate the:

1.1.1 Mean profit for the year (3)

1.1.2 Median profit for the year (1)

1.2 On the number line provided in the ANSWER BOOK, draw a box and whisker diagram to represent the data. (2)

1.3 Hence, determine the interquartile range of the data. (1)

1.4 Comment on the skewness in the distribution of the data. (1)

1.5 For the given data:

1.5.1 Calculate the standard deviation (1)

1.5.2 Determine the number of months in which the profit was less than one standard deviation below the mean (2)

[11]

QUESTION 2

It is said that the number of times that a cricket chirps in a minute gives a very good indication of the air temperature (in °C). The table below shows the information recorded during an observation study.

CHIRPS PER MINUTE	AIR TEMPERATURE IN °C
32	8
40	10
52	12
76	15
92	17
112	20
128	25
180	28
184	30
200	35

2.1 Represent the data above on the grid provided in the ANSWER BOOK. (3)

2.2 Explain why the claim, 'gives a very good indication', is TRUE. (1)

2.3 Determine the equation of the least squares regression line of the data. (3)

2.4 Predict the air temperature (in °C) if a cricket chirps 80 times a minute. (2)
[9]

March 2018

QUESTION 1

An organisation decided that it would set up blood donor clinics at various colleges. Students would donate blood over a period of 10 days. The number of units of blood donated per day by students of college X is shown in the table below.

DAYS	1	2	3	4	5	6	7	8	9	10
UNITS OF BLOOD	45	59	65	73	79	82	91	99	101	106

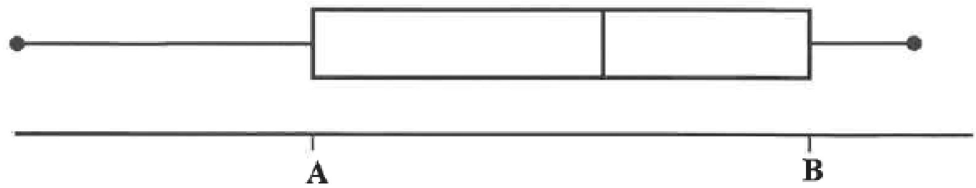
1.1 Calculate:

1.1.1 The mean of the units of blood donated per day over the period of 10 days (2)

1.1.2 The standard deviation of the data (2)

1.1.3 How many days is the number of units of blood donated at college X outside one standard deviation from the mean? (3)

1.2 The number of units of blood donated by the students of college X is represented in the box and whisker diagram below.



1.2.1 Describe the skewness of the data. (1)

1.2.2 Write down the values of **A** and **B**, the lower quartile and the upper quartile respectively, of the data set. (2)

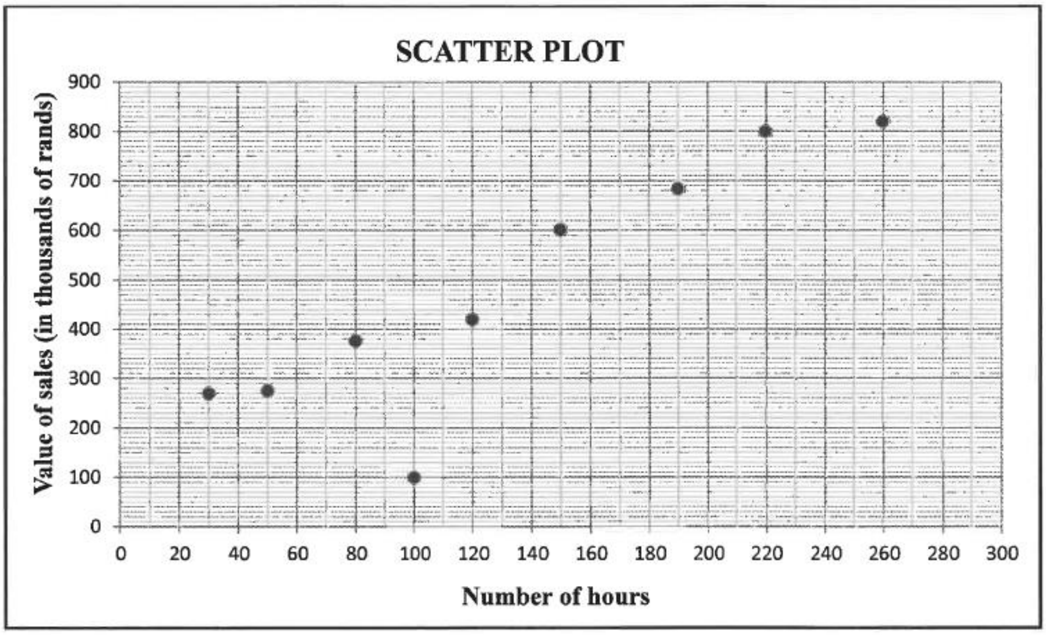
1.3 It was discovered that there was an error in counting the number of units of blood donated by college X each day. The correct mean of the data is 95 units of blood. How many units of blood were NOT counted over the ten days? (1)

[11]

QUESTION 2

The table below shows the number of hours that a sales representative of a company spent with each of his nine clients in one year and the value of the sales (in thousands of rands) for that client.

NUMBER OF HOURS	30	50	80	100	120	150	190	220	260
VALUE OF SALES (IN THOUSANDS OF RANDS)	270	275	376	100	420	602	684	800	820



2.1 Identify an outlier in the data above. (1)

2.2 Calculate the equation of the least squares regression line of the data. (3)

2.3 The sales representative forgot to record the sales of one of his clients. Predict the value of this client's sales (in thousands of rands) if he spent 240 hours with him during the year. (2)

2.4 What is the expected increase in sales for EACH additional hour spent with a client? (2)
[8]

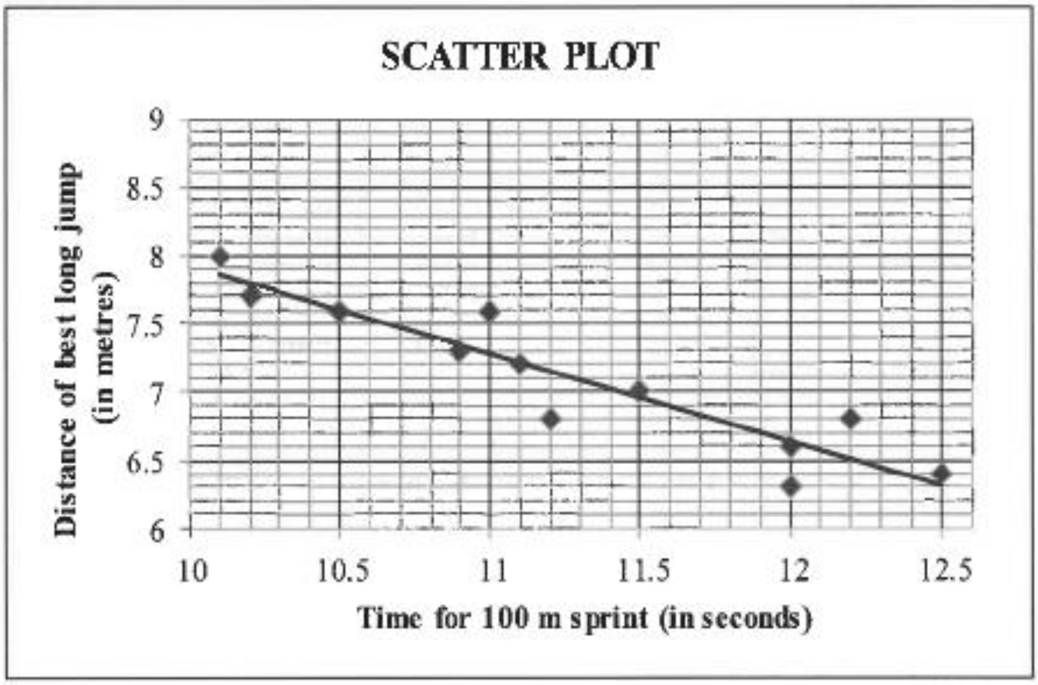
November 2017:

QUESTION 1

The table below shows the time (in seconds, rounded to ONE decimal place) taken by 12 athletes to run the 100 metre sprint and the distance (in metres, rounded to ONE decimal place) of their best long jump.

Time for 100 m sprint (in seconds)	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres)	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4

The scatter plot representing the data above is given below.



The equation of the least squares regression line is $\hat{y} = a + bx$.

1.1 Determine the values of a and b. (3)

1.2 An athlete runs the 100 metre sprint in 11,7 seconds. Use $\hat{y} = a + bx$ to predict the distance of the best long jump of this athlete. (2)

1.3 Another athlete completes the 100 metre sprint in 12,3 seconds and the distance of his best long jump is 7,6 metres. If this is included in the data, will the gradient of the least squares regression line increase or decrease? Motivate your answer without any further calculations. (2)

[7]

QUESTION 2

In an experiment, a group of 23 girls were presented with a page containing 30 coloured rectangles. They were asked to name the colours of the rectangles correctly as quickly as possible. The time, in seconds, taken by each of the girls is given in the table below.

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1 Calculate:

2.1.1 The mean of the data (2)

2.1.2 The interquartile range of the data (3)

2.2 The standard deviation of the times taken by the girls is 5,94. How many girls took longer than ONE standard deviation from the mean to name the colours? (2)

2.3 Draw a box and whisker diagram to represent the data on the number line provided in the ANSWER BOOK. (3)

2.4 The five-number summary of the times taken by a group of 23 boys in naming the colours of the rectangles correctly is (15 ; 21 ; 23,5 ; 26 ; 38).

2.4.1 Which of the two groups, girls or boys, had the lower median time to correctly name the colours of the rectangles? (1)

2.4.2 The first three learners who named the colours of all 30 rectangles correctly in the shortest time will receive a prize. How many boys will be among these three prizewinners? Motivate your answer. (2)

[13]

June 2017:

QUESTION 1

An IT company writes programs for apps. The time taken (in hours) to write the programs and the cost (in thousands of rands) are shown in the table below.

TIME TAKEN (IN HOURS)	5	7	5	8	10	13	15	20	18	25	23
COST (IN THOUSANDS OF RANDS)	10	10	15	12	20	25	28	32	28	40	30

1.1 Determine the equation of the least squares regression line. (3)

1.2 Use the equation of the least squares regression line to predict the cost, in rands, of an app that will take 16 hours to write. (2)

1.3 Calculate the correlation coefficient of the data. (1)

1.4 For each app that the company writes, there is a cost that is independent of the number of hours spent on writing the app. Calculate this cost (in rands). (2)

[8]

QUESTION 2

The commission earned, in thousands of rands, by the salesmen of a particular company in a certain month is shown in the table below.

COMMISSION EARNED (IN THOUSANDS OF RANDS)	FREQUENCY
$20 < x \leq 40$	7
$40 < x \leq 60$	6
$60 < x \leq 80$	8
$80 < x \leq 100$	10
$100 < x \leq 120$	4

2.1 Write down the modal class of the data. (1)

2.2 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

2.3 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (4)

2.4 A salesman receives a bonus if his commission is more than R90 000 for the month. Calculate how many of the salesmen received bonuses for this month. (2)

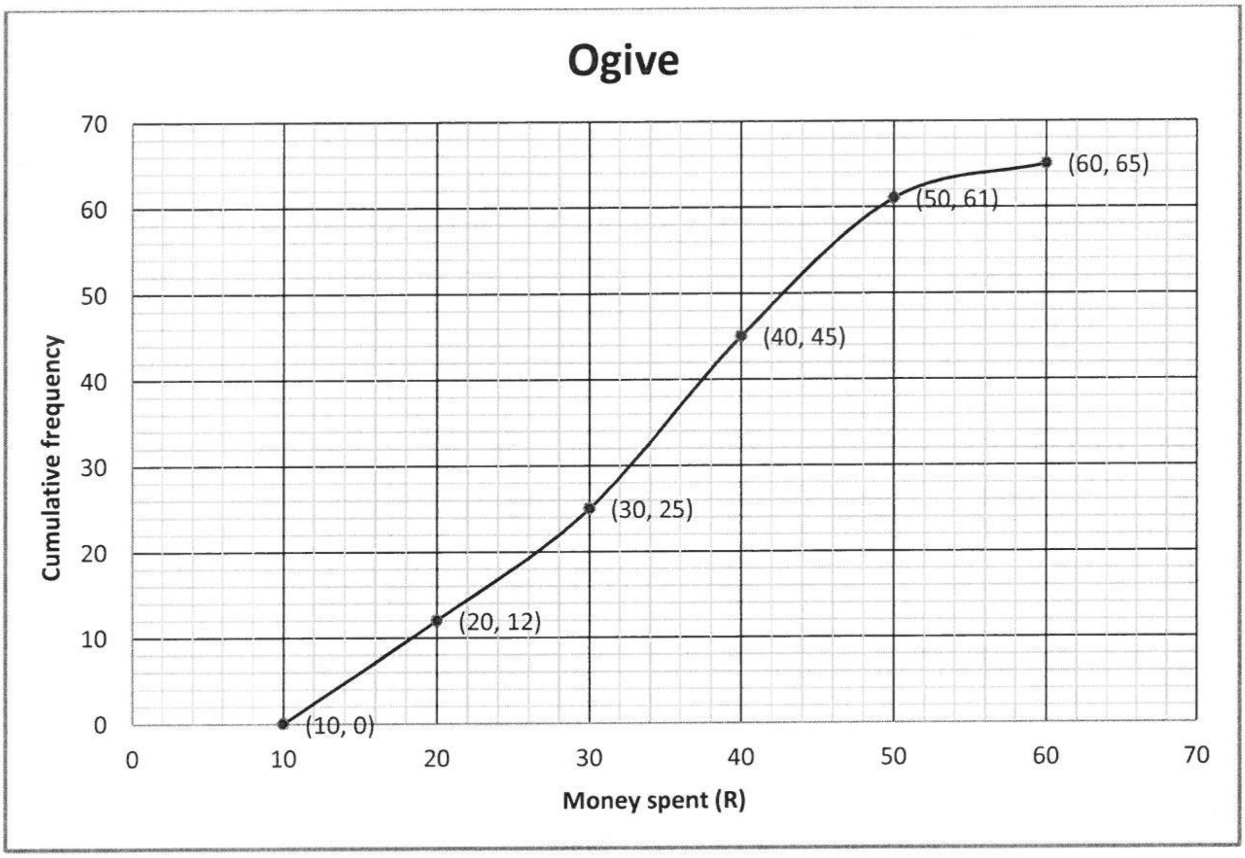
2.5 Determine the approximate mean commission earned by the salesmen in this month correct to the nearest thousand rand. (3)

[12]

March 2017:

QUESTION 1

The amount of money, in rands, that learners spent while visiting a tuck shop at school on a specific day was recorded. The data is represented in the ogive below.



An incomplete frequency table is also given for the data.

Amount of money (in R)	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$
Frequency	a	13	20	b	4

1.1 How many learners visited the tuck shop on that day? (1)

1.2 Write down the modal class of this data. (1)

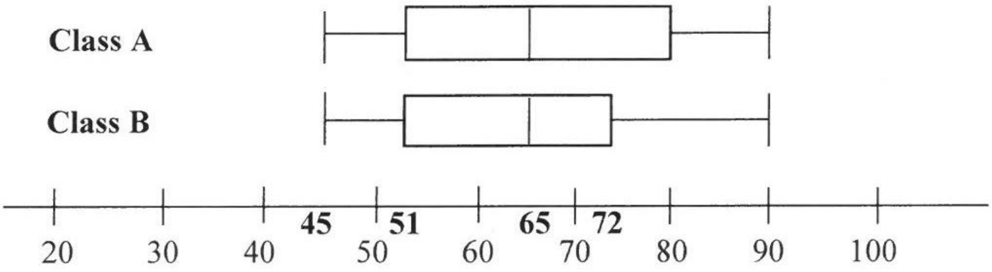
1.3 Determine the values of a and b in the frequency table. (2)

1.4 Use the ogive to estimate the number of learners who spent at least R45 on the day the data was recorded at the tuck shop. (2)

[6]

QUESTION 2

2.1 Mrs Smith has two classes, each having 30 learners. Their final marks (out of 100) for the year are represented in the box and whisker diagram below.



2.1.1 Determine the interquartile range of Class B. (2)

2.1.2 Explain the significance in the difference of the length of the boxes in the diagram. (2)

2.1.3 Mrs Smith studied the results and made the comment that there was no significant difference in the performance of the two classes. Give TWO reasons you think Mrs Smith will use to prove her statement. (2)

2.2 Eight couples entered a dance competition. Their performances were scored by two judges. The scores (out of 20) are given in the table below.

COUPLE	1	2	3	4	5	6	7	8
JUDGE 1	18	4	6	8	5	12	10	14
JUDGE 2	15	6	3	5	5	14	8	15

2.2.1 Determine the equation of the least squares regression line of the scores given by the two judges. (3)

2.2.2 A ninth couple entered late for the competition and received a score of 15 from JUDGE 1. Estimate the score that might have been assigned by JUDGE 2 to the nearest integral value. (2)

2.2.3 Are the judges consistent in assigning scores to the performance of the couples? Prove your answer and support it with relevant statistics. (2)

[13]

SESSION 5

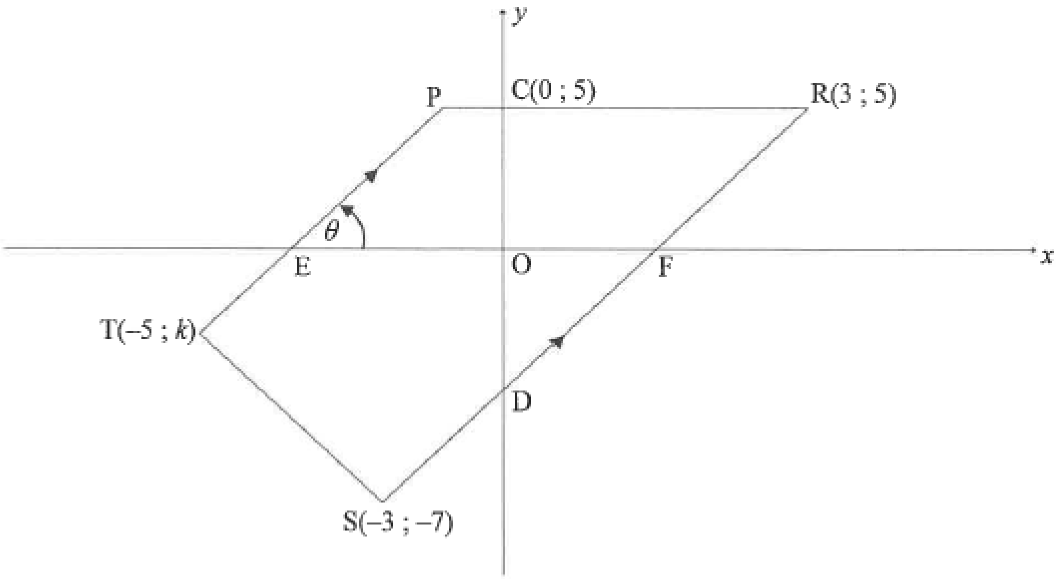
Coordinate Geometry Euclidean Geometry

Coordinate (Analytical) Geometry

November 2019

QUESTION 3

In the diagram, P, R(3 ; 5), S(–3 ; –7) and T(–5 ; k) are vertices of trapezium PRST and PT ∥ RS. RS and PR cut the y-axis at D and C(0 ; 5) respectively. PT and RS cut the x-axis at E and F respectively. $\hat{PEF} = \theta$.



3.1 Write down the equation of PR. (1)

3.2 Calculate the:

3.2.1 Gradient of RS (2)

3.2.2 Size of θ (3)

3.2.3 Coordinates of D (3)

3.3 If it is given that $TS = 2\sqrt{5}$, calculate the value of k. (4)

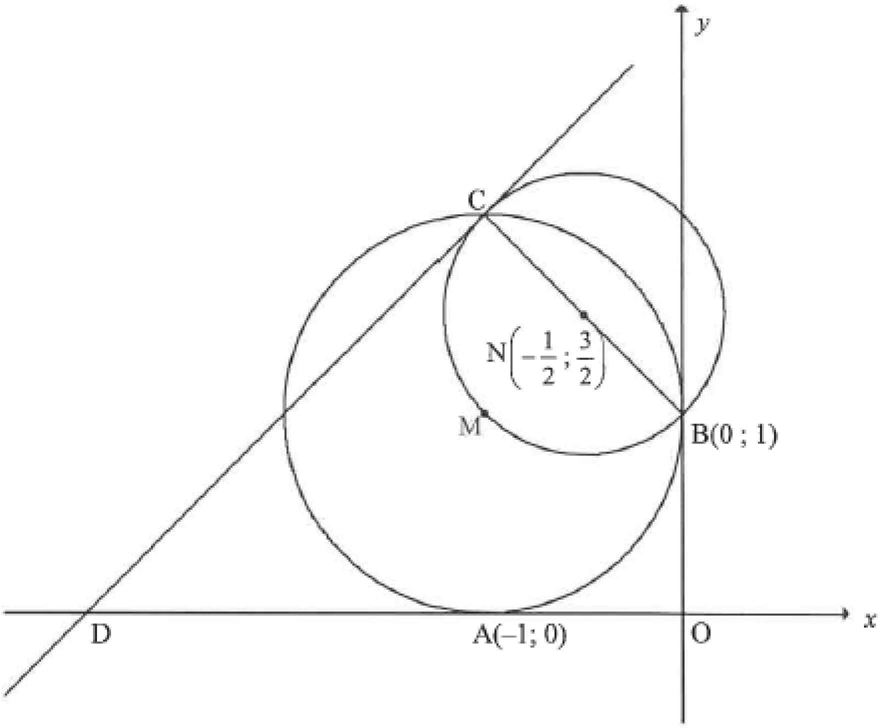
3.4 Parallelogram TDNS, with N in the 4^{th} quadrant, is drawn. Calculate the coordinates of N. (3)

3.5 ΔPRD is reflected about the y-axis to form $\Delta P'R'D'$. Calculate the size of $R\hat{D}R'$. (3)

[19]

QUESTION 4

In the diagram, a circle having centre M touches the x–axis at A(–1 ; 0) and the y-axis at B(0 ; 1). A smaller circle, centred at $N\left(-\frac{1}{2} ; \frac{3}{2}\right)$, passes through M and cuts the larger circle at B and C. BNC is a diameter of the smaller circle. A tangent drawn to the smaller circle at C, cuts the x-axis at D.



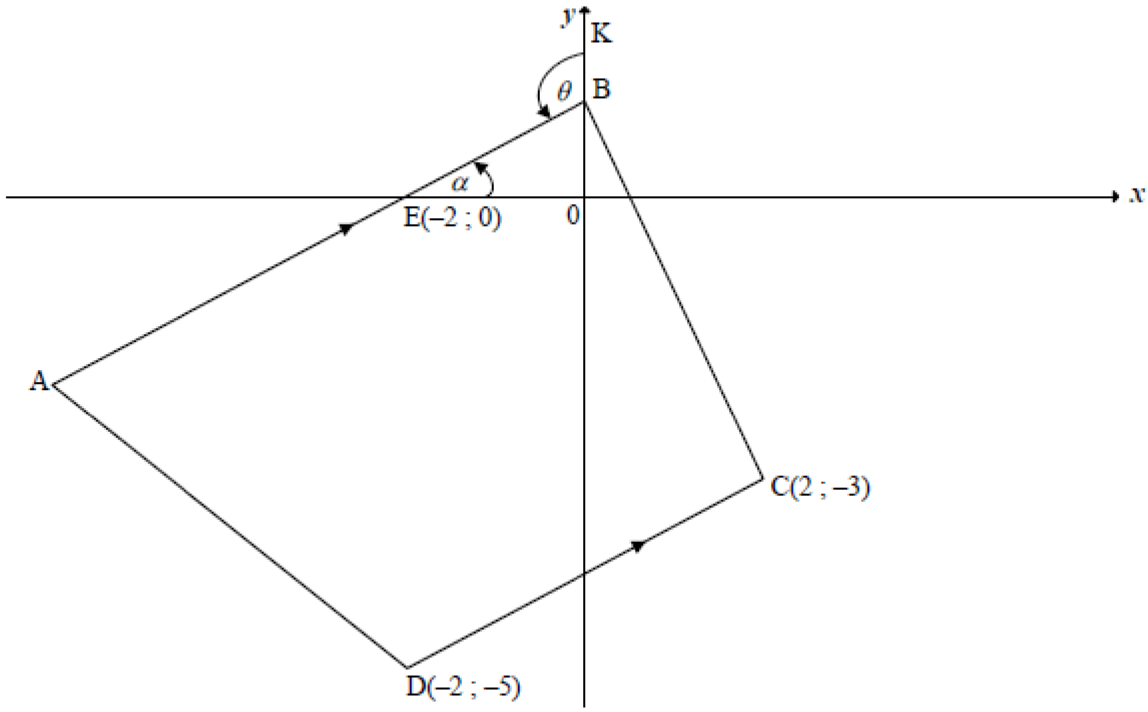
4.1	Determine the equation of the circle centred at M in the form $(x-a)^2+(y-b)^2=r^2$	(3)
4.2	Calculate the coordinates of C.	(2)
4.3	Show that the equation of the tangent CD is $y-x=3$.	(4)
4.4	Determine the values of t for which the line $y=x+t$ will NOT touch or cut the smaller circle.	(3)
4.5	The smaller circle centred at N is transformed such that point C is translated along the tangent to D. Calculate the coordinates of E, the new centre of the smaller circle.	(3)
4.6	If it is given that the area of quadrilateral OBCD is $2a^2$ square units and $a>0$, show that $a=\frac{\sqrt{7}}{2}$ units.	(5)

[20]

May-June 2019

QUESTION 3

In the diagram, A, B, C(2 ; –3) and D(–2 ; –5) are vertices of a trapezium with AB || DC. E(–2 ; 0) is the x-intercept of AB. The inclination of AB is α. K lies on the y-axis and $K\hat{B}E = \theta$.



3.1 Determine:

3.1.1 The midpoint of EC (2)

3.1.2 The gradient of DC (2)

3.1.3 The equation of AB in the form $y = mx + c$ (3)

3.1.4 The size of θ (3)

3.2 Prove that AB $\perp$ BC. (3)

3.3 The points E, B and C lie on the circumference of a circle. Determine:

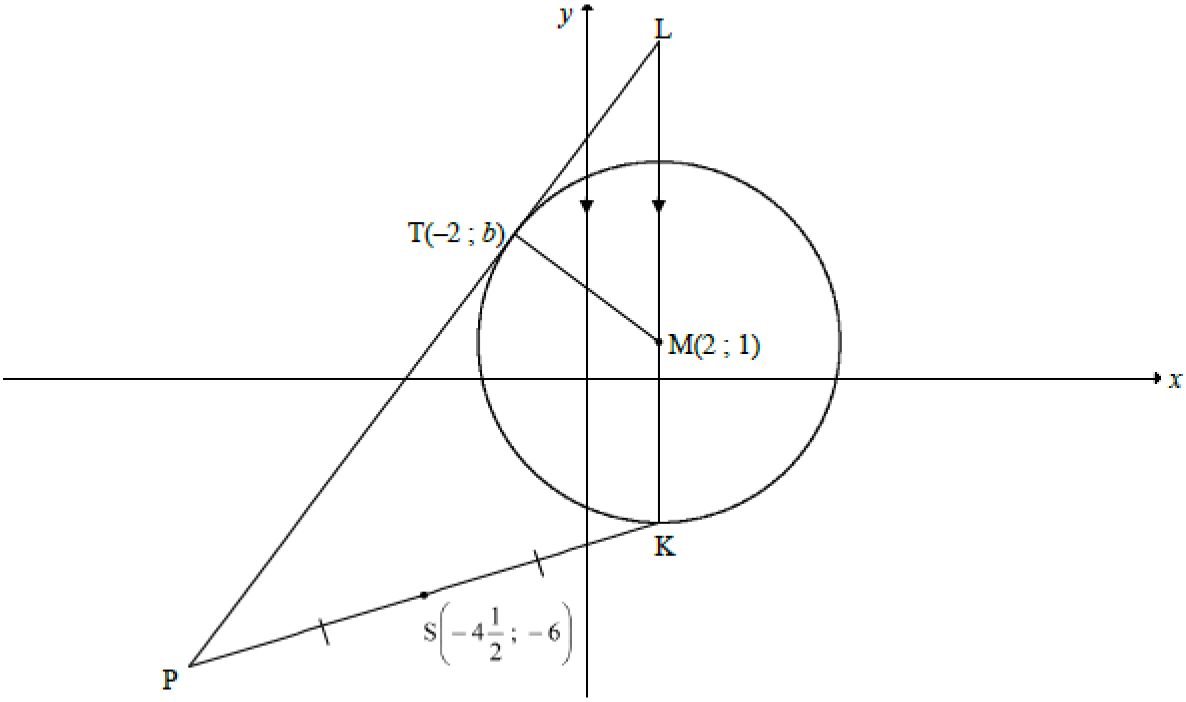
3.3.1 The centre of the circle (1)

3.3.2 The equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$ (4)

[18]

QUESTION 4

In the diagram, the circle is centred at M(2 ; 1). Radius KM is produced to L, a point outside the circle, such that KML ∥ y-axis. LTP is a tangent to the circle at T(–2 ; b). $S\left(-4\frac{1}{2}; -6\right)$ is the midpoint of PK.



4.1 Given that the radius of the circle is 5 units, show that $b = 4$. (4)

4.2 Determine:

4.2.1 The coordinates of K (2)

4.2.2 The equation of the tangent LTP in the form $y = mx + c$ (4)

4.2.3 The area of ΔLPK (7)

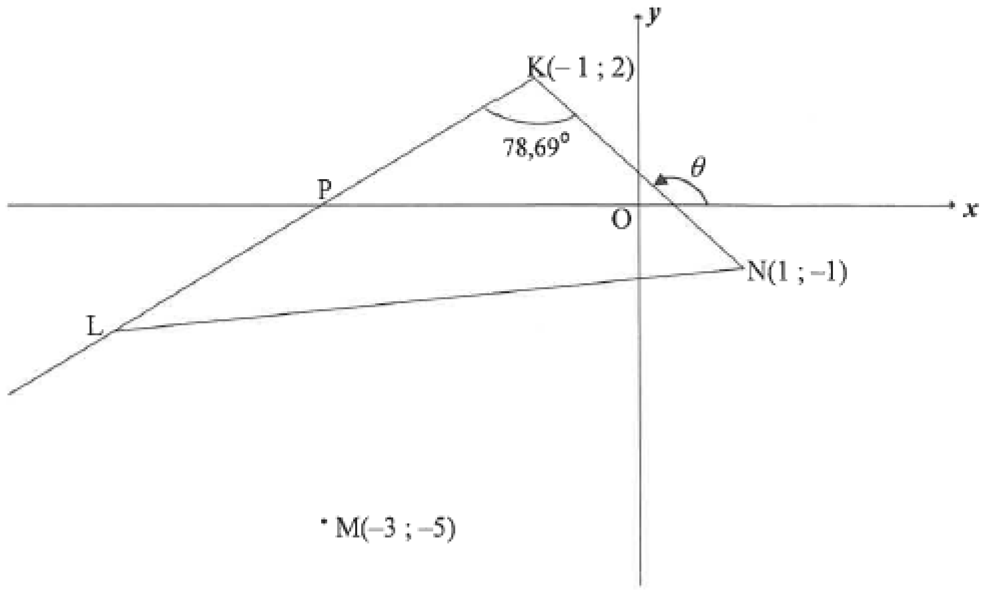
4.3 Another circle with equation $(x-2)^2 + (y-n)^2 = 25$ is drawn. Determine, with an explanation, the value(s) of n for which the two circles will touch each other externally. (4)

[21]

November 2018

QUESTION 3

In the diagram, K(– 1 ; 2), L and N(1 ; – 1) are vertices of ΔKLN such that $L\hat{K}N = 78{,}69°$. KL intersects the x-axis at P. KL is produced. The inclination of KN is θ. The coordinates of M are (–3 ; –5).



3.1 Calculate:

3.1.1 The gradient of KN (2)

3.1.2 The size of θ, the inclination of KN (2)

3.2 Show that the gradient of KL is equal to 1. (2)

3.3 Determine the equation of the straight line KL in the form $y = mx + c$. (2)

3.4 Calculate the length of KN. (2)

3.5 It is further given that KN = LM.

3.5.1 Calculate the possible coordinates of L. (5)

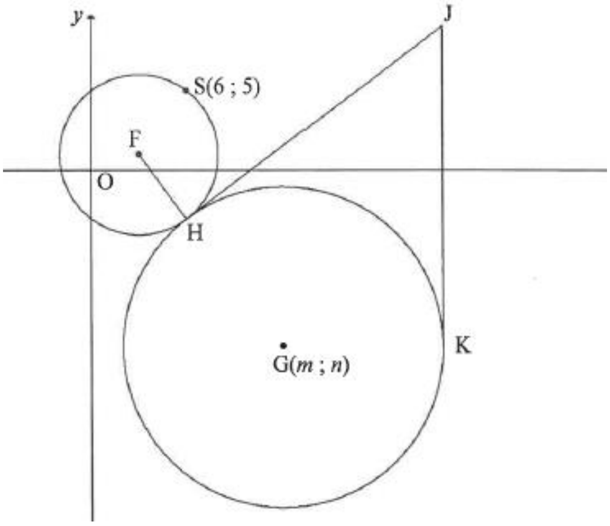
3.5.2 Determine the coordinates of L if it is given that KLMN is a parallelogram. (3)

3.6 T is a point on KL produced. TM is drawn such that TM = LM. Calculate the area of ΔKTN. (4)

[22]

QUESTION 4

In the diagram, the equation of the circle with centre F is $(x-3)^2+(y-1)^2=r^2$. S(6 ; 5) is a point on the circle with centre F. Another circle with centre G(m ; n) in the 4th quadrant touches the circle with centre F, at H such that FH : HG = 1 : 2. The point J lies in the first quadrant such that HJ is a common tangent to both these circles. JK is a tangent to the larger circle at K.

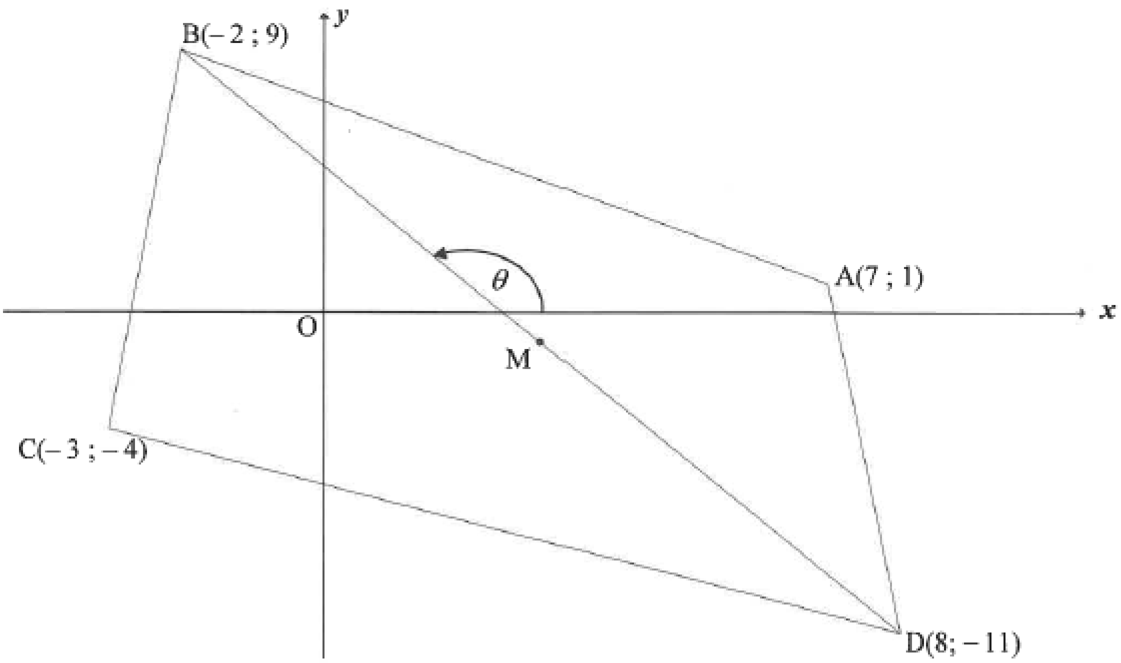


4.1	Write down the coordinates of F.	(2)
4.2	Calculate the length of FS.	(2)
4.3	Write down the length of HG.	(1)
4.4	Give a reason why JH = JK.	(1)
4.5	Determine:	
	4.5.1 The distance FJ, with reasons, if it is given that JK = 20	(4)
	4.5.2 The equation of the circle with centre G in terms of m and n in the form $(x-a)^2+(y-b)^2=r^2$	(1)
	4.5.3 The coordinates of G, if it is further given that the equation of tangent JK is $x = 22$	(7)
		[18]

June 2018

QUESTION 3

In the diagram, ABCD is a quadrilateral having vertices A(7 ; 1), B(– 2 ; 9), C(– 3 ; – 4) and D(8 ; – 11). M is the midpoint of BD.



3.1 Calculate the gradient of AC. (2)

3.2 Determine:

3.2.1 The equation of AC in the form $y = mx + c$ (2)

3.2.2 Whether M lies on AC (4)

3.3 Prove that BD $\perp$ AC. (3)

3.4 Calculate:

3.4.1 θ, the inclination of BD (2)

3.4.2 The size of $C\hat{B}D$ (3)

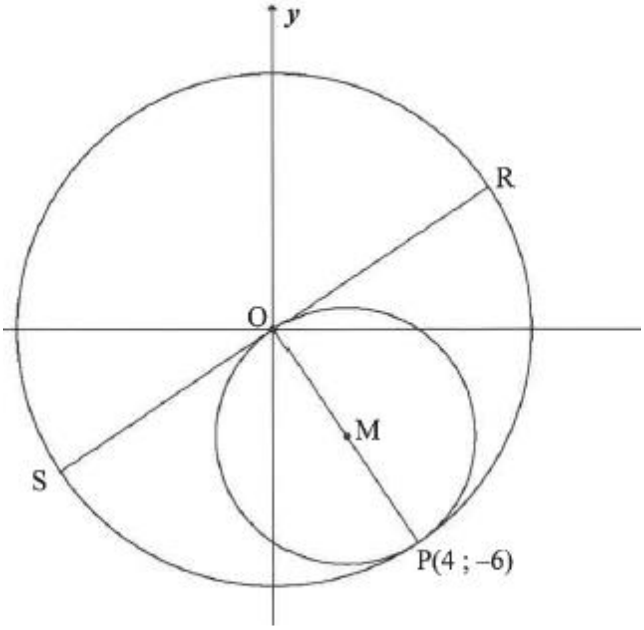
3.4.3 The length of AC (2)

3.4.4 The area of ABCD (5)

[23]

QUESTION 4

In the diagram, a circle having centre at the origin passes through P(4 ; – 6). PO is the diameter of a smaller circle having centre at M. The diameter RS of the larger circle is a tangent to the smaller circle at O.



4.1 Calculate the coordinates of M. (2)

4.2 Determine the equation of:

4.2.1 The large circle (2)

4.2.2 The small circle in the form $x^2 + y^2 + Cx + Dy + E = 0$ (3)

4.2.3 The equation of RS in the form $y = mx + c$ (3)

4.3 Determine the length of chord NR, where N is the reflection of R in the y-axis. (4)

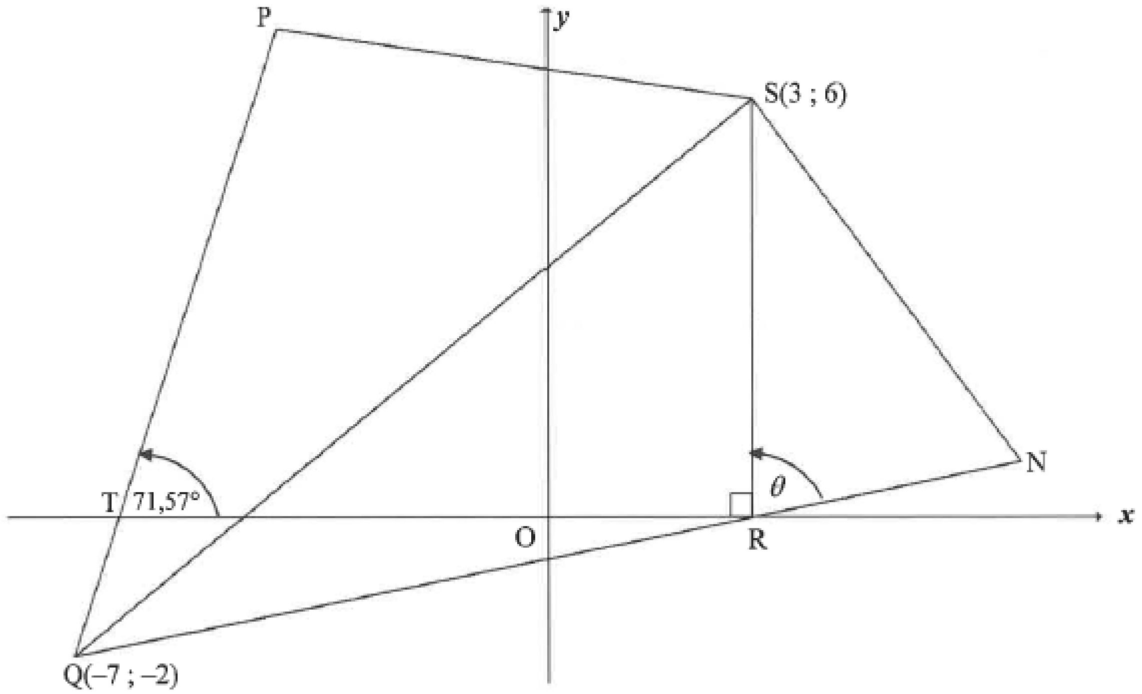
4.4 The circle with centre at M is reflected about the x-axis to form another circle centred at K. Calculate the length of the common chord of these two circles. (3)

[17]

March 2018

QUESTION 3

In the diagram, P, Q(–7 ; –2), R and S(3 ; 6) are vertices of a quadrilateral. R is a point on the x-axis. QR is produced to N such that QR = 2RN. SN is drawn. $P\hat{T}O = 71,57°$ and $S\hat{R}N = \theta$.

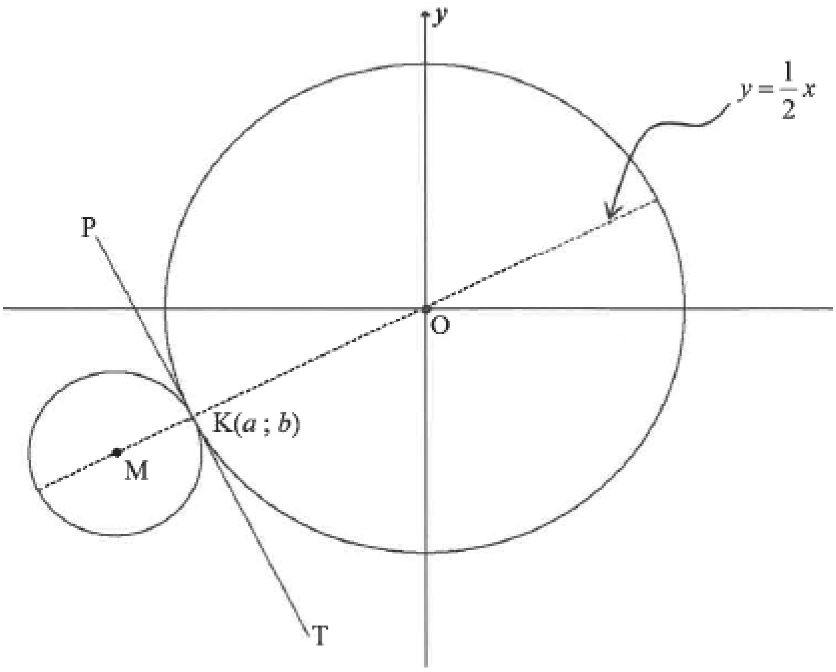


Determine:

3.1	The equation of SR	(1)
3.2	The gradient of QP to the nearest integer	(2)
3.3	The equation of QP in the form $y = mx + c$	(2)
3.4	The length of QR. Leave your answer **in surd form**.	(2)
3.5	$\tan(90° - \theta)$	(3)
3.6	The area of ΔRSN, **without using a calculator**	(6)
		[16]

QUESTION 4

In the diagram, PKT is a common tangent to both circles at K(a ; b). The centres of both circles lie on the line $y = \frac{1}{2}x$. The equation of the circle centred at O is $x^2 + y^2 = 180$. The radius of the circle is three times that of the circle centred at M.



4.1 Write down the length of OK **in surd form**. (1)

4.2 Show that K is the point (–12 ; –6). (4)

4.3 Determine:

4.3.1 The equation of the common tangent, PKT, in the form $y = mx + c$ (3)

4.3.2 The coordinates of M (6)

4.3.3 The equation of the smaller circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

4.4 For which value(s) of r will another circle, with equation $x^2 + y^2 = r^2$, intersect the circle centred at M at two distinct points? (3)

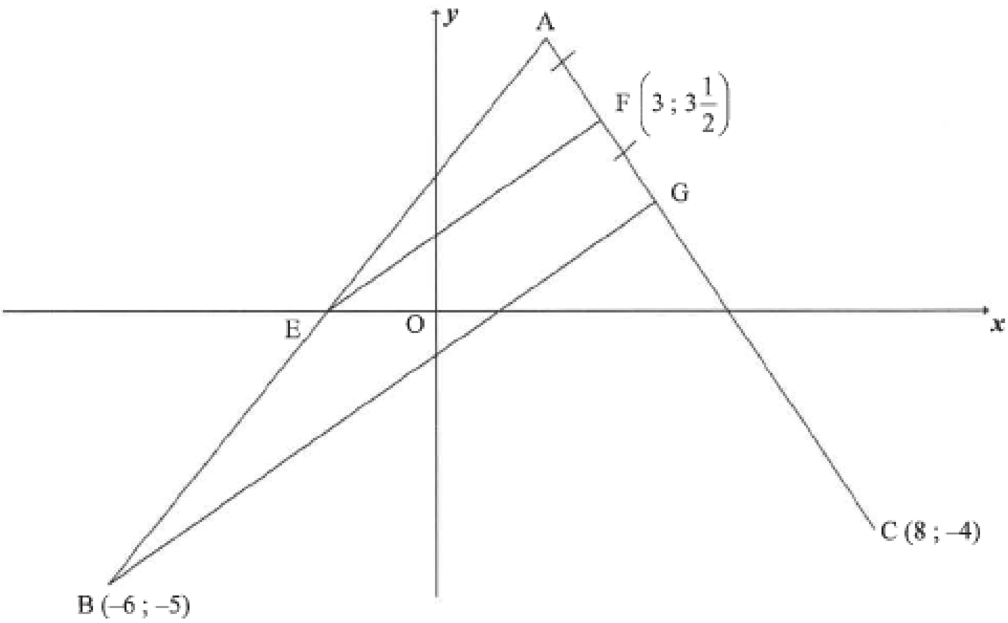
4.5 Another circle, $x^2 + y^2 + 32x + 16y + 240 = 0$, is drawn. Prove by calculation that this circle does NOT cut the circle with centre M(–16 ; –8). (5)

[24]

November 2017:

QUESTION 3

In the diagram, A, B(–6 ; –5) and C(8 ; –4) are points in the Cartesian plane. $F\left(3; 3\frac{1}{2}\right)$ and G are points on line AC such that AF = FG. E is the x-intercept of AB.



3.1 Calculate:

3.1.1 The equation of AC in the form $y = mx + c$ (4)

3.1.2 The coordinates of G if the equation of BG is $7x - 10y = 8$ (3)

3.2 Show by calculation that the coordinates of A is (2 ; 5). (2)

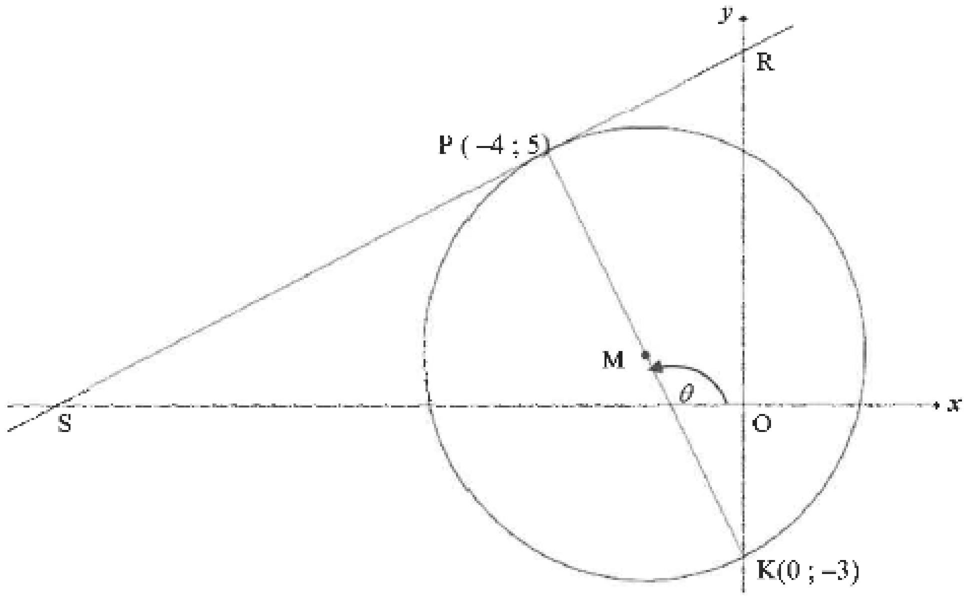
3.3 Prove that EF || BG. (4)

3.4 ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D. (4)

[17]

QUESTION 4

In the diagram, P(–4 ; 5) and K(0 ; –3) are the end points of the diameter of a circle with centre M. S and R are respectively the x- and y-intercept of the tangent to the circle at P. θ is the inclination of PK with the positive x-axis.



4.1 Determine:

4.1.1	The gradient of SR	(4)
4.1.2	The equation of SR in the form $y = mx + c$	(2)
4.1.3	The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$	(4)
4.1.4	The size of $P\hat{K}R$	(3)
4.1.5	The equation of the tangent to the circle at K in the form $y = mx + c$	(2)

4.2 Determine the values of t such that the line $y = \frac{1}{2}x + t$ cuts the circle at two different points. (3)

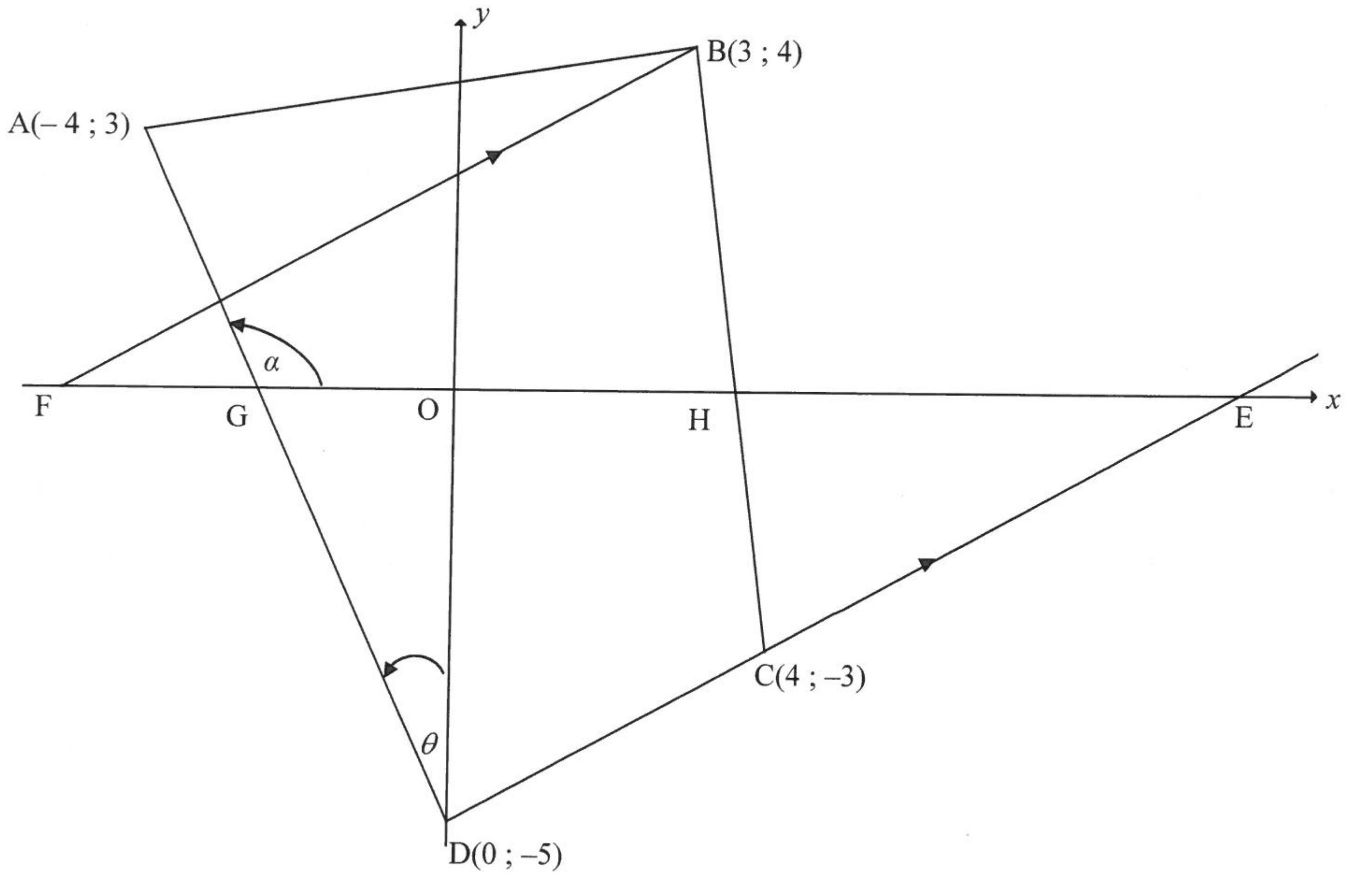
4.3 Calculate the area of ΔSMK. (5)

[23]

June 2017:

QUESTION 3

In the diagram, ABCD is a quadrilateral having vertices A(– 4 ; 3), B(3 ; 4), C(4 ; –3) and D(0 ; –5). DC produced cuts the x-axis at E, BC cuts the x-axis at H and AD cuts the x-axis at G. F is a point on the x-axis such that BF || DE. $A\hat{G}O = \alpha$ and $A\hat{D}O = \theta$.



3.1 Calculate the gradient of DC. (2)

3.2 Prove that AD ⊥ DC. (3)

3.3 Show by calculation that ΔABC is an isosceles. (4)

3.4 Determine the equation of BF in the form $y = mx + c$. (3)

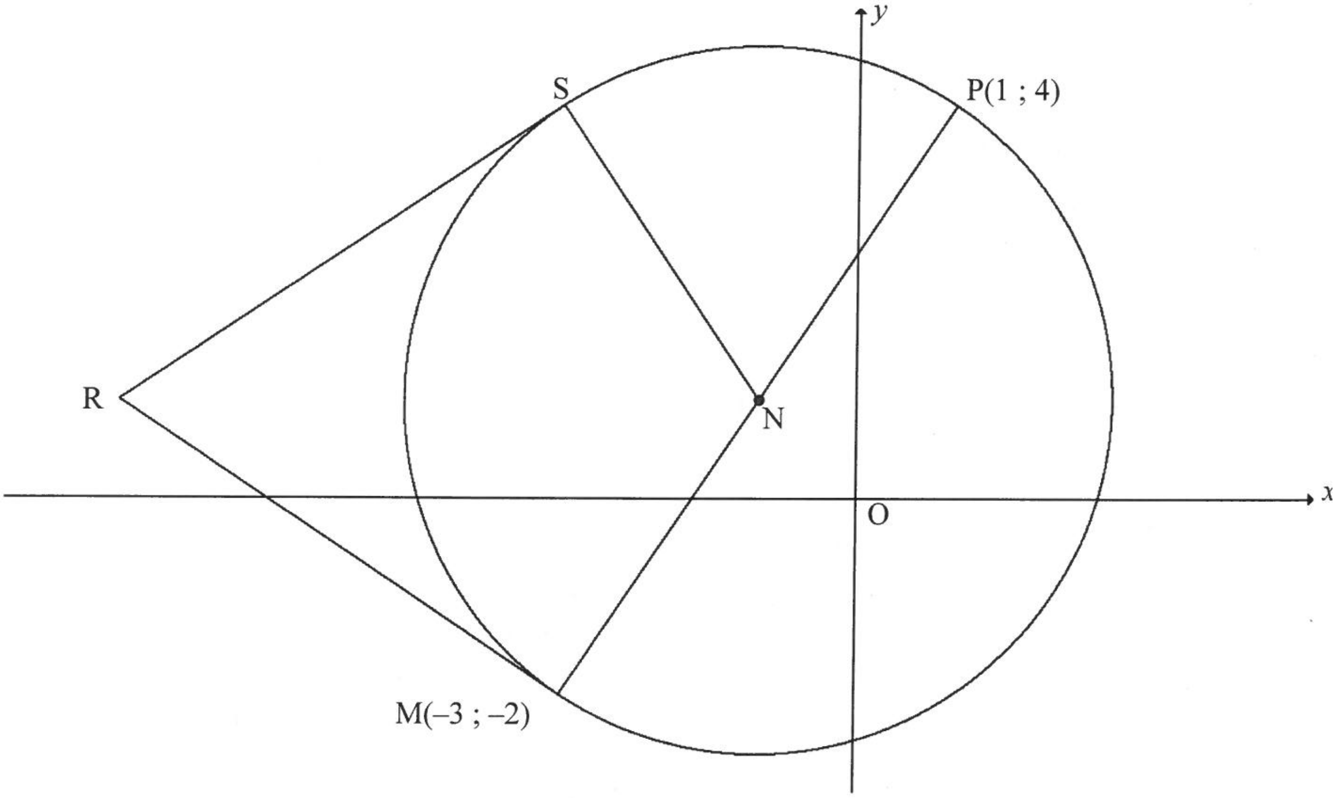
3.5 Calculate the size of θ. (3)

3.6 Determine the equation of the circle, with the centre as the origin and passing through point C, in the form $x^2 + y^2 = r^2$. (2)

[17]

QUESTION 4

In the diagram, N is the centre of the circle. M(–3 ; –2) and P(1 ; 4) are points on the circle. MNP is the diameter of the circle. Tangents drawn to circle N from point R, outside the circle, meet the circle at S and M respectively.

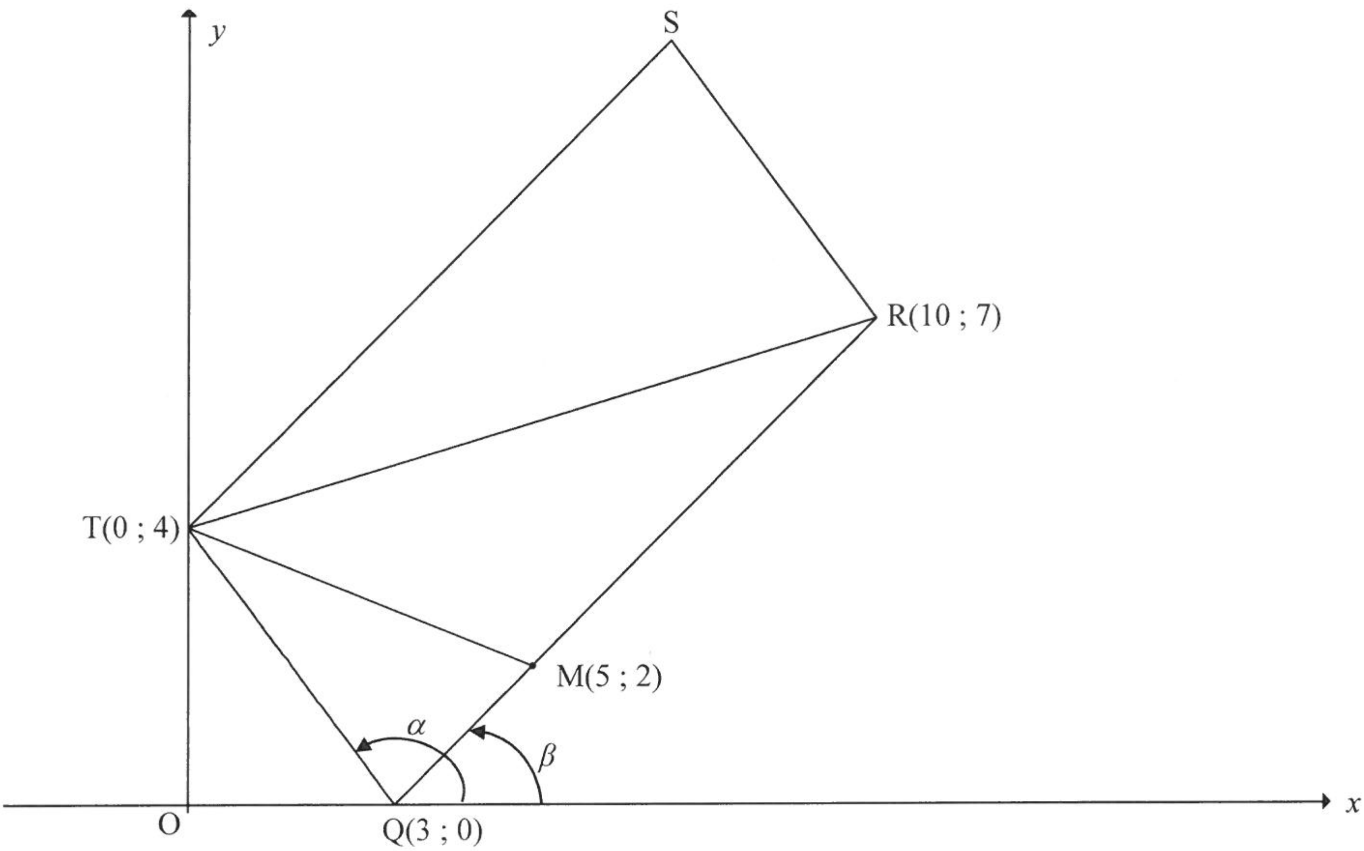


4.1	Determine the coordinates of N.	(3)
4.2	Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$.	(4)
4.3	Determine the equation of the tangent RM in the form $y = mx + c$.	(5)
4.4	If it is given that the line joining S to M is perpendicular to the x-axis, determine the coordinates of S.	(2)
4.5	Determine the coordinates of R, the common external point from which both tangents to the circle are drawn.	(4)
4.6	Calculate the area of RSNM.	(4)
		[22]

March 2017:

QUESTION 3

In the diagram, Q(3 ; 0), R(10 ; 7), S and T(0 ; 4) are the vertices of parallelogram QRST. From T a straight line is drawn to meet QR at M(5 ; 2). The angles of inclination of TQ and RQ are α and β respectively.



3.1 Calculate the gradient of TQ. (1)

3.2 Calculate the length of RQ. Leave your answer in surd form. (2)

3.3 F(k ; –8) is a point in the Cartesian plane such that T, Q and F are collinear. Calculate the value of k. (4)

3.4 Calculate the coordinates of S. (4)

3.5 Calculate the size of $T\hat{S}R$. (6)

3.6 Calculate, in the simplest form, the ratio of:

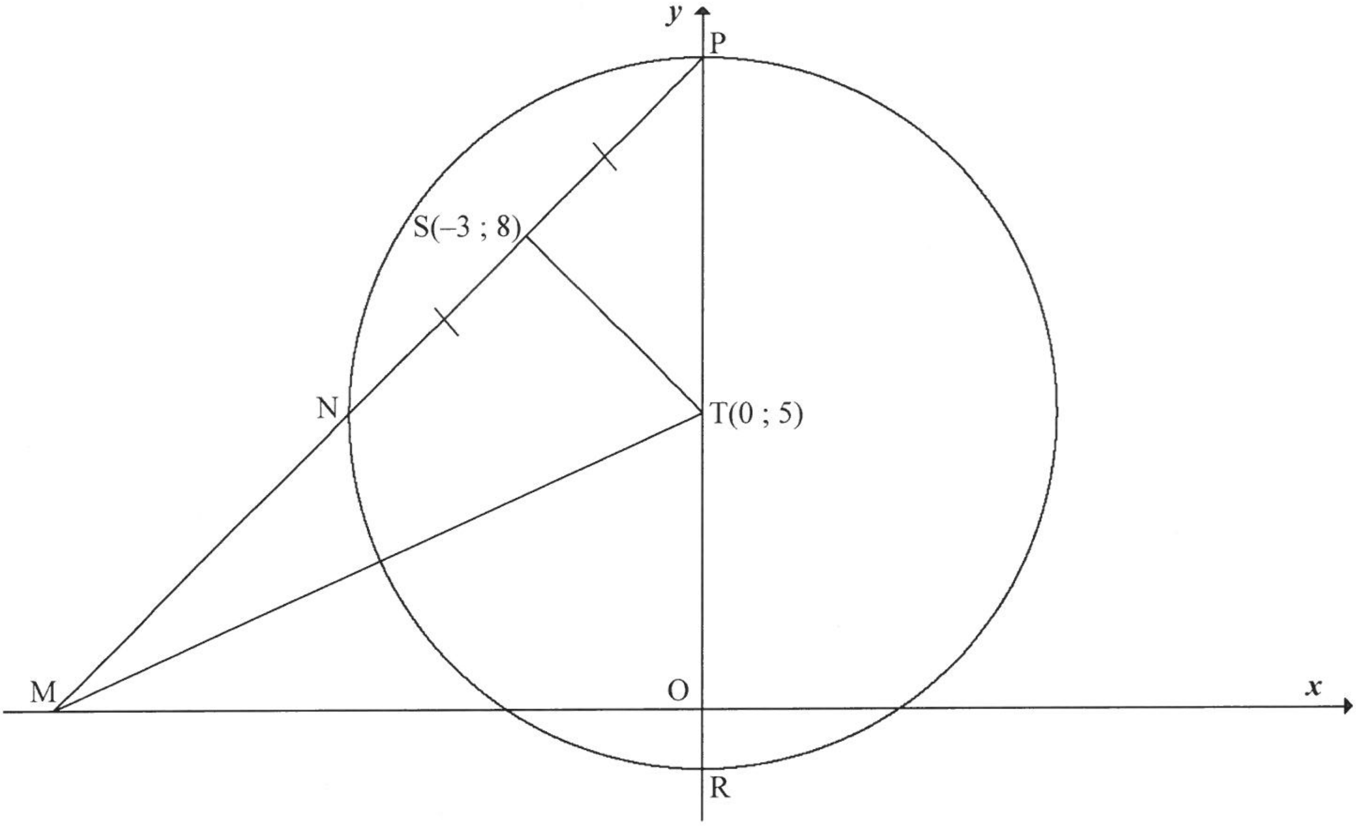
3.6.1 $\dfrac{MQ}{RQ}$ (3)

3.6.2 $\dfrac{\text{area of } \Delta TQM}{\text{area of parallelogram RQTS}}$ (3)

[23]

QUESTION 4

In the diagram, the circle, having centre T(0 ; 5), cuts the y-axis at P and R. The line through P and S(–3 ; 8) intersects the circle at N and the x-axis at M. NS = PS. MT is drawn.



4.1 Give a reason why TS ⊥ NP. (1)

4.2 Determine the equation of the line passing through N and P in the form $y = mx + c$. (5)

4.3 Determine the equations of the tangents to the circle that are parallel to the x-axis. (4)

4.4 Determine the length of MT. (4)

4.5 Another circle is drawn through the points S, T and M. Determine, with reasons, the equation of this circle STM in the form $(x-a)^2 + (y-b)^2 = r^2$. (5)

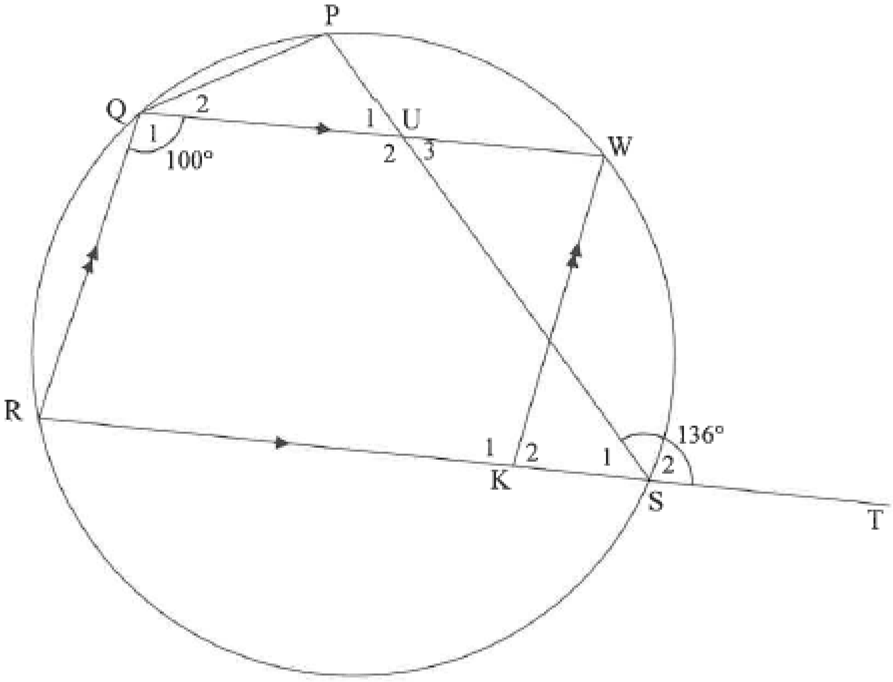
[19]

Euclidean Geometry

November 2019

QUESTION 8

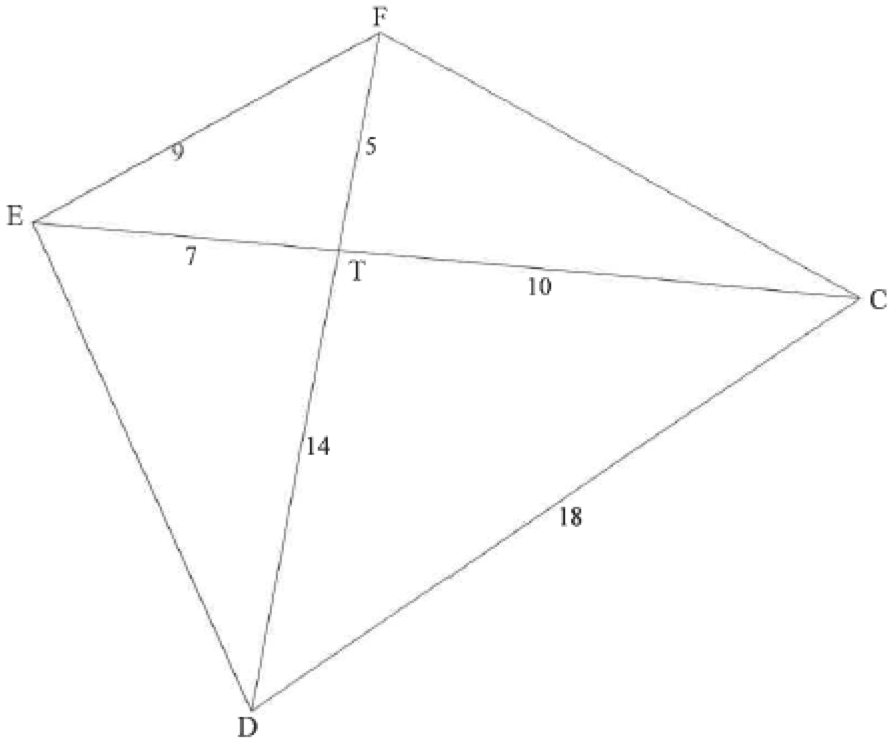
8.1 In the diagram, PQRS is a cyclic quadrilateral. Chord RS is produced to T. K is a point on RS and W is a point on the circle such that QRKW is a parallelogram. PS and QW intersect at U. $P\hat{S}T = 136°$ and $\hat{Q}_1 = 100°$.



Determine, with reasons, the size of:

8.1.1	$\hat{R}$	(2)
8.1.2	$\hat{P}$	(2)
8.1.3	$P\hat{Q}W$	(3)
8.1.4	$\hat{U}_2$	(2)

8.2 In the diagram, the diagonals of quadrilateral CDEF intersect at T.
EF = 9 units, DC = 18 units, ET = 7 units, TC = 10 units, FT = 5 units and TD = 14 units.



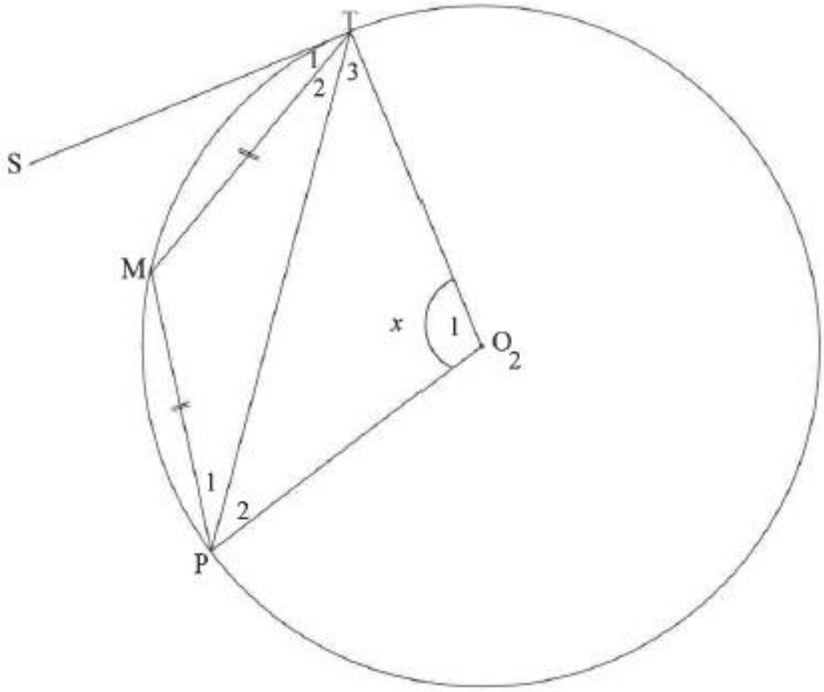
Prove, with reasons, that:

8.2.1 $E\hat{F}D = E\hat{C}D$ (4)

8.2.2 $D\hat{F}C = D\hat{E}C$ (3)

QUESTION 9

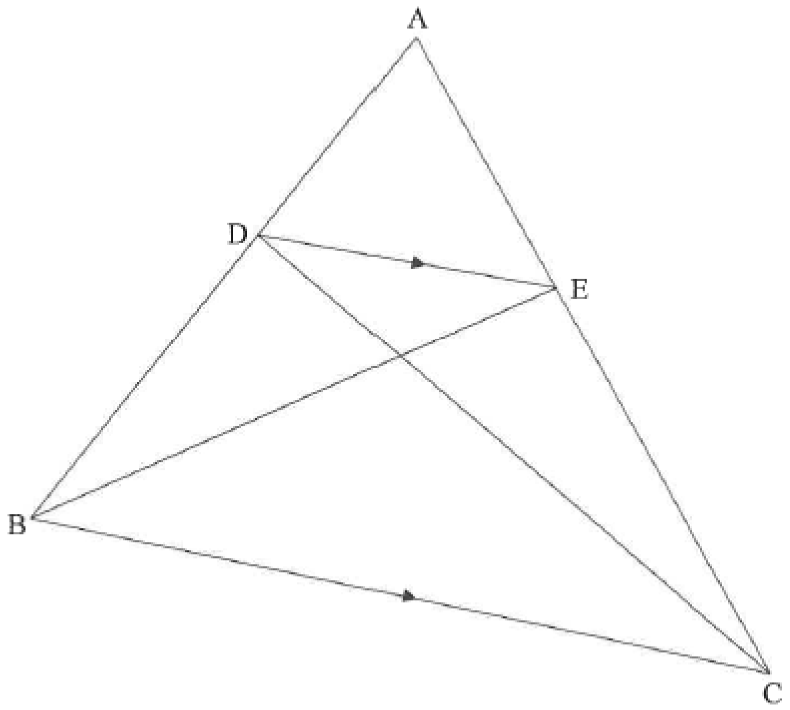
In the diagram, O is the centre of the circle. ST is a tangent to the circle at T. M and P are points on the circle such that TM = MP. OT, OP and TP are drawn. Let $\hat{O}_1 = x$.



Prove, with reasons, that $S\hat{T}M = \frac{1}{4}x$. **[7]**

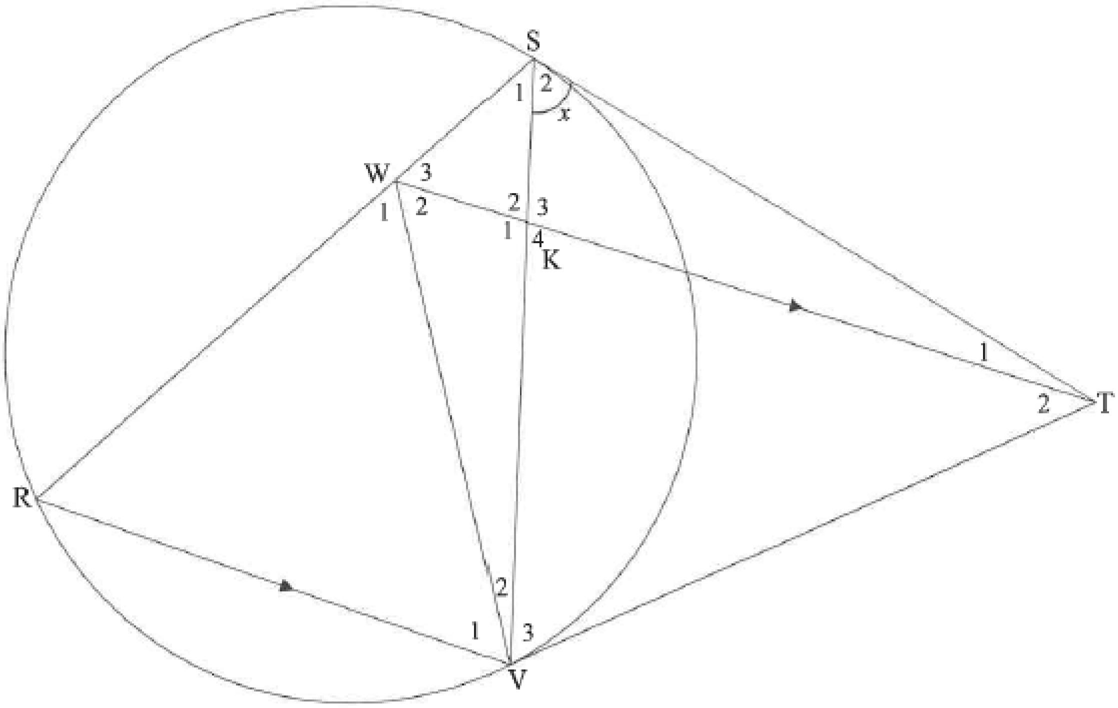
QUESTION 10

10.1 In the diagram, ΔABC is drawn. D is a point on AB and E is a point on AC such that $DE \parallel BC$. BE and DC are drawn.



Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, in other words prove that $\frac{AD}{DB} = \frac{AE}{EC}$ (6)

10.2 In the diagram, ST and VT are tangents to the circle at S and V respectively. R is a point on the circle and W is a point on chord RS such that WT is parallel to RV. SV and WV are drawn. WT intersects SV at K. Let $\hat{S}_2 = x$.



10.2.1 Write down, with reasons, THREE other angles EACH equal to x. (6)

10.2.2 Prove, with reasons, that:

(a) WSTV is a cyclic quadrilateral (2)

(b) ΔWRV is isosceles (4)

(c) $\Delta WRV \parallel\!| \Delta TSV$ (3)

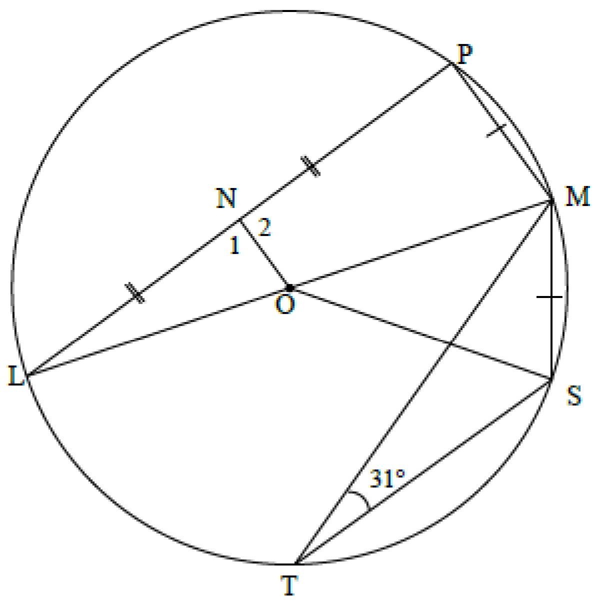
(d) $\frac{RV}{SR} = \frac{KV}{TS}$ (4)

[25]

<u>May-June 2019</u>

QUESTION 8

8.1 In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn. PM = MS and $\hat{MTS} = 31°$



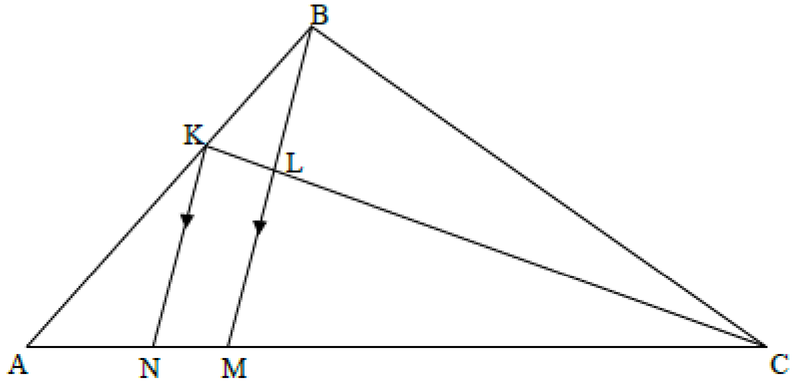
8.1.1 Determine, with reasons, the size of each of the following angles:

(a) $\hat{MOS}$ (2)

(b) $\hat{L}$ (2)

8.1.2 Prove that $ON = \frac{1}{2}MS$. (4)

8.2 In ΔABC in the diagram, K is a point on AB such that AK : KB = 3 : 2. N and M are points on AC such that KN || BM. BM intersects KC at L. AM : MC = 10 : 23.



Determine, with reasons, the ratio of:

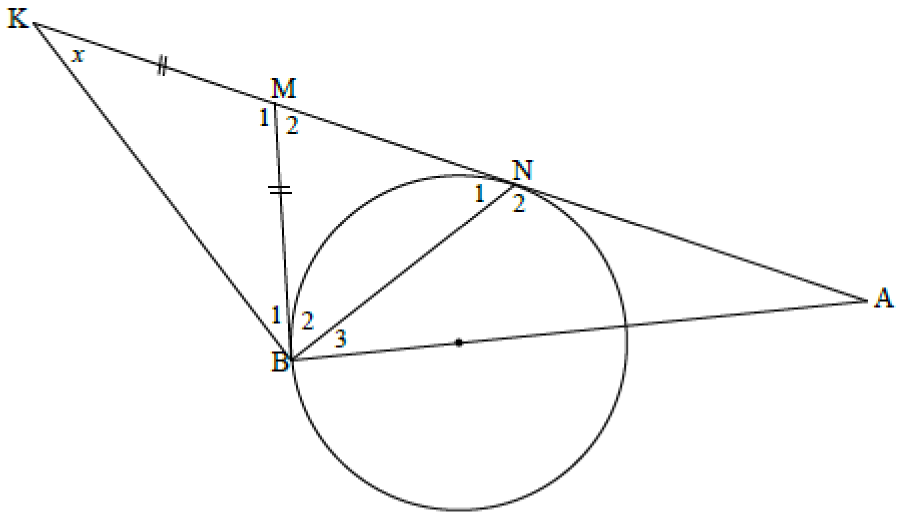
8.2.1 $\dfrac{AN}{AM}$ (2)

8.2.2 $\dfrac{CL}{LK}$ (3)

[13]

QUESTION 9

In the diagram, tangents are drawn from point M outside the circle, to touch the circle at B and N. The straight line from B passing through the centre of the circle meets MN produced in A. NM is produced to K such that BM = MK. BK and BN are drawn.
Let $\hat{K} = x$.

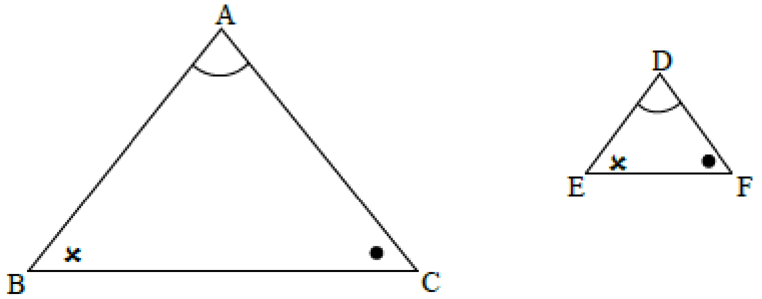


9.1 Determine, with reasons, the size of $\hat{N}_1$ in terms of x. (6)

9.2 Prove that BA is a tangent to the circle passing through K, B and N. (5)
[11]

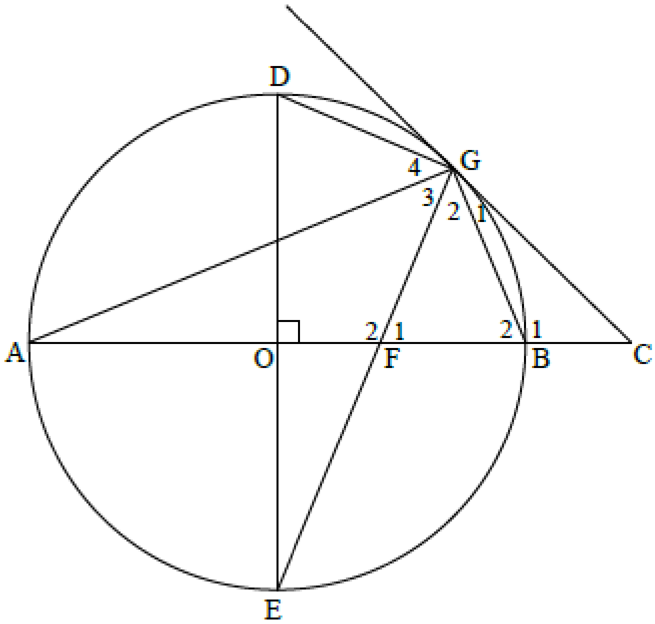
QUESTION 10

10.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE} = \frac{AC}{DF}$. (6)

10.2 In the diagram, O is the centre of the circle and CG is a tangent to the circle at G. The straight line from C passing through O cuts the circle at A and B. Diameter DOE is perpendicular to CA. GE and CA intersect at F. Chords DG, BG and AG are drawn.



10.2.1 Prove that:

(a) DGFO is a cyclic quadrilateral (3)

(b) GC = CF (5)

10.2.2 If it is further given that CO = 11 units and DE = 14 units, calculate:

(a) The length of BC (3)

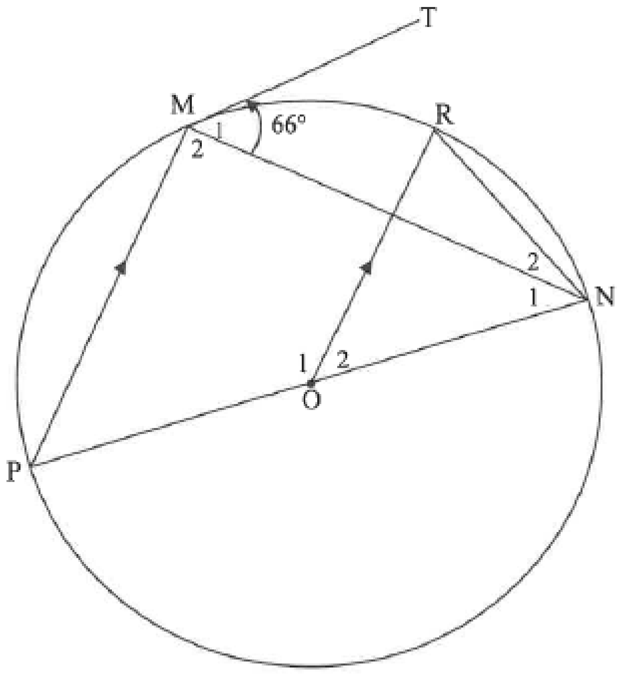
(b) The length of CG (5)

(c) The size of $\hat{E}$. (4)

November 2018

QUESTION 8

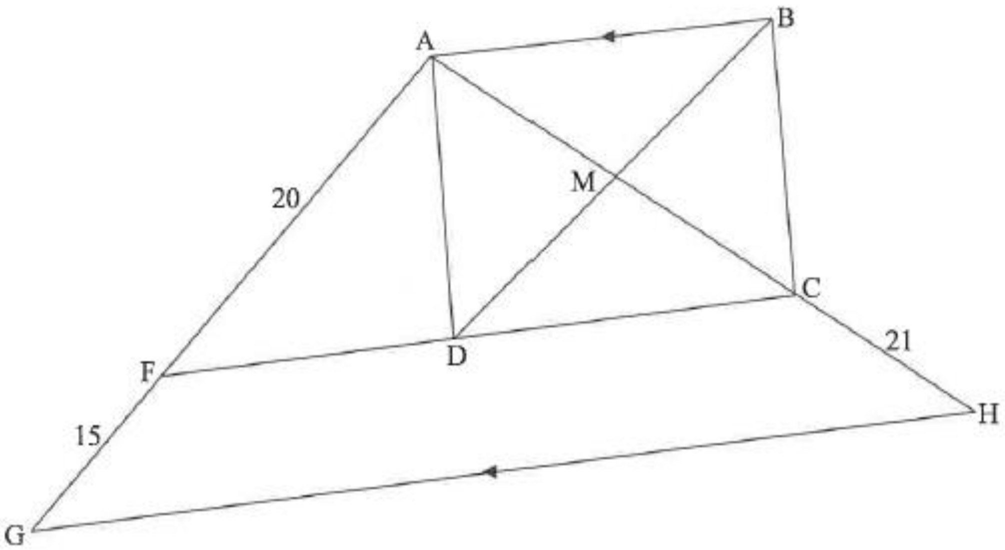
8.1 PON is a diameter of the circle centred at O. TM is a tangent to the circle at M, a point on the circle. R is another point on the circle such that OR ∥ PM. NR and MN are drawn. Let $\hat{M}_1 = 66°$.



Calculate, with reasons, the size of EACH of the following angles:

8.1.1	$\hat{P}$	(2)
8.1.2	$\hat{M}_2$	(2)
8.1.3	$\hat{N}_1$	(1)
8.1.4	$\hat{O}_2$	(2)
8.1.5	$\hat{N}_2$	(3)

8.2 In the diagram, ΔAGH is drawn. F and C are points on AG and AH respectively such that AF = 20 units, FG = 15 units and CH = 21 units. D is a point on FC such that ABCD is a rectangle with AB also parallel to GH. The diagonals of ABCD intersect at M, a point on AH.

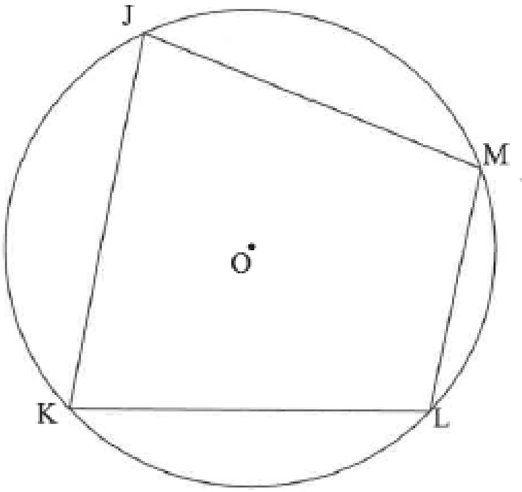


8.2.1 Explain why FC || GH. (1)

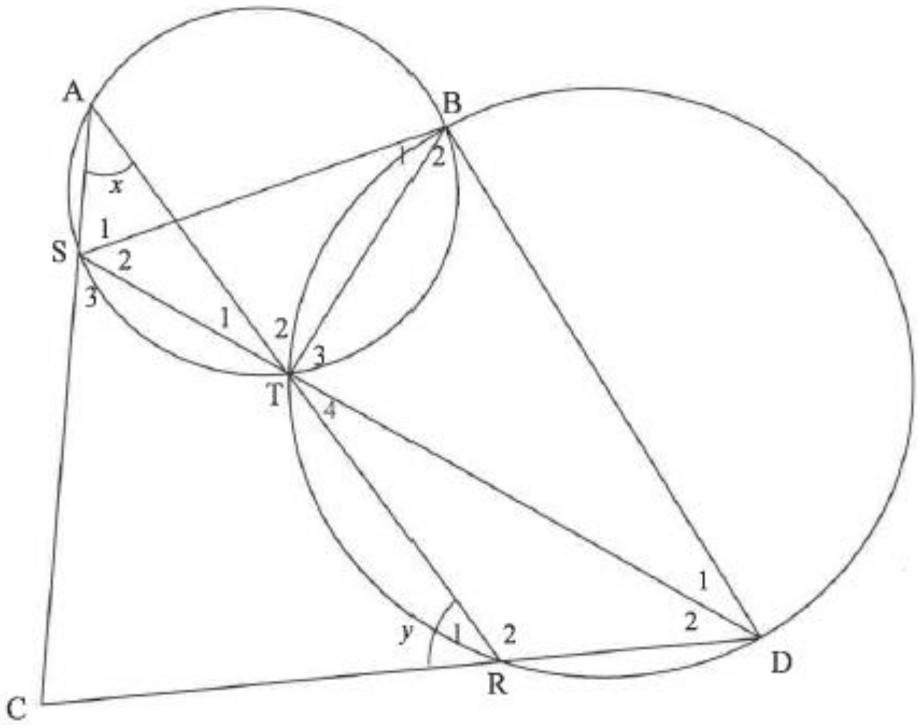
8.2.2 Calculate, with reasons, the length of DM. (5)
[16]

QUESTION 9

9.1 In the diagram, JKLM is a cyclic quadrilateral and the circle has centre O.
Prove the theorem which states that $\hat{J} + \hat{L} = 180°$. (5)



9.2 In the diagram, a smaller circle ABTS and a bigger circle BDRT are given. BT is a common chord. Straight lines STD and ATR are drawn. Chords AS and DR are produced to meet in C, a point outside the two circles. BS and BD are drawn.
$\hat{A} = x$ and $\hat{R}_1 = y$.



9.2.1 Name, giving a reason, another angle equal to:

(a) x (2)

(b) y (2)

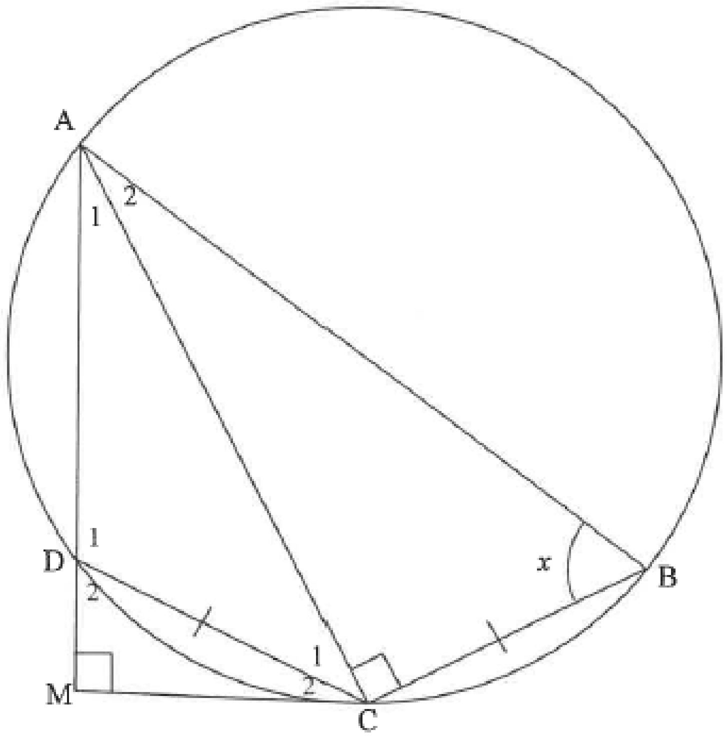
9.2.2 Prove that SCDB is a cyclic quadrilateral. (3)

9.2.3 It is further given that $\hat{D}_2 = 30°$ and $A\hat{S}T = 100°$.
Prove that SD is not a diameter of circle BDS. (4)

[16]

QUESTION 10

In the diagram, ABCD is a cyclic quadrilateral such that $AC \perp CB$ and $DC = CB$. AD is produced to M such that $AM \perp MC$. Let $\hat{B} = x$.



10.1 Prove that:

10.1.1 MC is a tangent to the circle at C (5)

10.1.2 $\Delta ACB \parallel\!| \Delta CMD$ (3)

10.2 Hence, or otherwise, prove that:

10.2.1 $\dfrac{CM^2}{DC^2} = \dfrac{AM}{AB}$ (6)

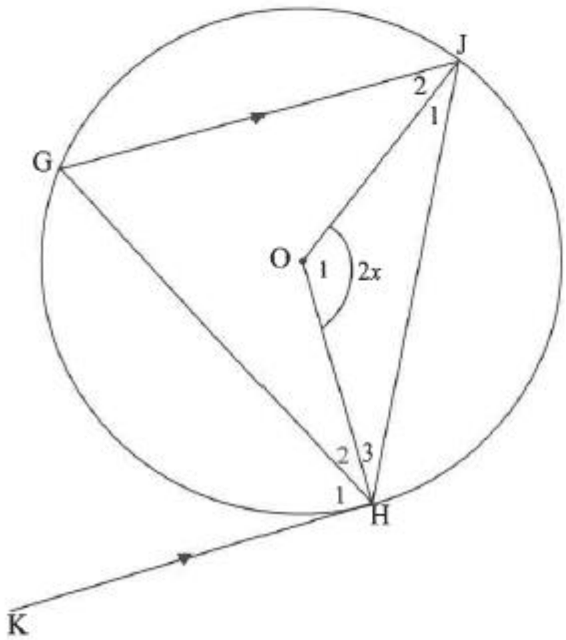
10.2.2 $\dfrac{AM}{AB} = \sin^2 x$ (2)

[16]

June 2018

QUESTION 8

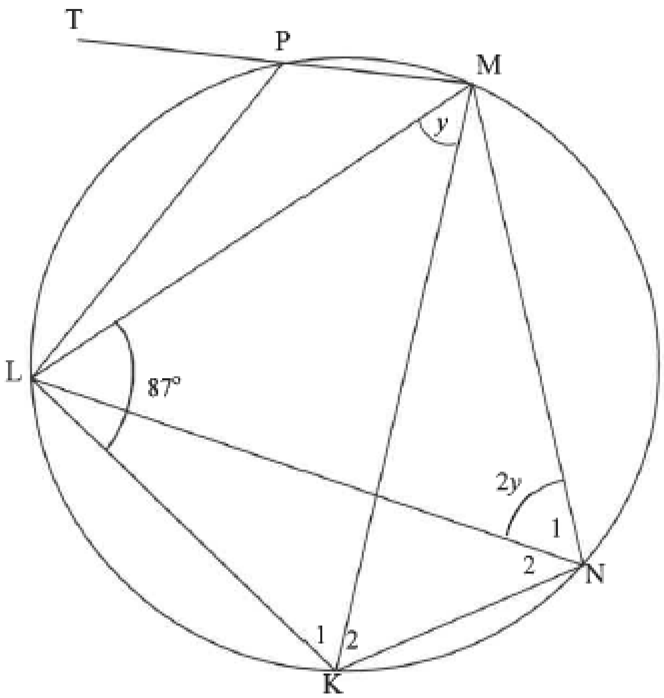
8.1 In the diagram, O is the centre of the circle. Radii OH and OJ are drawn. A tangent is drawn from K to touch the circle at H. ΔHGJ is drawn such that $GJ \parallel KH$. $\hat{O}_1 = 2x$.



8.1.1 Name, giving reasons, THREE angles, each equal to x. (5)

8.1.2 Prove that $\hat{H}_2 = \hat{H}_3$. (3)

8.2 In the diagram, KLMN is a cyclic quadrilateral with $K\hat{L}M = 87°$. Diagonals LN and MK are drawn. P is a point on the circle and MP is produced to T, a point outside the circle. Chord LP is drawn. $L\hat{M}K = y$ and $\hat{N}_1 = 2y$.



8.2.1 Name, giving a reason, another angle equal to y. (2)

8.2.2 Calculate, giving reasons, the size of:

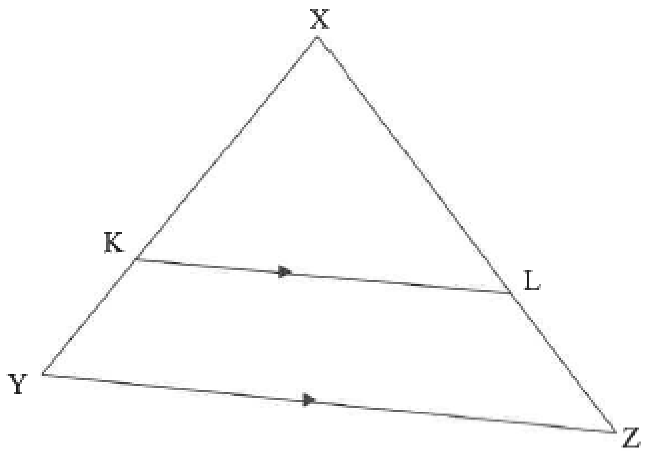
(a) y (3)

(b) $T\hat{P}L$ (2)

[15]

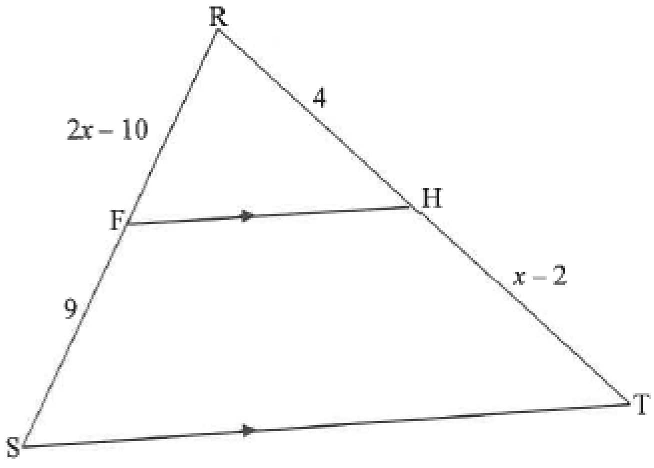
QUESTION 9

9.1 Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, that is prove that $\frac{XK}{KY} = \frac{XL}{LZ}$.



(5)

9.2 In ΔRST, F is a point on RS and H is a point on RT such that FH $\parallel$ ST.
RF = $2x - 10$, FS = 9, RH = 4 and HT = $x - 2$.



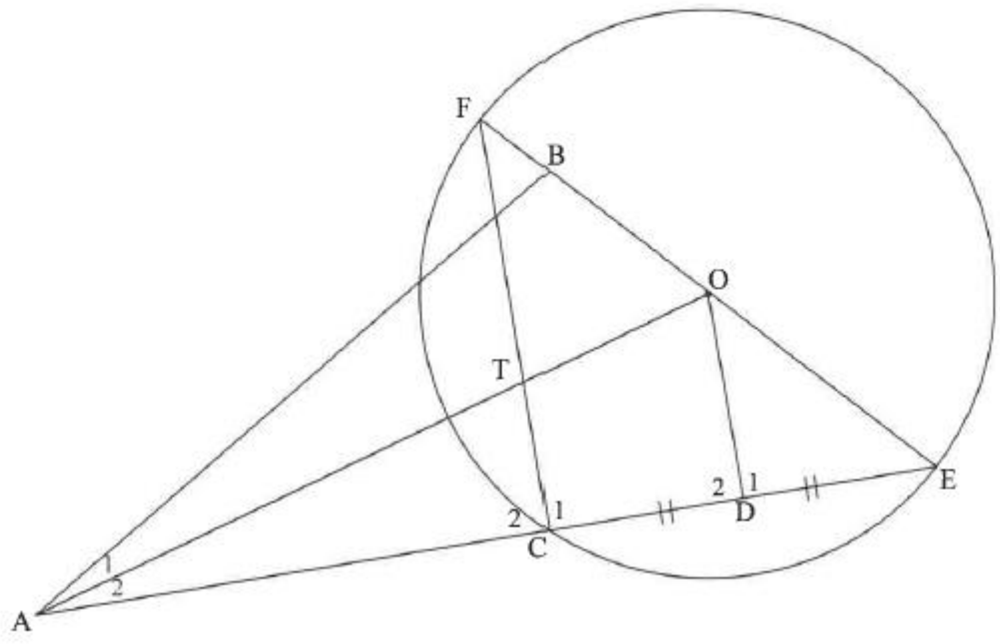
9.2.1 Determine, giving a reason, the value of x. (5)

9.2.2 Determine the ratio: $\dfrac{\text{area } \Delta RFH}{\text{area } \Delta RST}$. (4)

[14]

QUESTION 10

In the diagram, FBOE is a diameter of a circle with centre O. Chord EC produced meets line BA at A, outside the circle. D is the midpoint of CE. OD and FC are drawn. AFBC is a cyclic quadrilateral.



10.1 Prove, giving reasons, that:

10.1.1 $FC \parallel OD$ (5)

10.1.2 $D\hat{O}E = B\hat{A}E$ (4)

10.1.3 $AB \times OF = AE \times OD$ (7)

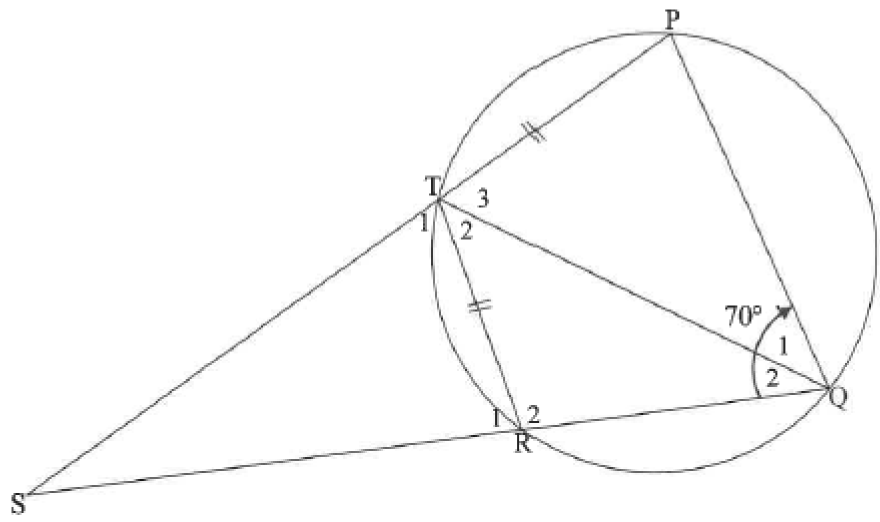
10.2 If it is further given that $AT = 3TO$, prove that $5CE^2 = 2BE.FE$ (5)

[21]

March 2018

QUESTION 7

In the diagram, PQRT is a cyclic quadrilateral in a circle such that PT = TR. PT and QR are produced to meet in S. TQ is drawn. $S\hat{Q}P = 70°$



7.1 Calculate, with reasons, the size of:

7.1.1 $\hat{T}_1$ (2)

7.1.2 $\hat{Q}_1$ (2)

7.2 If it is further given that PQ ∥ TR:

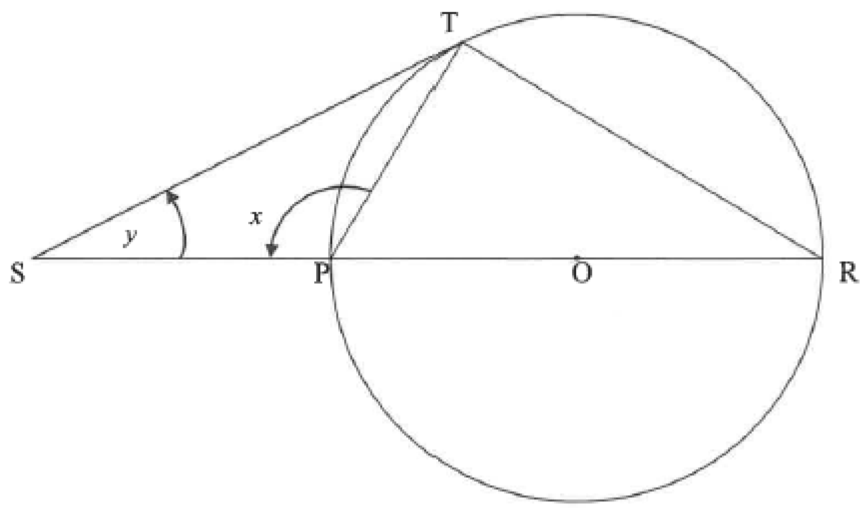
7.2.1 Calculate, with reasons, the size of $\hat{T}_2$ (2)

7.2.2 Prove that $\frac{TR}{TS} = \frac{RQ}{RS}$ (2)

[8]

QUESTION 8

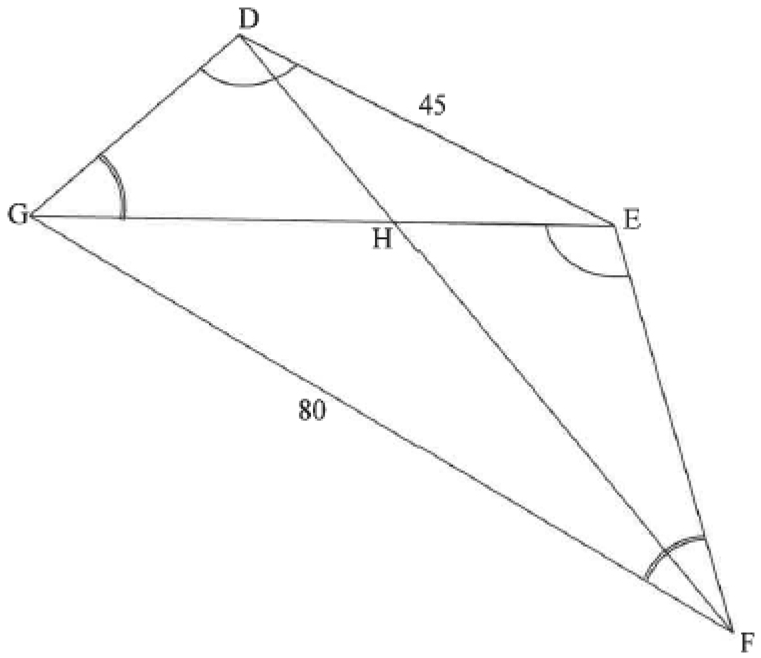
In the diagram, PR is a diameter of the circle with centre O. ST is a tangent to the circle at T and meets RP produced at S. $\hat{SPT} = x$ and $\hat{S} = y$.



Determine, with reasons, y in terms of x. **[6]**

QUESTION 9

In the diagram, DEFG is a quadrilateral with $DE = 45$ and $GF = 80$. The diagonals GE and DF meet in H. $G\hat{D}E = F\hat{E}G$ and $D\hat{G}E = E\hat{F}G$.



9.1 Give a reason why $\Delta DEG \,|||\, \Delta EGF$. (1)

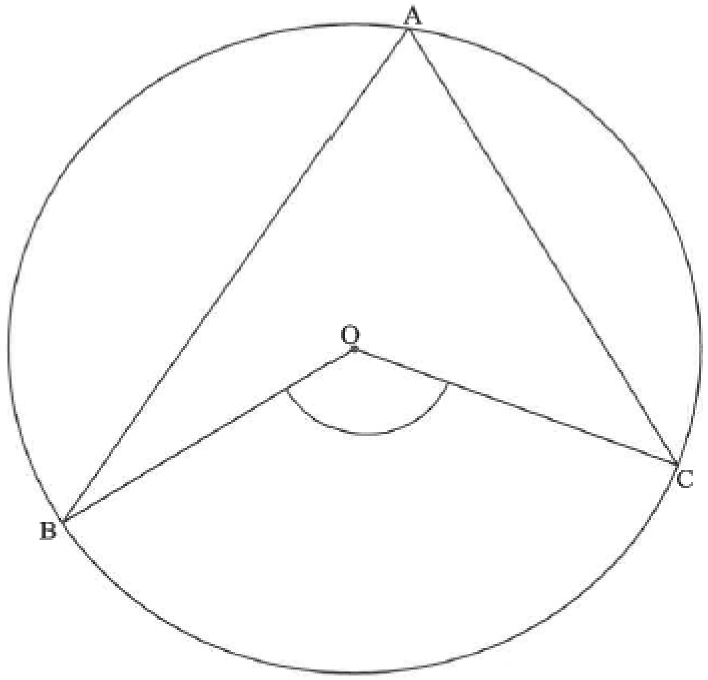
9.2 Calculate the length of GE. (3)

9.3 Prove that $\Delta DEH \,|||\, \Delta FGH$. (3)

9.4 Hence, calculate the length of GH. (3)
[10]

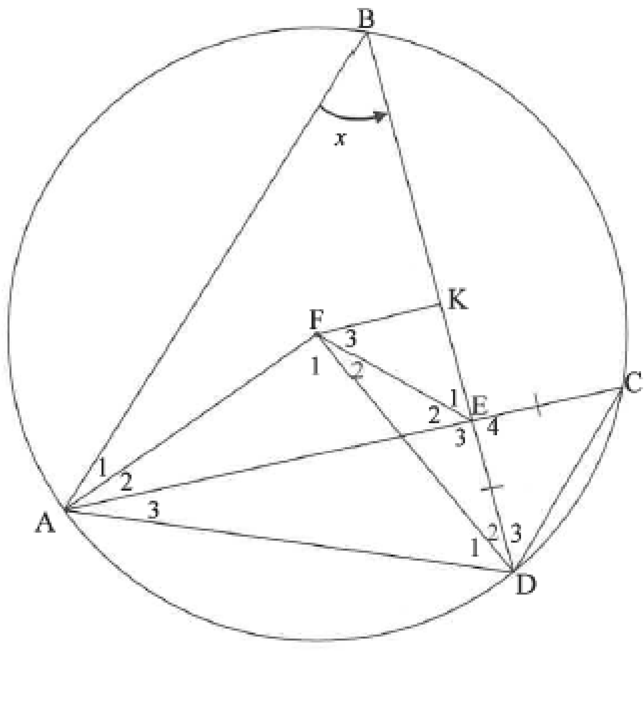
QUESTION 10

10.1 In the diagram, O is the centre of the circle with A, B and C drawn on the circle.



Prove the theorem which states that $B\hat{O}C = 2\hat{A}$. (5)

10.2 In the diagram, the circle with centre F is drawn. Points A, B, C and D lie on the circle. Chords AC and BD intersect at E such that EC = ED. K is the midpoint of chord BD. FK, AB, CD, AF, FE and FD are drawn. Let $\hat{B} = x$.



10.2.1 Determine, with reasons, the size of EACH of the following in terms of x:

(a) $\hat{F}_1$ (2)

(b) $\hat{C}$ (2)

10.2.2 Prove, with reasons, that AFED is a cyclic quadrilateral. (4)

10.2.3 Prove, with reasons, that $\hat{F}_3 = x$. (6)

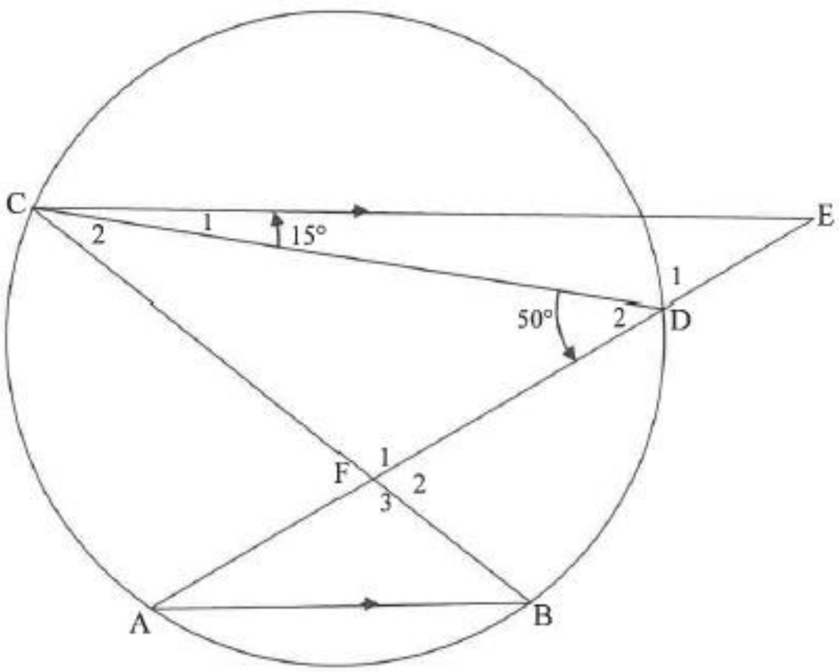
10.2.4 If area $\Delta AEB = 6{,}25 \times$ area ΔDEC, calculate $\frac{AE}{ED}$. (5)

[24]

<u>November 2017:</u>

QUESTION 8

In the diagram, points A, B, D and C lie on a circle. CE || AB with E on AD produced. Chords CB and AD intersect at F. $\hat{D}_2 = 50°$ and $\hat{C}_1 = 15°$.



8.1 Calculate, with reasons, the size of:

8.1.1 $\hat{A}$ (3)

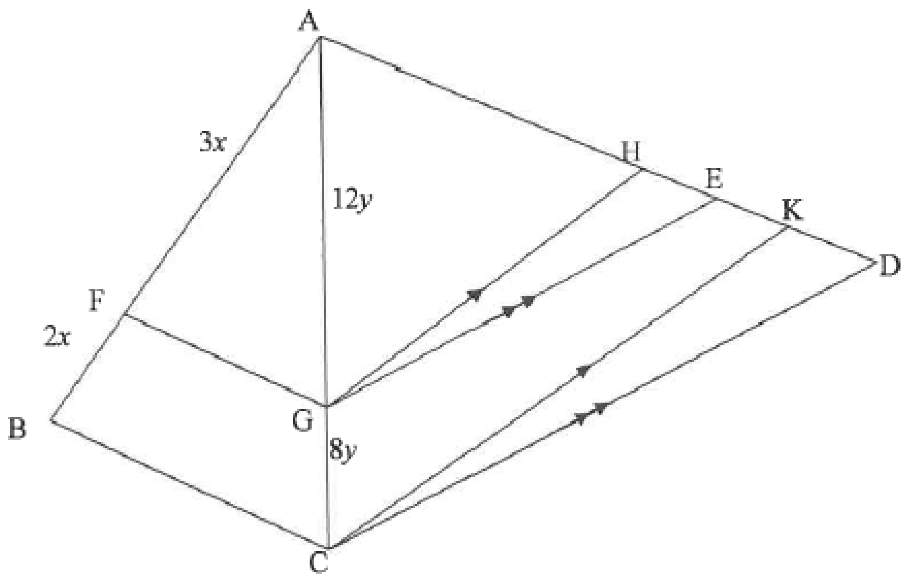
8.1.2 $\hat{C}_2$ (2)

8.2 Prove, with a reason, that CF is a tangent to the circle passing through points C, D and E. (2)

[7]

QUESTION 9

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that AF = $3x$, FB = $2x$, AG = $12y$ and GC = $8y$. H, E and K are points on side AD such that GH || CK and GE || CD.



9.1 Prove that:

9.1.1 FG || BC (2)

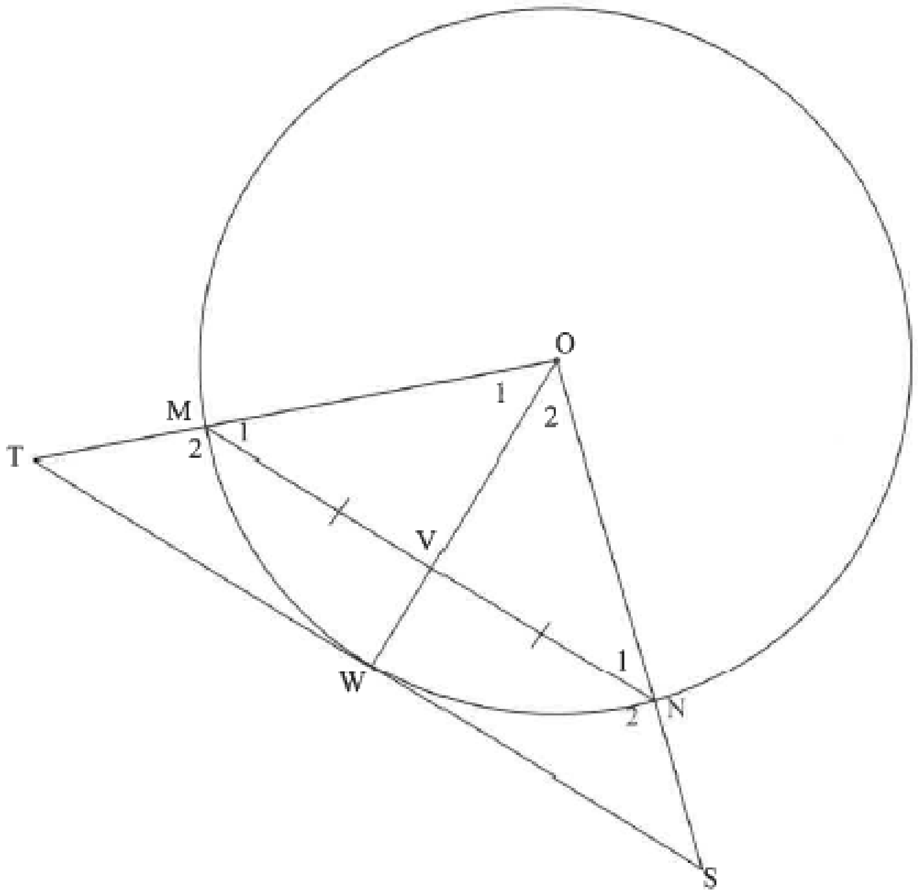
9.1.2 $\frac{AH}{HK} = \frac{AE}{ED}$ (3)

9.2 If it is further given that AH = 15 and ED = 12, calculate the length of EK. (5)

[10]

QUESTION 10

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.



10.1 Give a reason why OV ⊥ MN. (1)

10.2 Prove that:

10.2.1 MN || TS (2)

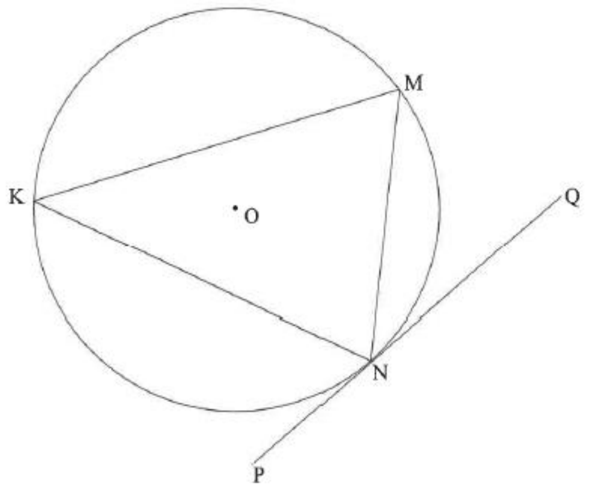
10.2.2 TMNS is a cyclic quadrilateral (4)

10.2.3 OS . MN = 2ON . WS (5)

[12]

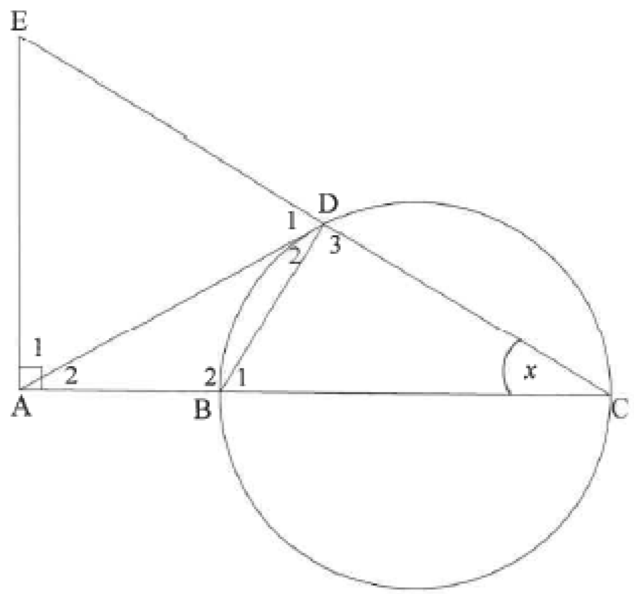
QUESTION 11

11.1 In the diagram, chords KM, MN and KN are drawn in the circle with centre O. PNQ is the tangent to the circle at N.



Prove the theorem which states that $M\hat{N}Q = \hat{K}$. (5)

11.2 In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that EA ⊥ AC. BD is drawn.
Let $\hat{C} = x$.



11.2.1 Give a reason why:

(a) $\hat{D}_3 = 90°$ (1)

(b) ABDE is a cyclic quadrilateral (1)

(c) $\hat{D}_2 = x$ (1)

11.2.2 Prove that:

(a) AD = AE (3)

(b) ΔADB ||| ΔACD (3)

11.2.3 It is further given that BC = 2AB = 2r.

(a) Prove that $AD^2 = 3r^2$ (2)

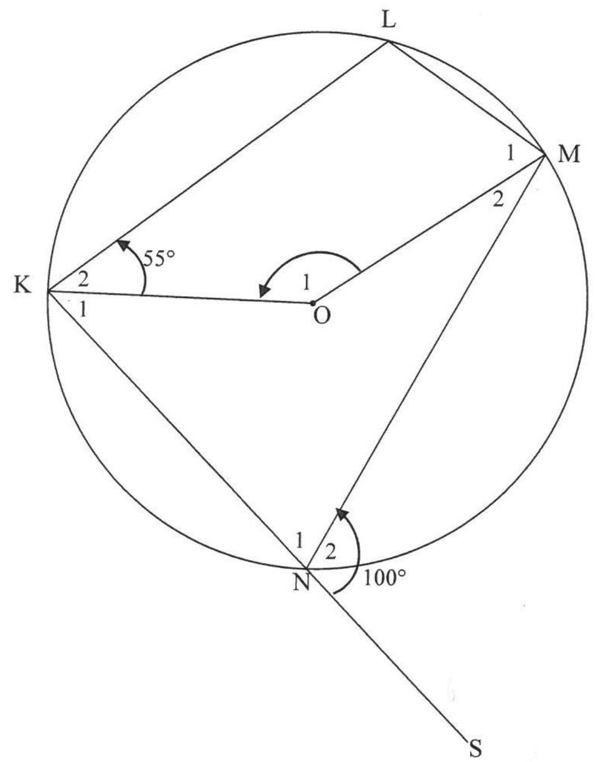
(b) Hence, prove that ΔADE is equilateral. (4)

[20]

June 2017:

QUESTION 8

In the diagram, O is the centre of circle KLMN and KO and OM are joined. Chord KN is produced to S. $\hat{K}_2 = 55°$ and $\hat{N}_2 = 100°$.



Determine, with reasons, the size of the following:

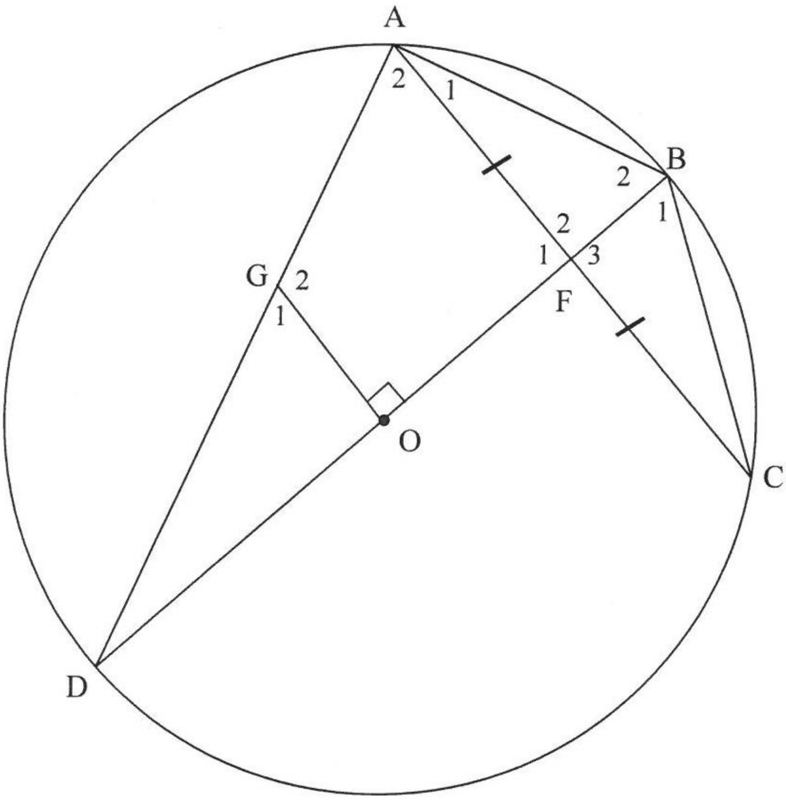
8.1 $\hat{L}$ (2)

8.2 $\hat{O}_1$ (3)

8.3 $\hat{M}_1$ (2)
[7]

QUESTION 9

In the diagram, O is the centre of circle ABCD and BOD is a diameter. F, the midpoint of chord AC, lies on BOD. G is a point on AD such that $GO \perp DB$.



9.1 Give a reason why:

9.1.1 $D\hat{A}B = 90°$ (1)

9.1.2 AGOB is a cyclic quadrilateral (1)

9.2 Prove that:

9.2.1 $AC \parallel GO$ (3)

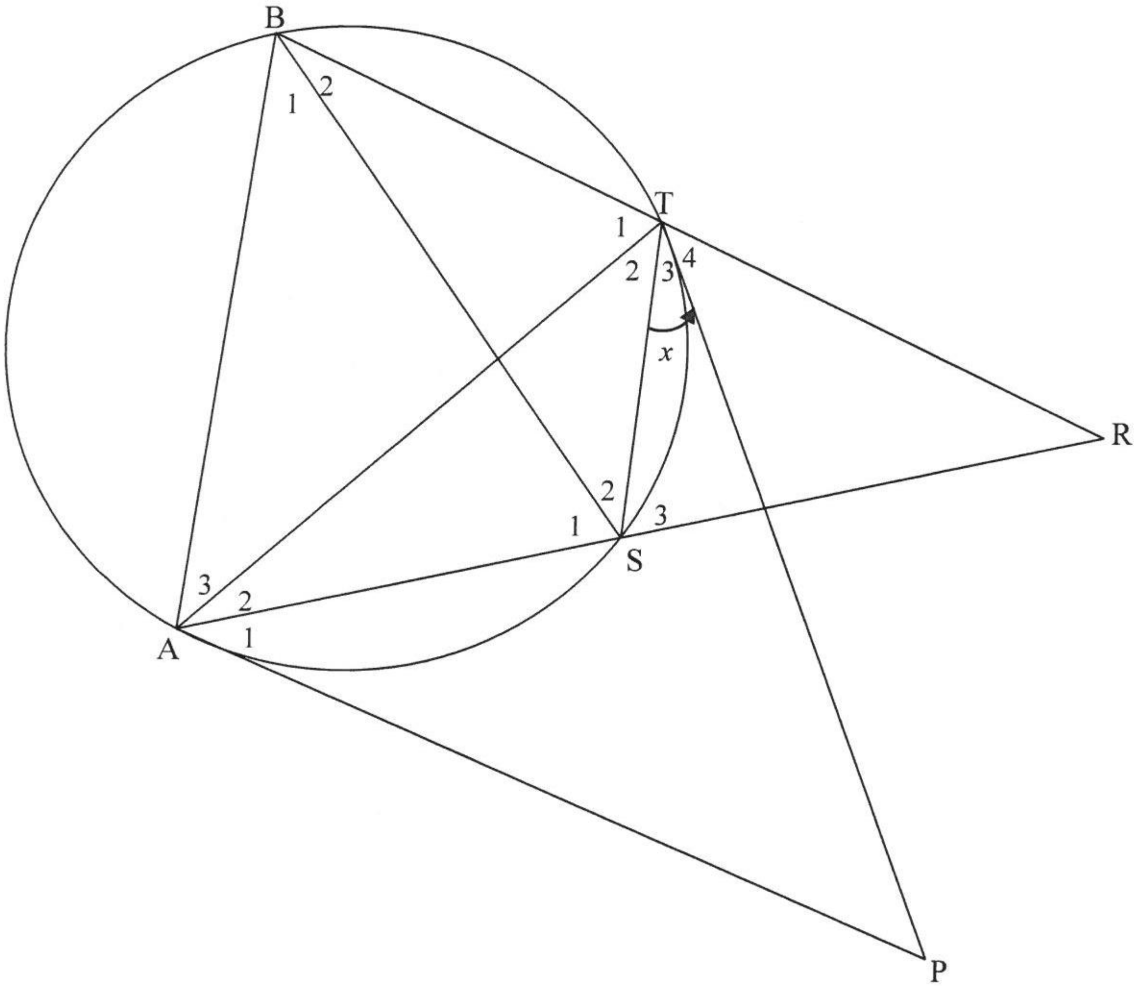
9.2.2 $\hat{G}_1 = \hat{B}_1$ (4)

9.3 If it is given that $FB = \frac{2}{5}r$, where r is the radius of the circle, determine, with reasons, the ratio of $\frac{DG}{DA}$. (3)

[12]

QUESTION 10

In the diagram, PA and PT are tangents to a circle at A and T respectively. B and S are points on the circle such that BT produced and AS produced meet at R and BR = AR.
BS, AT and TS are drawn. $\hat{T}_3 = x$.



10.1 Give a reason why $\hat{T}_3 = \hat{A}_2 = x$. (1)

10.2 Prove that:

10.2.1 AB || ST (5)

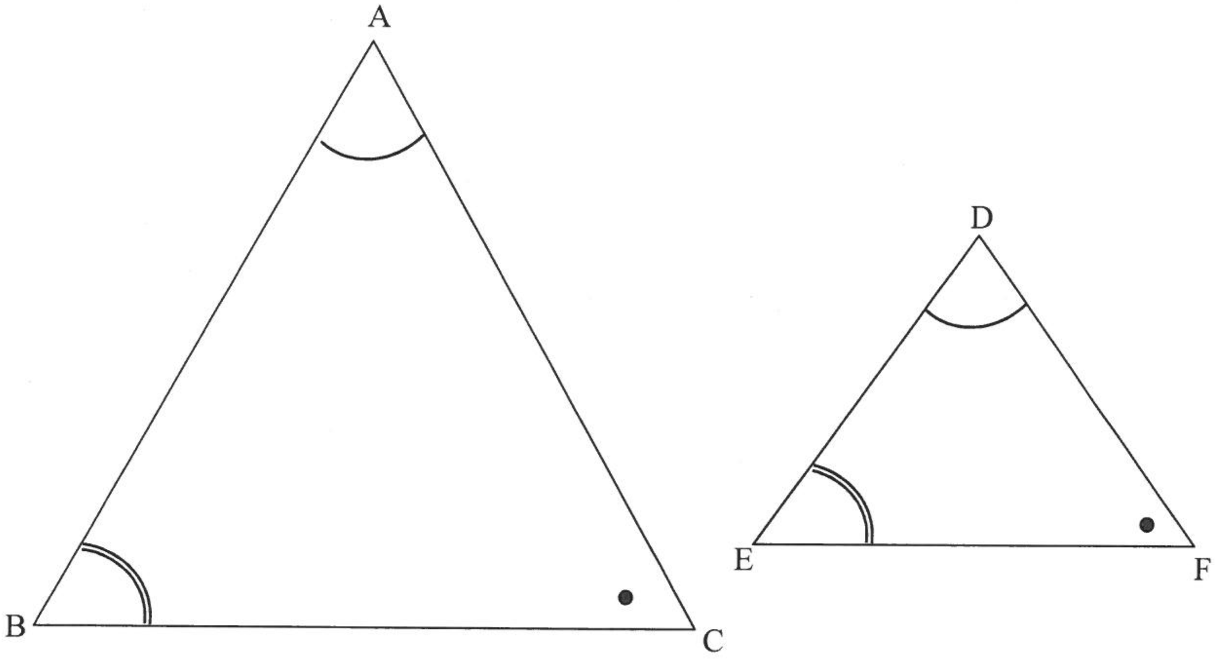
10.2.2 $\hat{T}_4 = \hat{A}_1$ (5)

10.2.3 RTAP is a cyclic quadrilateral (2)

[13]

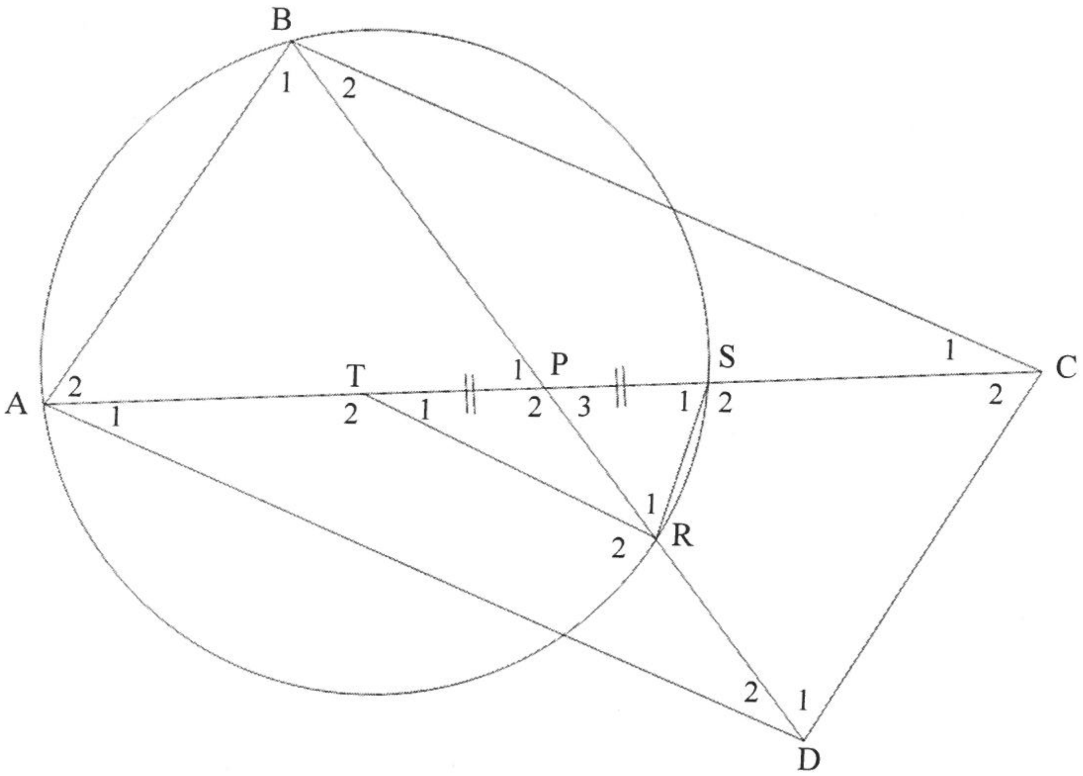
QUESTION 11

11.1 In the diagram, ΔABC and ΔDEF are drawn with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Prove the theorem which states that if two triangles, ΔABC and ΔDEF, are equiangular, then $\frac{DE}{AB} = \frac{DF}{AC}$. (6)

11.2 In the diagram, ABCD is a parallelogram with A and B on the circle. The diagonals BD and AC intersect in P. PC and PD intersect the circle at S and R respectively. T is a point on AP such that TP = PS. TR is drawn.



11.2.1 Prove that:

(a) AT = SC (2)

(b) ΔPSR ||| ΔPBA (5)

11.2.2 If it is further given that $\frac{PR}{PA} = \frac{TR}{AD}$, prove that:

(a) ΔRPT ||| ΔAPD (3)

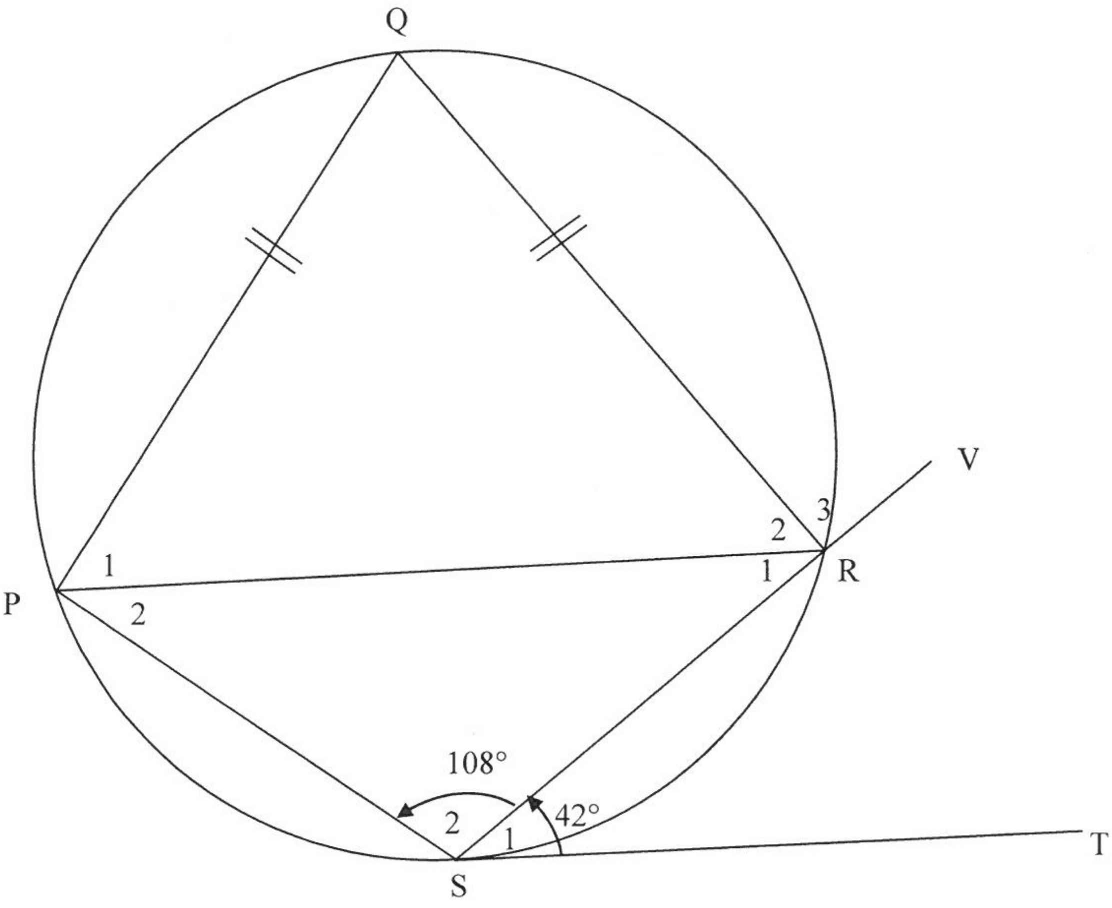
(b) ATRD is a cyclic quadrilateral (2)

[18]

March 2017:

QUESTION 8

In the diagram, PQRS is a cyclic quadrilateral. ST is a tangent to the circle at S and chord SR is produced to V. PQ = QR, $\hat{S}_1 = 42°$ and $\hat{S}_2 = 108°$.



Determine, with reasons, the size of the following angles:

8.1 $\hat{Q}$ (2)

8.2 $\hat{R}_2$ (2)

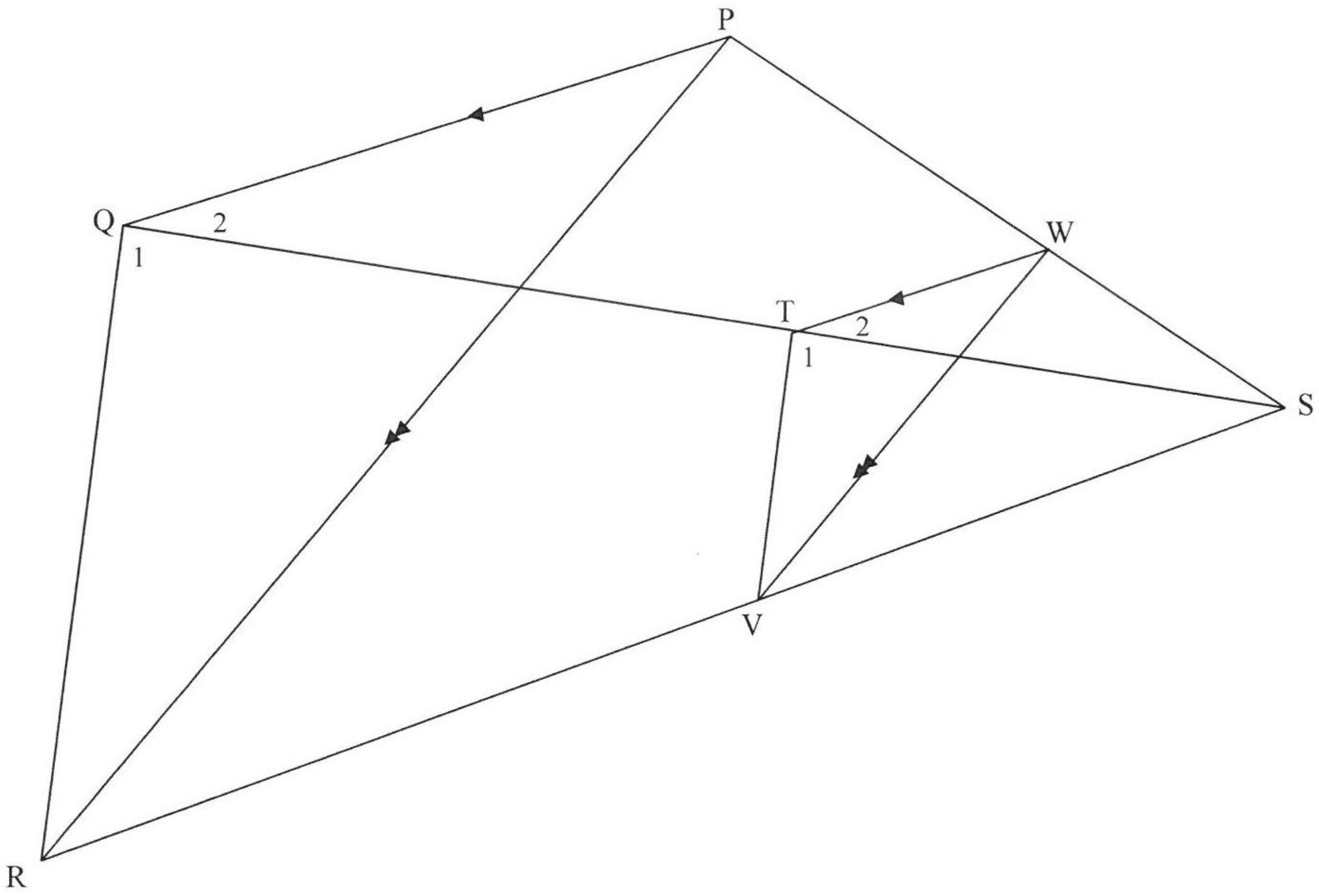
8.3 $\hat{P}_2$ (2)

8.4 $\hat{R}_3$ (2)

[8]

QUESTION 9

In the diagram, PQRS is a quadrilateral with diagonals PR and QS drawn. W is a point on PS. WT is parallel to PQ with T on QS. WV is parallel to PR with V on RS. TV is drawn. PW : WS = 3 : 2.



9.1 Write down the value of the following ratios:

9.1.1 $\frac{ST}{TQ}$ (2)

9.1.2 $\frac{SV}{VR}$ (1)

9.2 Prove that $\hat{T}_1 = \hat{Q}_1$. (4)

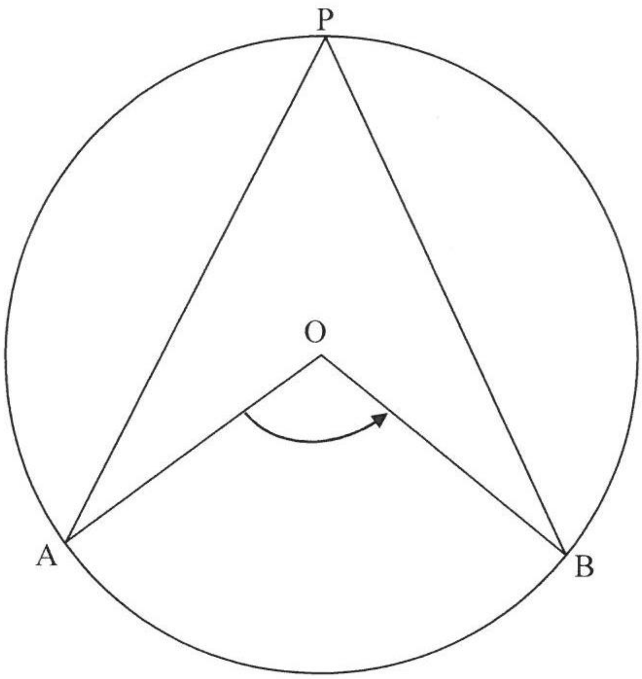
9.3 Complete the following statement: $\Delta VWS \, ||| \, \Delta \ldots$ (1)

9.4 Determine WV : PR. (2)

[10]

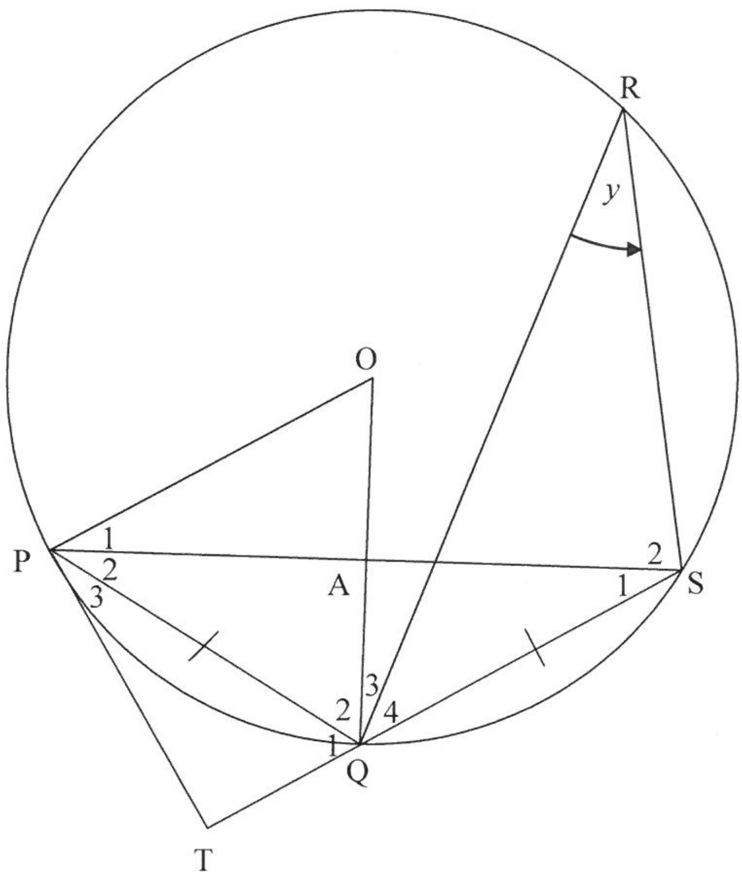
QUESTION 10

10.1 In the diagram, O is the centre of the circle and P is a point on the circumference of the circle. Arc AB subtends $A\hat{O}B$ at the centre of the circle and $A\hat{P}B$ at the circumference of the circle.



Use the diagram to prove the theorem that states that $A\hat{O}B = 2A\hat{P}B$. (5)

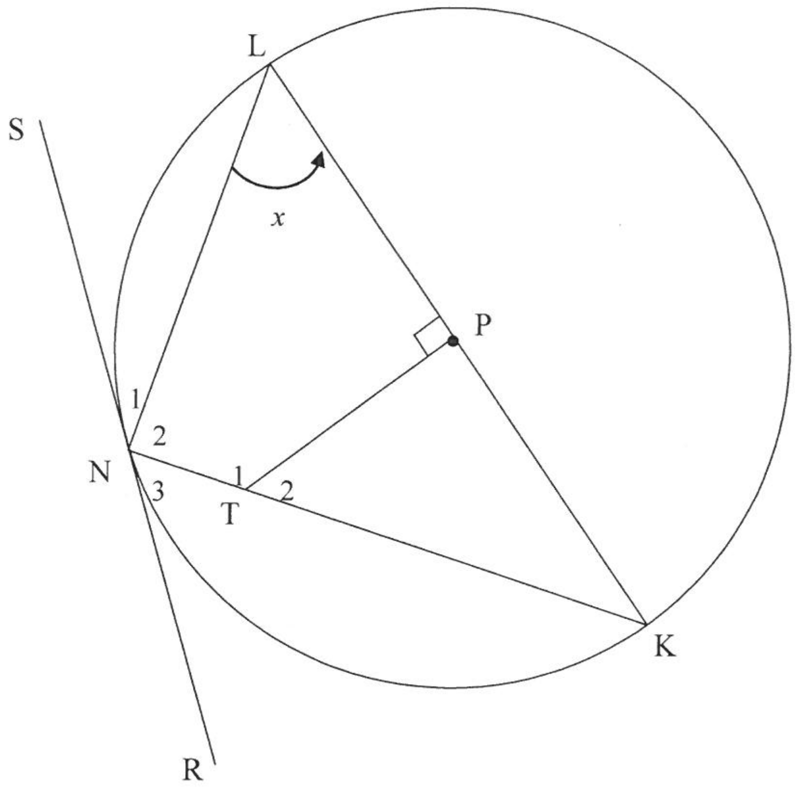
10.2 In the diagram, O is the centre of the circle and P, Q, S and R are points on the circle. PQ = QS and $Q\hat{R}S = y$. The tangent at P meets SQ produced at T. OQ intersects PS at A.



10.2.1	Give a reason why $\hat{P}_2 = y$.	(1)
10.2.2	Prove that PQ bisects $T\hat{P}S$.	(4)
10.2.3	Determine $P\hat{O}Q$ in terms of y.	(2)
10.2.4	Prove that PT is a tangent to the circle that passes through points P, O and A.	(2)
10.2.5	Prove that $O\hat{A}P = 90°$.	(5)
		[19]

QUESTION 11

In the diagram, LK is a diameter of the circle with centre P. RNS is a tangent to the circle at N. T is a point on NK and $TP \perp KL$. $P\hat{L}N = x$.



11.1 Prove that TPLN is a cyclic quadrilateral. (3)

11.2 Determine, giving reasons, the size of $\hat{N}_1$ in terms of x. (3)

11.3 Prove that:

11.3.1 $\Delta KTP \,|||\, \Delta KLN$ (3)

11.3.2 $KT \cdot KN = 2KT^2 - 2TP^2$ (5)

[14]

Answers to Questions

SESSION 1

Solving Equations, Inequalities and Simplifying Expressions

Nov 2019

Q1.1.1

$x^2+5x-6=0$

$(x+6)(x-1)=0$

$x=-6 \quad \text{or} \quad x=1$

Q1.1.2

$4x^2+3x-5=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-3\pm\sqrt{(3)^2-4(4)(-5)}}{2(4)}$$

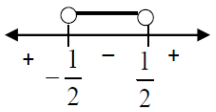
$$x=\frac{-3\pm\sqrt{89}}{8}$$

$x=-1{,}55 \quad \text{or} \quad x=0{,}8$

Q1.1.3

$4x^2-1<0$

$(2x+1)(2x-1)<0$



$\frac{-1}{2}<x<\frac{1}{2}$

Q1.1.4

$\sqrt{32-x^2}=x$

$32-x^2=x^2$

$-2x^2=-32$

$x^2=16$

$x=\pm4$

$\therefore x=4$

Q1.2

$y+x=12$

$y=-x+12 \text{..........}(1)$

$xy=14-3x \text{........}(2)$

Sub (1) into (2)

$x(-x+12)=14-3x$

$-x^2+12x-14+3x=0$

$-x^2+15x-14=0$

$x^2-15x+14=0$

$(x-14)(x-1)=0$

$x=14 \quad \text{or} \quad x=1$

$y=-2 \quad \text{or} \quad y=11$

Q1.3

3	6	9	12	15	18	21	24	27	30
3	3	3^2	3	3	3^2	3	3	3^3	3

$\therefore k=14$

May-June 2019

Q1.1.1

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x=6 \quad \text{or} \quad x=-1$

Q1.1.2

$(3x-1)(x-4)=16$

$3x^2-13x-12=0$

$$x=\frac{13\pm\sqrt{(-13)^2-4(3)(-12)}}{2(3)}$$

$$x=\frac{13\pm\sqrt{313}}{6}$$

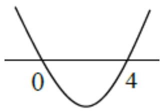
$x=5{,}12 \quad \text{or} \quad x=-0{,}78$

Q1.1.3

$x(4-x)\geq 0$

$-x(x-4)\geq 0$

$x(x-4)\leq 0$



$0\leq x\leq 4$

Q1.1.4

$$\frac{5^{2x}-1}{5^x+1}=4$$

$$\frac{(5^x+1)(5^x-1)}{5^x+1}=4$$

$5^x-1=4$

$5^x=5$

$x=1$

Q1.2

$x=2-3y \text{ ....}(1)$

$x^2+4xy-5=0 \text{ ....}(2)$

Substitute (1) in (2):

$(2-3y)^2+4y(2-3y)-5=0$

$4-12y+9y^2+8y-12y^2-5=0$

$-3y^2-4y-1=0$

$3y^2+4y+1=0$

$(3y+1)(y+1)=0$

$y=-\frac{1}{3} \quad \text{or} \quad y=-1$

$x=3 \quad \text{or} \quad x=5$

Q1.3

$ab=2\sqrt{10}$

$bc=3\sqrt{2}$

$ac=6\sqrt{5}$

$ab.bc.ac=2\sqrt{10}.6\sqrt{5}.3\sqrt{2}$

$(abc)^2=36\sqrt{100}$

$abc=\sqrt{360}=6\sqrt{10}$

Nov 2018

Q1.1.1

$x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=3 \quad \text{or} \quad x=1$

Q1.1.2

$5x^2-5x+1=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{5\pm\sqrt{25-4(5)(1)}}{2(5)}$

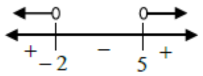
$=\dfrac{5\pm\sqrt{5}}{10}$

$x=0,72$ or $x=0,28$

Q1.1.3

$x^2-3x-10>0$

$(x-5)(x+2)>0$



$x<-2$ or $x>5$

Q1.1.4

$3\sqrt{x}=x-4$

$9x=x^2-8x+16$

$x^2-17x+16=0$

$(x-16)(x-1)=0$

$x=16$ or $x=1$

NA

Q1.2

$2y+9x^2=-1......(1)$

$3x-y=2 \quad ...... \quad (2)$

$y=3x-2 \quad ....(3)$

$2(3x-2)+9x^2=-1$

$6x-4+9x^2=-1$

$9x^2+6x-3=0$

$3x^2+2x-1=0$

$(3x-1)(x+1)=0$

$x=\dfrac{1}{3}$ or $x=-1$

$y=-1$ or $y=-5$

Q1.3

$\dfrac{\left(3^{x-1}\right)^3}{\sqrt{5}^{\sqrt{p}}}=\dfrac{3^{3x}.3^{-3}}{\left(5^{0.5}\right)^{\sqrt{p}}}$

$=\dfrac{3^{3x}.3^{-3}}{\left(5^{\sqrt{p}}\right)^{0.5}}$

$=\dfrac{4.3^{-3}}{\sqrt{64}}$

$=\dfrac{4.\dfrac{1}{27}}{8}=\dfrac{1}{54}$

June 2018

Q1.1.1

$x=\dfrac{1}{3}$ or $x=-4$

Q1.1.2

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{-9\pm\sqrt{9^2-4(2)(-14)}}{2(2)}$

$=\dfrac{-9\pm\sqrt{193}}{4}$

$x=1,22$ or $x=-5,72$

Q1.1.3

$\sqrt{3-26x}=3x$

$3-26x=9x^2$

$9x^2+26x-3=0$

$(9x-1)(x+3)=0$

$x=\dfrac{1}{9}$ or $x=-3$

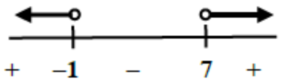
N/A

Q1.1.4

$x^2-5x+4>x+11$

$x^2-6x-7>0$

$(x-7)(x+1)>0$



$x<-1$ or $x>7$

Q1.2

$\dfrac{\sqrt{x^7}(4-5)}{\sqrt{x}}$

$=\sqrt{x^6}(-1)$

$=-x^3$

Q1.3

$x-2y-3=0$

$x=2y+3..............(1)$

$xy=9........................(2)$

Substitute (1) into (2)

$(2y+3)y=9$

$2y^2+3y=9$

$2y^2+3y-9=0$

$(2y-3)(y+3)=0$

$y=\dfrac{3}{2}$ or $y=-3$

$x=6$ or $x=-3$

Q1.4

$x^2+2xy+2y^2$

$=x^2+2xy+y^2+y^2$

$=(x+y)^2+y^2$

$(x+y)^2\geq 0$ and $y^2\geq 0$

Therefore $(x+y)^2+y^2\geq 0$

March 2018

Q1.1.1

$(x-8)(x+2)=0$

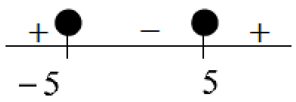
$x=-2$ or $x=8$

Q1.1.2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-7 \pm \sqrt{57}}{4}$$
$x = 0{,}14$ or $x = -3{,}64$

Q1.2

$$x^2 - 25 < 0$$
$$(x-5)(x+5) < 0$$



$$-5 < x < 5$$
$$x = \{-4\,;-3\,;-2\,;-1\,;0\,;1\,;2\,;3\,;4\}$$

Q1.3

$$x = 2y - 1$$
$$(2y-1)^2 - 7 - y^2 = -y$$
$$4y^2 - 4y + 1 - 7 - y^2 = -y$$
$$3y^2 - 3y - 6 = 0$$
$$y^2 - y - 2 = 0$$
$$(y-2)(y+1) = 0$$
$y = 2$ or $y = -1$

$x = 2(2) - 1$ or $x = 2(-1) - 1$

$x = 3$ or $x = -3$

Q1.4

$$\frac{3^{2018} + 3^{2016}}{3^{2017}}$$
$$= \frac{3^{2016}(3^2 + 1)}{3^{2017}}$$
$$= \frac{10}{3}$$

Q1.5.1

$3x - 5 \geq 0$ and $x \neq 3$

$x \geq \frac{5}{3}$ and $x \neq 3$

Q1.5.2

$$\frac{\sqrt{3x-5}}{x-3} = 1$$
$$\sqrt{3x-5} = x - 3$$
$$3x - 5 = (x-3)^2$$
$$3x - 5 = x^2 - 6x + 9$$
$$x^2 - 9x + 14 = 0$$
$$(x-7)(x-2) = 0$$
$x \neq 2$ or $x = 7$

Nov 2017:

Q1.1.1

$(x+7)(x+2) = 0$

$x = -7$ or $x = -2$

Q1.1.2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-9 \pm \sqrt{9^2 - 4(4)(-3)}}{2(4)}$$
$$= \frac{-9 \pm \sqrt{129}}{8}$$
$x = 0{,}29$ or $x = -2{,}54$

Q1.1.3

$$\sqrt{x^2 - 5} = 2\sqrt{x}$$
$$x^2 - 5 = 4x$$
$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$
$x = 5$ or $x = -1$

$x = 5$

Q1.2

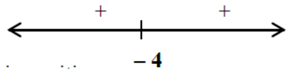
$$y = 3x - 4$$
$$x^2 + 2xy - y^2 = -2$$
$$x^2 + 2x(3x-4) - (3x-4)^2 = -2$$
$$x^2 + 6x^2 - 8x - (9x^2 - 24x + 16) = -2$$
$$7x^2 - 8x - 9x^2 + 24x - 16 = -2$$
$$-2x^2 + 16x - 14 = 0$$
$$x^2 - 8x + 7 = 0$$
$$(x-7)(x-1) = 0$$
$x = 1$ or $x = 7$

$y = 3(1) - 4$ $\quad$ $y = 3(7) - 4$

$y = -1$ or $y = 17$

Q1.3.1

$$x^2 + 8x + 16 > 0$$
$$(x+4)(x+4) > 0$$



The function values remain positive

$x \in R,\ x \neq -4$

Q1.3.2

$$x^2 + 8x + 16 = p$$
$$x^2 + 8x + 16 - p = 0$$
$$0 < 16 - p < 16$$
$$-16 < -p < 0$$
$$0 < p < 16$$

June 2017:

Q1.1.1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-10 \pm \sqrt{(10)^2 - 4(3)(6)}}{2(3)}$$
$x = -2{,}55$ or $x = -0{,}78$

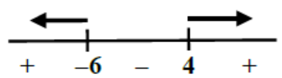
Q1.1.2

$$\sqrt{6x^2 - 15} = x + 1$$
$$6x^2 - 15 = (x+1)^2$$
$$6x^2 - 15 = x^2 + 2x + 1$$
$$5x^2 - 2x - 16 = 0$$
$$(5x+8)(x-2) = 0$$
$x = -\frac{8}{5}$ or $x = 2$

$\therefore x = 2$

Q1.1.3

$$x^2 + 2x - 24 \geq 0$$
$$(x+6)(x-4) \geq 0$$



$x \le -6$ or $x \ge 4$

Q1.2

$$y = -5x + 3$$
$$3x^2 - 2x(-5x+3) = (-5x+3)^2 - 105$$
$$3x^2 + 10x^2 - 6x = 25x^2 - 30x + 9 - 105$$
$$-12x^2 + 24x + 96 = 0$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x = -2 \text{ or } x = 4$$
$$y = 13 \text{ or } y = -17$$

Q1.3.1

$$p^2 - 48p - 49 = 0$$
$$(p-49)(p+1) = 0$$
$$p = -1 \text{ or } p = 49$$

Q1.3.2

$7^x = -1$ or $7^x = 49$

no solution $\quad x = 2$

<u>March 2017:</u>

Q1.1.1

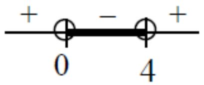
$$(x-3)(x+1) = 0$$
$$x = 3 \text{ or } x = -1$$

Q1.1.2

$$\sqrt{x^3} = 512$$
$$x^{\frac{3}{2}} = 512$$
$$\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = \left(8^3\right)^{\frac{2}{3}}$$
$$x = 64$$

Q1.1.3

$$x(x-4) < 0$$



$$0 < x < 4$$

Q1.2.1

$$x^2 - 5x + 2 = 0$$
$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$
$$x = \frac{5 \pm \sqrt{17}}{2}$$
$$x = 0{,}44 \text{ or } x = 4{,}56$$

Q1.2.2

$$f(x) = x^2 - 5x + 2$$
$$x^2 - 5x + 2 = c$$
$$x^2 - 5x + 2 - c = 0$$
$$b^2 - 4ac < 0$$
$$(-5)^2 - 4(1)(2-c) < 0$$
$$25 - 8 + 4c < 0$$
$$4c < -17$$
$$c < -\frac{17}{4}$$

Q1.3

$$x = 2y + 2$$
$$x^2 - 2xy + 3y^2 = 4$$
$$(2y+2)^2 - 2y(2y+2) + 3y^2 = 4$$
$$4y^2 + 8y + 4 - 4y^2 - 4y + 3y^2 = 4$$
$$3y^2 + 4y = 0$$
$$y(3y+4) = 0$$
$$y = 0 \text{ or } y = -\frac{4}{3}$$
$$x = 2 \qquad x = -\frac{2}{3}$$

<u>Arithmetic series and sequences</u>

<u>Nov 2019</u>

Q3.1

$$\sum_{y=3}^{10} \frac{1}{y-2} - \sum_{y=3}^{10} \frac{1}{y-1}$$
$$= \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{8}\right) - \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{8} + \frac{1}{9}\right)$$
$$= 1 - \frac{1}{9}$$
$$= \frac{8}{9}$$

Q3.2

$$\left(\frac{1}{3} \times \frac{2}{3}\right) + \left(\frac{2}{3} \times \frac{2}{3}\right) + \left(1 \times \frac{2}{3}\right) + \dots + \left(4 \times \frac{2}{3}\right)$$
$$= \frac{2}{9} + \frac{4}{9} + \frac{2}{3} + \dots + \frac{8}{3}$$
$$a = \frac{2}{9} \quad \text{and} \quad d = \frac{2}{3} - \frac{4}{9} = \frac{2}{9}$$
$$S_n = \frac{n}{2}(a + l)$$
$$S_{12} = \frac{12}{2}\left(\frac{2}{9} + \frac{8}{3}\right)$$
$$= \frac{52}{3}\text{m}^2$$

$\therefore$ for both sides $= 2 \times \frac{52}{3} = \frac{104}{3} = 34{,}67\text{m}^2$

<u>May-June 2019</u>

Q3.1.1

$$p + 6 - (2p + 3) = p - 2 - (p + 6)$$
$$-p + 3 = -8$$
$$p = 11$$

Q3.1.2

$T_n = 25 + (n-1)(-8) = 33 - 8n$

$33 - 8n < -55$

$-8n < -88$

$n > 11$

$\therefore$ Term 12 will be the first term smaller than -55

Q3.2

$S_6 = \frac{n}{2}[a+l] = \frac{6}{2}[(x-3)+(x-18)]$

$= 6x - 63$

$S_9 = \frac{n}{2}[a+l] = \frac{9}{2}[(x-3)+(x-27)]$

$= 9x - 135$

$6x - 63 = 9x - 135$

$3x = 72$

$x = 24$

$\therefore S_{15} = \frac{n}{2}[a+l] = \frac{15}{2}[(x-3)+(x-45)]$

$= \frac{15}{2}[2x - 48]$

$= \frac{15}{2}[2(24) - 48] = 0 = \text{RHS}$

Nov 2018

Q2.1.1

42

Q2.1.2

$2a = 6$

$a = 3$

$3a + b = 1$

$3(3) + b = 1$

$b = -8$

$a + b + c = 2$

$(3) + (-8) + c = 2$

$c = 7$

$T_n = 3n^2 - 8n + 7$

Q2.1.3

$T_{20} = 3(20)^2 - 8(20) + 7$

$= 1047$

Q2.2

$T_n = -7n + 42$

$-7n + 42 = -140$

$-7n = -182$

$n = 26$

Q2.3

$S_n = \frac{n}{2}(a + l)$

$S_n = \frac{n}{2}(35 - 7n + 42)$

$S_n = \frac{n}{2}(-7n + 77)$

$S_n = -\frac{7}{2}n^2 + \frac{77}{2}n$

$-\frac{7}{2}n^2 + \frac{77}{2}n = 3n^2 - 8n + 7$

$13n^2 - 93n + 14 = 0$

$(n-7)(13n-2) = 0$

$n = 7 \quad or \quad n = \frac{2}{13}$

NA

$\therefore n = 7$

June 2018

Q2.1.1

37 ; 50

Q2.1.2

$a = \frac{\text{second difference}}{2} = \frac{2}{2} = 1$

$3a + b = 5$

$3 + b = 5$

$b = 2$

$a + b + c = 5$

$1 + 2 + c = 5$

$c = 2$

$T_n = an^2 + bn + c$

$= n^2 + 2n + 2$

Q2.1.3

$n^2 + 2n + 2 = 1765$

$n^2 + 2n - 1763 = 0$

$(n+43)(n-41) = 0$

$n = -43 \text{ or } n = 41$

N/A

Q2.2

Sum of all multiples of 7 from 35 to 196:

$a = 35; \quad d = 7$

$S_n = \frac{n}{2}[a + \ell]$

$= \frac{24}{2}[35 + 196]$

$= 12[231]$

$= 2772$

Sum of all the natural numbers from 35 to 196:

$a = 35; \ d = 1; \ n = 162$

$S_n = \frac{n}{2}[a + \ell]$

$= \frac{162}{2}[35 + 196]$

$= 81[231]$

$= 18\ 711$

Sum of numbers not divisible by 7/
Som van getalle nie deelbaar deur 7

$= 18\ 711 - 2772$

$= 15\ 939$

March 2018

Q3.1

$-1 \ ; \ 2 \ ; \ 5$

$T_n = -1 + (n-1)(3)$

$= 3n - 4$

Q3.2

$T_{43} = 3(43) - 4$

$= 125$

Q3.3

$T_n = 3n - 4$

$\sum_{k=1}^{n} T_k = 3(1) - 4 + 3(2) - 4 + 3(3) - 4 + \ldots\ldots\ldots + 3n - 4$

$= 3(1 + 2 + 3 + \ldots\ldots + n) - 4n$

$= \frac{3n(n+1)}{2} - 4n$

$= \frac{3n^2 - 5n}{2}$

Q3.4

$T_{11} = (T_{11} - T_{10}) + (T_{10} - T_9) + (T_9 - T_8) + .$

$.. + (T_3 - T_2) + (T_2 - T_1) + T_1$

$125 = 29 + 26 + 23 + \ldots 2 + T_1$

$= \frac{10}{2}(29+2) + T_1$

$= 155 + T_1$

$T_1 = -30$

November 2017:

Q2.2.1

$2k-7\ ;\ k+8$ and $2k-1$

$k+8-(2k-7) = 2k-1-(k+8)$

$-k+15 = k-9$

$2k = 24$

$k = 12$

$2k-7; k+8$ and $2k-1$

$17\ ;\ 20\ ;\ 23 \ldots\ldots$

$d = 3$

$T_{15} = 17 + 14(3)$

$= 59$

Q2.2.2

Sequence is 17 ; 20 ; 23 ; 26 ; 29 ; 32 …….

Every alternate term of the sequence will be even /

Elke tweede term van die ry sal ewe wees

$20 + 26 + 32 + \ldots\ldots$

$S_{30} = \frac{30}{2}[2(20)+(29)(6)]$

$= 15[40+174]$

$= 3210$

June 2017:

Q2.2.1

$T_n = a + (n-1)d$

$T_{18} = 100 + (18-1)(150)$

$= R\ 2\,650$

Q2.2.2

$S_n = \frac{n}{2}[2a + (n-1)d]$

$30\,500 = \frac{n}{2}[2(100) + (n-1)(150)]$

$61\,000 = n(150n + 50)$

$61\,000 = 150n^2 + 50n$

$3n^2 + n - 1\,220 = 0$

$(3n+61)(n-20) = 0$

$n = -\frac{61}{3}$ or $n = 20$

N/A

$x = 100 + (20-1)(150)$

$= R\ 2\,950$

SESSION 2

Geometric Sequences and Series

Nov 2019

Q2.2.1

$a = \frac{5}{8}\ ;\ r = \frac{1}{2}\ ;\ n = 21$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_{21} = \frac{\frac{5}{8}\left(1-\left(\frac{1}{2}\right)^{21}\right)}{1-\frac{1}{2}}$

$= 1{,}2499\ldots$

$= 1{,}25$

Q2.2.2

$T_n > \frac{5}{8192}$

$ar^{n-1} > \frac{5}{8192}$

$\frac{5}{8}\left(\frac{1}{2}\right)^{n-1} > \frac{5}{8192}$

$\left(\frac{1}{2}\right)^{n-1} > \frac{1}{1024}$

$\left(\frac{1}{2}\right)^{n-1} > \left(\frac{1}{2}\right)^{10}$

$\therefore n-1 < 10$

$n < 11$

$\therefore n = 10$

May-June 2019

Q2.2.1

$r = \frac{-18}{36} = -\frac{1}{2}$

Q2.2.2

$T_n = 36\left(-\frac{1}{2}\right)^{n-1}$

$\frac{9}{4096} = 36\left(-\frac{1}{2}\right)^{n-1}$

$\frac{1}{16384} = \left(-\frac{1}{2}\right)^{n-1}$

$\left(-\frac{1}{2}\right)^{14} = \left(-\frac{1}{2}\right)^{n-1}$

$14 = n-1$

$n = 15$

Q2.2.3

$S_\infty = \frac{a}{1-r}$

$= \frac{36}{1-\left(-\frac{1}{2}\right)}$

$= 24$

Q2.2.4

$\frac{T_1 + T_3 + T_5 + T_7 + \ldots + T_{499}}{T_2 + T_4 + T_6 + T_8 + \ldots + T_{500}}$

$= \frac{a + ar^2 + ar^4 + \ldots + ar^{498}}{ar + ar^3 + ar^5 + \ldots + ar^{499}}$

$= \frac{a + ar^2 + ar^4 + \ldots + ar^{498}}{r(a + ar^2 + ar^4 + \ldots + ar^{498})}$

$= \frac{1}{r}$

$= -2$

Nov 2018

Q3.1

$r=\frac{1}{2}$ and $S_\infty=6$

$S_\infty=\frac{a}{1-r}$

$6=\frac{a}{1-\frac{1}{2}}$

$a=3$

Q3.2

$T_n=ar^{n-1}$

$T_8=3\left(\frac{1}{2}\right)^7$

$T_8=\frac{3}{128}$

Q3.3

$\sum_{k=1}^{n}3(2)^{1-k}=5{,}8125$

$3+\frac{3}{2}+\frac{3}{4}+\ldots=5{,}8125$

$S_n=\frac{a(1-r^n)}{1-r}=5{,}8125$

$\frac{3\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}=5{,}8125$

$6\left[1-\left(\frac{1}{2}\right)^n\right]=5{,}8125$

$\left(\frac{1}{2}\right)^n=\frac{1}{32}=0{,}03125$

$2^{-n}=2^{-5}$

$n=5$

Q3.4

$\sum_{k=1}^{20}3(2)^{1-k}=p$

$\sum_{k=1}^{20}6(2)^{-k}=p$

$\therefore\sum_{k=1}^{20}24(2)^{-k}=4p$

June 2018

Q3.1

$r=0{,}94;\quad a=100$

$T_3=ar^2$

$=100(0{,}94)^2$

$=88{,}36\text{ km}$

Q3.2

$S_n=\frac{a(r^n-1)}{r-1}$

$750=\frac{100(0{,}94^n-1)}{0{,}94-1}$

$\frac{750(-0{,}06)}{100}=0{,}94^n-1$

$0{,}94^n=1-\frac{9}{20}$

$0{,}94^n=0{,}55$

$n=\frac{\log 0{,}55}{\log 0{,}94}$

$=9{,}66$

He will pass halfway mark on the tenth day.

Q3.3

$S_\infty=\frac{a}{1-r}$

$1500<\frac{100}{1-r}$

$1-r<\frac{100}{1500}$

$r>\frac{14}{15}$ or $93{,}33\%$

March 2018

Q2.1.1

$30\ ;\ 10\ ;\ \frac{10}{3}\ldots\ldots$

$a=30\qquad r=\frac{1}{3}$

$T_n=ar^{n-1}$

$\frac{10}{729}=30\left(\frac{1}{3}\right)^{n-1}$

$\frac{1}{2187}=3^{1-n}$

$3^{-7}=3^{1-n}$

$-7=1-n$

$n=8$

Q2.1.2

$S_\infty=\frac{a}{1-r}$

$=\frac{30}{1-\frac{1}{3}}$

$=45$

Q2.2

$S_n=a+(a+d)+\ldots\ldots+(a+(n-2)d)+T_n$

$S_n=T_n+(T_n-d)+(T_n-2d)+\ldots\ldots\ldots+a$

$2S_n=(a+T_n)+(a+T_n)+(a+T_n)+\ldots..+(a+T_n)$

$S_n=\frac{n}{2}(a+T_n)$

but $Tn=a+(n-1)d$

$S_n=\frac{n}{2}[2a+(n-1)d]$

November 2017:

Q3.1

$a+ar=2$

$a(1+r)=2$

$a=\frac{2}{1+r}$

Q3.2

$$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$$

$$S_\infty = 2 + \frac{1}{4}$$

$$\frac{a}{1-r} = 2 + \frac{1}{4}$$

$$\frac{a}{1-r} = \frac{9}{4}$$

$$\left(\frac{2}{1+r}\right) \times \left(\frac{1}{1-r}\right) = \frac{9}{4}$$

$$\frac{2}{1-r^2} = \frac{9}{4}$$

$$8 = 9 - 9r^2$$

$$9r^2 = 1$$

$$r = \frac{1}{3}$$

$$a = \frac{3}{2}$$

June 2017:

Q2.1.1

$3;\ 2;\ k;\ \ldots$

$$r = \frac{2}{3}$$

Q2.1.2

$$r = \frac{T_3}{T_2}$$

$$T_3 = r \times T_2$$

$$= \frac{2}{3} \times 2$$

$$= \frac{4}{3}$$

Thus $k = \frac{4}{3}$

Q2.1.3

$$T_n = a.r^{n-1}$$

$$\frac{128}{729} = 3 \times \left(\frac{2}{3}\right)^{n-1}$$

$$\left(\frac{2}{3}\right)^{n-1} = \frac{128}{2187}$$

$$\left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^{7}$$

$$n - 1 = 7$$

$$n = 8$$

March 2017:

Q2.1

For geometric:

$$-\frac{1}{4}\ ;\ b\ ;\ -1\ ;\ \ldots\ldots$$

$$\frac{b}{-\frac{1}{4}} = -\frac{1}{b}$$

$$b^2 = \frac{1}{4}$$

$$b = \pm\frac{1}{2}$$

Q2.2

$$-\frac{1}{4};\ \frac{1}{2};\ -1;\ \ldots\ldots$$

$$r = -2$$

$$T_{19} = ar^{18}$$

$$= \left(-\frac{1}{4}\right)(-2)^{18}$$

$$= \left(-\frac{2^{18}}{2^2}\right)$$

$$= -2^{16}$$

$$= -65536$$

Q2.3

The series is:

$$-\frac{1}{4};\ \frac{1}{2};\ -1;\ 2;\ -4;\ 8;\ \ldots$$

The new positive series is:

$$\frac{1}{2};\ 2\ ;\ 8\ ;\ 32\ ;\ 128$$

$$a = \frac{1}{2} \qquad r = 4$$

$$\sum_{n=1}^{20}\left(\frac{1}{2}\right)(4)^{n-1}$$

Or you could write it as:

$$\sum_{p=0}^{19}\left(\frac{1}{2}\right)(4)^{p}$$

Q2.4

No, the series is not convergent

$r = 4$ and for convergence $-1 < r < 1$

QUADRATIC PATTERNS:

Nov 2019

Q2.1.1

$209\ ;\ 186$

Q2.1.2

$321\ ;\ 290\ ;\ 261\ ;\ 234$

1st diff: $-31 \quad -29 \quad -27$

2nd diff: $2 \quad 2$

$2a = 2$	$3a + b = -31$	$a + b + c = 321$
$a = 1$	$3(1) + b = -31$	$1 + (-34) + c = 321$
	$b = -34$	$c = 354$

$$T_n = n^2 - 34n + 354$$

Q2.1.3

$$n^2 - 34n + 354 = 74$$

$$n^2 - 34n + 280 = 0$$

$$(n-14)(n-20) = 0$$

$$n = 14 \quad \text{or} \quad n = 20$$

Q2.1.4

$$f^{/}(n) = 0$$
$$2n - 34 = 0$$
$$2n = 34$$
$$n = 17$$

Term 17 will have the smallest value

<u>May-June 2019</u>

Q2.1.1

59

Q2.1.2

15 29 41 51

14 12 10

–2 –2

$$2a = -2$$
$$a = -1$$
$$3(-1) + b = 14$$
$$b = 17$$
$$(-1) + (17) + c = 15$$
$$c = -1$$
$$T_n = -n^2 + 17n - 1$$

Q2.1.3

$$T_{27} = -(27)^2 + 17(27) - 1$$
$$= -271$$

<u>November 2017:</u>

Q2.1.1

5 –4 –19 –40

–9 –15 –21

–6 –6

first differences: –9; –15; –21
second difference = – 6

Q2.1.2

$$T_n = an^2 + bn + c$$

$$a = \frac{\text{second difference}}{2} = -3$$

$$3a + b = -9$$
$$3(-3) + b = -9$$
$$b = 0$$
$$a + b + c = 5$$
$$-3 + 0 + c = 5$$
$$c = 8$$
$$T_n = -3n^2 + 8$$

Q2.1.3

$$-3n^2 + 8 = -25\,939$$
$$-3n^2 = -25947$$
$$n^2 = 8649$$
$$n = -93 \quad or \quad n = 93$$

The 93rd term has a value of –25 939

<u>June 2017:</u>

Q3.1

First differences: 17; 15
Second difference: –2

$$T_n = an^2 + bn + c$$

$$a = \frac{\text{second difference}}{2} = \frac{-2}{2} = -1$$

$$3a + b = 17$$
$$3(-1) + b = 17$$
$$b = 20$$
$$a + b + c = 0$$
$$-1 + 20 + c = 0$$
$$c = -19$$
$$T_n = -n^2 + 20n - 19$$

Q3.2

$$56 = -n^2 + 20n - 19$$
$$n^2 - 20n + 75 = 0$$
$$(n-15)(n-5) = 0$$
$$n = 5 \text{ or } n = 15$$

Q3.3

$$\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$$
$$= T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} - T_{11} - T_{12} - T_{13} - T_{14} - T_{15}$$
$$= (T_5 - T_{15}) + (T_6 - T_{14}) + \ldots + (T_9 - T_{13}) + T_{10}$$
$$= T_{10}$$

because by symmetry $T_5 = T_{15}$; $T_6 = T_{14}$ ...

$$T_{10} = -(10)^2 + 20(10) - 19$$
$$= 81$$

<u>March 2017:</u>

Q3.1.1

24

Q3.1.2

6 6 9 15

0 3 6

3 3

$$2a = 3 \qquad 3a + b = 0$$

$$a = \frac{3}{2} \qquad b = -\frac{9}{2}$$

$$T_n = \frac{3}{2}n^2 - \frac{9}{2}n + 9$$

$$a + b + c = 6$$

$$c = 9$$

Q3.1.3

$$\frac{3}{2}n^2 - \frac{9}{2}n + 9 = 3249$$
$$3n^2 - 9n + 18 = 6498$$
$$3n^2 - 9n - 6480 = 0$$
$$n^2 - 3n - 2160 = 0$$
$$(n+45)(n-48) = 0$$
$$n \neq -45 \quad or \quad n = 48$$

Q3.2

$$-1 \,;\, 2\sin 3x \,;\, 5; \ldots$$
$$2\sin 3x + 1 = 5 - 2\sin 3x$$
$$4\sin 3x = 4$$
$$\sin 3x = 1$$
$$3x = 90^\circ$$
$$x = 30^\circ$$

FINANCIAL MATHEMATICS

Nov 2019

Q6.1

Kuda: $A = P(1+in)\times 1{,}04$

$= 5\ 000(1+0{,}083\times 4)\times 1{,}04$

$= R6\ 926{,}40$

Thabo: $A = P(1+i)^n$

$= 5\ 000\left(1+\frac{0{,}081}{12}\right)^{12\times 4}$

$= R6\ 905{,}71$

Kuda will have a better investment

Q6.2.1

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$525\ 000 = \frac{6\ 000\left[1-\left(1+\frac{0{,}1}{12}\right)^{-n}\right]}{\frac{0{,}1}{12}}$$

$$\frac{35}{48} = 1-\left(1+\frac{0{,}1}{12}\right)^{-n}$$

$$-n\log\left(1+\frac{0{,}1}{12}\right) = \log\frac{13}{48}$$

$$-n = \frac{\log\frac{13}{48}}{\log\left(1+\frac{0{,}1}{12}\right)}$$

$n = 157{,}40$

$n = 158$ payments

Q6.2.2

$$F = \frac{x[(1+i)^n-1]}{i}$$

$$F = \frac{6000\left[\left(1+\frac{0{,}1}{12}\right)^{108}-1\right]}{\frac{0{,}1}{12}}$$

$= R1\ 044\ 322{,}28$

$$F = \frac{5\ 066{,}36\left[\left(1+\frac{0{,}1}{12}\right)^{108}-1\right]}{\frac{0{,}1}{12}}$$

$F = R881\ 818{,}77.....$

Amount available for withdrawal

$= R1\ 044\ 322{,}28 - R\ 881\ 818{,}77$

$= R162\ 503{,}51$

May-June 2019

Q6.1.1

$A = P(1-i)^n$

$79866{,}96 = 180\,000(1-0{,}15)^n$

$(1-0{,}15)^n = \frac{79866{,}96}{180\,000}$

$$n = \frac{\log\left(\frac{79866{,}96}{180\,000}\right)}{\log(1-0{,}15)}$$

$n = 4{,}999\ldots$ years

$n \approx 5$ years

Q6.1.2

$A = P(1+i)^n$

$= 49\ 000\left(1+\frac{0{,}1}{4}\right)^{20}$

$=$ R80 292,21

The money will be enough to buy the car.

Q6.2.1

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$P = \frac{7853{,}15\left[1-\left(1+\frac{0{,}1025}{12}\right)^{-234}\right]}{\frac{0{,}1025}{12}}$$

$P = R793\ 749{,}25$

Q6.2.2

$A = P(1+i)^n$

$= 793749{,}25\left(1+\frac{0{,}1025}{12}\right)^3$

$= R814\ 263{,}3052$

New instalment/*Nuwe paaiement*:

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$814263{,}3052 = \frac{x\left[1-\left(1+\frac{0{,}1025}{12}\right)^{-231}\right]}{\frac{0{,}1025}{12}}$$

$x = R8\ 089{,}20$

November 2018

Q7.1.1

$$F = \frac{x[(1+i)^n-1]}{i}$$

$$F = \frac{15\ 000\left[\left(1+\frac{0{,}088}{4}\right)^{16}-1\right]}{\frac{0{,}088}{4}}$$

$F = R283\ 972{,}28$

Q7.1.2

$A = R283\ 972{,}28 - 100\,000\left(1+\frac{0{,}088}{4}\right)^4$

$= R\ 174\ 877{,}60$

Q7.2.1

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$1\ 500\ 000 = \frac{x\left[1-\left(1+\frac{0{,}105}{12}\right)^{-12\times 20}\right]}{\frac{0{,}105}{12}}$$

$x = R14\ \ 975{,}70$

Q7.2.2

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$P = \frac{14\ \ 975{,}70\left[1-\left(1+\frac{0{,}105}{12}\right)^{-12\times 8}\right]}{\frac{0{,}105}{12}}$$

$P = R969\ 927{,}74$

June 2018

Q6.1

$$A = P(1-i)^n$$

$$0{,}5P = P(1-0{,}15)^n$$

$$(1-0{,}15)^n = 0{,}5$$

$$(0{,}85)^n = 0{,}5$$

$$n = \frac{\log 0{,}5}{\log 0{,}85} \text{ or } \log_{0{,}85} 0{,}5$$

$$= 4{,}27 \text{ years}$$

Q6.2

$$F = \frac{x(1+i)[(1+i)^n - 1]}{i}$$

$$= \frac{1500\left(1+\frac{0{,}092}{12}\right)\left[\left(1+\frac{0{,}092}{12}\right)^{384} - 1\right]}{\frac{0{,}092}{12}}$$

$$= R3\,505\,289{,}91$$

Q6.3

a = amount invested at 8,4% p.a. compounded quarterly
bedrag belê teen 8,4% p.a. kwartaalliks saamgestel
b = amount invested at 9,6% p.a. compounded monthly
bedrag belê teen 9,6% p.a. maandeliks saamgestel

a + b = 150 000
a = 150 000 – b

$$(150000-b)\left(1+\frac{0{,}084}{4}\right)^{48} = b\left(1+\frac{0{,}096}{12}\right)^{144}$$

$$150000\left(1+\frac{0{,}084}{4}\right)^{48} = b\left[\left(1+\frac{0{,}096}{12}\right)^{144} + \left(1+\frac{0{,}084}{4}\right)^{48}\right]$$

b = R69 390,95
a = R80 609,05

March 2018

Q7.1

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$= \frac{2500\left[\left(1+\frac{0{,}06}{12}\right)^{60} - 1\right]}{\frac{0{,}06}{12}}$$

$$= R174\,425{,}08$$

Q7.2.1

After eleven months, Genevieve will owe/
Na elf maande skuld Genevieve

$$A = 82\,000\left(1+\frac{0{,}15}{12}\right)^{11}$$

$$= R\,94006{,}79$$

Q7.2.2

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$94006{,}79 = \frac{3\,200\left[1-\left(1+\frac{0{,}15}{12}\right)^{-n}\right]}{\frac{0{,}15}{12}}$$

$$\frac{94\,006{,}79}{3\,200}\times\frac{0{,}15}{12} = 1-\left(1+\frac{0{,}15}{12}\right)^{-n}$$

$$\left(1+\frac{0{,}15}{12}\right)^{-n} = 1-0{,}3672147...$$

$$-n\log\left(1+\frac{0{,}15}{12}\right) = \log 0{,}6327852...$$

$$-n = -36{,}8382...$$

$$n = 36{,}84$$

She pays 36 instalments of R3200 each

November 2017:

Q6.1

$$A = P(1+i)^n$$

$$12\;146{,}72 = 10\;000\left(1+\frac{r}{12}\right)^{36}$$

$$\left(1+\frac{r}{12}\right)^{36} = 1{,}214672$$

$$1+\frac{r}{12} = \sqrt[36]{1{,}214672}$$

$$= 1{,}005416$$

$$\frac{r}{12} = 0{,}005416$$

$$r = 0{,}06500$$

$$r = 6{,}5\%$$

Q6.2.1

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$235\;000 = \frac{x\left[1-\left(1+\frac{0{,}11}{12}\right)^{-54}\right]}{\frac{0{,}11}{12}}$$

$$x = \frac{235\,000\times\frac{0{,}11}{12}}{\left[1-\left(1+\frac{0{,}11}{12}\right)^{-54}\right]}$$

$$= R5\,536{,}95$$

Q6.2.2

Amount pd for the year:

$$(5\;536{,}95\times 12) = R66\;443{,}40$$

Balance:

$$= 235\;000\left(1+\frac{0{,}11}{12}\right)^{12} - \frac{5\;536{,}95\left[\left(1+\frac{0{,}11}{12}\right)^{12} - 1\right]}{\frac{0{,}11}{12}}$$

$$= 192\;296{,}17$$

$$\text{Interest} = (5\;536{,}95\times 12)-(235\;000-192\;296{,}17)$$

$$= 66\;443{,}40 - 42\;703{,}83$$

$$= 23\;739{,}57$$

June 2017:

Q7.1

$$A = P(1-i)^n$$

$$331527 = 500000(1-i)^3$$

$$(1-i)^3 = \frac{331527}{500000}$$

$$1-i = \sqrt[3]{\frac{331527}{500000}}$$

$$i = 0{,}12800...$$

$$= 12{,}8\%$$

Q7.2

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$46\;000 = \frac{1900\left[1-\left(1+\frac{0{,}24}{12}\right)^{-n}\right]}{\frac{0{,}24}{12}}$$

$$\frac{46}{95} = 1-\left(1+\frac{0{,}24}{12}\right)^{-n}$$

$$\left(1+\frac{0,24}{12}\right)^{-n}=\frac{49}{95}$$

$$n=-\log_{\left(1+\frac{0,24}{12}\right)}\frac{49}{95}$$

$$=33,43276544...\text{ months}$$

It will take him 34 months to pay back the loan.

Q7.3

$$F=\frac{x[(1+i)^n-1]}{i}$$

$$=\frac{3500\left[\left(1+\frac{0,075}{4}\right)^{4\times 6,5}-1\right]}{\frac{0,075}{4}}$$

$$=R\ 115\ 902,69$$

$$A=P(1+i)^n$$

$$=115\ 902,69\left(1+\frac{0,075}{4}\right)^{4\times 3,5}$$

$$=R\ 150\ 328,12$$

March 2017:

Q6.1.1

$$A=150\ 000(1-0,2)^2$$

$$=R96\ 000$$

Q6.1.2

$$150\ 000(1-0,2)^n=49\ 152$$

$$(0,8)^n=\frac{1024}{3125}$$

$$n\log(0,8)=\log\frac{1024}{3125}$$

$$n=5$$

The machine will need to be replaced at the beginning of 2020

Q6.1.3

R280 000 – R49 152
=R230 848

$$230\ 848=\frac{x\left[\left(1+\frac{0,085}{4}\right)^{20}-1\right]}{\frac{0,085}{4}}$$

$$x=R9\ 383,26$$

Q6.2

$$P_v=\frac{x[1-(1+i)^{-n}]}{i}$$

$$=\frac{9\ 000\left[1-\left(1+\frac{0,11}{12}\right)^{-180}\right]}{\frac{0,11}{12}}$$

$$=R791\ 837,43$$

Lerato qualifies for a loan of R 791 000 under the given conditions

SESSION 3

Functions

Nov 2019

Q4.1

$$p=-1$$

Q4.2

$$y=\frac{a}{x-1}$$

$$-3=\frac{a}{0-1}$$

$$a=3$$

$$y=x^2+bx-3$$

$$0=(1)^2+(1)b-3$$

$$b=2$$

Q4.3

$$\frac{dy}{dx}=0$$

$$2x+2=0$$

$$x=-1$$

$$y=(-1)^2+2(-1)-3=-4$$

$$C(-1;-4)$$

Q4.4

$$y\geq -4$$

Q4.5

$$m=\tan 45°=1$$

$$y=mx+c$$

$$-4=(1)(-1)+c$$

$$c=-3$$

$$y=x-3$$

Q4.6

No, the line passes through C and D

OR

No, a tangent through turning point C will have a gradient of 0

Q4.7

$$f(m-x)=f[-(x-m)]$$

f is reflected in the y-axis and translated 1 unit to the left and 4 units upwards.

Therefore: $m=-1$
$q=4$

OR

Substitute $x=0$ and $q=4$ for one x- intercept

$$h(x)=(m-x)^2+2(m-x)-3+q$$

$$h(0)=(m-0)^2+2(m-0)-3+4$$

$$0=m^2+2m+1$$

$$0=(m+1)^2$$

$$m=-1$$

$$q=4$$

Q5.1

$$f(x)=k^x$$

$$16=k^4$$

$$k=2$$

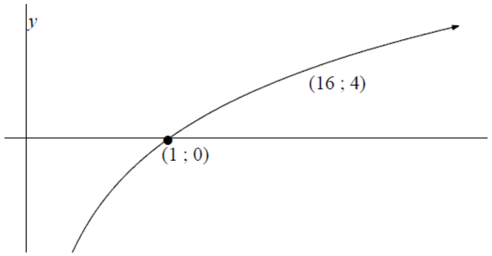
Q5.2

$f:\quad y=2^x$

$f^{-1}:\quad x=2^y$

$y=\log_2 x$

Q5.3



Q5.4.1

$$x>1$$

Q5.4.2

$$0 < x \le \frac{1}{2}$$

Q5.5

$$2^x - 2^{-x} = \frac{15}{4}$$

$$2^x - \frac{1}{2^x} = \frac{15}{4}$$

$$2^{2x} - 1 = \frac{15}{4} \times 2^x$$

$$4.2^{2x} - 4 = 15 \times 2^x$$

$$4.2^{2x} - 15.2^x - 4 = 0$$

$$(4.2^x + 1)(2^x - 4) = 0$$

$$4.2^x + 1 = 0 \text{ or } 2^x - 4 = 0$$

$$2^x = \frac{-1}{4} \text{ or } 2^x = 2^2$$

$$\text{N/A} \qquad x = 2$$

May-June 2019

Q4.1

$y > 0$

Q4.2

$$g: y = \left(\frac{1}{2}\right)^x$$

$$g^{-1}: x = \left(\frac{1}{2}\right)^y$$

$$y = \log_{\frac{1}{2}} x$$

Q4.3

Yes. The vertical line test cuts g^{-1} once

Q4.4.1

$$y = -\log_2 x$$

$$2 = -\log_2 a$$

$$a = 2^{-2} = \frac{1}{4}$$

Q4.4.2

$$M'\left(2; \frac{1}{4}\right)$$

Q4.5

$$M''\left(-1; \frac{9}{4}\right)$$

Qs.1.1

$x = -2$

$y = 3$

Q5.1.2

$$\left(0\,;\frac{7}{2}\right)$$

Q5.1.3

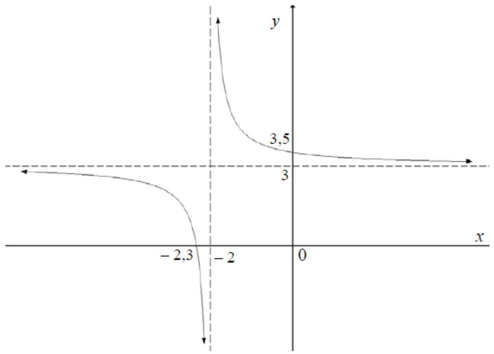
$$\frac{1}{x+2} + 3 = 0$$

$$1 + 3(x+2) = 0$$

$$3x = -7$$

$$x = -\frac{7}{3}$$

Q5.1.4



Q5.2.1

$$-2x + 4 = 0$$

$$2x = 4$$

$$x = 2$$

$$\therefore S(2\,;0)$$

Q5.2.2

Equation of k:

$$y = a(x+1)^2 + 18$$

$$0 = a(2+1)^2 + 18$$

$$9a = -18$$

$$a = -2$$

$$y = -2(x+1)^2 + 18$$

Q5.2.3

$$-2x^2 - 4x + 16 = -2x + 4$$

$$-2x^2 - 2x + 12 = 0$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x = -3 \text{ or } x = 2$$

$$y = -2(-3) + 4 = 10$$

T(–3 ; 10)

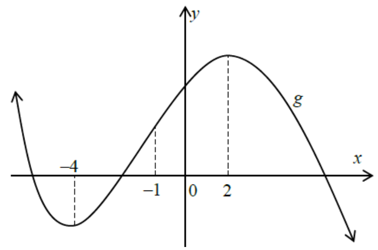
Q5.2.4

$x < -3$ or $x > 2$

Q5.2.5a

$x < -1$

Q5.2.5b



November 2018

Q4.1

Yes, it is a one-to-one mapping

Q4.2

R(-12;-6)

Q4.3

$f(x) = ax^2$ substitute $(-6\,;-12)$

$$-12 = a(-6)^2$$

$$a = \frac{-1}{3}$$

Q4.4

$$f: y = -\left(\frac{1}{3}\right)x^2$$

$$f^{-1}: x = -\left(\frac{1}{3}\right)y^2$$

$$y^2 = -3x$$

$$y = \pm\sqrt{-3x}$$

Only $y = -\sqrt{-3x}$ and $x \le 0$

Q5.1

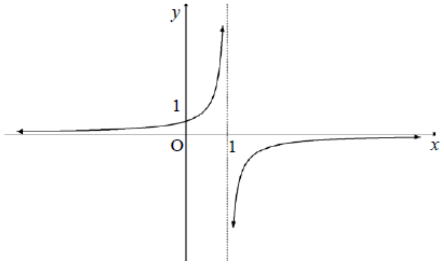
Domain: $x \in R\,;\ x \ne 1$

Q5.2

$x = 1$

$y = 0$

Q5.3



Q5.4

$x \geq 0 \; ; \; x \neq 1$

Q6.1

$y = mx + c$

$5 = m(4) + 1$

$m = 1$

$g(x) = x + 1$

Q6.2

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$

A$(-1\,;\,0)$ B$(3\,;\,0)$

Q6.3

$x = \frac{-1+3}{2}$

$x = 1$

$f(x) = x^2 - 2x - 3$

$y = (1)^2 - 2(1) - 3$

$y = -4$

$y \geq -4$

Q6.4.1

MN: $y = (x^2 - 2x - 3) - (x + 1)$

$= x^2 - 3x - 4$

$6 = x^2 - 3x - 4$

$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$x = 5$ or $x = -2$

OT = 2 or OT = 5
NA

Q6.4.2

$y = x + 1$ substitute $x = -2$

$= (-2) + 1$

$= -1$

N$(-2\,;\,-1)$

Q6.5

$f'(x) = 2x - 2$

$2x - 2 = 1$

$x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = \frac{-15}{4}$

$y + \frac{15}{4} = 1\left(x - \frac{3}{2}\right)$

$y = x - \frac{21}{4}$

Q6.6

$k < \frac{-21}{4}$

June 2018

Q4.1

$0 < x \leq 1$

Q4.2

$p = \log_{\frac{4}{3}} \frac{16}{9}$

$\left(\frac{4}{3}\right)^p = \frac{16}{9}$

$\left(\frac{4}{3}\right)^p = \left(\frac{4}{3}\right)^2$

$p = 2$

Q4.3

$f : y = \log_{\frac{4}{3}} x$

$f^{-1} : x = \log_{\frac{4}{3}} y$

$y = \left(\frac{4}{3}\right)^x$

Q4.4

$y > 0$

Q4.5

$\left(-2\,;\,\frac{16}{9}\right)$

Q5.1

$x \in R; \quad x \neq -1$

Q5.2

x-intercept of f:

$0 = \frac{2}{x+1} + 4$

$\frac{2}{x+1} = -4$

$2 = -4x - 4$

$4x = -6$

$x = -\frac{3}{2}$

Q5.3

$y = \frac{2}{x+1} + 4$

$\frac{14}{3} = \frac{2}{k+1} + 4$

$\frac{2}{k+1} = \frac{14}{3} - 4$

$\frac{2}{k+1} = \frac{2}{3}$

$2k + 2 = 6$

$k + 1 = 3$

$k = 2$

Q5.4

C$(2\,;\,4)$

Q5.5

$$y = a(x+p)^2 + q$$
$$= a(x-2)^2 + 4$$

Substitute (0 ; 0):

$$0 = a(0-2)^2 + 4$$
$$0 = 4a + 4$$
$$a = -1$$
$$y = -(x-2)^2 + 4$$

Q5.6

$$x \leq -\frac{3}{2} \quad \text{or} \quad -1 < x < 0 \quad \text{or} \quad x > 4$$

Q5.7

$\frac{2}{x} - 5$: f shifted 1 unit to the right and 9 units down.

f is 1 eenheid na regs en 9 eenhede afgeskuif.

$-(x-3)^2 - 5$: g shifted 1 unit to the right and 9 units down.

g is 1 eenheid na regs en 9 eenhede afgeskuif.

Therefore the shift of both graphs took place relative to each other/*Dus het die skuif van die grafieke relatief tot mekaar plaasgevind.*

They only intersect in the third quadrant.

Hulle sny mekaar slegs in die derde kwadrant.

Therefore there is only one point of intersection.

Daar is dus slegs een snypunt.

March 2018

Q4.1

E(4 ; –9)

Q4.2

$$f(x) = (x-4)^2 - 9$$
$$(x-4)^2 - 9 = 0$$
$$(x-4)^2 = 9$$
$$x - 4 = \pm 3$$
$$x = 7 \quad \text{or} \quad x = 1$$

A(1 ; 0)

Q4.3

C(0 ; 7)

M(8 ; 7)

Q4.4

C(0 ; 7)

D(4 ; 0)

$$m = \frac{7-0}{0-4}$$
$$m = -\frac{7}{4}$$
$$y - 0 = -\frac{7}{4}(x-4)$$
$$y = -\frac{7}{4}x + 7$$

Q4.5

$$g : y = -\frac{7}{4}x + 7$$
$$g^{-1} : x = -\frac{7}{4}y + 7$$
$$4x = -7y + 28$$
$$7y = -4x + 28$$
$$y = -\frac{4}{7}x + 4$$

Q4.6

$$x \cdot f(x) \leq 0$$

$$\therefore x \leq 0 \text{ or } 1 \leq x \leq 7$$

Q5.1

$$a^0 = 1$$

T(0 ; 1)

Q5.2

$$g(x) = a^x$$
$$9 = a^2$$
$$a = 3$$

Q5.3

$$y = \left(\frac{1}{3}\right)^x \quad \text{or} \quad y = 3^{-x}$$

Q5.4

$$3^0 < 3^{\log_3 x} < 3^1$$
$$1 < x < 3$$

Q6.1

$$q = 1$$

Q6.2

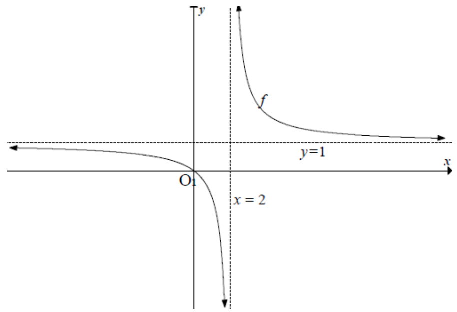
Subs (0;0) $\quad 0 = \frac{a}{0+p} + 1$

$$\frac{a}{p} = -1$$
$$a = -p$$

Subs P:

$$\sqrt{2} + 1 = \frac{a}{\sqrt{2} + 2 + p} + 1$$
$$\sqrt{2} = \frac{a}{\sqrt{2} + 2 + p}$$
$$2 + 2\sqrt{2} + \sqrt{2}p = a$$
$$2 + 2\sqrt{2} = a - p\sqrt{2} = a + a\sqrt{2}$$
$$2(1+\sqrt{2}) = a(1+\sqrt{2})$$
$$a = 2 \; ; \; p = -2$$

Q6.3



November 2017:

Q4.1

$$f(x) = -ax^2 + bx + 6$$
$$f'(x) = -2ax + b$$
$$-2ax + b = 3$$

at $x = -1$

$2a+b=3 \qquad [1]$

$f(-1)=\frac{7}{2}$

$-a-b+6=\frac{7}{2}$

$-2a-2b+12=7$

$2a+2b=5 \qquad [2]$

$[2]-[1]$

$b=2$

$2a+2=3$

$a=\frac{1}{2}$

Q4.2

$f(x)=-\frac{1}{2}x^2+2x+6$

$x-\text{intercepts}:$

$-\frac{1}{2}x^2+2x+6=0$

$-x^2+4x+12=0$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$(-2\,;0) \quad (6\,;0)$

Q4.3

$f'(x)=0$

$-x+2=0$

$x=2$

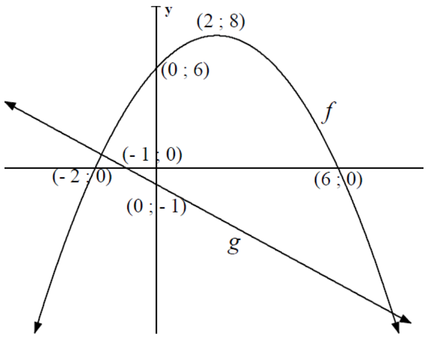
$y=-\frac{1}{2}(2)^2+2(2)+6$

$=-2+4+6$

$=8$

$\text{TP }(2\,;8)$

Q4.4 and Q4.6



Q4.5

$0<x<4$

Q4.7

$x\le -2 \text{ or } -1\le x\le 6$

Q5.1

$y\in R\,;y\neq -1$

Q5.2

$D(2\,;-1)$

$g(x)=\frac{2}{x-2}-1$

Q5.3

$f(x)=\log_3 x.$

$\log_3 t=1$

$t=3$

Q5.4

$x=\log_3 y$

$y=3^x$

Q5.5

$3^x<3^1$

$x<1$

Q5.6

Equation of the axis of symmetry: $y=-x+1$

x-intercept of the axis of symmetry is at $x=1$

f has an x-intercept at B(1 ; 0) which is the same as the x-intercept of the axis of symmetry

Point of intersection: B $(1\,;0)$

June 2017:

Q4.1

A (4; 3)

Q4.2

$y=\frac{6}{-4}+3$

$=\frac{3}{2}$

$B\left(0\,;\frac{3}{2}\right)$

Q4.3

$0=\frac{6}{x-4}+3$

$-3=\frac{6}{x-4}$

$-3(x-4)=6$

$-3x+12=6$

$x=2$

$C(2\,;0)$

Q4.4

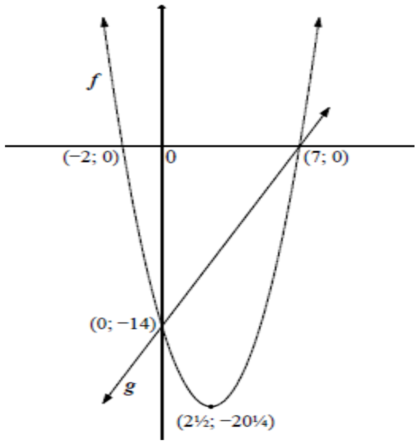
$\text{Average gradient}=\frac{0-\frac{3}{2}}{2-0}$

$=-\frac{3}{4}$

Q4.5

$y=-x+7$

Q5.1



Q5.2

$y=-20\frac{1}{4}$

Q5.3

$-20\frac{1}{4}<k<-14$

Q5.4

Reflecting in the x-axis: $y=-2x+14$

Shifting 7 units to the left:

$$y=-2(x+7)+14$$
$$=-2x-14+14$$
$$=-2x$$

Q6.1

$f: y=b^x$

$f^{-1}: x=b^y$

$y=\log_b x$

Q6.2

$y=x$

Q6.3

P(0; 1)

Q6.4

T(1; 0)

$y=mx+c$

$y=-x+1$

Q6.5

At point R, PT and OR intersect:

$-x+1=x$

$2x=1$

$x=\frac{1}{2}$

$y=\frac{1}{2}$

Substitute $\left(\frac{1}{2};\frac{1}{2}\right)$ into the equation of f:

$y=b^x$

$\frac{1}{2}=b^{\frac{1}{2}}$

$b=\left(\frac{1}{2}\right)^2=\frac{1}{4}$

March 2017:

Q4.1

U(1; 0)

Q4.2

$x=1$

$y=1$

Q4.3

$\frac{2}{x-1}+1=0$

$2=-x+1$

$x=-1$

$T(-1;0)$

Q4.4

$f(x)=\log_5 x$

$h: x=\log_5 y$

$y=5^x$

Q4.5

y=0

Q4.6

$V(\sqrt{2}+1;\sqrt{2}+1)$

$V(2{,}41;2{,}41)$

Q4.7

$T^{/}(3;2)$

Q5.1.1

C(0 ; –3)

Q5.1.2

$f(x)=x^2-2x-3$

$(x-3)(x+1)=0$

$x=-1$ or $x=3$

$AB=3-(-1)$

$AB=4$ units

Q5.1.3

$x=\frac{2}{2(1)}$

$=1$

$y=(1)^2-2(1)-3$

$=-4$

$D(1;-4)$

Q5.1.4

AV gradient =

$=\frac{-4+3}{1-0}$

$=-1$

Q5.1.5

$OC=OB=3$

$O\hat{C}B=45°$

Q5.1.6

$-4<k<-3$

Q5.1.7

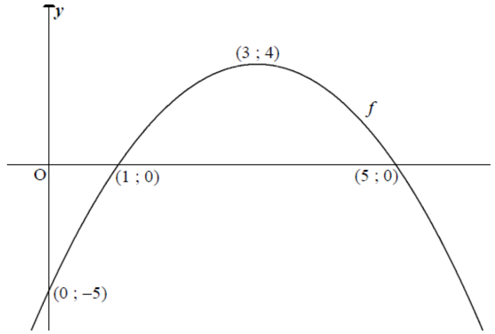
$f^{/}(x).f^{//}(x)>0$

$(2x-2).2>0$

$2x-2>0$

$x>1$

Q5.2



$f(x)=a(x-1)(x-5)$

$4=a(3-1)(3-5)$

$4=-4a$

$a=-1$

$f(x)=-x^2+6x-5$

DIFFERENTIAL CALCULUS

Nov 2019

Q7.1

$f(x) = 4 - 7x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{4 - 7(x+h) - (4 - 7x)}{h}$$

$$= \lim_{h \to 0} \frac{h(-7)}{h}$$

$$= -7$$

7.2

$$y = 4x^8 + \sqrt{x^3}$$

$$= 4x^8 + x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = 32x^7 + \frac{3}{2}x^{\frac{1}{2}}$$

Q7.3.1

$$y = ax^2 + a$$

$$\frac{dy}{dx} = 2ax + 0$$

$$\frac{dy}{dx} = 2ax$$

Q7.3.2

$$y = ax^2 + a$$

$$\frac{dy}{da} = x^2 + 1$$

Q7.4

Substitute $(2 \,;\, b)$ in $y = x + \frac{12}{x}$

$$b = 2 + \frac{12}{2}$$

$$b = 8$$

$$m_{\text{tangent}} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{12}{x^2}$$

$$m_{\text{tangent}} = 1 - \frac{12}{2^2} = -2$$

$$m_{\text{perp}} = \frac{1}{2}$$

Equation of perpendicular line:

$$y - y_1 = m(x - x_1)$$

$$y - 8 = \frac{1}{2}(x - 2)$$

$$y = \frac{1}{2}x + 7$$

Q8.1

36cm

Q8.2

$\therefore t = 6$

only once

Q8.3

$$h(t) = -2t^3 + 15t^2 - 24t + 36$$

$$h'(t) = -6t^2 + 30t - 24$$

$$-6t^2 + 30t - 24 = 0$$

$$t^2 - 5t + 4 = 0$$

$$(t-4)(t-1) = 0$$

$t = 4$ or $t = 1$

Only $t = 4$ because maximum value required

$$h = -2(4)^3 + 15(4)^2 - 24(4) + 36 = 52\ cm$$

Q9.1

$$f'(x) = 9x^2$$

$$3x^3 = 9x^2$$

$$3x^3 - 9x^2 = 0$$

$$3x^2(x - 3) = 0$$

$x = 0$ or $x = 3$

Q9.2.1

For f and f'

Q9.2.2

The point $(0 \,;\, 0)$ is :

A point of inflection of f

A turning point of f'

Q9.3

$$f''(x) = 18x$$

$$\text{Distance} = f''(1) - f'(1)$$

$$= 18(1) - 9(1)^2$$

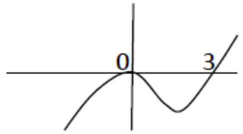
$$= 9$$

Q9.4

$$3x^3 - 9x^2 < 0$$

$$3x^2(x - 3) < 0$$

but $3x^2 > 0$



$$\therefore x - 3 < 0$$

$$\therefore x < 3 \,,\, x \neq 0$$

May-June 2019

Q7.1

$$f(x) = x^2 + 2$$

$$f(x+h) = (x+h)^2 + 2$$

$$= x^2 + 2xh + h^2 + 2$$

$$f(x+h) - f(x) = x^2 + 2xh + h^2 + 2 - (x^2 + 2)$$

$$= 2xh + h^2$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$= \lim_{h \to 0}(2x + h)$$

$$= 2x$$

Q7.2.1

$$y = 4x^3 + 2x^{-1}$$

$$\frac{dy}{dx} = 12x^2 - 2x^{-2}$$

Q7.2.2

$$y = 4\sqrt[3]{x} + (3x^3)^2$$

$$= 4x^{\frac{1}{3}} + 9x^6$$

$$\frac{dy}{dx}=\frac{4}{3}x^{-\frac{2}{3}}+54x^5$$

Q7.3

Point of contact: (1 ; 5)

$m=2$

$y-y_1=m(x-x_1)$

$y-5=2(x-1)$

$y=2x+3$

Q8.1

$h(x)=-2(x+\frac{3}{2})(x-1)(x+3)$

$h(x)=-(2x+3)(x^2+2x-3)$

$h(x)=-2x^3-7x^2+9$

Q8.2

$h'(x)=-6x^2-14x$

$-6x^2-14x=0$

$-2x(3x+7)=0$

$x=0$ or $x=-\frac{7}{3}$

Q8.3

$x<-\frac{7}{3}$ or $x>0$

Q8.4

$y=4x+7$

$-6x^2-14x=4$

$0=6x^2+14x+4$

$0=3x^2+7x+2$

$0=(3x+1)(x+2)$

$x=-\frac{1}{3}$ or $x=-2$

Q9.1

Volume of Sphere

$=\frac{4}{3}\pi(8)^3$ or $=\frac{2048\pi}{3}$

Q9.2

$r^2+x^2=8^2$ (Pythagoras)

$r^2=64-x^2$

Q9.3

$$V_{cone}=\frac{1}{3}\pi r^2h$$

$$=\frac{1}{3}\pi(64-x^2)(8+x)$$

$$=\frac{\pi}{3}(512+64x-8x^2-x^3)$$

$$\frac{dV}{dx}=\frac{64\pi}{3}-\frac{16\pi}{3}x-\frac{3\pi}{3}x^2$$

$$0=64-16x-3x^2$$

$$0=(8-3x)(x+8)$$

$$x=\frac{8}{3} \qquad x\neq-8$$

$$\frac{V_{cone}}{V_{sphere}}=\frac{\frac{1}{3}\pi\left(\frac{512}{9}\right)\left(\frac{32}{3}\right)}{\frac{2048\pi}{3}}$$

$$=\frac{8}{27}=0{,}3$$

__**November 2018**__

Q8.1

$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to0}\frac{x^2+2xh+h^2-5-x^2+5}{h}$$

$$=\lim_{h\to0}\frac{h(2x+h)}{h}$$

$$=\lim_{h\to0}(2x+h)$$

$$=2x$$

Q8.2.1

$$\frac{dy}{dx}=9x^2+12x+1$$

Q8.2.2

$y(x-1)=2x(x-1)$

$y=\frac{2x(x-1)}{x-1}$ if $x\neq1$

$y=2x$

$\frac{dy}{dx}=2$

Q9.1.1

$g(x)=(x+5)(x-x_1)^2$

$20=5(x_1)^2$

$x_1{}^2=4$

$x_1=2$

$g(x)=(x+5)(x-2)^2$

$g(x)=(x+5)(x^2-4x+4)$

$g(x)=x^3+x^2-16x+20$

Q9.1.2

$g(x)=x^3+x^2-16x+20$

$g'(x)=3x^2+2x-16$

$3x^2+2x-16=0$

$(3x+8)(x-2)=0$

$x=\frac{-8}{3}$ or $x=2$

$R\left(\frac{-8}{3};\frac{1372}{27}\right)$ or $R(-2{,}67;50{,}81)$

$P(2;0)$

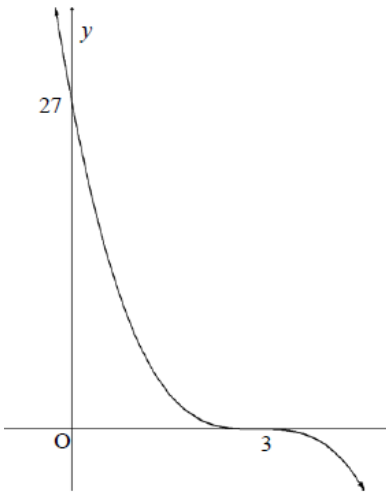
Q9.1.3

$g''(x)=6x+2$

$g''(0)=2$

$\therefore$ concave up

Q9.2



Q10.1

$$\frac{AH}{HG}=\frac{3}{2}$$

Q10.2

Area of a parallelogram $=$ base $\times \perp$ height

$$\text{Area }=\frac{3}{5}(5-t).\frac{2}{5}t$$

$$\text{Area }=\frac{6}{25}(5-t)t$$

$$A(t)=-\frac{6}{25}t^2+\frac{6}{5}t$$

$$A'(t)=-\frac{12}{25}t+\frac{6}{5}$$

$$-\frac{12}{25}t+\frac{6}{5}=0$$

$$12t-30=0$$

$$t=\frac{30}{12}\text{ or }\frac{5}{2}$$

June 2018

Q7.1

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{2-3(x+h)^2-(2-3x^2)}{h}$$

$$=\lim_{h\to 0}\frac{2-3x^2-6xh-3h^2-(2-3x^2)}{h}$$

$$=\lim_{h\to 0}\frac{-6xh-3h^2}{h}$$

$$=\lim_{h\to 0}\frac{h(-6x-3h)}{h}$$

$$=\lim_{h\to 0}(-6x-3h)$$

$$=-6x$$

Q7.2.1

$$D_x\left[(4x+5)^2\right]$$

$$=D_x\left(16x^2+40x+25\right)$$

$$=32x+40$$

Q7.2.2

$$y=\sqrt[4]{x}+\frac{x^2-8}{x^2}$$

$$y=x^{\frac{1}{4}}+1-8x^{-2}$$

$$\frac{dy}{dx}=\frac{1}{4}x^{-\frac{3}{4}}+16x^{-3}$$

Q8.1

$C(0;12)$

Q8.2

$$-x^3+13x+12=0$$

$$x^3-13x-12=0$$

$$(x+1)(x^2-x-12)=0$$

$$(x+1)(x-4)(x+3)=0$$

$A(-3;0)$

$B(4;0)$

Q8.3

$$g(x)=x^3-13x-12$$

$$g'(x)=3x^2-13$$

$$g''(x)=6x$$

$$6x=0$$

$$x=0$$

$$(0;-12)$$

Q8.4

$$f'(x)=-3x^2+13$$

$$-3x^2+13=-14$$

$$-3x^2=-27$$

$$x^2=9$$

$$x=3\quad\text{or}\quad x=-3$$

Q9.1.1

$AC=\ t-30$

Q9.1.2

$30^2=(t-30)^2+p^2$ [Pythagoras]

$$p^2=900-(t-30)^2$$

$$p^2=900-(t^2-60t+900)$$

$$p^2=900-t^2+60t-900$$

$$p^2=60t-t^2$$

Q9.2

$$V(t)=\frac{1}{3}\pi r^2t$$

$$=\frac{1}{3}\pi(60t-t^2)t$$

$$=20\pi t^2-\frac{1}{3}\pi t^3$$

Q9.3

$$V(t)=20\pi t^2-\frac{1}{3}\pi t^3$$

$$V'(t)=40\pi t-\pi t^2$$

$$40\pi t-\pi t^2=0$$

$$t(40\pi-t\pi)=0$$

$t=0$ OR $t=40$ cm

N/A

Q9.4

Volume of cone/keël

$$=20(\pi)(40)^2-\frac{1}{3}\pi(40)^3$$

$=10\ 666{,}67\pi$ or $33510{,}33211$

Volume of sphere/sfeer

$$=\frac{4}{3}\pi r^3$$

$$=\frac{4}{3}\pi(30)^3$$

$=36000\pi$ or $113097{,}3355$

$$\frac{10666{,}67\pi}{36000\pi}$$

$=0{,}296296$

$\approx 29{,}63\%$

March 2018

Q8.1

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\left[\frac{4(x+h)^2 - 4x^2}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{4x^2 8xh + 4h^2 - 4x^2}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{8xh + 4h^2}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{h(8x+4h)}{h}\right]$$

$$= 8x$$

Q8.2.1

$$D_x\left[\frac{x^2-2x-3}{x-1}\right]$$

$$= D_x\left[\frac{(x-3)(x+1)}{x+1}\right]$$

$$= D_x(x-3)$$

$$= 1$$

Q8.2.2

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

$$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$$

Q9.1

$$f(x) = (x+2)(x-1)(x-4)$$

$$= (x^2+x-2)(x-4)$$

$$= x^3 + x^2 - 2x - 4x^2 - 4x + 8$$

$$= x^3 - 3x^2 - 6x + 8$$

$$b = -3 \;;\; c = -6 \;;\; d = 8$$

Q9.2

$$f(x) = x^3 - 3x^2 - 6x + 8$$

$$f'(x) = 0$$

$$3x^2 - 6x - 6 = 0$$

$$x^2 - 2x - 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{12}}{2}$$

$$x = -0{,}73$$

Q9.3

$$f(x) = x^3 - 3x^2 - 6x + 8$$

$$f(-1) = (-1)^3 - 3(-1)^2 - 6(-1) + 8$$

$$= 10$$

$$f'(-1) = 3(-1)^2 - 6(-1) - 6$$

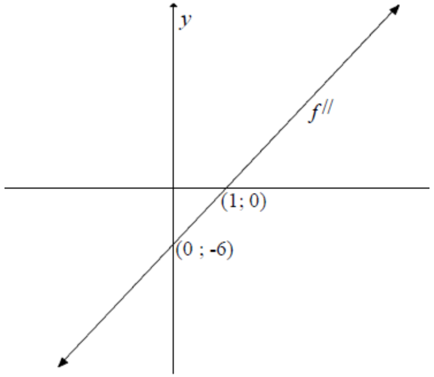
$$= 3$$

$$y - 10 = 3(x+1)$$

$$y = 3x + 13$$

Q9.4

$$f''(x) = 6x - 6$$



Q9.5

f concave upwards

$$f''(x) > 0$$

$$6x - 6 > 0$$

$$x > 1$$

Q10

$$f(x) = -3x^3 + x$$

$$-9x^2 + 1 = 0$$

$$x = \frac{1}{3} \quad \text{or} \quad x = -\frac{1}{3}$$

Maximum of f will be at $x = \frac{1}{3}$

$$f\left(\frac{1}{3}\right) = -3\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)$$

$$= \frac{2}{9}$$

Maximum of $f(x)+q$ will also be at $x = \frac{1}{3}$

$$f\left(\frac{1}{3}\right) + q = \frac{8}{9}$$

$$\frac{2}{9} + q = \frac{8}{9}$$

$$q = \frac{6}{9}$$

$$= \frac{2}{3}$$

For $f(x)+q$ to have a maximum of $\frac{8}{9}$ the value of q has to be $\frac{2}{3}$.

November 2017:

Q7.1

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{2(x+h)^2 - (x+h) - (2x^2 - x)}{h}$$

$$= \lim_{h\to 0}\frac{2x^2 + 4xh + 2h^2 - x - h - 2x^2 + x}{h}$$

$$= \lim_{h\to 0}\frac{4xh + 2h^2 - h}{h}$$

$$= \lim_{h\to 0}\frac{h(4x + 2h - 1)}{h}$$

$$= \lim_{h\to 0}(4x + 2h - 1)$$

$$= 4x - 1$$

Q7.2.1

$$D_x[(x+1)(3x-7)]$$

$$= D_x(3x^2 - 4x - 7)$$

$$= 6x - 4$$

Q7.2.2

$$y=\sqrt{x^3}-\frac{5}{x}+\frac{1}{2}\pi$$

$$y=x^{\frac{3}{2}}-5x^{-1}+\frac{1}{2}\pi$$

$$\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}+5x^{-2}$$

Q8.1

$$f(x)=x^3-6x^2+9x$$

$$f'(x)=3x^2-12x+9$$

$$f''(x)=6x-12=0$$

$$x=2$$

$$f''(0)=6(0)-12$$

$$=-12$$

$$f''(3)=6(3)-12$$

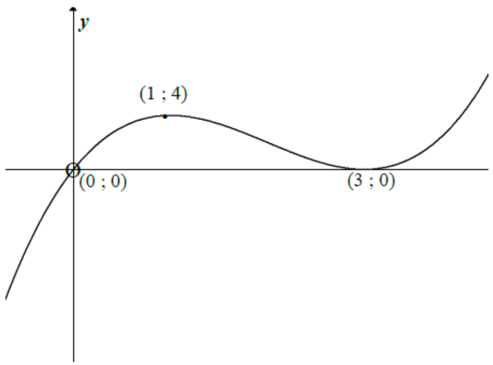
$$=6$$

$f''(x)<$ | $f''(x)>0$

2

Point of inflection at $x=2$

Q8.2



Q8.3

f concave up for $x>2$

$y=-f(x)$ will be concave down for $x>2$

Q8.4.1

$(3;7)$

Q8.4.2

$$f'(2)=3(2)^2-12(2)+9$$

$$=-3$$

$$\neq 1$$

Q9:

$$y=x^2+2$$

$$\mathrm{P}(x\,;x^2+2)$$

$$\mathrm{B}(0\,;3)$$

$$\mathrm{PB}^2=(x-0)^2+(x^2+2-3)^2$$

$$=x^2+x^4-2x^2+1$$

$$=x^4-x^2+1$$

PB will be a minimum if PB^2 is a minimum

$$\frac{d(\mathrm{PB}^2)}{dx}=4x^3-2x$$

$$4x^3-2x=0$$

$$x(2x^2-1)=0$$

$$x=0 \text{ or } x^2=\frac{1}{2}$$

$$x=\frac{1}{\sqrt{2}}$$

$$\mathrm{PB}^2=\left(\frac{1}{\sqrt{2}}\right)^4-\left(\frac{1}{\sqrt{2}}\right)^2+1$$

$$=\frac{1}{4}-\frac{1}{2}+1$$

$$=\frac{3}{4}$$

$$\mathrm{PB}=\frac{\sqrt{3}}{2}=0{,}87$$

<u>June 2017:</u>

Q8.1

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{3-2(x+h)^2-(3-2x^2)}{h}$$

$$=\lim_{h\to 0}\frac{3-2x^2-4xh-2h^2-3+2x^2}{h}$$

$$=\lim_{h\to 0}\frac{-4xh-2h^2}{h}$$

$$=\lim_{h\to 0}\frac{h(-4x-2h)}{h}$$

$$=\lim_{h\to 0}(-4x-2h)$$

$$=-4x$$

Q8.2

$$y=\frac{12x^2+2x+1}{6x}$$

$$=2x+\frac{1}{3}+\frac{1}{6x}$$

$$=2x+\frac{1}{3}+\frac{1}{6}x^{-1}$$

$$\frac{dy}{dx}=2-\frac{1}{6}x^{-2}$$

$$=2-\frac{1}{6x^2}$$

Q8.3

$$y=x^3+bx^2+cx-4$$

$$y'=3x^2+2bx+c$$

$$y''=6x+2b$$

At point of inflection:

$$y''=6x+2b=0$$

Substitute $x=2$:

$$6(2)+2b=0$$

$$2b=-12$$

$$b=-6$$

$$y=x^3-6x^2+cx-4$$

Substitute $(2;\,4)$:

$$4=2^3-6(2)^2+c(2)-4$$

$$2c=24$$

$$c=12$$

$$y=x^3-6x^2+12x-4$$

Q9.1

$(0\,;\,1)$

Q9.2

$f(x) = x^3 - x^2 - x + 1$

$f(x) = x^2(x-1) - (x-1)$

$f(x) = (x-1)(x^2-1)$

$f(x) = (x-1)(x-1)(x+1)$

$f(x) = 0$

$(x-1)(x-1)(x+1) = 0$

x-intercepts: $(-1;\ 0);\ (1;\ 0)$

Q9.3

$f(x) = x^3 - x^2 - x + 1$

$f'(x) = 3x^2 - 2x - 1$

$f'(x) = 0$

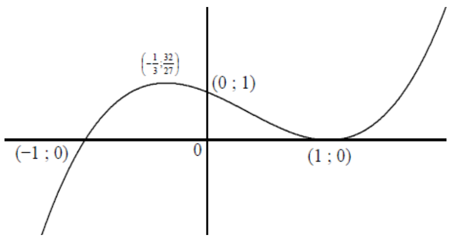
$(3x+1)(x-1) = 0$

$x = -\frac{1}{3}$ or $x = 1$

$y = \frac{32}{27}$ $\quad y = 0$

$\left(-\frac{1}{3};\frac{32}{27}\right)$ $(1;0)$

Q9.4



Q9.5

$f'(x) < 0$

$-\frac{1}{3} < x < 1$

Q10.1

$60 = 2b + 2r + \frac{1}{2}(2\pi r)$

$2b = 60 - 2r - \pi r$

$b = 30 - r - \frac{1}{2}\pi r$

Q10.2

$A(r) = \text{length} \times \text{breadth} + \frac{1}{2}(\text{area of circle})$

$= (2r)\left(30 - r - \frac{1}{2}\pi r\right) + \frac{1}{2}(\pi r^2)$

$= 60r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$

$= 60r - 2r^2 - \frac{1}{2}\pi r^2$

$= 60r - \left(2 + \frac{1}{2}\pi\right)r^2$

For a maximum,

$A'(r) = 0$

$60 - 2\left(2 + \frac{1}{2}\pi\right)r = 0$

$60 - (4 + \pi)r = 0$

$r = \frac{60}{4+\pi}$

$= 8{,}40 \text{ m}$

<u>March 2017:</u>

Q7.1

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 5 - (x^2 - 5)}{h}$

$= \lim_{h\to 0} \frac{2xh + h^2}{h}$

$= \lim_{h\to 0} \frac{h(2x+h)}{h}$

$= \lim_{h\to 0}(2x+h)$

$= 2x$

Q7.2

$g(x) = 5x^2 - \frac{2x}{x^3}$

$= 5x^2 - 2x^{-2}$

$g'(x) = 10x + 4x^{-3}$

$= 10x + \frac{4}{x^3}$

Q7.3

$h(x) = ax^2,\ x > 0$

$h^{-1}:\ x = ay^2 \quad y > 0$

$y = \sqrt{\frac{x}{a}}$

$h^{-1}(8) = \sqrt{\frac{8}{a}}$

$h'(x) = 2ax$

$h'(4) = 2a(4)$

$= 8a$

$\sqrt{\frac{8}{a}} = 8a$

$64a^2 = \frac{8}{a}$

$a^3 = \frac{1}{8}$

$a = \frac{1}{2}$

Q8.1

$f'(x) = 0$

$6x^2 - 10x + 4 = 0$

$3x^2 - 5x + 2 = 0$

$(3x-2)(x-1) = 0$

$x = \frac{2}{3}$ or $x = 1$

$y = 2\left(\frac{2}{3}\right)^3 - 5\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)$

$= \frac{28}{27}$

or y = 1

Turning points are $\left(\frac{2}{3};\frac{28}{27}\right)$ and $(1;1)$

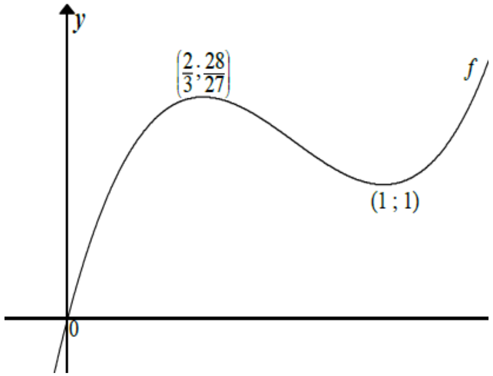
Q8.2

$$2x^3 - 5x^2 + 4x = 0$$
$$x(2x^2 - 5x + 4) = 0$$
$$x = 0 \quad \text{or} \quad x = \frac{5 \pm \sqrt{25 - 4(2)(4)}}{4}$$
$$= \frac{5 \pm \sqrt{-7}}{4}$$

No real roots / *Geen reële wortels*

Q8.3

$$f(x) = 2x^3 - 5x^2 + 4x$$
$$x(2x^2 - 5x + 4) = 0$$



Q8.4

$$f(x) = 2x^3 - 5x^2 + 4x$$
$$f'(x) = 6x^2 - 10x + 4$$
$$f''(x) = 12x - 10$$
$$f''(x) > 0$$
$$12x - 10 > 0$$
$$x > \frac{5}{6}$$

Q9

Length of one side of the square

$$= \frac{x}{4}$$

Length of the rectangle

$$2l + x + \frac{x}{4} = 6$$
$$l = \frac{6 - \frac{5x}{4}}{2}$$
$$= \frac{24 - 5x}{8}$$
$$A = \left(\frac{x}{4}\right)^2 + \frac{x}{4}\left(\frac{24 - 5x}{8}\right)$$
$$= \frac{x^2}{16} + \frac{24x - 5x^2}{32}$$
$$= \frac{24x - 3x^2}{32}$$
$$\frac{dA}{dx} = 0$$
$$\frac{dA}{dx} = \frac{24 - 6x}{32}$$
$$6x = 24$$
$$x = 4$$

PROBABILITY:

Nov 2019

Q10.1

$$\text{P(same day)} = \frac{4}{16} \text{ or } \frac{1}{4} \text{ or } 0{,}25 \text{ or } 25\%$$

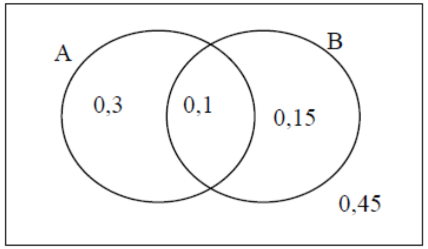
Q10.2

$$\text{P(2 consecutive days)} = \frac{3 \times 2}{16} = \frac{3}{8}$$

Q11.1.1

$P(A) \times P(B)$ independent events

$= 0{,}40 \times 0{,}25 = 0{,}1$



Q11.1.2

$$P(\text{A or not B}) = P(\text{A}) + P(\text{not B}) - P(\text{A and not B})$$
$$= 0{,}4 + 0{,}75 - 0{,}3$$
$$= 0{,}85$$

OR

$$P(\text{A or not B}) = 1 - P(\text{only B})$$
$$= 1 - 0{,}15$$
$$= 0{,}85$$

OR

From Venn diagram:

$0{,}3 + 0{,}1 + 0{,}45 = 0{,}85$

Q11.2

$(5 \times 1 \times 5) + (5 \times 1 \times 6) + (5 \times 1 \times 6) + (5 \times 1 \times 5) = 110$

$110 \times 5 = 550 > 500$

Not possible, because not enough space

OR

$(5 \times 2 \times 5) + (5 \times 2 \times 6) = 110$

$110 \times 5 = 550 > 500$

Not possible because not enough space

OR

$5 \times 4 \times 6 = 120$

$5 \times 2 = 10$

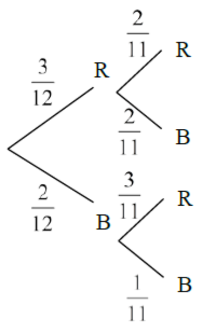
$\therefore 120 - 10 = 110$

$110 \times 5 = 550 > 500$

Not possible because not enough space

May-June 2019

Q10.1



P(One Red and One Blue)

= P(Red, Blue) + P(Blue, Red)

$$= \left(\frac{3}{12}\right) \times \left(\frac{2}{11}\right) + \left(\frac{2}{12}\right) \times \left(\frac{3}{11}\right)$$
$$= \frac{1}{11}$$

Q10.2.1

$$a = 0{,}48 \times 250$$
$$a = 120$$

Q10.2.2

$b = 150$

$P(S) \times P(F)$

$= \frac{200}{250} \times \frac{150}{250}$

$= 0{,}48$

$= P(S \text{ and } F)$

These events are independent /

Hierdie gebeurtenisse is onafhanklik

Q11.1

10×9

$= 90$

Q11.2.1

$10!$

$= 3\ 628\ 800$

Q11.2.2

$2! \times 2! \times 2! \times 2! \times 2! \times 4!$

$= 768$

November 2018

Q11.1.1

$7^5 = 16\ 807$

Q11.1.2

$7 \times 6 \times 5 \times 4 \times 3$

$= \frac{7!}{2!} = 2520$

Q11.2

$2 \times 7 \times 1 = 14$

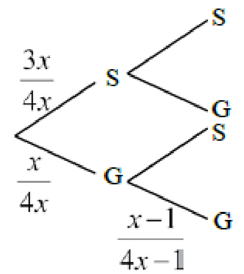
Q12.1

$P(A \text{ or } B) = P(A) + P(B)$

$0{,}74 = 0{,}45 + y$

$y = 0{,}29$

Q12.2



Let the number of mystery gift bags $= x$

The total number of bags $= 4x$

$$\left(\frac{x}{4x}\right) \times \left(\frac{x-1}{4x-1}\right) = \frac{7}{118}$$

$$\frac{1}{4} \times \frac{x-1}{4x-1} = \frac{7}{118}$$

$$\frac{x-1}{4x-1} = \frac{28}{118}$$

$118x - 118 = 112x - 28$

$x = 15$

June 2018

Q10.1

$10!$

$= 3\ 628\ 800$

Q10.2

$4! \times 7!$

$= 120\ 960$

Q10.3

$\frac{6!}{10!}$

$= \frac{1}{5040}$

Q11.1

$P(\text{tennis}) \times P(\leq 35 \text{ years}) = P(\text{tennis and} \leq 35 \text{ years})$

$\frac{21}{140} \times \frac{80}{140} = \frac{a}{140}$

$a = 12$

Q11.2

$P(\text{gym or} \leq 35 \text{ years})$

$= P(\text{gym}) + P(\leq 35 \text{ years}) - P(\text{gym and} \leq 35 \text{ years})$

$= \frac{70}{140} + \frac{80}{140} - \frac{40}{140}$

$= \frac{110}{140}$

$= \frac{11}{14}$ or $0{,}79$

March 2018

Q11.1.1

Let the event Veli arrive late for school be V.

Let the event Bongi arrive late for school be B. /

Laat V die gebeurtenis wees dat Veli Laat B die gebeurtenis wees dat Bongi laatkom

$P(V \text{ or } B) = 1 - 0{,}7$

$= 0{,}3$

Q11.1.2

$P(V \text{ or } B) = P(V) + P(B) - P(V \text{ and } B)$

$0{,}3 = 0{,}25 + P(B) - 0{,}15$

$P(B) = 0{,}2$

Q11.1.3

$P(V) \times P(B) = 0{,}25 \times 0{,}2$

$= 0{,}05$

$P(V) \times P(B) \neq P(V \text{ and } B)$

V and B are NOT independent/

V en B is NIE onafhanklik nie.

Q11.2.1

$6! = 720$

Q11.2.2

Number of arrangements

$= 3! \times 3! \times 2$

$= 72$

Q11.2.3

$P(\text{hearts next to each other})$

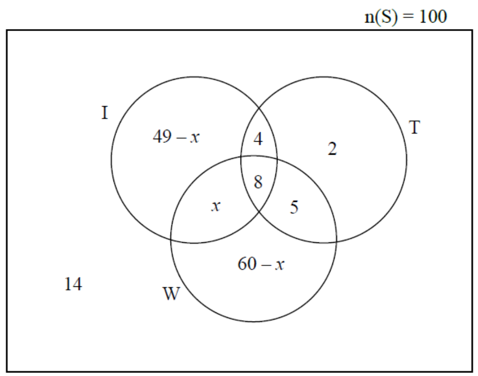
$= \frac{3! \times 4!}{6!}$

$= \frac{144}{720}$

$= \frac{1}{5}$ or $0{,}2$ or 20%

November 2017:

Q10.1



Q10.2

$(49 - x) + x + 8 + 4 + 5 + 2 + (60 - x) + 14 = 100$

$-x + 142 = 100$

$x = 42$

Q10.3

$$\text{P (use only one application)} = \frac{7+2+18}{100}$$

$$= \frac{27}{100} \text{ or } 27\%$$

Q11.1

$$5 \times 5 \times 10 \times 9$$

$$= 2250$$

Q11.2

No of digits used	Letters	Digits	Total
1	5 x 5	10	250
2	5 x 5	10 x 9	2 250
3	5 x 5	10 x 9 x 8	18 000
4	5 x 5	10 x 9 x 8 x7	126 000
5	5 x 5	10 x 9 x 8 x7 x 6	756 000

Codes of two letters and five digits will ensure unique numbers for 700 000 clients.

June 2017:

Q11.1

$$8 \times 7 \times 6 \times 5 \times 4 \quad \text{or} \quad \frac{8!}{3!}$$

$$= 6720$$

Q11.2

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$= 0{,}4 \times 0{,}35$$

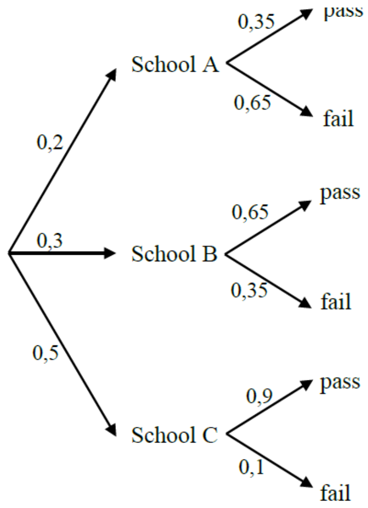
$$= 0{,}14$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= 0{,}4 + 0{,}35 - 0{,}14$$

$$= 0{,}61$$

Q11.3



Q11.3.1

$$100\% - 20\% \qquad \text{or/of} \qquad 1 - 0{,}2$$

$$= 80\% \qquad\qquad = 0{,}8$$

Q11.3.2

$$0{,}3 \times 0{,}35 = 0{,}105$$

$$= 10{,}5\%$$

Q11.3.3

$$(0{,}2 \times 0{,}35) + (0{,}3 \times 0{,}65) + (0{,}5 \times 0{,}9)$$

$$= 0{,}715$$

$$= 71{,}5\%$$

March 2017:

Q10.1.1

$$P(S \text{ and } T) = P(S) \times P(T)$$

$$\frac{1}{6} = \left(\frac{1}{4}\right) \times P(T)$$

$$P(T) = \frac{2}{3}$$

Q10.1.2

$$P(S \text{ or } T) = P(S) + P(T) - P(S \text{ and } T)$$

$$= \left(\frac{1}{4}\right) + \left(\frac{2}{3}\right) - \frac{1}{6}$$

$$= \frac{3}{4}$$

Q10.2.1

$$5!$$

$$= 120$$

Q10.2.2

$$5^5$$

$$= 3125$$

Q10.3

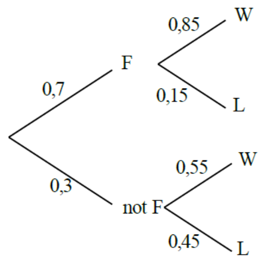
$$n(E) = 5! \times 2! \times 2!$$

$$n(S) = 7!$$

$$P(E) = \frac{5! \times 2! \times 2!}{7!}$$

$$= \frac{2}{21}$$

Q11



P(F and W) = 0,595

P(F and L) = 0,105

P(not F and W) = 0,165

P(not F and L) = 0135

$$P(\text{Win}) = P(F \text{ and } W) + P(\text{not } F \text{ and } W)$$

$$= 0{,}7 \times 0{,}85 + 0{,}3 \times 0{,}45$$

$$= 0{,}595 + 0{,}165$$

$$= 0{,}76$$

$$= 76\%$$

$$= \frac{19}{25}$$

SESSION 4

Trigonometry

Nov 2019

Q5.1

$$\frac{\sin x}{\cos x . \tan x} + \sin(180° + x)\cos(90° - x)$$

$$= \frac{\sin x}{\cos x . \frac{\sin x}{\cos x}} + (-\sin x)\sin x$$

$$= 1 - \sin^2 x$$

$$= \cos^2 x$$

Q5.2

$$\frac{\sin^2 35° - \cos^2 35°}{4 \sin 10° \cos 10°}$$

$$= \frac{-(\cos^2 35° - \sin^2 35°)}{2(2 \sin 10° \cos 10°)}$$

$$= \frac{-\cos 70°}{2 \sin 20°}$$

$$= \frac{-\cos 70°}{2 \cos 70°} \quad \textbf{OR} \quad = \frac{-\sin 20°}{2 \sin 20°} = -\frac{1}{2}$$

Q5.3

$$2\sin^2 77° = 2[\sin(90° - 13°)]^2$$
$$= 2\cos^2 13°$$
$$= 2\cos^2 13° - 1 + 1$$
$$= \cos 26° + 1$$
$$= m + 1$$

Q5.4.1

$$\sin(x+25°)\cos 15° - \cos(x+25°)\sin 15° = \tan 165°$$
$$\sin(x+25°-15°) = -0{,}2679...\ OR\ -2+\sqrt{3}$$
$$\sin(x+10°) = -0{,}2679...OR\ -2+\sqrt{3}$$

$x + 10° = 195{,}54° + k.360°$
$x = 185{,}54° + k.360°;\ k \in Z$ or

or $x + 10° = 344{,}46° + k.360°$
$x = 334{,}46° + k.360°; k \in Z$

Q5.4.2

$$f(x) = \sin(x+10°)$$

For minimum value of $\sin x$: $x = 270°$
For minimum value of $\sin(x+10°)$: $x = 260°$

Q6.1

Range of f: $y \in [-2\ ; 0]$

Q6.2

$x \in (90°\ ;\ 270°)$

Q6.3

$$PQ = \cos 2x - (\sin x - 1)$$
$$= 1 - 2\sin^2 x - \sin x + 1$$
$$= -2\sin^2 x - \sin x + 2$$

$$\sin x = -\frac{b}{2a}$$
$$= \frac{-(-1)}{2(-2)}$$
$$\sin x = -\frac{1}{4}$$
$$\therefore x = 194{,}48° \text{ or } x = 345{,}52°$$

Q7.1

$$\sin 60° = \frac{AK}{x}$$

$$AK = x\sin 60° \text{ or } \frac{\sqrt{3}}{2}x \text{ or } 0{,}866x$$

Q7.2

$$K\hat{C}F = 120°$$

Q7.3

$$KF^2 = CF^2 + CK^2 - 2CF.CK\cos K\hat{C}F$$
$$= x^2 + \left(\frac{x}{2}\right)^2 - 2x\left(\frac{x}{2}\right)\cos 120°$$
$$= x^2 + \frac{x^2}{4} - x^2\left(-\frac{1}{2}\right)$$
$$= \frac{7x^2}{4}$$
$$KF = \frac{\sqrt{7}x}{2}$$

$$A\hat{K}F = y$$

$$\text{Area } \Delta AKF = \frac{1}{2}.AK.KF\sin A\hat{K}F$$
$$= \frac{1}{2}.\frac{\sqrt{3}x}{2}.\frac{\sqrt{7}x}{2}\sin y$$
$$= \frac{x^2\sqrt{21}\sin y}{8}$$

May-June 2019

Q5.1.1

$$\sin 191°$$
$$= -\sin 11°$$

Q5.1.2

$$\cos 22°$$
$$= \cos(2 \times 11°)$$
$$= 1 - 2\sin^2 11°$$

Q5.2

$$\cos(x - 180°) + \sqrt{2}\sin(x + 45°)$$
$$= -\cos x + \sqrt{2}(\sin x\cos 45° + \cos x\sin 45°)$$
$$= -\cos x + \sqrt{2}\left(\sin x\left(\frac{1}{\sqrt{2}}\right) + \cos x\left(\frac{1}{\sqrt{2}}\right)\right)$$
$$= -\cos x + \sin x + \cos x$$
$$= \sin x$$

Q5.3

$$\sin P + \sin Q = \sin P + \cos P$$
$$(\sin P + \cos P)^2 = \left(\frac{7}{5}\right)^2$$
$$\sin^2 P + 2\sin P\cos P + \cos^2 P = \frac{49}{25}$$
$$2\sin P\cos P = \frac{49}{25} - 1$$
$$\sin 2P = \left(\frac{49}{25} - \frac{25}{25}\right)$$
$$= \frac{24}{25}$$

Q6.1

$$\cos(x - 30°) = 2\sin x$$
$$\cos x\cos 30° + \sin x\sin 30° = 2\sin x$$
$$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 2\sin x$$
$$\frac{\sqrt{3}}{2}\cos x = \frac{3}{2}\sin x$$
$$\tan x = \frac{\sqrt{3}}{3}$$
$$x = 30° + k.180°;\quad k \in Z$$

Q6.2.1a

$A(120°\ ;\ 0)$

Q6.2.1b

$C(-150°\ ;\ -1)$

Q6.2.2a

$-90° < x < 30°$

Q6.2.2b

$-160° < x < 20°$

Q6.2.3

Range of $y = 2\sin x$: $y \in [-2\ ;\ 2]$ **OR** $-2 \le y \le 2$
Range of $y = 2\sin x + 3$: $y \in [1\ ;\ 5]$ **OR** $1 \le y \le 5$
Range: $y = 2^{2\sin x + 3}$: $y \in [2\ ;\ 32]$ **OR** $2 \le y \le 32$

Q7.1.1

$$\sin\theta = \frac{x}{AC}$$
$$AC = \frac{x}{\sin\theta}$$

Q7.1.2

$$\cos 60° = \frac{x+2}{CE}$$

$$CE = \frac{x+2}{\cos 60°}$$

$$= \frac{x+2}{\frac{1}{2}} = 2(x+2)$$

Q7.2

$$\text{Area } \Delta ACE = \frac{1}{2} AC.EC.\sin A\hat{C}E$$

$$= \frac{1}{2}\left(\frac{x}{\sin\theta}\right)(2(x+2))\sin 2\theta$$

$$= \frac{x(x+2)\times 2\sin\theta\cos\theta}{\sin\theta}$$

$$= 2x(x+2)\cos\theta$$

Q7.3

$$EC = 2(12+2) = 28$$

$$AE^2 = AC^2 + EC^2 - 2(AC)(EC)\cos A\hat{C}E$$

$$= \left(\frac{12}{\sin 55°}\right)^2 + 28^2 - 2\left(\frac{12}{\sin 55°}\right)(28)\cos 110°$$

$$AE = 35{,}77m$$

November 2018

Q5.1.1

$$k^2 = (\sqrt{5})^2 - 1^2$$

$$= 4$$

$$k = -2$$

Q5.1.2a

$$\tan\theta = -\frac{1}{2}$$

Q5.1.2b

$$\cos(180° + \theta) = -\cos\theta$$

$$= \frac{2}{\sqrt{5}}$$

Q5.1.2c

$$\sin(\theta + 60°) = \frac{a+b}{\sqrt{20}}$$

$$LHS = \sin\theta\cos 60° + \cos\theta\sin 60°$$

$$= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{2}\right) + \left(-\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1-2\sqrt{3}}{2\sqrt{5}}$$

$$= \frac{1-2\sqrt{3}}{\sqrt{20}}$$

Q5.1.3

$$\tan\theta = -\frac{1}{2}$$

$$\therefore \theta = 180° - 26{,}57°$$

$$\therefore \theta = 153{,}43°$$

$$\tan(2\theta - 40°) = \tan[(2\times 153{,}43°) - 40°]$$

$$= \tan 266{,}87°$$

$$= 18{,}3$$

Q5.2

$$LHS = \frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} \qquad RHS = 2\tan 2x$$

$$= \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$$

$$= \frac{\cos^2 x + 2\sin x\cos x + \sin^2 x - \cos^2 x + 2\sin x\cos x - \sin^2 x}{\cos^2 x - \sin^2 x}$$

$$= \frac{2(2\sin x\cos x)}{\cos^2 x - \sin^2 x}$$

$$= \frac{2\sin 2x}{\cos 2x}$$

$$= 2\tan 2x$$

$$= RHS$$

Q5.3

$$\sum_{A=38°}^{52°} \cos^2 A$$

$$= \cos^2 38° + \cos^2 39° + \cos^2 40° + \ldots + \cos^2 51° + \cos^2 52°$$

$$= \sin^2 52° + \sin^2 51° + \sin^2 50° + \ldots + \cos^2 51° + \cos^2 52°$$

$$= 7(1) + \cos^2 45°$$

$$= 7 + \left(\frac{\sqrt{2}}{2}\right)^2 \quad \text{or} \quad = 7 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 7\frac{1}{2}$$

Q6.1

$$\text{Period} = 120°$$

Q6.2

$$2 = -2\tan\frac{3}{2}x$$

$$\tan\left(\frac{3}{2}t\right) = -1$$

$$\frac{3}{2}t = 135° + k.180°$$

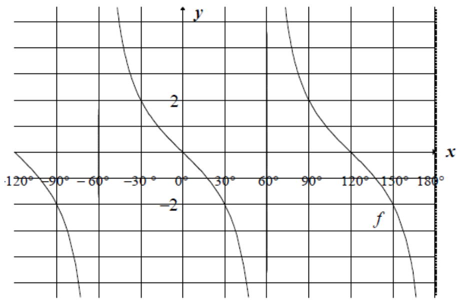
$$t = 90° + k.120° \quad ; \quad k \in Z$$

OR

$$\frac{3}{2}t = -45° + k.180°$$

$$t = -30° + k.120° \quad ; \quad k \in Z$$

Q6.3



Q6.4

$$-60° < x \leq -30° \text{ or } 60° < x \leq 90°$$

Q6.5

$$g(x) = -2\tan\left[\frac{3}{2}(x + 40°)\right] = f(x + 40°)$$

Translation of 40° to the left / skuif met 40° links

Q7.1

$$A\hat{B}D = 30°$$

$$\sin 30° = \frac{h}{AB}$$

$$AB = \frac{h}{\sin 30°}$$

$$AB = 2h$$

Q7.2

$$BC^2 = AB^2 + AC^2 - 2AB.AC\cos B\hat{A}C$$

$$= (2h)^2 + (3h)^2 - 2(2h)(3h)\cos 2x$$

$$= 13h^2 - 12h^2(2\cos^2 x - 1)$$

$$= 13h^2 - 24h^2\cos^2 x + 12h^2$$

$$= 25h^2 - 24h^2\cos^2 x$$

$$BC = h\sqrt{25 - 24\cos^2 x}$$

June 2018

Q5.1.1

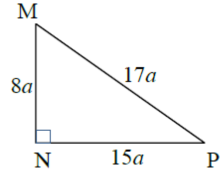
$$\text{Given}: \sin M = \frac{15}{17}$$

$$MN^2 = 17^2 - 15^2$$

$$= 64$$

$$MN = 8$$

OR



$$\therefore \tan M = \frac{15}{8}$$

Q5.1.2

$$\sin M = \frac{NP}{MP}$$

$$\frac{NP}{51} = \frac{15a}{17a}$$

$$\therefore NP = 45$$

Q5.2

$$\cos(x - 360°).\sin(90° + x) + \cos^2(-x) - 1$$

$$= \cos x \,.\, \cos x + \cos^2 x - 1$$

$$= \cos^2 x + \cos^2 x - 1$$

$$= 2\cos^2 x - 1$$

$$= \cos 2x$$

Q5.3.1

$\sin(2x+40°)\cos(x+30°)-\cos(2x+40°)\sin(x+30°)$
$=\sin[(2x+40°)-(x+30°)]$
$=\sin(x+10°)$

Q5.3.2

$\sin(2x+40°)\cos(x+30°)-\cos(2x+40°)\sin(x+30°)=\cos(2x-20°)$
$\therefore \cos(2x-20°)=\sin(x+10°)$
$\cos(2x-20°)=\cos[90°-(x+10°)]$
$2x-20°=80°-x+k.360°$ or $2x-20°=360°-(80°-x)+k.360°$
$3x=100°+k.360°$ or $2x-20°=280°+x+k.360°$
$x=33{,}33°+k.120°$ or $x=300°+k.360°\ ;\ k\in Z$

Q6.1

Period $=720°$

Q6.2

$-2\le y\le 2$

Q6.3

$f(-120°)-g(-120°)$
$=-3\sin\left(-\dfrac{120°}{2}\right)-2\cos(-120°-60°)$
$=\dfrac{4+3\sqrt{3}}{2}$ or 4,60 (4,5980...)

Q6.4.1

x-intercepts of g at $-90°+60°=-30°$
and $90°+60°=150°$
$-30°<x<150°$

Q6.4.2

$-180°\le x<-120°$
$-30°<x<60°$
$150°<x\le 180°$

Q7.1

In PMQ : $\tan\theta=\dfrac{x}{\text{QM}}$
$\therefore \text{QM}=\dfrac{x}{\tan\theta}$

Q7.2

In PMR : $\tan\theta=\dfrac{x}{\text{MR}}$

OR PMQ ≡ PMR [AAS/HHS]

$\therefore \text{MR}=\dfrac{x}{\tan\theta}=\text{QM}$

$Q\hat{M}R=180°-2\beta$

$\dfrac{\sin\beta}{\text{MR}}=\dfrac{\sin Q\hat{M}R}{12x}$

$\sin\beta\times\dfrac{\tan\theta}{x}=\dfrac{\sin(180°-2\beta)}{12x}$

$\tan\theta=\dfrac{\sin 2\beta}{12x}\times\dfrac{x}{\sin\beta}$

$\tan\theta=\dfrac{2\sin\beta\cos\beta}{12x}\times\dfrac{x}{\sin\beta}$

$\tan\theta=\dfrac{\cos\beta}{6}$

Q7.3

$\dfrac{x}{\text{QM}}=\dfrac{\cos\beta}{6}$ [both equal $\tan\theta$]

$x=\dfrac{60\cos 40}{6}$

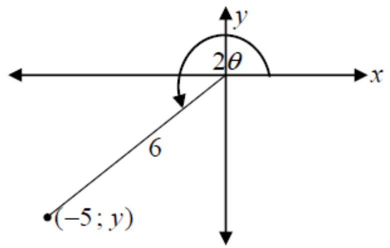
$x=7{,}66$

The height of the lighthouse is 8 metres

March 2018

Q5.1.1

$\cos 2\theta=-\dfrac{5}{6}$, where $2\theta\in[180°;270°]$



$y^2=6^2-(-5)^2$ [Pythagoras]
$y=\pm\sqrt{11}$
(5 ; y) is in 3rd quadrant:
$\therefore y=-\sqrt{11}$

$\sin 2\theta=-\dfrac{\sqrt{11}}{6}$

Q5.1.2

$\cos 2\theta=1-2\sin^2\theta$
$2\sin^2\theta=1-\cos 2\theta$

$\sin^2\theta=\dfrac{1-\left(-\dfrac{5}{6}\right)}{2}$

$=\dfrac{11}{6}\times\dfrac{1}{2}$

$=\dfrac{11}{12}$

Q5.2

$\sin(180°-x).\cos(-x)+\cos(90°+x).\cos(x-180°)$
=sinx.cosx + sinxcosx
=2sinxcosx
=sin2x

Q5.3

$\sin 3x.\cos y+\cos 3x.\sin y$
$\sin(3x+y)$
$=\sin 270°$
$=-1$

Q5.4.1

$2\cos x=3\tan x$
$2\cos x=\dfrac{3\sin x}{\cos x}$
$2\cos^2 x=3\sin x$
$2(1-\sin^2 x)=3\sin x$
$2-2\sin^2 x=3\sin x$
$2\sin^2 x+3\sin x-2=0$

Q5.4.2

$2\sin^2 x+3\sin x-2=0$
$(2\sin x-1)(\sin x+2)=0$
$\sin x=\dfrac{1}{2}$ or $\sin x=-2$ (no solution)

$x=30°+k.360°$ or $x=150°+k.360°\ ;\ k\in Z$

Q5.4.3

$5y=30°+k.360°$ or $5y=150°+k.360°$
$y=6°+k.72°$ or $y=30°+k.72°$
$\therefore y=144°+6°$ or $y=144°+30°$
$y=150°$ or $y=174°$

Q5.5.1

$g(x)=-4\cos(x+30°)$
maximum value = 4

Q5.5.2

range of/*waardeversameling van* g(x):
$-4\le y\le 4$ **OR/*OF*** $y\in[-4\ ;\ 4]$

$\therefore$range of/*waardeversameling van* g(x) + 1:
$-3\le y\le 5$ **OR/*OF*** $y\in[-3\ ;\ 5]$

Q5.5.3

$y = -4\cos(x + 30°)$

shifted to the left/*skuif na links*:

$y = -4\cos(x + 30° + 60°)$

$= -4\cos(x + 90°)$

$= 4\sin x$

$\therefore h(x) = -4\sin x$

Q6.1.1

$\tan\theta = \frac{PQ}{QR} = \frac{PQ}{x}$

$\therefore PQ = x\tan\theta$

Q6.1.2

$\frac{AR}{\sin A\hat{Q}R} = \frac{QR}{\sin Q\hat{A}R}$

$AR = \frac{x\sin(90° + \theta)}{\sin\theta}$

Q6.2

$\sin 2\theta = \frac{AB}{AR}$

$AB = AR\sin 2\theta$

$= \frac{x\sin(90° + \theta).\sin 2\theta}{\sin\theta}$

$= \frac{x\cos\theta.\sin 2\theta}{\sin\theta}$

$= \frac{x\cos\theta.2\sin\theta\cos\theta}{\sin\theta}$

$= 2x\cos^2\theta$

Q6.3

$\frac{AB}{QP} = \frac{2x\cos^2 12°}{x\tan 12°}$

$= 9$

<u>November 2017:</u>

Q5.1

$\frac{\sin(A - 360°).\cos(90° + A)}{\cos(90° - A).\tan(-A)}$

$= \frac{\sin A(-\sin A)}{\sin A(-\tan A)}$

$= \frac{\sin A}{\left(\frac{\sin A}{\cos A}\right)}$

$= \cos A$

Q5.2.1

$t^2 = (\sqrt{34})^2 - (3)^2$

$\therefore t = -5$

Q5.2.2

$\tan\beta = \frac{-5}{3}$

Q5.2.3

$\cos 2\beta = 2\cos^2\beta - 1$

$= 2\left(\frac{3}{\sqrt{34}}\right)^2 - 1$

$= 2\left(\frac{9}{34}\right) - 1$

$= -\frac{16}{34}$ OR $-\frac{8}{17}$

Q5.3.1

LHS = sin(A + B) – sin(A – B)

= sin A.cos B + cos A.sin B – (sin A. cos B – cos A. sin B)

= sin A.cos B + cos A.sin B – sin A. cos B + cos A. sin B

= 2cos A.sin B

= RHS

Q5.3.2

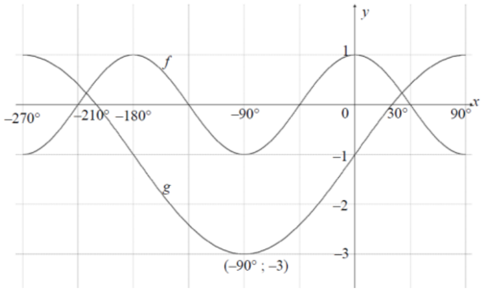
$\sin 77° - \sin 43° = \sin(60° + 17°) - \sin(60° - 17°)$

$= 2\cos 60°.\sin 17°$

$= 2 \times \frac{1}{2} \times \sin 17°$

$= \sin 17°$

Q6.1



Q6.2

$\cos 2x = 2\sin x - 1$

$1 - 2\sin^2 x = 2\sin x - 1$

$2\sin^2 x + 2\sin x - 2 = 0$

$\sin^2 x + \sin x - 1 = 0$

$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$

$\sin x = \frac{-1 + \sqrt{5}}{2}$

since $\sin x = \frac{-1 - \sqrt{5}}{2} < -1$ has no solution

Q6.3

$\sin x = \frac{-1 + \sqrt{5}}{2} = 0{,}618...$

Reference $\angle = 38{,}17°$

$\therefore x = 38{,}17° + k.360°$ or $x = 141{,}83° + k.360°$; $k \in Z$

$\therefore x = 38{,}17°$ or $-218{,}17°$

$y = 0{,}24$

$\therefore$ Points of intersection/*snypunte*:

$(38{,}17° ; 0{,}24)$ and $(-218{,}17° ; 0{,}24)$

Q7.1

$A\hat{B}C = 90°$

Q7.2

In Δ ABE:

$\frac{AB}{BE} = \tan y$

$AB = k\tan y$

In Δ ABC:

$\frac{AB}{AC} = \sin x$

$AC = \frac{AB}{\sin x}$

$= \frac{k\tan y}{\sin x}$

Q7.3

$A\hat{D}C = A\hat{C}D = \frac{180° - 2x}{2} = 90° - x$

$\frac{DC}{\sin 2x} = \frac{AC}{\sin(90° - x)}$

$\frac{DC}{2\sin x\cos x} = \frac{AC}{\cos x}$

$DC = \frac{AC(2\sin x\cos x)}{\cos x}$

$= \frac{k\tan y}{\sin x}.\frac{2\sin x\cos x}{\cos x}$

$= 2k\tan y$

<u>June 2017:</u>

Q5.1.1

$$\tan A = \frac{\sin A}{\cos A}$$
$$= \frac{2p}{p}$$
$$= 2$$

Q5.1.2

$\sin^2 A + \cos^2 A = 1$

$(2p)^2 + p^2 = 1$

$4p^2 + p^2 = 1$

$5p^2 = 1$

$p^2 = \frac{1}{5}$

$\therefore p = -\frac{1}{\sqrt{5}}$

Q5.2

$2\sin^2 x - 5\sin x + 2 = 0$

$(2\sin x - 1)(\sin x - 2) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = 2$ (no solution)

ref $\angle = 30°$

$\therefore x = 30° + k.360°$ or $x = 150° + k.360°\ ; k \in Z$

Q5.3.1

$\sin(x + 300°) = \sin x \cos 300° + \cos x \sin 300°$

Q5.3.2

$\sin(x + 300°) - \cos(x - 150°)$

$= \sin x \cos 300° + \cos x \sin 300° - (\cos x \cos 150° + \sin x \sin 150°)$

$= \sin x \cos 60° - \cos x \sin 60° - (-\cos x \cos 30° + \sin x \sin 30°)$

$= \sin x \cos 60° - \cos x \sin 60° + \cos x \cos 30° - \sin x \sin 30°$

$= \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$

$= 0$

Q5.4

Consider: $\frac{\tan x + 1}{\sin x \tan x + \cos x} = \sin x + \cos x$

$$\text{LHS} = \frac{\left(\frac{\sin x}{\cos x} + 1\right)}{\left(\sin x.\frac{\sin x}{\cos x} + \cos x\right)} = \frac{\left(\frac{\sin x + \cos x}{\cos x}\right)}{\left(\frac{\sin^2 x + \cos^2 x}{\cos x}\right)}$$

$$= \frac{\frac{\sin x + \cos x}{\cos x}}{\frac{1}{\cos x}}$$

$$= \frac{\sin x + \cos x}{\cos x} \times \frac{\cos x}{1}$$

$= \sin x + \cos x$

$=$ RHS

Q5.5.1

$\left(\sqrt{1+k}\right)^2 = (\sin x + \cos x)^2$

$1 + k = \sin^2 x + 2\sin x \cos x + \cos^2 x$

$1 + k = 1 + \sin 2x$

$k = \sin 2x$

Q5.5.2

From 5.5.1

$\sin x + \cos x = \sqrt{1 + \sin 2x}$

$\therefore$ max value: $\sin x + \cos x = \sqrt{1+1}$

$= \sqrt{2}$

Q6.1

Period $= 180°$

Q6.2

$-75°$

Q6.3

$\sin 2x \le \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x$

$\sin 2x \le \cos 45°.\cos x + \sin 45°.\sin x$

$\sin 2x \le \cos(x - 45°)$

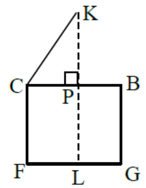
$x \in [-75°\ ; 165°]$

Q7.1

KC = 6 cm

Q7.2

Let P be the point of intersection of KL and CB



$\frac{KP}{KC} = \sin 60°$

$KP = 6\sin 60°$

$KP = 3\sqrt{3}$ or 5,20

$\therefore KL = 8 + 3\sqrt{3}$ or 13,20 cm

Q7.3

$DK^2 = 6^2 + 12^2$

$DK = \sqrt{180}$ or $6\sqrt{5}$ or 13,42 cm

$\frac{\sin K\hat{D}L}{KL} = \frac{\sin D\hat{L}K}{DK}$

$\frac{\sin K\hat{D}L}{\sin D\hat{L}K} = \frac{KL}{DK}$

$= \frac{8 + 3\sqrt{3}}{6\sqrt{5}}$ or $\frac{13,20}{13,42}$ or 0,98

March 2017:

Q5.1

$a = -1$

$b = 2$

Q5.2

$f(3x) = -\sin 3x$

Period of $f(3x) = \frac{360°}{3}$

$= 120°$

Q5.3

$x \in [90°\ ;\ 135°) \cup \{180°\}$

Q6.1.1

$\sin(360° - 36°) = -\sin 36°$

Q6.1.2

$\cos 72° = \cos(2 \times 36°)$

$= 1 - 2\sin^2 36°$

Q6.2

$$\text{LHS} = \frac{1 + \tan^2\theta - \tan^2\theta}{1 + \tan^2\theta}$$

$$= \frac{1}{1 + \frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{1}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}$$

$$= \frac{1}{\frac{1}{\cos^2\theta}}$$

$= \cos^2\theta$

$=$ RHS

Q6.3

$$\cos^2 \frac{1}{2}x = \frac{1}{4}$$

$$\cos \frac{1}{2}x = \frac{1}{2} \text{ or } -\frac{1}{2}$$

$$\frac{1}{2}x = 60° + k.360° \quad \text{or} \quad \frac{1}{2}x = 300° + k.360° \quad \text{or}$$

$$\frac{1}{2}x = 120° + k.360° \quad \text{or} \quad \frac{1}{2}x = 240° + k.360°$$

$$x = 120° + k.720° \quad \text{or} \quad x = 600° + k.720° \quad \text{or}$$

$$x = 240° + k.720° \quad \text{or} \quad x = 480° + k.720°; \quad k \in Z$$

Q6.4.1

$$\begin{aligned}\sin(A - B) &= \cos[90° - (A - B)]\\ &= \cos[(90° + B) - A]\\ &= \cos(90° + B)\cos A + \sin(90° + B)\sin A\\ &= -\sin B\cos A + \cos B\sin A\\ &= \sin A\cos B - \cos A\sin B\end{aligned}$$

Q6.4.2

$$\begin{aligned}&\sin(x + 64°)\cos(x + 379°) + \sin(x + 19°)\cos(x + 244°)\\ &= \sin(x + 64°)\cos(x + 19°) + \sin(x + 19°)[-\cos(x + 64°)]\\ &= \sin(x + 64°)\cos(x + 19°) - \cos(x + 64°)\sin(x + 19°)\\ &= \sin[x + 64° - (x + 19°)]\\ &= \sin 45°\\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

Q7.1

$$\sin 27° = \frac{CD}{8,6}$$

$$CD = 8,6\sin 27°$$

$$CD = 3,90\text{ m}$$

Q7.2

$$\cos 40° = \frac{10}{AE}$$

$$AE = \frac{10}{\cos 40°}$$

$$AE = 13,05\text{ m}$$

Q7.3

$$AC^2 = CE^2 + AE^2 - 2\,CE.AE(\cos A\hat{E}C)$$

$$= (8,6)^2 + (13,05)^2 - 2(8,6)(13,05)(\cos 70°)$$

$$= 167,49$$

$$AC = 12,94\text{ m}$$

STATISTICAL REASONING:

Nov 2019

Q1.1

$$a = -1946,875\ldots = -1946,88$$

$$b = 0,41$$

$$\hat{y} = -1946,88 + 0,41x$$

Q1.2

$$\hat{y} = -1946,88 + 0,41(14000)$$

$$\approx R3\ 793,12$$

Q1.3

$$r = 0,946\ \ldots \approx 0,95$$

Q1.4

Not to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. **OR** D

Q2.1

Number people paid R200 or less = 19

Q2.2

$$7 + 12 + a + 35 + b + 6 = 100$$

$$a = 40 - b$$

$$309 = \frac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times b)+(550\times6)}{100}$$

$$309 = \frac{(50\times7)+(150\times12)+(250\times(40-b))+(350\times35)+(450\times b)+(550\times6)}{100}$$

$$350 + 1800 + 10000 - 250b + 12250 + 450b + 3300 = 30900$$

$$200b = 3200$$

$$b = 16$$

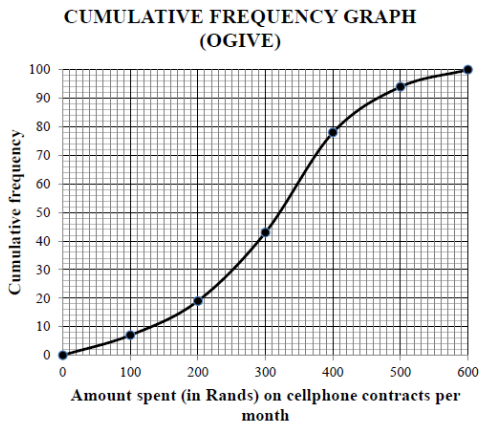
$$a = 24$$

Q2.3

Modal Class:

$$300 < x \leq 400$$

Q2.4



Q2.5

18 people paid more than R420 per month

May-June 2019

Q1.1

45 children

Q1.2

$$\bar{x} = \frac{\sum fx}{n}$$

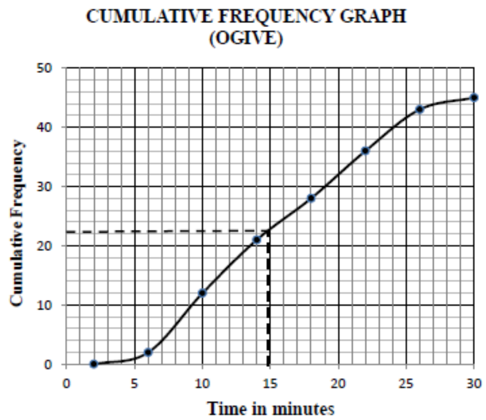
$$= \frac{(4\times2)+(8\times10)+(12\times9)+(16\times7)+(20\times8)+(24\times7)+(28\times2)}{45}$$

$$\bar{x} = 15,38 \text{ minutes}$$

Q1.3

Time taken (t) (in minutes)	Number of children	Cumulative frequency
$2 < t \leq 6$	2	2
$6 < t \leq 10$	10	12
$10 < t \leq 14$	9	21
$14 < t \leq 18$	7	28
$18 < t \leq 22$	8	36
$22 < t \leq 26$	7	43
$26 < t \leq 30$	2	45

Q1.4



Q1.5

On graph at the y-value of 22,5 or 23

Median = ± 15 minutes.

Q2.1

$$a = 12,44$$

$$b = 0,98$$

$$y = 12,44 + 0,98x$$

Q2.2.1

$$\text{Percentage} = \frac{15}{50}\times100$$

$$= 30\%$$

Q2.2.2

$\hat{y} = 12,44 + 0,98x$

$\hat{y} = 12,44 + 0,98(30)$

$\hat{y} = 41,84$

$= 42$

Q2.3.1

standard deviation =13,88

Q2.3.2

$x = 50,67 - 45,67$

$= 5\%$

November 2018

Q1.1.1

140 items

Q1.1.2

$20 \le x < 30$ minutes

Q1.1.3

Number of minutes taken = 20 minutes

Q1.1.4

140 – 126 [Accept: 124 to 128]

14 orders (12 to 16)

Q1.1.5

75^{th} percentile is at 105 items

=37 minutes [accept 36 – 38 minutes]

Q1.1.6

Lower quartile is at 35 items

=21,5 min [accept 21 – 23 min]

IQR = 37 – 21,5

= 15,5 min [accept 13 – 17 min]

Q1.2.1a

$$\bar{x} = \frac{1420}{15}$$

$= R94,666.. = R94,67$

Q1.2.1b

$\sigma = R22,691... = R22,69$

Q1.2.2a

They both collected the **same (equal) amount** in tips, i.e. R1 420 over the 15-day period.

Q1.2.2b

Mary's standard deviation is smaller than Reggie's which suggests that there was **greater variation in the amount of tips that Reggie collected** each day compared to the number of tips that Mary collected each day.

Q2.1

251 km/h

Q2.2.1

$r = 0,52$

Q2.2.2

The points are **fairly scattered** and the least squares regression line is increasing.

Q2.3

There is a weak positive relation hence the height could have an influence

Q2.4

For (0 ; 27,07), it means that the player has a height of 0 m but can serve at a speed of 27,07 km/h.

It is impossible for a person to have a height of 0 m.

June 2018

Q1.1.1

$$\text{Mean}/\textit{Gemiddelde} = \frac{2283}{12}$$

$= 190,25$

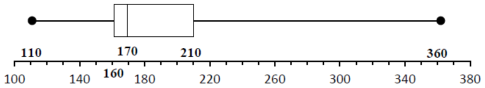
Mean profit/ = R190 250,00

Q1.1.2

$$\text{Median} = \frac{169 + 171}{2} = 170 \text{ thousand rands}$$

= R170 000

Q1.2



Q1.3

$IQR = Q_3 - Q_1$

= 210 – 160 thousand rands

= R50 000

Q1.4

Skewed to the right or positively skewed.

Q1.5.1

$\sigma = 67,04118759$ thousand rands

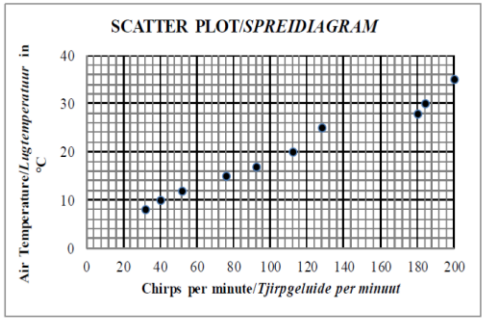
= R67 041,19

Q1.5.2

$\bar{x} - \sigma = 123,21$ thousand rands

For **2** months the profit was less than one standard deviation below the mean.

Q2.1



Q2.2

$r = 0,99$ so there is a very strong positive relationship between the number of chirps per minute and the temperature of the air.

Q2.3

$a = 3,97$

$b = 0,15$

$\hat{y} = 3,97 + 0,15x$

Q2.4

$\hat{y} \approx 3,97 + 0,15(80)$

$\approx 15,97°C$

March 2018

Q1.1.1

$$\bar{x} = \frac{800}{10}$$

$= 80$

Q1.1.2

$\sigma = 18,83$

Q1.1.3

(61,17 ; 98,83)

Days 1, 2, 8, 9 and 10 lie outside 1 Sd from the mean.

$\therefore$ 5 days

Q1.2.1

Skewed to the left or negatively skewed.

Q1.2.2

A = 65

B = 99

Q1.3

New total = 95 × 10 = 950

$\therefore$ Units not counted = 950 – 800 = 150

Q2.1

Outlier/*Uitskieter*: (100 ; 100)

Q2.2

$a = 94,50273...$

$b = 2,913729...$

$\hat{y} = 94,50 + 2,91x$

Q2.3

$\hat{y} = 2{,}91(240) + 94{,}50$ (CA from 2.1)

$= 792{,}90$

Value = R793 000

Q2.4

$b = 2{,}913729\ldots$

$\therefore$ R2 914 **OR/*OF*** R2 910 (calculator)

November 2017:

Q1.1

$a = 14{,}343\ldots = 14{,}34$

$b = -0{,}642\ldots = -0{,}64$

Q1.2

$y = 14{,}34 - 0{,}64(11{,}7)$

$= 6{,}85$

Q1.3

The gradient increases

The point (12,3 ; 7,6) lies some distance above the current data

Q2.1.1

$\bar{x} = \frac{472}{23}$

$\bar{x} = 20{,}52$ seconds

Q2.1.2

$Q_1 = 16$

$Q_3 = 24$

IQR/*IKO* $= Q_3 - Q_1$

$= 24 - 16 = 8$

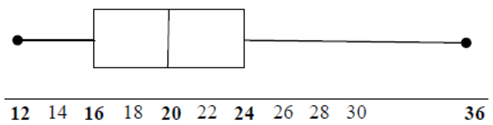
Q2.2

$20{,}52 + 5{,}94 = 26{,}46$

$\therefore > 26{,}46$

$\therefore$ 4 girls/*dogters*

Q2.3



Q2.4.1

Girls

Q2.4.2

Five-number summary of boys:

(15 ; 21 ; 23,5 ; 26 ; 38)

None of the boys

5 girls completed in less than 15 seconds which was the minimum time taken by the boys.

June 2017:

Q1.1

$a = 4{,}806\ldots = 4{,}81$

$b = 1{,}323\ldots = 1{,}32$

$y = 4{,}81 + 1{,}32x$

Q1.2

$y = 4{,}81 + 1{,}32(16)$

$y = 25{,}93$

Cost = R25 930

Q1.3

$r = 0{,}949\ldots = 0{,}95$

Q1.4

$x = 0$

$y = 4{,}81$ **OR** (4,80647)

$\therefore$ R4 810 **OR** R4806,47

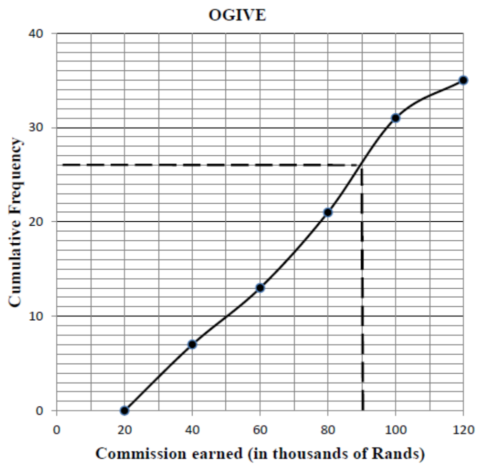
Q2.1

modal class: $80 < x \leq 100$

Q2.2

Commission earned (in thousands of Rands)	Frequency	Cumulative Frequency
$20 < x \leq 40$	7	7
$40 < x \leq 60$	6	13
$60 < x \leq 80$	8	21
$80 < x \leq 100$	10	31
$100 < x \leq 120$	4	35

Q2.3



Q2.4

No. of salesmen awarded bonuses: 35 – 26 = 9 salesmen

Q2.5

Estimated mean $= \frac{(30\times7)+(50\times6)+(70\times8)+(90\times10)+(110\times4)}{35}$

$= \frac{2410}{35}$

= 68,86 thousand rand or R68 857,14

= R69 000 or 69 thousand rand

March 2017:

Q1.1

65 learners

Q1.2

Modal class

$30 \leq x < 40$

Q1.3

$a = 12$

$b = 61 - 45$

$= 16$

Q1.4

No. of learners = 65 – 54 = 11

Q2.1.1

IQR of Class B

$= Q_3 - Q_1$

$= 72 - 51$

$= 21$ marks/*punte*

Q2.1.2

Although the boxes contain the same number of data points, the marks for Class A are more widely spread

Q2.1.3

Medians are the same

Ranges are the same **OR** Maximum and minimum values are the same

75% of both classes obtained 51 and above

Q2.2.1

$a = -0{,}03$

$b = 0{,}93$

$\hat{y} = -0{,}03 + 0{,}93x$

Q2.2.2

$\hat{y} = -0{,}03 + 0{,}93(15)$

$= 13{,}92$ **OR/*OF*** $13{,}85$

≈ 14

Q2.2.3

Yes

because r = 0,9.

SESSION 5

Analytical Geometry

Nov 2019

Q3.1

Equation of PR: y = 5

Q3.2.1

$$m_{RS} = \frac{5-(-7)}{3-(-3)} = \frac{12}{6}$$
$$= 2$$

Q3.2.2

$$m_{RS} = m_{PT} \quad [PT \parallel RS]$$
$$\tan\theta = 2$$
$$\theta = 63{,}43°$$

Q3.2.3

$$m_{RS} = m_{RD} = m_{DS}$$
$$2 = \frac{5-y}{3-0} = \frac{y+7}{0-(-3)}$$
$$\therefore y = -1$$
$$\therefore D(0\,;-1)$$

Q3.3

$$ST = 2\sqrt{5} = \sqrt{[-5-(-3)]^2 + (k-(-7))^2}$$
$$20 = 4 + (k+7)^2$$
$$(k+7)^2 = 16$$
$$k+7 = \pm 4$$
$$k = -11 \text{ or } k = -3$$
$$\therefore k = -3$$

Q3.4

Method: translation

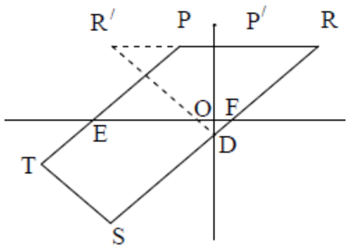
T→S:

$$(x\,;y) \to (x+2\,;y-4)$$

∴ by symmetry: D→N:

$$D(0\,;-1) \to N(0+2\,;-1-4)$$
$$\therefore N(2\,;-5)$$

Q3.5



β is the inclination of RS ∴ $\beta = 63{,}434...°$

$O\hat{F}D = 63{,}434...°$ [vert opp ∠s]

$O\hat{D}F = 90° - 63{,}434...° = 26{,}565...°$

$R\hat{D}R' = 2(26{,}565...°) = 53{,}13°$

Q4.1

$$M(-1;1)$$
$$(x+1)^2 + (y-1)^2 = 1$$

Q4.2

Midpoint of CB, N: (– 0,5 ; 1,5)

$$\therefore \frac{x_C + 0}{2} = -\frac{1}{2} \text{ and } \frac{y_C + 1}{2} = \frac{3}{2}$$
$$\therefore C(-1\,;2)$$

Q4.3

$$m_{radius} = \frac{2-1}{-1-0}$$
$$= -1$$
$$\therefore m_{tangent} = 1$$
$$y - y_1 = m(x - x_1)$$
$$y - y_1 = 1(x - x_1)$$
$$y - 2 = 1(x-(-1))$$
$$y - 2 = x + 1$$
$$\therefore y = x + 3$$
$$y - x = 3$$

Q4.4

Tangents to circle:

$y = x + 3$ and $y = x + 1$

$\therefore t > 3$ or $t < 1$

Q4.5

$$D(-3\,;0)$$

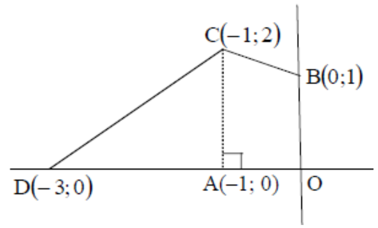
C→N:

$$(x\,;y) \to (x+0{,}5\,;y-0{,}5)$$

D→E:

$$D\,(x\,;y) \to E(x+0{,}5\,;y-0{,}5)$$
$$\therefore E(-3+0{,}5\,;0-0{,}5)$$
$$\therefore E(-2{,}5\,;-0{,}5)$$

Q4.6



area of trapezium AOBC = $\frac{1}{2}(1+2)(1)$

= $1\frac{1}{2}$ square units

area of ΔACD = $\frac{1}{2}(2)(2)$

= 2 square units

area of quadrilateral OBCD = $3\frac{1}{2}$ square units

$$\therefore 2a^2 = \frac{7}{2}$$
$$a^2 = \frac{7}{4}$$
$$a = \frac{\sqrt{7}}{2}$$

May-June 2019

Q3.1.1

Midpoint of EC:

$$= \left(\frac{-2+2}{2}\,;\frac{0+(-3)}{2}\right) = \left(0\,;\frac{-3}{2}\right)$$

Q3.1.2

$$m_{DC} = \frac{-3-(-5)}{2-(-2)}$$
$$= \frac{2}{4} = \frac{1}{2}$$

Q3.1.3

$$m_{AB} = \frac{1}{2}$$
$$y = \frac{1}{2}x + c$$
$$0 = \frac{1}{2}(-2) + c$$
$$c = 1$$
$$\therefore y = \frac{1}{2}x + 1$$

Q3.1.4

$\tan\alpha = m_{AB} = \frac{1}{2}$

$\alpha = 26{,}57°$

$\theta = 90° + 26{,}57°$

$= 116{,}57°$

Q3.2

B(0 ; 1)

$m_{BC} = \frac{1-(-3)}{0-2}$

$= -2$

$m_{AB} \times m_{BC} = \frac{1}{2} \times -2$

$= -1$

$\therefore AB \perp BC$

Q3.3.1

$A\hat{B}C = 90°$

∴ EC is diameter [converse: ∠ in semi circle]

∴ centre of circle $= \left(0 ; -\frac{3}{2}\right)$

Q3.3.2

$(x-0)^2 + \left(y+\frac{3}{2}\right)^2 = r^2$

$(-2-0)^2 + \left(0+\frac{3}{2}\right)^2 = r^2$

$\therefore\ r^2 = \frac{25}{4}$ or $r = \frac{5}{2}$

$x^2 + \left(y+\frac{3}{2}\right)^2 = \frac{25}{4}$

Q4.1

$(x-2)^2 + (y-1)^2 = 25$

$(-2-2)^2 + (b-1)^2 = 25$

$(b-1)^2 = 9$

$b-1 = \pm 3$

$\therefore b = 4$ or $b \neq -2$

Q4.2.1

K(2 ; 1 – 5)

∴ K(2 ; –4)

Q4.2.2

$m_{MT} = \frac{4-1}{-2-2} = -\frac{3}{4}$

$m_{PL} = \frac{4}{3}$ [radius ⊥ tangent]

$y - y_1 = \frac{4}{3}(x - x_1)$

$y - 4 = \frac{4}{3}(x+2)$

$y = \frac{4}{3}x + \frac{20}{3}$

Q4.2.3

$y_L = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$

$L\left(2 ; \frac{28}{3}\right)$ and $K(2 ; -4)$: $LK = \frac{28}{3} - (-4) = \frac{40}{3}$

Coordinates of P:

$\frac{x+2}{2} = -4\frac{1}{2}$ and $\frac{y-4}{2} = -6$

$\therefore x = -11$ $y = -8$

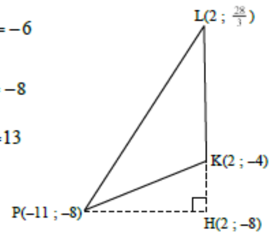
$\therefore P(-11 ; -8)$

$\perp$height $(PH) = 2-(-11) = 13$

Area $\Delta PKL = \frac{1}{2}(LK)(PH)$

$= \frac{1}{2}\left(\frac{40}{3}\right)(13)$

$= \frac{260}{3}$ **OR** 86,67 square units



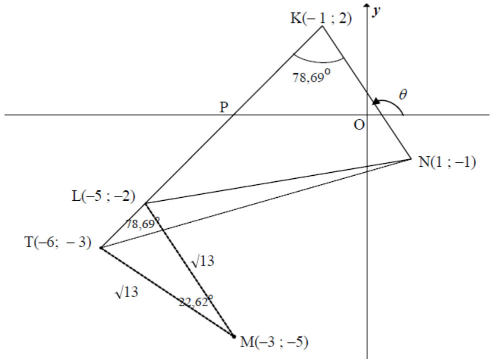
Q4.3

The centres of the two circles lie on the same vertical line $x = 2$. and the sum of the radii = 10

$n - 1 = 10$ or $1 - n = 10$

$n = 11$ $n = -9$

<u>November 2018</u>



Q3.1.1

$m_{KN} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{KN} = \frac{2-(-1)}{-1-1}$

$= -\frac{3}{2}$

Q3.1.2

$\tan\theta = m_{KN} = -\frac{3}{2}$

$\theta = 180° - 56{,}31°$

$\theta = 123{,}69°$

Q3.2

Inclination KL = 123,69° – 78,69° = 45°

$\tan 45° = m_{KL} = 1$

Q3.3

$y - y_1 = 1(x - x_1)$

$y - 2 = 1(x - (-1))$

$y = x + 3$

Q3.4

$KN = \sqrt{(1+1)^2 + (-1-2)^2}$

$KN = \sqrt{13}$ or 3,61

Q3.5.1

$(x+3)^2 + (y+5)^2 = 13 \quad \ldots(1)$

L is a point on KL

$y = x + 3 \quad \ldots(2)$

(2) in (1):

$(x+3)^2 + (x+3+5)^2 = 13$

$x^2 + 6x + 9 + x^2 + 16x + 64 = 13$

$2x^2 + 22x + 60 = 0$

$x^2 + 11x + 30 = 0$

$(x+5)(x+6) = 0$

$x = -5$ or $x = -6$

$y = -2$ or $y = -3$

L(–5 ; –2) or (–6 ; –3)

Q3.5.2

Midpoint of KM: (– 2 ; – 1,5)

$\therefore \frac{x_L + 1}{2} = -2$ and $\frac{y_L - 1}{2} = -\frac{3}{2}$

∴ L(– 5 ; – 2)

Q3.6

T(–6 ; –3) (from Question 3.5.1)

$KT = \sqrt{(-1-(-6))^2 + (2-(-3))^2}$

$= \sqrt{50}$

$KN = \sqrt{13}$ (CA from 3.4)

Area of $\Delta KTN = \frac{1}{2} KT.KN \sin L\hat{K}N$

$= \frac{1}{2}\sqrt{50}.\sqrt{13}\sin 78{,}69°$

= 12,50 square units

Q4.1

F(3 ; 1)

Q4.2

$FS = \sqrt{(6-3)^2 + (5-1)^2}$

$FS = 5$

Q4.3

FH(FS) : HG = 1 : 2

$\therefore$ HG = 2 FH

= 10

Q4.4

Tangents from common/same point

Q4.5.1

$F\hat{H}J = 90°$

$FJ^2 = 20^2 + 5^2$

$FJ = \sqrt{425}$ or $5\sqrt{17}$ or 20,62

[tan ⊥ radius / *rkl ⊥ radius*]

[Pyth theorem/*stelling*]

Q4.5.2

$(x-m)^2 + (y-n)^2 = 100$

Q4.5.3

K(22; n)

GK = HG = 10

FH = FS = 5

$m = 22 - 10$

$m = 12$

Let J(22 ; y):

$FJ^2 = (22-3)^2 + (y-1)^2$

$425 = 361 + y^2 - 2y + 1$

$0 = y^2 - 2y - 63$

$0 = (y-9)(y+7)$

$\therefore y = 9$ or/of $y \neq -7$

$\therefore n = 9 - 20 = -11$

$\therefore$ G(12; −11)

<u>June 2018</u>

Q3.1

$$m_{AC} = \frac{1-(-4)}{7-(-3)} = \frac{5}{10} = \frac{1}{2}$$

Q3.2.1

$$y - y_1 = \frac{1}{2}(x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 7)$$

$$y - 1 = \frac{1}{2}x - \frac{7}{2}$$

$$y = \frac{1}{2}x - 2\frac{1}{2}$$

Q3.2.2

$$M\left(\frac{-2+8}{2} ; \frac{9+(-11)}{2}\right)$$

$\therefore$ M(3 ; −1)

Equation of AC: $y = \frac{1}{2}x - 2\frac{1}{2}$

$y = \frac{1}{2}(3) - 2\frac{1}{2}$

$y = -1$

$\therefore$ M lies on AC

Q3.3

$$m_{BD} = \frac{9-(-11)}{-2-8} = -2$$

$$m_{BD} \times m_{AC} = \frac{1}{2} \times -2 = -1$$

$\therefore BD \perp AC$

Q3.4.1

$\tan\theta = m_{BD} = -2$

$\therefore \theta = 116{,}57°$

Q3.4.2

$\tan\beta = m_{BC}$

$$m_{BC} = \frac{9-(-4)}{-2-(-3)} = 13$$

$\beta = 85{,}6°$

$\therefore C\hat{B}D = 116{,}57° - 85{,}60°$ [ext ∠ of Δ]

$= 30{,}97°$

Q3.4.3

$$AC = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

$$= \sqrt{(7-(-3))^2 + (1-(-4))^2}$$

$$= \sqrt{100+25}$$

$$= \sqrt{125} = 5\sqrt{5} = 11{,}58$$

Q3.4.4

$$BM = \sqrt{((-2)-3)^2 + (9-(-1))^2}$$

$$= \sqrt{125} = 5\sqrt{5}$$

Area of ΔABC $= \frac{1}{2}$ base × ⊥ height

$= \frac{1}{2}(\sqrt{125})(\sqrt{125})$

= 62,5 square units

Area of ABCD = 2 × 62,5

= 125 square units

Q4.1

$$M\left(\frac{0+4}{2} ; \frac{0+(-6)}{2}\right)$$

$\therefore$ M(2 ; − 3)

Q4.2.1

$x^2 + y^2 = 4^2 + (-6)^2$

$= 52$

$\therefore x^2 + y^2 = 52$

Q4.2.2

$$(x-2)^2 + (y+3)^2 = \left(\frac{\sqrt{52}}{2}\right)^2 = 13$$

$x^2 - 4x + 4 + y^2 + 6y + 9 - 13 = 0$

$x^2 + y^2 - 4x + 6y = 0$

Q4.2.3

$$m_{OP} = \frac{-6}{4} = -\frac{3}{2}$$

$m_{RS} \times m_{OP} = -1$

$\therefore m_{RS} = \frac{2}{3}$

$\therefore y = \frac{2}{3}x$

Q4.3

$$x^2+y^2=52 \quad and \quad y=\frac{2}{3}x$$

$$x^2+\left(\frac{2}{3}x\right)^2=52$$

$$x^2+\frac{4}{9}x^2=52$$

$$1\frac{4}{9}x^2=52$$

$$x^2=36$$

$$x=6$$

$\therefore$ R(6 ; 4) and N(–6 ; 4)

$\therefore$ NR = 12 units

Q4.4

Let T(x ; 0) be the other x intercept of the small circle
Then OT is the common chord

$$\therefore (x-2)^2+(0+3)^2=13$$

$$(x-2)^2=13-9=4$$

$$x-2=\pm 2$$

$$x=2\pm 2$$

$$x=4 \quad or \quad 0$$

$\therefore$length of common chord = OT = 4 units

March 2018

Q3.1

x=3

Q3.2

$$m_{QP}=\tan 71{,}57°$$
$$=3$$

Q3.3

$$y-y_1=m(x-x_1)$$
$$y+2=3(x+7)$$
$$y=3x+19$$

Q3.4

R(3 ; 0)

$$QR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(-7-3)^2+(-2-0)^2}$$
$$=\sqrt{104} \text{ or } 2\sqrt{26}$$

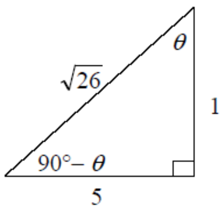
Q3.5

$$\tan(90°-\theta)=m_{QR}$$
$$=\frac{0-(-2)}{3-(-7)}$$
$$=\frac{1}{5}$$

Q3.6

$$RN=\frac{1}{2}.2\sqrt{26}=\sqrt{26}$$

SR = 6



$$\text{Area } \Delta RSN=\frac{1}{2}SR\ .\ RN.\ \sin\theta$$
$$=\frac{1}{2}\times 6\times\sqrt{26}\times\frac{5}{\sqrt{26}}$$
$$=15 \text{ square units}$$

Q4.1

$$OK=\sqrt{180} \quad \text{or} \quad 6\sqrt{5}$$

Q4.2

$$a^2+b^2=180$$
$$a=2b$$
$$(2b)^2+b^2=180$$
$$5b^2=180$$
$$b^2=36 \qquad \therefore b=-6$$
$$a=2(-6)$$

K (–12 ; – 6) (given)

Q4.3.1

$$m_{OK}=\frac{1}{2}$$
$$m_{PT}=-2$$
$$y=mx+c$$
$$-6=-2(-12)+c$$
$$c=-30$$
$$y=-2x-30$$

Q4.3.2

3MK = OK

$$\Rightarrow OM=\frac{4}{3}OK$$

$$M=\frac{4}{3}(-12;-6)$$

$\therefore$M(–16 ; –8)

Q4.3.3

$$(x-(-16))^2+(y-(-8))^2=\left(\frac{1}{3}\sqrt{180}\right)^2$$
$$(x+16)^2+(y+8)^2=20$$

Q4. 4

$$OK<r<OK+2KM$$
$$\sqrt{180}<r<\sqrt{180}+\frac{2}{3}\sqrt{180}$$
$$6\sqrt{5}<r<10\sqrt{5}$$

Q4.5

$$x^2+32x+(16)^2+y^2+16y+(8)^2=256+64-240$$
$$(x+16)^2+(y+8)^2=80$$

New circle/*nuwe sirkel*:
Centre/*middelpt* $(-16;-8)$ &
$r=4\sqrt{5}$

Original circle/*oorspronklike sirkel*:
M$(-16;-8)$ & $r=2\sqrt{5}$

This circle will never cut the circle with centre M as they have the **same centre (concentric circles) but unequal radii**/*Hierdie sirkel sal nooit die sirkel met middelpt M sny nie, want hulle is* ***konsentries, want het dieselfde middelpunt met verskillende radii***.

November 2017:

Q3.1.1

$$m_{FC}=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{3\frac{1}{2}-(-4)}{3-8}$$
$$=-\frac{3}{2}$$

$$(y-(-4))=-\frac{3}{2}(x-8)$$
$$y+4=-\frac{3}{2}x+12$$
$$y=-\frac{3}{2}x+8$$

Q3.1.2

AC: $3x + 2y = 16$ and BG: $7x - 10y = 8$

$15x + 10y = 80$

$\underline{7x - 10y = 8}$

$22x = 88$

$x = 4$

$3(4) + 2y = 16$

$y = 2$

$\therefore G(4\ ;\ 2)$

Q3.2

$\dfrac{x_A + 4}{2} = 3$ and $\dfrac{y_A + 2}{2} = 3\dfrac{1}{2}$

$\therefore A(2\ ;\ 5)$

Q3.3

The coordinates of the midpt of AB:

$\left(\dfrac{2+(-6)}{2} ; \dfrac{5+(-5)}{2}\right) = (-2\ ;\ 0)$

But the y-coordinate of E is 0

$\therefore$ E(–2 ; 0) is the midpoint of AB

$\therefore$ EF || BG [midpoint theorem

Q3.4

Midpoint of AC $= \left(5\ ;\ \dfrac{1}{2}\right)$

$\dfrac{x_D + (-6)}{2} = 5$ and $\dfrac{y_D + (-5)}{2} = \dfrac{1}{2}$

$\therefore D(16\ ;\ 6)$

Q4.1.1

$m_{PK} = \dfrac{5-(-3)}{-4-0}$

$= -2$

PK ⊥ SR [radius ⊥ tangent

$\therefore m_{PK} \times m_{RS} = -1$

$\therefore m_{RS} = \dfrac{1}{2}$

Q4.1.2

$(y-5) = \dfrac{1}{2}(x-(-4))$

$(y-5) = \dfrac{1}{2}x + 2$

$y = \dfrac{1}{2}x + 7$

Q4.1.3

$M\left(\dfrac{-4+0}{2} ; \dfrac{5+(-3)}{2}\right)$

$\therefore M(-2\ ;\ 1)$

$r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$r^2 = (-2+4)^2 + (1-5)^2$

$\therefore r^2 = 20$

$\therefore (x+2)^2 + (y-1)^2 = 20$ *or* $(\sqrt{20})^2$

Q4.1.4

$\tan\theta = m_{PK} = -2$

$\therefore \theta = 180° - 63{,}43°$

$= 116{,}57°$

$P\hat{K}R = 116{,}57° - 90°$ [ext ∠of ΔMOK]

$= 26{,}57°$

Q4.1.5

RS || tangent at K(0 ; – 3)

$\therefore m_{PS} = m_{tang} = \dfrac{1}{2}$

$\therefore y = \dfrac{1}{2}x - 3$

Q4.2

$t \in (-3\ ;\ 7)$

Q4.3

RS: $y = \dfrac{1}{2}x + 7$ $\therefore S(-14\ ;\ 0)$

$SP = \sqrt{(-14-(-4))^2 + (0-5)^2} = \sqrt{100+25} = \sqrt{125}$

Area ΔSMK $= \dfrac{1}{2}$. MK . SP

$= \dfrac{1}{2}(\sqrt{20})(\sqrt{125})$

= 25 square units

<u>June 2017:</u>

Q3.1

$m_{CD} = \dfrac{-3-(-5)}{4-0}$

$= \dfrac{-3+5}{4-0}$

$= \dfrac{1}{2}$

Q3.2

$m_{AD} = \dfrac{-5-3}{0-(-4)}$

$= -2$

$m_{CD} \times m_{AD} = \dfrac{1}{2} \times -2$

$= -1$

$\therefore$ AD ⊥ DC

Q3.3

$AB = \sqrt{(3+4)^2 + (4-3)^2} = \sqrt{50} = 5\sqrt{2}$

$BC = \sqrt{(4-3)^2 + (-3-4)^2} = 5\sqrt{2}$

AB = BC

$\therefore$ ΔABC is an iscosceles triangle

Q3.4

[BF || DC]

$y - 4 = \dfrac{1}{2}(x-3)$

$y - 4 = \dfrac{1}{2}x - 1\dfrac{1}{2}$

$y = \dfrac{1}{2}x + 2\dfrac{1}{2}$

Q3.5

$\tan\alpha = -2$

$\therefore \alpha = 116.57°$

$\alpha = 90° + \theta$

$\therefore \theta = 26{,}57°$

Q3.6

$$x^2+y^2=r^2$$
$$(4)^2+(-3)^2=25$$
$$x^2+y^2=25$$

Q4.1

$$N\left(\frac{1+(-3)}{2};\frac{4+(-2)}{2}\right)$$

$N(-1;1)$ is the centre of the circle

Q4.2

$$r=\sqrt{(1-(-1))^2+(4-1)^2}$$
$$r=\sqrt{13}=\text{radius}$$
$$(x+1)^2+(y-1)^2=13$$

Q4.3

$$m_{NM}\times m_{MR}=-1$$
$$m_{NM}=\frac{1-(-2)}{-1-(-3)}$$
$$=\frac{3}{2}$$
$$m_{MR}=-\frac{2}{3}$$
$$y-y_1=-\frac{2}{3}(x-x_1)$$
$$y+2=-\frac{2}{3}(x+3)$$
$$y=-\frac{2}{3}x-4$$

Q4.4

Symmetry of a kite: S(–3 ; 4)

Q4.5

$(SR)^2=(RM)^2$...Tangents from common pt

$$(x+3)^2+(y-4)^2=(x+3)^2+(y+2)^2$$
$$y^2-8y+16=y^2+4y+4$$
$$-12y=-12$$
$$y=1$$
$$\frac{2}{3}x=-4-1$$
$$x=-\frac{15}{2}$$
$$\therefore R\left(-7\frac{1}{2};1\right)$$

Q4.6

$$RS=\sqrt{(-3+7{,}5)^2+(4-1)^2}$$
$$RS=\frac{3\sqrt{13}}{2}=5{,}41$$

area of RSNM = 2area of ΔRSN

$$=2\left(\frac{1}{2}\right)(\sqrt{13})\left(\frac{3\sqrt{13}}{2}\right)$$
$$=\frac{39}{2}$$

March 2017:

Q3.1

$$m_{TQ}=\frac{4-0}{0-3}$$
$$=-\frac{4}{3}$$

Q3.2

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$RQ=\sqrt{(10-3)^2+(7-0)^2}$$
$$RQ=\sqrt{98}=7\sqrt{2}$$

Q3.3

$$m_{FQ}=m_{TQ}$$
$$\frac{-8}{k-3}=-\frac{4}{3}$$
$$4k-12=24$$
$$k=9$$

Q3.4

Using transformation

$\therefore S(7;11)$

Q3.5

$T\hat{S}R=T\hat{Q}R$ [opp ∠s of ||m

$$T\hat{Q}R=\alpha-\beta$$
$$\tan\alpha=m_{TQ}=-\frac{4}{3}$$
$$\therefore \alpha=180°-53{,}13°=126{,}87°$$
$$\tan\beta=m_{RQ}=\frac{7}{7}=1$$
$$\therefore \beta=45°$$
$$T\hat{Q}R=126{,}87°-45°$$
$$=81{,}87°$$
$$T\hat{S}R=81{,}87°$$

Q3.6.1

$$MQ=\sqrt{(5-3)^2+(2-0)^2}$$
$$MQ=\sqrt{8}$$
$$\frac{MQ}{RQ}=\frac{\sqrt{8}}{\sqrt{98}}$$
$$=\frac{2}{7}\quad\text{or}\quad 0{,}29$$

Q3.6.2

$$\frac{\text{area of }\Delta TQM}{\text{area of }\Delta TQR}=\frac{\frac{1}{2}.QM.\perp h}{\frac{1}{2}.QR.\perp h}\quad[\perp h\text{ same}$$
$$=\frac{QM}{QR}=\frac{2}{7}$$
$$\frac{\text{area of }\Delta TQM}{\text{area of parm }RQTS}=\frac{\text{area of }\Delta TQM}{2\times\text{area of }\Delta TQR}$$
$$=\frac{1}{2}\left(\frac{2}{7}\right)=\frac{1}{7}$$

Q4.1

line from centre to **midpt of chord**

Q4.2

$$m_{ST}=\frac{8-5}{-3-0}$$
$$=-1$$
$$m_{ST}\times m_{NP}=-1$$
$$\therefore m_{NP}=1$$
$$y-y_1=1(x-x_1)$$
$$y-8=1(x+3)$$
$$y=x+11$$

Q4.3

P(0 ; 11) [*y*-intercept of chord NP]

∴ radius is 6 units

R(0 ; –1)

Equations of the tangents to the circle parallel to the *x*-axis

$y = 11$ and $y = -1$

Q4.4

M(–11 ; 0) [x–intercept

$$MT = \sqrt{(0-11)^2 + (5-0)^2}$$

$$MT = \sqrt{146} = 12,08$$

Q4.5

MT = diameter/*middellyn* [conv∠ in $\frac{1}{2}$ circle

radius $= \frac{\sqrt{146}}{2}$ units

Centre of circle/*Middelpunt v sirkel*

= Midpoint MT /*Middelpunt MT*

$= \left(\frac{-11}{2} ; \frac{5}{2}\right)$

Equation of circle through S, T and M:

$$\left(x + \frac{11}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{146}{4}$$

EUCLIDEAN GEOMETRY

Nov 2019

Q8.1.1

$\hat{R} = 80°$ [co-int ∠s/*ko-binne* ∠*e*; QW || RK]

Q8.1.2

$\hat{P} = 100°$ [opp ∠s of cyclic quad

Q8.1.3

$P\hat{Q}R = 136°$

$\hat{Q}_2 = 36°$ ext ∠ of cyclic quad

Q8.1.4

$\hat{U}_2 = \hat{S}_2 = 136°$

[alt ∠s/*verwiss* ∠*e* ; QW || RK]

Q8.2.1

In ΔEFT and ΔDCT:

$$\frac{EF}{CD} = \frac{9}{18} = \frac{1}{2}$$

$$\frac{FT}{TC} = \frac{5}{10} = \frac{1}{2}$$

$$\frac{ET}{TD} = \frac{7}{14} = \frac{1}{2}$$

∴ ΔEFT ||| ΔDCT

∴ $E\hat{F}D = E\hat{C}D$ Sides of Δ in prop

Q8.2.2

$E\hat{F}D = E\hat{C}D$

E, F, C and D are concyclic

EFCD is a cyclic quad

converse ∠s in the same segment

∴ $D\hat{F}C = D\hat{E}C$ [∠s in the same segment

Q9

$\hat{O}_2 = 360° - x$ [∠s round a pt/∠*e om 'n punt*]

∴ $\hat{M} = 180° - \frac{1}{2}x$ [∠ at centre = 2 ×∠ at circumference/ *middelpunts∠ = 2 × omtreks∠*]

∴ $\hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$ [sum of ∠s in Δ/*som ∠e van* Δ]

∴ $\hat{T}_2 = \hat{P}_1 = \frac{1}{4}x$ [∠s opp equal sides/∠*e teenoor gelyke sye*]

∴ $S\hat{T}M = \hat{P}_1 = \frac{1}{4}x$ [tan chord theorem/*raaklyn koordstelling*]

Q10.1

Constr: Draw h_1 from E ⊥ AD and h_2 from D ⊥ AE

Konstr: Trek h_1 *vanaf* E ⊥ AD *en* h_2 *vanaf* D ⊥ AE

Proof/*Bewys*:

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1} = \frac{AD}{DB}$$

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{\frac{1}{2}AE \times h_2}{\frac{1}{2}EC \times h_2} = \frac{AE}{EC}$$

But area ΔBDE = area ΔDEC [same base & height or DE || BC/ *dies basis & hoogte*; of DE || BC]

$$\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

Q10.2.1

$\hat{V}_3 = x$ [Tans from same point/*raaklyne vanaf dieselfde pt*]

$\hat{R} = x$ [tan chord theorem/*raaklyn koordstelling*]

$\hat{W}_3 = x$ [corresp ∠s/*ooreenkomstige* ∠*e*; WT || RV]

Q10.2.2a

$\hat{V}_3 = \hat{W}_3 = x$ [proved in 10.2.1]

W, S, T and V are concyclic/*is konsiklies*

WSTV is a cyclic quad [converse ∠s in the same segment/ *Omgekeerde ∠e in dieselfde segment*]

Q10.2.2b

$\hat{W}_2 = \hat{S}_2 = x$ [∠s in the same segment/∠*e in dies segment*]

$\hat{V}_1 = \hat{W}_2 = x$ [alt ∠s/*verwiss* ∠*e* ; WT || RV]

But $\hat{R} = x$ [proved in 10.2.1]

∴ $\hat{R} = \hat{V}_1 = x$

∴ WR = WV [sides opp equal ∠s/*sye teenoor gelyke* ∠*e*]

ΔWRV is isosceles/*is gelykbenig*

Q10.2.2c

In ΔWRV and/*en* ΔTSV

$\hat{R} = \hat{S}_2 = x$ [proved **OR** tan chord theorem]

$\hat{V}_1 = \hat{V}_3 = x$ [proved]

∴ ΔWRV ||| ΔTSV [∠, ∠, ∠]

Q10.2.2d

$\frac{RV}{SV} = \frac{WR}{TS}$ [ΔWRV ||| ΔTSV]

∴ WR × SV = RV × TS

$\frac{WR}{SR} = \frac{KV}{SV}$ [prop theorem/*eweredighst*; WT || RV]

∴ WR × SV = KV × SR

∴ RV × TS = KV × SR

$\therefore \frac{RV}{SR} = \frac{KV}{TS}$

May-June 2019

Q8.1.1a

$M\hat{O}S = 62°$ [∠ at centre = 2× ∠ at circumf/

Q8.1.1b

$\hat{L} = 31°$ [equal chords; equal ∠s

Q8.1.2

LN = NP and LO = OM

∴ ON = $\frac{1}{2}$PM [midpoint theorem/*middelpuntstelling*]

∴ ON = $\frac{1}{2}$MS [PM = MS]

Q8.2.1

$\frac{AN}{AM} = \frac{AK}{AB}$ [line || one side of Δ **OR** prop theorem; KN ||BM/ *lyn || sy van Δ* **OR** *eweredigheidst; KN||BM*]

$$\frac{AN}{AM} = \frac{3y}{5y} = \frac{3}{5}$$

Q8.2.2

$\frac{AM}{MC} = \frac{10x}{23x}$ [given]

AM = 5y = 10x ∴ y = 2x

$\frac{LC}{KL} = \frac{MC}{NM}$ [line || one side of Δ **OR** prop theorem; KN ||LM/ *lyn || sy van Δ* **OR** *eweredigheidst; KN||BM*]

$$= \frac{23x}{2y} = \frac{23x}{4x} = \frac{23}{4}$$

Q9.1

$\hat{B}_1 = x$ [$\angle$'s opp = sides/$\angle$e *teenoor = sye*]

$\hat{M}_2 = 2x$ [ext $\angle$ of Δ] **OR** $\hat{M}_1 = 180° - 2x$ [$\angle$s of Δ]

BM = MN [2 tans from a common point/*raaklyne vanuit dieselfde punt*]

$\hat{N}_1 = \frac{180° - 2x}{2} = 90° - x$ [$\angle$'s opp = sides/$\angle$e *teenoor = sye*]

OR

NM = BM [2 tans from a common point/*raaklyne vanuit dieselfde punt*]

$\hat{B}_2 = \hat{N}_1$ [$\angle$'s opp = sides/$\angle$e *teenoor = sye*]

$\hat{B}_1 = x$ [$\angle$'s opp = sides/$\angle$e *teenoor = sye*]

In Δ KBN:

$x + x + \hat{B}_2 + \hat{N}_1 = 180°$ [sum of $\angle$'s of Δ]

$2x + 2\hat{N}_1 = 180°$

$x + \hat{N}_1 = 90°$

$\hat{N}_1 = 90° - x$

Q9.2

$M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90°$ [tangent$\perp$diameter/*raaklyn$\perp$middellyn*]

$\hat{B}_3 = 90° - \hat{B}_2$

$= 90° - (90° - x) = x$

$\hat{B}_3 = \hat{K} = x$

$\therefore$ AB is a tangent/*raaklyn* converse tan-chord theorem/ *omgekeerde raakl koordst*]]

Q10.1

Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN.

Konst: Merk M en N op AB en AC onderskeidelik af sodanig dat AM = DE en AN = DF. Verbind MN.

Proof:

In Δ AMN and Δ DEF

AM = DE [Constr]

AN = DF [Constr]

$\hat{A} = \hat{D}$ [Given]

$\therefore \Delta AMN \equiv \Delta DEF$ (SAS)

$\therefore A\hat{M}N = \hat{E} = \hat{B}$

MN $\parallel$ BC [corresp $\angle$'s are equal/*ooreenkomstige $\angle$e =*]

$\frac{AB}{AM} = \frac{AC}{AN}$ [line $\parallel$ one side of Δ **OR** prop theorem; MN $\parallel$BC]

$\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [AM=DE and AN=DF]

Q10.2.1a

$D\hat{O}B = 90°$

$D\hat{G}F = \hat{G}_3 + \hat{G}_4 = 90°$ [$\angle$ in semi-circle/$\angle$ *in halfsirkel*]

$D\hat{O}B + D\hat{G}F = 180°$

$\therefore$ DGFO is a cyclic quad. [converse: opp $\angle$s of cyclic quad/ *omgekeerde teenoorst $\angle$e v koordevh*]

OR

$\angle$s of quad = 180°/$\angle$*e van koordevh = 180°*]

OR

$E\hat{O}B = 90°$

$D\hat{G}F = \hat{G}_3 + \hat{G}_4 = 90°$ [$\angle$ in semi-circle/$\angle$ *in halfsirkel*]

$E\hat{O}B = D\hat{G}F$

$\therefore$ DGFO is a cyclic quad. [converse: ext $\angle$ = opp int $\angle$/ *omgekeerde buite$\angle$ = teenoorst $\angle$*]

OR

ext$\angle$ of quad = opp int $\angle$/*buite$\angle$ v vh = teenoorst $\angle$*]

Q10.2.1b

$\hat{F}_1 = \hat{D}$ [ext $\angle$of cyclic quad/buite$\angle$ v koordevh]

$\hat{G}_1 + \hat{G}_2 = \hat{D}$ [tan-chord theorem/raakl koordst]

$\therefore \hat{F}_1 = \hat{G}_1 + \hat{G}_2$

$\therefore$ GC = CF [sides opp equal $\angle$s/sye teenoor = $\angle$e]

Q10.2.2a

AB = DE = 14 [diameters/*middellyne*]

$\therefore$ OB = 7 units

$\therefore$ BC = OC – OB = 11 – 7

= 4 units

Q10.2.2b

In Δ CGB and Δ CAG

$\hat{G}_1 = \hat{A} = x$ [tan-chord theorem/*raakl koordst*]

$\hat{C} = \hat{C}$ [common]

$\Delta CGB \parallel\parallel \Delta CAG$ [$\angle, \angle, \angle$]

$\frac{CG}{CA} = \frac{CB}{CG}$

$\frac{CG}{18} = \frac{4}{CG}$

$CG^2 = 72$

$CG = \sqrt{72}$ or $6\sqrt{2}$ *or* 8,49 units

Q10.2.2c

OF = OC – FC

$= 11 - \sqrt{72}$

$\tan E = \frac{OF}{OE}$

$= \frac{11 - \sqrt{72}}{7} = 0{,}36$

$\hat{E} = 19{,}76°$

November 2018

Q8.1.1

$\hat{P} = \hat{M}_1 = 66°$

[tan chord theorem/*raaklyn koordst*]

Q8.1.2

$\hat{M}_2 = 90°$

[$\angle$ in semi circle/$\angle$ *in halfsirkel*]

Q8.1.3

$\hat{N}_1 = 180° - (90° + 66°)$

$= 24°$

[sum of $\angle$s of /*som van $\angle$e* MNP] Δ

Q8.1.4

$\hat{O}_2 = \hat{P} = 66°$

[corres. $\angle$s;/*ooreenk $\angle$e*, PM $\parallel$ OR]

Q8.1.5

$\hat{R} + \hat{N}_1 + \hat{N}_2 = 180° - 66°$

$= 114°$

[sum of $\angle$s of/*som van $\angle$e* ΔRNO]

$\hat{R} = \hat{N}_1 + \hat{N}_2 = 57°$

$\therefore \hat{N}_2 = 33°$

[$\angle$s opposite = radii/$\angle$*e teenoor = radii*]

Q8.2.1

FC $\parallel$ AB $\parallel$ GH [opp sides of **rectangle**

Q8.2.2

$\frac{AC}{CH} = \frac{AF}{FG}$ [line $\parallel$ one side of Δ]

$\frac{AC}{21} = \frac{20}{15}$

$AC = \frac{20 \times 21}{15}$

$= 28$

DB = AC = 28 [diags of rectangle

$DM = \frac{1}{2}DB = 14$ [diags of rectangle bisect

Q9.1

Constr/*Konstr*.: Draw KO and MO/*Trek KO en MO*

Proof:

$\hat{O}_1 = 2\hat{J}$ [$\angle$ at centre = twice $\angle$ at circumference] [*midpts$\angle$ = 2 × omtreks$\angle$*]

$\hat{O}_2 = 2\hat{L}$ [$\angle$ at centre = twice $\angle$ at circumference]

$\hat{O}_1 + \hat{O}_2 = 360°$ [$\angle$s around a point /$\angle$*e om 'n punt*]

$\therefore 2\hat{J} + 2\hat{L} = 360°$

$\therefore 2(\hat{J} + \hat{L}) = 360°$

$\therefore \hat{J} + \hat{L} = 180°$

Q9.2.1a

$\hat{B}_1 = x$ [$\angle$s in same seg

Q9.2.1b

$\hat{B}_2 = y$ [ext $\angle$ of cyclic quad

Q9.2.2

$\hat{C} = 180° - (x + y)$ [sum of $\angle$s of/*som v $\angle$e*, ΔACR]

$S\hat{B}D + \hat{C} = x + y + 180° - (x + y)$

$S\hat{B}D + \hat{C} = 180°$

SCDB is a cyclic quad [converse opp angles of cyclic quad] [*omgekeerde teenoorst $\angle$e koordevh*]

Q9.2.3

$\hat{T}_4 = y - 30°$ [ext $\angle$ of/*buite $\angle$* ΔTDR]

$\hat{T}_1 = y - 30°$ [vert opp $\angle$s =/*regoorst $\angle$e* =]

$y - 30° + x + 100° = 180°$ [sum of $\angle$s of/*som v $\angle$e*, ΔAST]

$\therefore x + y = 110°$

$S\hat{B}D = 110°$

$\therefore$ SD not diameter [line does not subtend 90° $\angle$]

SD nie 'n middellyn [lyn onderspan nie 90°$\angle$]

Q10.1.1

$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s
= *kde onderspan* = ∠*e*]
$\hat{D}_2 = x$ [exterior angle of cyclic quad/*buite* ∠ *koordevh.*]
$\therefore \hat{C}_2 = 90° - x$ [sum of ∠s of/*som v* ∠*e*, ΔDCM]
$\therefore \hat{C}_2 = \hat{A}_1 = 90° - x$
∴ MC is a tangent to the circle at C [converse: tan chord th]
MC is 'n raaklyn by C [omgekeerde raakl koordst]

Q10.1.2

In ΔACB and/*en* ΔCMD
$\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior ∠ of cyclic quad.]
*[bewys **OF** buite ∠ v koordevh]*
$\hat{A}_2 = \hat{C}_2 = 90° - x$ [proved **OR** sum of ∠s in Δ]
*[Bewys **OF** som v ∠e in Δ]*
ΔACB ||| ΔCMD [∠, ∠, ∠]

Q10.2.1

$\frac{BC}{MD} = \frac{AB}{DC}$ [ΔACB ||| ΔCMD]

$\frac{DC}{MD} = \frac{AB}{DC}$ [BC = DC]

$\therefore DC^2 = AB \times MD$

In ΔAMC and/*en* ΔCMD
$\hat{M}$ is common/*gemeen*
$\hat{A}_1 = \hat{C}_2$ [tan chord th /*raaklyn koordst*]

Q10.2.2

In ΔDMC:

$\frac{CM}{DC} = \sin x$

$\frac{CM^2}{DC^2} = \sin^2 x \quad \frac{AC}{AB} = \frac{CM}{DC}$

$\therefore \frac{AM}{AB} = \sin^2 x$

June 2018

Q8.1.1

$\hat{G} = x$ [∠ centre = 2 × circumference / *midpts*∠ = 2 × *omtreks*∠]
$\hat{H}_1 = x$ [alt ∠s / *verwiss* ∠*e*; KH || GJ]
$G\hat{J}H = x$ [tan chord theorem / *raaklyn koordstelling*]

Q8.1.2

$\hat{J}_1 + \hat{H}_3 = 180° - 2x$ [sum of ∠s in Δ / *som van* ∠*e in* Δ]
$\therefore \hat{J}_1 = \hat{H}_3 = 90° - x$ [∠s opp equal sides / ∠*e teenoor gelyke sye*]
$\therefore x + \hat{H}_2 = 90°$ **OR** [tan ⊥ radius / *raaklyn* ⊥ *radius*]
$\hat{H}_2 = 90° - x$
$\therefore \hat{H}_2 = \hat{H}_3$

Q8.2.1

$\hat{N}_2 = y$ [∠s in the same seg

Q8.2.2a

$2y + y + 87° = 180°$ [opp ∠s of cyclic quad
$3y = 93°$
$y = 31°$

Q8.2.2b

$T\hat{P}L = 62°$ [ext. ∠ of cyclic quad

Q9.1

Constr: Join KZ and LY and draw h_1 from K ⊥ XL and h_2 from L ⊥ XK
Konstr: Verbind KZ en LY en trek h_1 vanaf K ⊥ XL en h_2 vanaf L ⊥ XK

Proof / *Bewys*:

$\frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\frac{1}{2} XK \times h_1}{\frac{1}{2} KY \times h_1} = \frac{XK}{KY}$

$\frac{\text{area } \Delta XKL}{\text{area } \Delta KLZ} = \frac{\frac{1}{2} XL \times h_2}{\frac{1}{2} LZ \times h_2} = \frac{XL}{LZ}$

area ΔXKL = area ΔXKL [common / *gemeenskaplik*]

But area ΔLYK = area ΔKLZ [same base & height ; LK || YZ / *dies basis & hoogte* ; LK || YZ]

$\therefore \frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\text{area } \Delta XKL}{\text{area } \Delta KLZ}$

$\therefore \frac{XK}{KY} = \frac{XL}{LZ}$

Q9.2.1

$\frac{RF}{FS} = \frac{RH}{HT}$ [line || one side of Δ **OR** prop theorem; FH || ST]
*[Lyn || een sy van Δ **OF** eweredigh. st; FH|| ST]*

$\frac{2x - 10}{9} = \frac{4}{x - 2}$
$(2x - 10)(x - 2) = 4 \times 9$
$2x^2 - 14x - 16 = 0$
$x^2 - 7x - 8 = 0$
$(x - 8)(x + 1) = 0$
$\therefore x = 8 \quad (x \neq -1)$

Q9.2.2

$$\frac{\text{area } \Delta RFH}{\text{area } \Delta RST} = \frac{\frac{1}{2} RF \times RH \sin\hat{R}}{\frac{1}{2} RS \times RT \sin\hat{R}} = \frac{\frac{1}{2} \times 6 \times 4 \times \sin\hat{R}}{\frac{1}{2} \times 15 \times 10 \times \sin\hat{R}} = \frac{24}{150} = \frac{4}{25}$$

Q10.1.1

$\hat{C}_1 = 90°$ [∠ in semi circle / ∠ *in halfsirkel*]
$\hat{D}_1 = 90°$ [line from centre to midpt of chord / *lyn vanaf midpt na midpt van koord*]
$\therefore \hat{C}_1 = \hat{D}_1$
∴ FC || OD [corresp ∠s = / *ooreenkomstige* ∠*e* =]

OR/OF

FO = OE [radii]
CD = DE [given / *gegee*]
∴ FC || OD [midpoint theorem / *middelpuntstelling*]

Q10.1.2

$D\hat{O}E = \hat{F}$ [corresp ∠s =; FC || OD]
$B\hat{A}E = \hat{F}$ [∠s in the same seg]
$\therefore D\hat{O}E = B\hat{A}E$

Q10.1.3

In ΔABE and ΔFCE:
$\hat{E}$ is common
$B\hat{A}E = \hat{F}$ [proved in 10.1.2]
$\therefore A\hat{B}E = \hat{C}_1$ [sum of ∠s in Δ]
∴ ΔABE ||| ΔFCE [∠∠∠]

$\frac{AB}{FC} = \frac{AE}{FE}$ [||| Δs]
AB × FE = AE × FC
But FE = 2 OF [*d* = 2*r*]
And FC =2 OD [midpoint theorem]
AB × 2OF = AE × 2OD
∴ AB × OF = AE × OD

Q10.2

$\frac{AT}{TO} = \frac{AC}{CD} = \frac{3}{1}$ [line || one side of Δ
But CD = DE

$\frac{AE}{CE} = \frac{5}{2} \quad \therefore AE = \frac{5}{2} CE$

$\frac{BE}{CE} = \frac{AE}{FE}$ [||| Δs]

$\frac{BE}{CE} = \frac{\frac{5}{2} CE}{FE}$

$BE \times FE = \frac{5}{2} CE^2$

$\therefore 5CE^2 = 2BE.FE$

March 2018

Q7.1.1

$\hat{T}_1 = 70°$ [ext ∠ of cyclic quad

Q7.1.2

$\hat{Q}_1 = \hat{Q}_2 = 35°$ [equal chords;equal∠s

Q7.2.1

$\hat{T}_2 = \hat{Q}_1 = 35°$ [alt ∠s/*verwiss* ∠*e*; PQ || TR]

Q7.2.2

$\frac{PT}{TS}=\frac{QR}{RS}$ [prop theorem

$\therefore \frac{TR}{TS}=\frac{QR}{RS}$ [PT = TR]

Q8

$P\hat{T}R = 90°$ [∠ in semi-circle/*halfsirkel*]

$x = 90° + \hat{R}$ [ext/*buite* ∠ of/*van* Δ]

$\therefore \hat{R} = x - 90°$

$S\hat{T}P = x - 90°$ [tan chord theorem/*raakl koordstelling*]

$x + x - 90° + y = 180°$ [sum of/*som van* ∠s/e in Δ]

$\therefore y = 270° - 2x$

Q9.1

equiangular Δs

Q9.2

$\therefore \frac{GE}{GF}=\frac{DE}{GE}$ [||| Δs]

$GE^2 = 45 \times 80$

$GE = 60$

Q9.3

In ΔDEH and ΔFGH:

$D\hat{H}E = F\hat{H}G$ [vert opp ∠s =/*regoorst* ∠e =]

$D\hat{E}H = F\hat{G}H$ [||| Δs]

$E\hat{D}H = G\hat{F}H$ [sum of/*som van* ∠s/e in Δ]

∴ΔDEH ||| ΔFGH

Q9.4

$\frac{GH}{EH}=\frac{FG}{DE}$ [||| Δs]

$\frac{GH}{60-GH}=\frac{80}{45}$ [EH = 60 – GH]

45 GH = 80(60 – GH)

45 GH = 4800 – 80 GH

125 GH = 4800

GH = 38,4

Q10.1

Construction:
AO is drawn and produced to M

$\hat{O}_1 = \hat{A}_1 + \hat{B}$ [ext ∠ of Δ/*buite* ∠ *van* Δ]

But $\hat{A}_1 = \hat{B}$ [∠s opp = radii/∠e *teenoor* =*radii*]

$\therefore \hat{O}_1 = 2\hat{A}_1$

Similarly/*Netso*: $\hat{O}_2 = 2\hat{A}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{A}_1 + 2\hat{A}_2$

$= 2(\hat{A}_1 + \hat{A}_2)$

$B\hat{O}C = 2B\hat{A}C$

Q10.2.1a

$\hat{F}_1 = 2x$ [∠ centre = 2∠ at circum

Q10.2.1b

$\hat{C} = x$ [∠s in the same seg

Q10.2.2

$\hat{D}_3 = x$ [∠s opp equal sides/∠e *teenoor* = *sye*]

$\hat{E}_3 = 2x$ [ext ∠ of Δ/*buite* ∠ *van* Δ]

$\therefore \hat{F}_1 = \hat{E}_3 = 2x$

∴ AFED is a cyclic quadrilateral [converse ∠s in the same seg]/
Is 'n koordevierhoek [*omgekeerde* ∠*e in dieselfde segm*]

Q10.2.3

$\hat{A}_2 + \hat{A}_3 + \hat{D}_1 + \hat{F}_1 = 180°$ [sum of ∠s in Δ/*som van* ∠*e in* Δ]

$\hat{A}_2 + \hat{A}_3 = \hat{D}_1$ [∠s opp = sides/∠e *teenoor* = *sye*]

$\therefore \hat{A}_2 + \hat{A}_3 = 90° - x$

$\hat{E}_1 = \hat{A}_2 + \hat{A}_3$ [ext∠of cyclic quad/*buite*∠ *v koordevh*]

$= 90° - x$

$F\hat{K}E = 90°$ [line from centre bisects chord]/
[*lyn van midpt halveer koord*]

$\hat{F}_3 = x$ [sum of ∠s in Δ/*som van* ∠*e in* Δ]

Q10.2.4

$B\hat{A}C = \hat{D}_3$ [∠s in the same seg/∠*e in dieselfde segm*]

AE = BE [sides opp equal ∠s/*sye teenoor* =∠*e*]

$\frac{\text{area } \Delta AEB}{\text{area } \Delta DEC} = \frac{\frac{1}{2}(BE)(AE).\sin A\hat{E}B}{\frac{1}{2}(EC)(ED).\sin D\hat{E}C}$

$6,25 = \frac{AE^2}{ED^2}$

$\therefore \frac{AE}{ED} = 2,5$

<u>November 2017:</u>

Q8.1.1

$\hat{E} = 50° - 15° = 35°$ [ext ∠ of Δ/*buite* ∠ *van* Δ]

$\hat{A} = 35°$ [alt ∠s / *verwiss* ∠*e*; CE || AB]

Q8.1.2

$\hat{C}_2 = 35°$ [∠s in same segment

Q8.2

$\hat{C}_2 = \hat{E}$ [from 8.1.1 and 8.1.2]

Thus CF is a tangent to the circle
[converse tan chord theorem

Q9.1.1

$\frac{AF}{BF}=\frac{3x}{2x}=\frac{3}{2}$ & $\frac{AG}{CG}=\frac{12y}{8y}=\frac{3}{2}$

$\therefore \frac{AF}{BF}=\frac{AG}{CG}$

∴ FG || BC [conv prop th/*omg eweredigh st.* **OR** line divides 2 sides of Δ in prop/*lyn verdeel 2 sye v* Δ *in dies verh*]

Q9.1.2

$\frac{AG}{GC}=\frac{AH}{HK}$ [prop theorem/*eweredigh st*; <u>GH || CK</u> **OR** line || to 1 side of Δ/*lyn* || *1 sy van* Δ]

$\frac{AG}{GC}=\frac{AE}{ED}$ [prop theorem/*eweredigh st*; <u>GE || CD</u>]

$\therefore \frac{AH}{HK}=\frac{AE}{ED}$

Q9.2

$\frac{AE}{ED}=\frac{3}{2}$ and $\frac{AH}{HK}=\frac{3}{2}$

$\frac{AE}{12}=\frac{3}{2}$ and $\frac{15}{HK}=\frac{3}{2}$

∴ AE = 18 and HK = 10

∴ HE = AE – AH
= 18 – 15
= 3

∴ EK = HK – HE
= 10 – 3
= 7

OR/*OF*

AD = 30

KD = AD – AH – HK
= 30 – 15 – 10
= 5

EK = ED – KD
= 12 – 5
= 7

Q10.1

Line from centre to midpoint of chord

Q10.2.1

$O\hat{W}T = O\hat{W}S = 90°$ [radius ⊥ tangent/*raaklyn*]

∴MN || TS [corresp ∠s =/*ooreenkomstige* ∠*e* =
OR co-int ∠s 180°/*ko-binne*∠*e 180°*
OR alternate ∠s/*verwiss* ∠*e*]

Q10.2.2

$\hat{M}_1 = \hat{N}_1$ [∠s opp = sides/ ∠*e teenoor* = *sye*]

$\hat{M}_1 = \hat{T}$ [corresp ∠s/*ooreenk* ∠*e*; MN || TS]

$\therefore \hat{N}_1 = \hat{T}$

∴TMNS is a cyclic quadrilateral [conv: ext∠ cyclic quad]
TMNS is 'n koordevierhoek [*omgek: buite*∠ *kdvh*]

Q10.2.3

In ΔOVN and ΔOWS

$\hat{O}_2 = \hat{O}_2$ [common/*gemeenskaplik*]

$O\hat{V}N = O\hat{W}S = 90°$ [from 10.1]

$O\hat{N}V = O\hat{S}W$ [sum ∠s Δ/*som* ∠*e* Δ]

∴ΔOVN ||| ΔOWS [∠, ∠, ∠]

$\therefore \frac{VN}{WS}=\frac{ON}{OS}$

But $VN = \frac{1}{2}MN$ [given]

$\therefore \frac{\frac{1}{2}MN}{WS}=\frac{ON}{OS}$

∴OS.MN = 2ON. WS

Q11.1

Construction: Draw diameter NR and draw RM
Konstruksie: Trek middellyn NR en verbind RM

$O\hat{N}M + M\hat{N}Q = 90°$ [radius ⊥ tangent/*raaklyn*]

$N\hat{M}R = 90°$ [∠ in semi circle/*semi-sirkel*]

$\therefore M\hat{R}N = 180° - (90° + 90° - M\hat{N}Q)$ [sum ∠s Δ]

$= M\hat{N}Q$

but $M\hat{R}N = M\hat{K}N$ [∠s same segment/∠*e dieselfde segment*]

$\therefore M\hat{N}Q = \hat{K}$

Q11.2.1(a)

Angle in a semi circle

Q11.2.1(b)

Exterior ∠of quad = opp interior ∠

Q11.2.1(c)

tangent chord theorem

Q11.2.2(a)

In ΔAEC

$\hat{E} = 180° - (90° + x)$ [sum ∠s Δ]

$= 90° - x$

$\hat{D}_1 = 180° - (90° + x)$ [∠s on a straight line]

$= \hat{E} = 90° - x$

∴ AD = AE [sides opp = ∠s/ *sye teenoor = ∠e*]

Q11.2.2(b)

In ΔADB and ΔACD

$\hat{A}_2 = \hat{A}_2$ [common]

$\hat{D}_2 = \hat{C}$ [proven]

$\hat{B}_2 = \hat{D}_2 + \hat{D}_3$ [sum ∠^{e} Δ]

∴ΔADB ||| ΔACD

Q11.2.3(a)

$\frac{AD}{AC} = \frac{AB}{AD}$ [||| Δs]

$AD^2 = AC \cdot AB$

$= 3r \times r$

$= 3r^2$

Q11.2.3(b)

AD = AE = $\sqrt{3}r$ [from 11.2.2(a) &11.2.3(a)]

AB = r and BC = $2r$ ∴AC = $3r$

<u>In ΔACE:</u>

$\tan \hat{E} = \frac{AC}{AE}$

$= \frac{3r}{\sqrt{3}r} = \sqrt{3}$

∴ $\hat{E} = 60°$

∴ $\hat{D}_1 = 60°$ [from 11.2.2(a)]

∴ $\hat{A}_1 = 60°$ [∠s of Δ = 180°]

∴ ΔADE is equilateral/*is gelyksydig*

<u>June 2017:</u>

Q8.1

$\hat{L} = 100°$ [ext ∠ cyclic quad = int opp ∠

Q8.2

$\hat{N}_1 = 80°$ [∠s on straight line

∴$\hat{O}_1 = 160°$ [∠ at centre = 2×∠ at circumference

Q8.3

$\hat{M}_1 = 360° - (100° + 55° + 160°)$ [sum ∠*s of* quad

∴$\hat{M}_1 = 45°$

Q9.1.1

∠ in semi-circle

Q9.1.2

Opp ∠s of quad = 180°

Q9.2.1

OF ⊥ AC [line from centre bisects chord

∴ AC || GO [co-interior

Q9.2.2

$\hat{G}_1 = \hat{B}_2$ [ext ∠ cyclic quad

but ΔABF ≡ ΔCBF [s, ∠, s]

∴$\hat{B}_2 = \hat{B}_1$

∴$\hat{G}_1 = \hat{B}_1$

Q9.3

OF : FB = 3 : 2

∴ DO = $5k$ and DF = $8k$ **OR/*OF*** DB = $2r$; DF = $2r - \frac{2}{5}r = \frac{8}{5}r$

∴ $\frac{DG}{DA} = \frac{DO}{DF} = \frac{r}{\frac{8}{5}r}$ [line || side of Δ/*lyn* || *sy v* Δ]

∴ $\frac{DG}{DA} = \frac{5}{8}$

Q10.1

Tangent-chord theorem

Q10.2.1

$\hat{A}_2 + \hat{A}_3 = \hat{B}_1 + \hat{B}_2$ [∠^{s} opp = sides/*∠e teenoor = sye*]

$\hat{S}_3 = \hat{B}_1 + \hat{B}_2$ [ext ∠ cyclic quad/*buite ∠ koordevh*]

∴ $\hat{S}_3 = \hat{A}_2 + \hat{A}_3$

∴ AB || ST [corresp/*ooreenk* ∠^{s} =]

Q10.2.2

$\hat{B}_2 = x$ [tan chord theorem/*raakl – koordst*]

$x + \hat{T}_4 = \hat{B}_1 + \hat{B}_2$ [corresp/*ooreenk* ∠^{s} ; AB // ST]

∴$\hat{T}_4 = \hat{B}_1$

$\hat{B}_1 = \hat{A}_1$ [tan chord theorem/*raakl – koordst*]

∴ $\hat{T}_4 = \hat{A}_1$

Q10.2.3

$\hat{T}_4 = \hat{A}_1$ [proven/*bewys in* 10.2.2]

∴ RTAP is a cyclic quadrilateral [line subtends = ∠^{s}]

Is 'n koordevierhoek [lyn onderspan = ∠e]

Q11.1

Constr: On sides AB and AC of ΔABC, mark points G and H respectively such that AG = DE and AH = DF. Draw GH/*Merk punt G en H op sy AB en AC van ΔABC onderskeidelik af sodanig dat AG = DE en AH = DF. Trek GH.*

Proof/*Bewys*:

ΔAGH ≡ ΔDEF [s, ∠, s]

∴ A$\hat{G}$H = $\hat{E}$

= $\hat{B}$ [$\hat{B} = \hat{E}$, given/*gegee*]

∴ GH || BC [corresp/*ooreenk* ∠^{s} =]

∴ $\frac{AG}{AB} = \frac{AH}{AC}$ [line || side of Δ / *lyn* || *sye v* Δ]

∴ $\frac{DE}{AB} = \frac{DF}{AC}$ [constr/*konstruksie*]

Q11.2.1(a)

AP = PC [diag ||^m bisect each other

But TP = PS [given/*gegee*]

AP – TP = PC – PS

∴ AT = SC

Q11.2.1(b)

In ΔPSR and ΔPBA :

$\hat{P}_1 = \hat{P}_3$ [vertically opp ∠^{s} / *regoorst ∠e*]

$\hat{B}_1 = \hat{S}_1$ [∠^{s} in same segment/*∠e in dies segment*]

∴ ΔPSR ||| ΔPBA [∠, ∠, ∠]

Q11.2.2(a)

$\frac{PR}{PA} = \frac{PS}{PB}$ [||| Δs]

∴ $\frac{PR}{PA} = \frac{TR}{AD} = \frac{PS}{PB}$ [given $\frac{PR}{PA} = \frac{TR}{AD}$]

∴ $\frac{PR}{PA} = \frac{TR}{AD} = \frac{TP}{PD}$ [PS = TP; PB = PD]

∴ ΔRPT ||| ΔAPD [sides of Δ in prop

Q11.2.2(b)

$\hat{T}_1 = \hat{D}_2$ [||| Δs]

∴ ATRD is a cyclic quad [converse: ext ∠ of cyclic quad/ *Omgekeerde buite ∠ v* koordevh]

<u>March 2017:</u>

Q8.1

$\hat{Q} = 72°$ [opp ∠s of cyclic quad

Q8.2

$\hat{R}_2 = \hat{P}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*]

$\hat{R}_2 = \frac{180° - 72°}{2}$ [sum of ∠s in Δ/*som v ∠e in Δ*]

$= 54°$

Q8.3

$\hat{P}_2 = 42°$ [tan chord theorem

Q8.4

$\hat{R}_3 = \hat{P}_1 + \hat{P}_2$ [ext ∠ of cyclic quad

$= 54° + 42°$

$= 96°$

Q9.1.1

$\frac{ST}{TQ} = \frac{SW}{WP}$ [prop theorem

$= \frac{2}{3}$

Q9.1.2

$\frac{SV}{VR} = \frac{SW}{WP}$ [prop theorem

$= \frac{2}{3}$

Q9.2

$\frac{ST}{TQ} = \frac{SV}{VR}$ [both equal/*beide gelyk* $\frac{WS}{PW}$]

$\therefore$ TV ∥ QR [line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*]

$\therefore \hat{T}_1 = \hat{Q}_1$ [corresp/*ooreenkomst* ∠s/e; TV ∥ QR]

Q9.3

ΔVWS ||| ΔRPS

Q9.4

$\frac{WV}{PR} = \frac{SW}{SP}$

$= \frac{2}{5}$

Q10.1

Constr/*Konst*:

Draw line PO and extend /*Trek lyn PO en verleng*

Proof/*Bewys*:

OP = OA [radii]

$\therefore \hat{P}_1 = \hat{A}$ [∠s opp/*teenoor* = sides/*sye*]

but $\hat{O}_1 = \hat{P}_1 + \hat{A}$ [ext ∠ of Δ]

$\therefore \hat{O}_1 = 2\hat{P}_1$

Similarly/*Netso*, $\hat{O}_2 = 2\hat{P}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$

i.e. $A\hat{O}B = 2A\hat{P}B$

Q10.2.1

∠s in the same segment

Q10.2.2

$\hat{P}_2 = \hat{S}_1 = y$ [∠s opp equal sides

$\hat{S}_1 = \hat{P}_3 = y$ [tan chord theorem/

$\therefore \hat{P}_2 = \hat{P}_3$

$\therefore$ PQ bisects $T\hat{P}S$

Q10.2.3

$P\hat{O}Q = 2\hat{S}_1 = 2y$ [∠at centre = 2×∠at circ

Q10.2.4

$T\hat{P}A = \hat{P}_2 + \hat{P}_3 = 2y$ [proved/*bewys* in 11.2.2]

$\therefore T\hat{P}A = P\hat{O}Q$ [proved/*bewys* in 11.2.3]

$\therefore$ PT = tangent [converse tan chord theorem/*omgek raakl-koordst*]

Q10.2.5

$O\hat{P}Q + O\hat{Q}P = 180° - 2y$ [sum of/*sum v* ∠s/e in Δ]

$\therefore O\hat{Q}P = 90° - y$ [∠s opp equal sides/∠e *to = sye*; OP = OQ]

In ΔPAQ:

$O\hat{Q}P + \hat{P}_2 + Q\hat{A}P = 180°$

$90° - y + y + Q\hat{A}P = 180°$ [sum of/*sum v* ∠s/e in Δ]

$Q\hat{A}P = 90°$

$\therefore O\hat{A}P = 90°$ [∠s/e on straight line/*op reguitlyn*]

Q11.1

$\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*]

$\therefore$ TPLN is a cyclic quad/ 'n *koordevh* [opp ∠s of quad is suppl/ *teenoor∠e v vh is suppl*]

OR

$\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*]

$\therefore$ TPLN is a cyclic quad [ext ∠ = int opp ∠/*buite∠ = to binne ∠*]

Q11.2

$\hat{T}_2 = P\hat{L}N = x$ [ext ∠ of cyclic quad/*buite∠ van koordevh*]

$\hat{K} = 90° - x$ [sum of/*som v* ∠s/e in Δ]

$\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*]

Q11.3.1

In ΔKTP and ΔKLN:

$P\hat{K}T = L\hat{K}N$ [common/*gemeen*]

$K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*]

$\therefore$ Δ KTP ||| Δ KLN [∠∠∠]

Q11.3.2

$\frac{KT}{KL} = \frac{KP}{KN}$ [||| Δs]

$\therefore$ KT . KN = KP . KL

But KL = 2KP [radii: PK = LP]

$\therefore$ KT . KN = KP . 2KP

$= 2KP^2$

$= 2(KT^2 - TP^2)$ [Theorem of Pythagoras]

$= 2KT^2 - 2TP^2$