KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

CURRICULUM GRADE 10 -12 DIRECTORATE

NCS (CAPS) SUPPORT

JUST IN TIME LEARNER REVISION DOCUMENT

MATHEMATICS

GRADE 12

2023

This document has been compiled by the FET Mathematics Subject Advisors together with Lead Teachers.

TABLE OF CONTENTS

TOPIC **ALGEBRA**

GUIDELINES, SUMMARY NOTES, & STRATEGIES

Summary	**Strategies**
Factorisation	Common factor, solve Transpose, factorise and solve Remove brackets, transpose, factorise and solve
Quadratic formula	Standard form, formula, substitution and answers correct to TWO decimal places, unless stated otherwise. Remove the brackets, transpose, standard form and answers correct to TWO decimal places, unless stated otherwise.
Surds	Isolate the radical sign, square both sides, solve and validate (check the validity of the answers).
Simultaneous equations	Choose the simplest equation, make x or y the subject of the formula, substitution, standard form, factorise (or use a quadratic formula), solve , substitution for the other variable.
Exponents	Laws of exponents, write as a power, factorise, equate exponents, solve Same base, equate exponents, solve. Split, factorize, simplify and solve Standard form, factorise, solve, laws of exponents and solve again. OR use a k -method.
Inequalities	Critical values, method, solution Standard form, factorise, critical values, method and solve Remove brackets, standard form, factorise, critical values, method and solve.
Nature of the roots	$\Delta = b^2 - 4ac$

Values of discriminant (Δ)	Nature of the roots
$\Delta \geq 0$	Real
$\Delta > 0$	Real, unequal, rational/ irrational
$\Delta = 0$	Real and equal
$\Delta < 0$	Non-real

Standard form, discriminant, substitution, solution

REVISION QUESTIONS

1. NW Question 1

1.1 Solve for x:

1.1.1 $x(2-x)=0$ (2) **L1**

1.1.2 $2x^2-3x-7=0$ (correct to TWO decimal places) (3) **L1**

1.1.3 $2-x=\sqrt{2-7x}$ (4) **L2**

1.1.4 $(3-x)(x+1)<0$ (3) **L2**

1.2 Solve for x and y:

$2x-y=3$

$x^2+5xy+y^2=15$ (6) **L2**

1.3 Determine the value of x given that $6^x+6^x+6^x+6^x+6^x+6^x=6^{6x}$ (3) **L4**

2. EC, June 2023 Question 1

2.1 Solve for x:

2.1.1 $x^2-9=0$ (2) **L1**

2.1.2 $x-5+\frac{2}{x}=0$ (4) **L1**

2.1.3 $x=1+\sqrt{7-x}$ (5) **L2**

2.1.4 $x^2+2x-15\geq 0$ (3) **L2**

2.2 Solve simultaneously for x and y in:

$y+2x=3$ **L2**

$y^2-y=3x^2-5x$ (6)

2.3 Simply completely, WITHOUT the use of a calculator: $\sqrt[n]{\frac{10^n+2^{n+2}}{5^{2n}+4\left(5^n\right)}}$ (4) **L2**

3. **FS, March 2022 Question 1**

3.1 Solve for x:

3.1.1 $(x-5)(x+2)=6$ (4) **L2**

3.1.2 $(x+4)(x-3)=3$ (correct to one decimal place) (4) **L2**

3.1.3 $2^x(x-5)\leq 0$ (3) **L3**

3.1.4 $x-3\sqrt{x+2}=2$ (4) **L2**

3.1.5 $3^{3x+1}=9^{2x-4}$ (3) **L1**

3.2 Solve both x and y: (4) **L4**

$(3x-y)^2+(x-5)^2=0$

4.

4.1 Solve for x:

4.1.1 $(x-1)^2=2(1-x)$ (4) **L2**

4.1.2 $-x^2-2x+1=0$ (correct to two decimal places) (3) **L1**

4.1.3 $x-3\sqrt{x}-4=0$ (4) **L2**

4.1.4 $x=\frac{\sqrt{10^{1009}}}{\sqrt{10^{1011}}-\sqrt{10^{1007}}}$ (3) **L3**

4.2 Solve both x and y: **L2**

$2^{y-3x}=\frac{1}{16}$

$x^2+xy=24$ (7)

5

5.1 Solve for x:

5.1.1 $(x-2)(5+x)=0$ (2) **L1**

5.1.2 $x-\frac{3}{x}=-2$ (4) **L2**

5.1.3 $(x-1)(x-2)\leq 6$ (4) **L2**

5.1.4 $2^{x+2}+2^x=40$ (3) **L2**

5.2 Solve for both x and y: **L2**

$x-2y=3$

$3x^2-5xy=24+16y$ (6)

5.3 The roots of a quadratic equation are: $\frac{-3\pm\sqrt{4-p}}{2}$

For which values of p are the roots real? (2) **L1**

6. **Answer Series, Grade 12**

6.1 Solve for x:

6.1.1 $(x+2)^2=3x(x-2)$ correct to one decimal digit. (4) **L2**

6.1.2 $x^2-9x\geq 36$ (4) **L2**

6.1.3 $3^x-3^{x-2}=72$ (4) **L2**

6.1.4 $(\sqrt{x-1}-3)(\sqrt{x-1}+2)=0$ (3) **L2**

7. **FS/September 2022**

Solve for x: (1) **L1**

7.1.1 $3x-4=\frac{2}{x}$ (correct to two decimal places) (4) **L1**

7.1.2 $x^2-8x+16>0$ (3) **L2**

7.1.3 $3^{x+1}+m.3^x=2m+6$ (correct to two decimal places) (4) **L3**

8. **Answer Series 12**

8.1 Solve for x:

8.1.1 $(3-x)(x+4)\geq 0$ (4) **L2**

8.1.2 $x^2-6x+9>0$ (3) **L2**

8.1.3 $x^{\frac{1}{2}}+3x^{\frac{1}{4}}-18=0$ **L2**

8.1.4 $9.3^{2x}+1=6.3^x$ **L2**

8.2 Simplify: $\frac{a^{-2}-b^{-2}}{a^{-1}-b^{-1}}$ **L2**

L1

9. **Kevin Smith, Maths Handbook And Study Guide, Grade11**

9.1 Solve for x, rounded off to TWO decimal places where necessary:

9.1.1 $x(x-1)=2$ (3) **L1**

9.1.2 $2x^2-3x=8$ (4) **L1**

9.1.3 $3x^2+x-2\geq 0$ (4) **L2**

9.2 Given the equation $x^2+2xy-8y^2=0$:

9.2.1 Determine the values of the ratio $\frac{x}{y}$. (3) **L3**

9.2.2 Hence, determine the values of x and y if $x+y=6$ (5) **L2**

9.3 Simplify the following without the use of a calculator. Show all workings: **L3**

$$\left(\frac{\sqrt{3^{2011}}-\sqrt{3^{2009}}}{\sqrt{3^{2008}}}+\sqrt{3}\right)^2$$

(4)

10 The roots of a quadratic equation are given by $x=\frac{-5\pm\sqrt{20+8k}}{6}$, where $k\in\{-3;-2;-1;0;1;2;3\}$.

10.1.1 Write down TWO values of k for which the roots will be rational. (2) **L1**

10.1.2 Write down ONE value of k for which the roots will be no-real. (1) **L1**

10.2 Calculate a and b if $\sqrt{\frac{7^{2014}-7^{2012}}{12}}=a(7^b)$ and a is not a multiple of 7. **L4** (4)

11

11.1 Solve for x:

11.1.1 $x^2-x-6=0$ (3) **L1**

11.1.2 $x(x+6)+1=0$ (correct to TWO decimal places) (4) **L1**

11.1.3 $6x-2x^2\leq 0$ (3) **L2**

11.1.4 $\left(\sqrt{\sqrt{2}-x}\right)\left(\sqrt{\sqrt{x}+2}\right)=x$ **L3** (5)

11.2 Given: $f(x)=3(x-1)^2+5$ and $g(x)=3$

11.2.1 Is it possible for $f(x)=g(x)$? Justify your answer. (2) **L3**

11.2.2 Determine the value(s) of k for which $f(x)=g(x)+k$ has TWO unequal roots. (2) **L3**

12

12.1 Solve for x:

12.1.1 $x^2+5x-6=0$ (3) **L1**

12.1.2 $4x^2+3x-5=0$ (correct to TWO decimal places) (3) **L1**

12.1.3 $4x^2-1<0$ (3) **L2**

12.1.4 $3^{x+1}-3^{x-1}-24=0$ (4) **L2**

12.2 Solve simultaneously for x and y: **L2**

$3x-4y=5$ and $2x^2-5xy+3y^2=4$ (5)

12.3 Consider the product: $1\times2\times3\times4\times...\times30$ **L4**

Determine the largest value of k such that 2^k is a factor of this product. (4)

13

13.1 Solve for x:

13.1.1 $2x(3-x)=0$ (2) **L1**

13.1.2 $5x^2-4x=2$ (Rounded off to TWO decimal places) (4) **L1**

13.1.3 $\sqrt{7+3x}+2x=0$ (5) **L2**

13.1.4 $3^{x+2}+3^{2-x}=82$ (5) **L2**

13.2 The equations $x^2+ax+b=0$ and $x^2+bx+a=0$ both have real and equal roots. **L4**
Solve for a and b, where $a>0$ and $b>0$. (7)

Limpopo SEPTEMBER 2022

14 Solve for x:

14.1.1 $2x^2-6x+1=0$ (correct to two decimal places) (3) **L1**

14.1.2 $-x^2-3x>0$ (3) **L1**

14.1.3 $x+3-2\sqrt{5-x}=0$ (5) **L2**

14.2 Solve the following equations simultaneously: **L2**
$2^x-2^{y+2}=0$ and $x^2+2xy+y^2=36$ (6)

14.3 A square has sides x cm each. If each side of the square is increased by 4 cm, the area is increased by 392 cm^2. Calculate x. **L3** (4)

Mpumalanga June 2023

15 Solve for x:

15.1.1 $(2+x)(x-4)=0$ (2) **L1**

15.1.2 $3x^2=2x+4$ (correct to two decimal places) (4) **L1**

15.1.3 $x-2\sqrt{x-1}=4$ (6) **L2**

15.1.4 $-x-12>-x^2$ (4) **L2**

15.2 Solve the following equations simultaneously
$x^2-xy-5y^2=-5$ and $x+2y=1$ (6) **L2**

15.3 Determine the values of t for which the equation **L3**
$5^x=2-t$ will have real roots (3)

16 **Study and Master**

16.1 Solve for x: $\sqrt{3x-1}+1=\dfrac{6}{\sqrt{3x-1}}$ (6) **L3**

16.2 Solve for x in terms of a if $x^2-2a(x-1)=1$ (5) **L2**

16.3 The solutions of quadratic equations is: $x=-2\pm\dfrac{\sqrt{13-2k}}{3}$
Determine the largest values of k for which this x-value will be rational (4) **L3**

17 **NC Last push 2022**

17.1 Solve for x:

17.1.2 $(x-1)(x+4)\geq 6$ (3) **L2**

17.2 Given $2^{x+1}+2^x=3^{y+2}-3^y$, where x and y are integers. Determine the values of x and y. (3) **L3**

TOPIC SEQUENCE AND SERIES

GUIDELINES, SUMMARY NOTES, & STRATEGIES

Quadratic Pattern:

Definition: Second difference are equal where the first term forms an arithmetic sequence.

General term: $T_n = an^2 + bn + c$

To calculate the values of a, b and c:

	$n=1$		$n=2$		$n=3$		$n=4$
T_n	$a+b+c$		$4a+2b+c$		$9a+3b+c$		$16a+4b+c$
1st difference		$3a+b$		$5a+b$		$7a+b$	
2nd difference			$2a$		$2a$		

NB: For a **MINIMUM** or **MAXIMUM** term: $n = \frac{-b}{2a}$ or $\frac{dT_n}{dn} = 0$ i.e. First derivative

Arithmetic number patterns:

Definition: All first differences are equal, i.e. you always add or subtract a constant difference

N:B $T_2 - T_1 = T_3 - T_2$

General term: $Tn = a + (n-1)d$

$d = T_2 - T_1$

Sum of n Terms: $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{n}{2}(a + l)$

Where l is the last term or T_n

Geometric number patterns:

Definition: There exists constant ratio, i.e. you multiply by the same ratio.

N:B $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ (common ratio; r)

General term: $T_n = ar^{n-1}; r = \frac{T_2}{T_1}$

Sum of n terms: $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_n = \frac{a(1 - r^n)}{1 - r}$

Sum to infinity: $S_\infty = \frac{a}{1 - r}$

REVISION QUESTIONS

DBE/MAY-JUNE 2023

1.1 Given the geometric series: $\frac{1}{5} + \frac{1}{15} + \frac{1}{45} + ...$

1.1.1 Is this a convergence geometric series? Justify your answer with necessary calculations. (2) **L1**

1.1.2 Calculate the sum of infinity of this series. (2) **L1**

1.2 An arithmetic and geometric sequence are combined to form the Pattern, which is given by: $P_n = x;\frac{1}{3};2x;\frac{1}{9};3x;\frac{1}{27};...$

1.2.1 Write down the next TWO terms of the pattern (2) **L1**

1.2.2 Determine the general term (T_n) for the odd terms. Write down your answer in terms of x (2) **L2**

1.2.3 Calculate the value of P_{26}. (3) **L3**

1.2.4 If $\sum_{n=1}^{21} P_n = 33,5$, determine the value of x (6) **L4**

2 A quadratic sequence has the following properties:

- The second difference is 10
- The first two terms are equal, i.e. $T_1 = T_2$
- $T_1 + T_2 + T_3 = 28$

2.1 Show that the general term of the sequence is $T_n = 5n^2 - 15n + 16$ (6) **L3**

2.2 Is 216 a term in this sequence? Justify your answer with the necessary calculations (3) **L2**

EASTERN CAPE MAY/JUNE /2023

3.1 Given the following geometric series: $\frac{24}{x} + 12 + 6x + 3x^2 + ...$

3.1.1 Determine the value of *r,* the common ratio, in terms of x (1) **L1**

3.1.2 Determine the value of x for which this series converges (2) **L1**

3.1.3 If $x = 4$, determine the sum of the series to 15 terms. (3) **L2**

3.2 Calculate: $\sum_{n=1}^{\infty} 6(2)^{-n}$ (3) **L2**

3.3 The sum of the first n terms of an arithmetic series is given $S_n = -n^2 + 8n$

3.3.1 Calculate the sum of the first 15 terms (2) **L1**

3.3.2 Calculate the value of T_{15} (2) **L2**

3.3.3 If the first term of the series is 7, which term of the series will have a value of -169? (4) **L2**

Consider the following quadratic number pattern :95; 72; *y*; 32;…

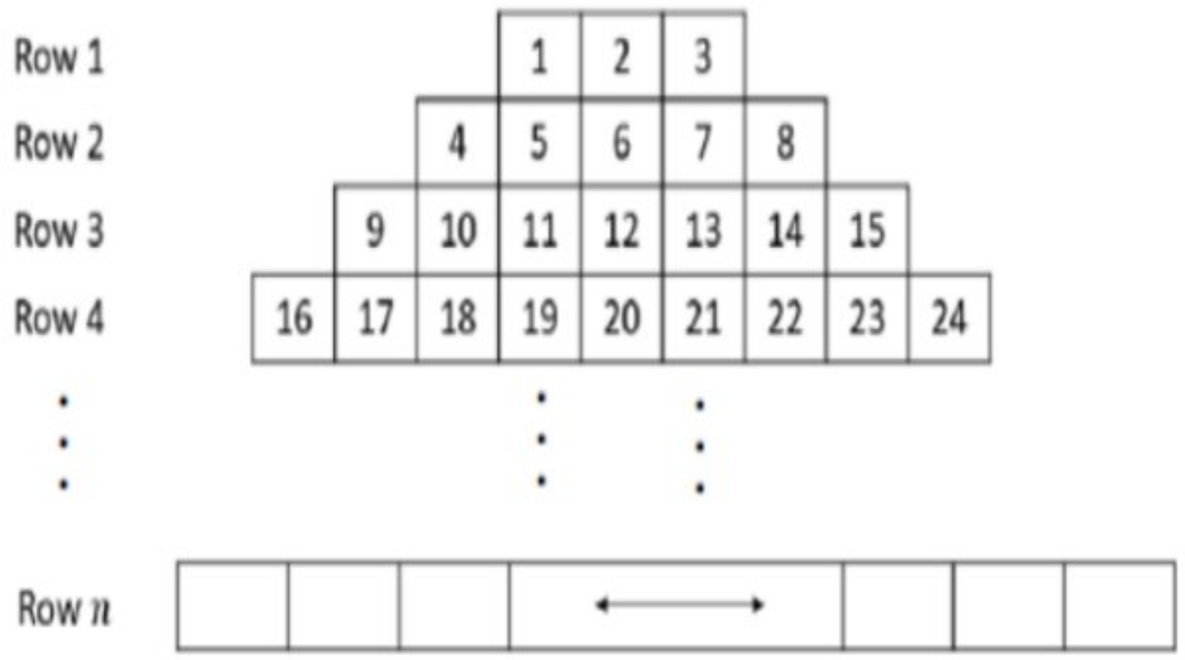
4.1. Determine the value of y (2) **L1**

4.2 If y= 51, determine the general term of the number pattern in the form $T_n = an^2 + bn + c$ (4) **L2**

4.3 Determine T_{22}. (1) **L1**

4.4 Which term in the number pattern will be equal to 1040? (4) **L2**

In a storage room there are sets of drawers that are used for storage. These drawers are packed in rows and numbered as shown in the diagrams below



5.1 The number of the last drawer in each row form a sequence (4) **L2**
Determine the general term of this sequence.

5.2 How many drawers are there in row 8? (2) **L2**

5.3 The manager bought 255 handles for the drawers. (4) **L 3**
Determine how many complete rows will have handles.

DBE/NOV 2020

6.1 $7; x; y; -11; \ldots$ is an arithmetic sequence. Determine the values of x and y (4) **L2**

6.2 Given the quadratic number pattern $-3; 6; 27; 60; \ldots$

6.2.1 Determine the general term of the pattern in the form $T_n = an^2 + bn + c$ (4) **L2**

6.2.2 Calculate the value of the 50^{th} term of the pattern. (2) **L1**

6.2.3 Show that the sum of the first n first -differences of this pattern can be given by (3) **L3**
$S_n = 6n^2 + 3n$

6.2.4 How many consecutive first -differences were added to the first term of the (4) **L4**
quadratic number pattern to obtain a term in the quadratic number pattern that has a value of 21060?

NORTH WEST SEPTEMBER 2020

7.1 Consider the series: $64 + 32 + 16 + \ldots$

7.1.1 Determine the ninth term in the sequence (3) **L1**

7.1.2 Determine the sum to infinity (2) **L2**

7.2 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 2. The general term of the first differences in given by $6n + 8$

7.2.1 Show that $a = 3$ (3) **L2**

7.2.2 Determine the general term T_n. (3) **L2**

7.3 Given the series: $17p^8k^{15} + 20p^9k^{14} + 23p^{10}k^{13} + \ldots + 53p^{20}k^3$
Write the series in sigma notation (4) **L3**

DBE/MAY- JUNE 2019

8.1 The first three terms of an arithmetic sequence are $2p + 3;\ p + 6$ and $p - 2$

8.1.1 Show that $p = 11$ (2) **L1**

8.1.2 Calculate the smallest value of n which $T_n < -55$ (3) **L2**

8.2 Given that $\sum_{k=1}^{6}(x - 3k) = \sum_{k=1}^{9}(x - 3k)$, prove that $\sum_{k=1}^{15}(x - 3k) = 0$ (5) **L3**

FREE STATE PROVINCE SEPTEMBER 2019

Given that: $\sum_{n=1}^{\infty} 63p^{n-1} = \frac{189}{2}$

9.1 Solve for p. (4) **L3**

9.2 If it is further given that $p = \frac{1}{3}$, determine the smallest value of n such that $T_n < \frac{1}{6561}$ (5) **L3**

10.1 Given the sequence: 2; 5; 8; …

10.1.1 If the pattern continues, then write down the next two terms. (1) **L1**

10.1.2 Prove that none of the terms of this sequence are perfect squares. (5) **L3**

10.2 1; 3; 5 are the first three terms of the first differences of a quadratic sequence
The 7^{th} term of the quadratic sequence is 35.

10.2.1 Determine the 5^{th} and 6^{th} terms of the quadratic sequence. (4) **L2**

10.2.2 Determine the n^{th} term of the quadratic sequence. (5) **L2**

10.3 Prove that the sum to n terms of a geometric sequence is given by: (4) **L2**

$$\frac{a(r^n - 1)}{r - 1}\ ; r \neq 1$$

10.4 Calculate the value of n if: (5) **L2**

$$\sum_{k=1}^{n} 2(3)^{k-1} = 531440$$

11.1 The sum to n terms of a sequence of numbers is given as: $S_n = \frac{n}{2}(5n + 9)$

11.1.1 Calculate the sum to 23 terms of the sequence. (2) **L1**

11.1.2 Hence calculate the 23^{rd} term of the sequence. (3) **L2**

11.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three terms of the arithmetic sequence.

Determine TWO possible values for the common ratio, r, of the geometric sequence. (6) **L4**

12.1 Evaluate:

$$\sum_{n=3}^{20} (15 - 4n)$$ (4) **L2**

12.2 A water tank contains 216 litres of water at the end of day 1. Because of a leak, the tank loses one-sixth of the previous day's contents each day. How many litres of water will be in the tank by the end of:

12.2.1 the 2^{nd} day? (2) **L2**

12.2.2 the 7^{th} day? (3) **L2**

12.3 Consider the geometric series: $2(3x-1) + 2(3x-1)^2 + 2(3x-1)^3 + \ldots$

12.3.1 For which values of x is the series convergent? (3) **L1**

12.3.2 Calculate the sum to infinity of the series if $x = \frac{1}{2}$ (4) **L2**

12.4 $2; x; 12; y; \ldots$ are the first four terms of a quadratic sequence. If the second difference is 6, calculate the values of x and y. (5) **L3**

NORTH WEST SEPTEMBER 2021

13 Consider the series: $\cos\theta + \sin 2\theta + 4\sin^2\theta.\cos\theta + \ldots$ where θ is an acute angle.

13.1 Prove that it is a geometric series. (4) **L2**

13.2 Calculate for which values of θ it will be a converging series. (3) **L2**

FREE STATE PROVINCE SEPTEMBER 2022

14.1 The nth term of a quadratic pattern is $T_n = an^2 + bn + c$ and the third term of the pattern is -1.

The value $-12; -8; -4; \ldots$ are the first differences of this pattern.

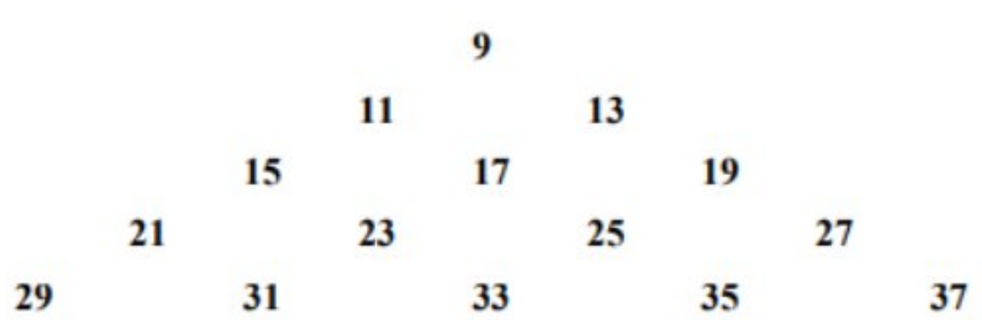
14.1.1 Write down the values of the first two terms of the pattern. (2) **L1**

14.1.2 Determine the values of a, b, and c (3) **L2**

14.1.3 Which term of the pattern is equal to 139? (4) **L2**

DURBAN GIRLS HIGH SCHOOL MAY 2019

15.1 A pyramid of odd numbers has been arranged below, starting with the number 9
In the first row, Five rows of numbers are shown below .



15.1.1 Determine the value of the last number in the next (sixth) row. (1) **L2**

15.1.2 Determine the value of the first number in row fifty. (5) **L3**

16. If a= - 1 and n is an odd positive integer, determine the value of the following: (2) **L3**
$a + a^2 + a^3 + a^4 + \ldots + a^n$

17. The sixteenth term of an arithmetic sequence is eleven times the first term. If the Common difference is six, determine first term (a) and hence T_{16} (4) **L3**

18 Given: $1 + \frac{1}{3} + \frac{1}{9} + \ldots$

Calculate the difference between S_∞ and S_7 (4) **L2**

19 Given the first three terms of an arithmetic sequence:
$x+5;\ 37-x\ ;\ x+13$

19.1 Determine the value of x (3) **L1**

19.2 Determine the general T_n of the sequence (2) **L2**

KUTLWANONG MARCH 2018

20. 20.1 A Mathematics exercise has 100 questions. The mathematics teacher asks Matthew to answer each question numbered $5n-2$
How many questions did Matthew answer? (2) **L2**

20.2 In an arithmetic sequence, the third term is -4 and the seventh term is -20. How many terms will add up to -140? (8) **L2**

20.3 The seventh term of a geometric sequence is $\frac{3645}{64}$ and the fourth term is $\frac{135}{8}$, determine the first term. (4) **L2**

21 The first three terms of a sequence are 4, k and 16. Calculate the value(s) of k if the sequence is:

21.1 arithmetic. (2) **L1**

21.2 geometric. (3) **L1**

22. Given the following series: $-11-4+3+.....+220$

22.1 Calculate the sum of the series (4) **L2**

22.2 Write the series in sigma notation (3) **L2**

TOPIC FUNCTIONS

GUIDELINES, SUMMARY NOTES, & STRATEGIES

FUNCTIONS

- A function is a special kind of relation where one input (x) value maps with exactly one output (y) value in an output.
- One-to-one function: one x-value corresponds with exactly one y-value, e.g. Hyperbola and straight line.
- Many-to-one function: more than one x-value corresponds with only one y-value that relation is called a function, e.g. parabola.

1. STRAIGHT LINE: $y = mx + c$

- Given two points
- Given gradient ($\boldsymbol{m}$) and one point
- Given line parallel or perpendicular to the line for which an equation is to be determined, and one point
- Domain : $x \in R$ Range : $y \in R$

2. QUADRATIC FUNCTION (PARABOLA)

$y = a(x+p)^2 + q$

- Turning point: $(-p;q)$
- Axis of symmetry: $x = -p$
- Maximum/minimum value: $y = q$

$y = ax^2 + bx + c$

- Axis of symmetry: $x = \dfrac{-b}{2a}$
- Derivative: $y' = 2ax + b$

3. HYPERBOLA: $y = \dfrac{a}{x+p} + q$

- Equations of asymptotes: $x = -p$ **(vertical asymptote)** $y = q$ **(horizontal asymptote)**
- Line of symmetry equation: $\begin{cases} \text{with positive gradient: } y = x + c \\ \text{with negative gradient: } y = -x + c \end{cases}$ **OR** $\begin{cases} y = (x-p) + q \\ y = -(x-p) + q \end{cases}$

  substitute point of intersection of asymptotes

- **Domain:** $x \in R,\ x \neq -p$
- **Range:** $y \in R;\ y \neq q$

4. EXPONENTIAL FUNCTION: $y = a.b^{x+p} + q$, $b > 0$ **and** $b \neq 1$

- **One asymptote:** $y = q$
- **Domain:** $x \in R$ **Range:** $y > q$ if $a > 0$

INVERSE FUNCTION

Indicated as f^{-1}

- Swop x and y in the given function
- Make y subject of the formula in the new function
- The graph of the given function and the graph of its inverse are reflected about the line $y = x$

1. STRAIGHT LINE: $y = mx + c$

- **Inverse:** $x = my + c$ **and** $y = \dfrac{x}{m} - \dfrac{c}{m}$ **… is a function**
- **Domain:** $x \in R$ **Range:** $y \in R$

2. **PARABOLA:** $y = ax^2$

- **Inverse**: $x = ay^2$ **and** $y = \pm\sqrt{\frac{x}{a}}$ **… not a function**
- **Restricting domain**: $\left\{\begin{matrix} x \geq 0 \\ x \leq 0 \end{matrix}\right\}$ **… inverse will be a function**
- **Domain:** $x \geq 0$ or $x \leq 0$ **Range:** $y > 0$ **if** $a > 0$ $y < 0$ if $a < 0$

3. **EXPONENTIAL:** $y = b^x$

- **Inverse:** $x = b^y$ **and** $y = \log_b x$ **… is a function**
- **Domain:** $x > 0$ **Range:** $y \in R$

Point(s) of intersection of two graphs: Graph 1 = Graph 2 (Equate equations of graphs and solve)
A line between two graphs: Graph above – graph below

REVISION QUESTIONS

DBE /NOV 2017

1 Given: $f(x) = -ax^2 + bx + 6$

1.1 The gradient of the tangent to the graph of f at the point $\left(-1;\frac{7}{2}\right)$ is 3. (5) **L3**

Show that $a = \frac{1}{2}$ and $b = 2$.

1.2 Calculate the x-intercepts of f. (3) **L2**

1.3 Calculate the coordinates of the turning point of f. (3) **L2**

1.4 Sketch the graph of f. Clearly indicate ALL intercepts with the axes and the turning point. (4) **L2**

1.5 Use the graph to determine the values of x for which $f(x) > 6$. (3) **L3**

1.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as f. Clearly indicate ALL intercepts with the axes. (2) **L2**

1.7 Write down the values of x for which $f(x).g(x) \leq 0$ (3) **L3**

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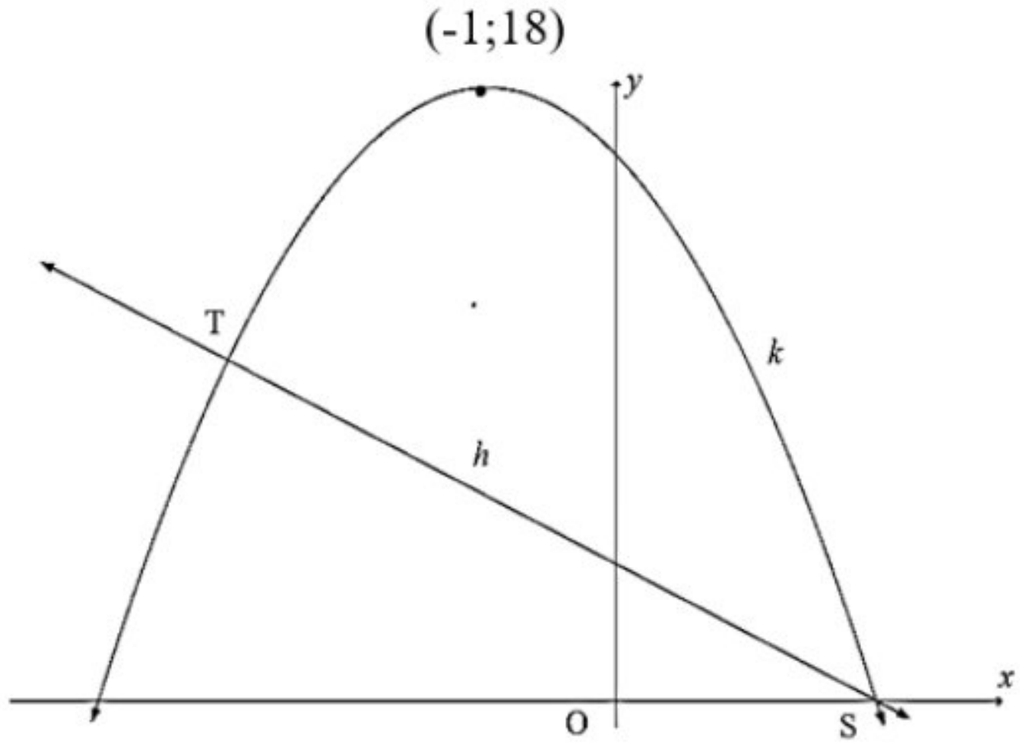
2 Sketched alongside are the graphs of $k(x) = ax^2 + bx + c$ and $h(x) = -2x + 4$. Graph k has a turning point at (-1;18). S is the x-intercept of h and k. Graphs h and k also intersect at T.



2.1 Calculate the coordinates of S. (2) **L1**

2.2 Determine the equation of k in the form $y = a(x + p)^2 + q$ (3) **L2**

2.3 If $k(x) = -2x^2 - 4x + 16$, determine the coordinates of T. (5) **L3**

2.4 Determine the value(s) of x for which $k(x) < h(x)$. (2) **L3**

2.5 It is further given that k is the graph of $g'(x)$.

2.5.1 For which values of x will the graph of g be concave up? (2) **L2**

2.5.2 Sketch the graph of g, showing clearly the x-values of the turning points and the point of inflection. (3) **L3**

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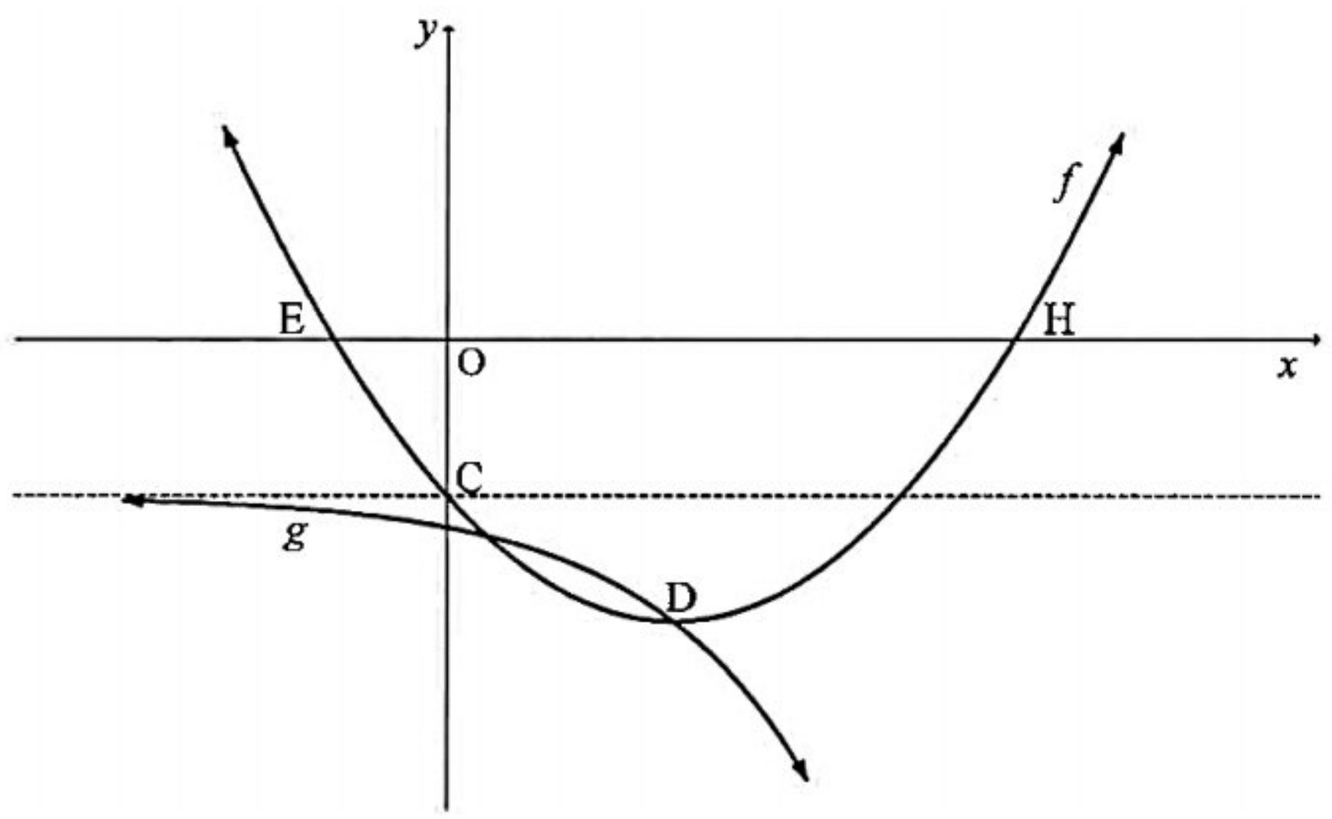
3 The graphs of $f(x) = x^2 - 4x - 5$ and $g(x) = a.2^x + q$ are sketched below.

- E and H are the x-intercepts of f.
- C is the y-intercept of f and lies on the asymptote of g.
- The two graphs intersect at D, the turning point of f.



3.1 Write down the y-co-ordinate of C. (1) **L1**

3.2 Determine the coordinates of D. (2) **L2**

3.3 Determine the values of a and q. (3) **L2**

3.4 Write down the range of g. (1) **L1**

3.5 Determine the values of k for which the value of $f(x) - k$ will always be positive. (2) **L3**

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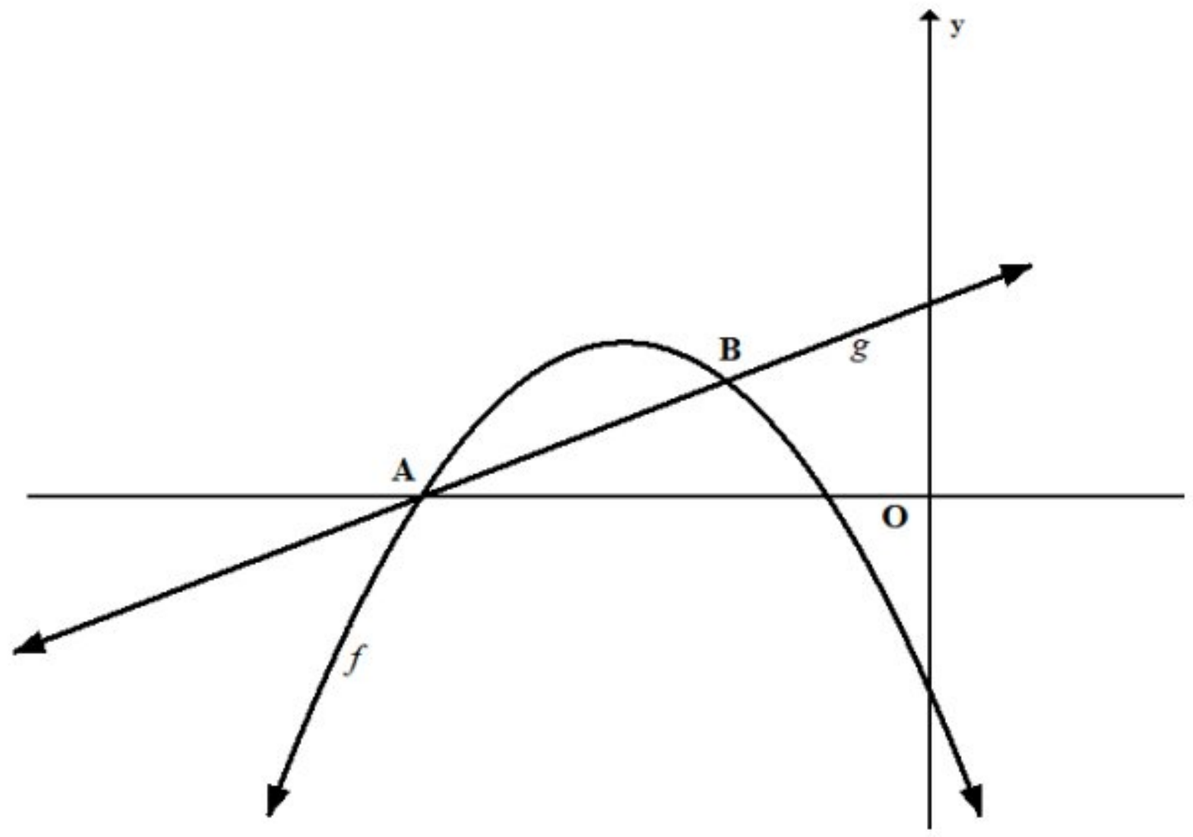
4 The graphs of the functions $f(x) = -(x+3)^2 + 4$ and $g(x) = x + 5$ are drawn alongside. The graphs intersect at A and B.



4.1 Write down the coordinates of the turning point of f. (2) **L1**

4.2 Write down the range of f. (1) **L1**

4.3 Show that the x-coordinates of A and B are -5 and -2 respectively. (4) **L2**

4.4 Hence, determine the values of c for which the equation $-(x+c+3)^2+4=(x+c)+5$ has ONE negative and ONE positive root (2) **L3**

4.5 The maximum distance between f and g in the interval $x_A < x < x_B$ is k. If $h(x)=g(x)+k$, determine the equation of h in the form $h(x)=...$ (5) **L4**

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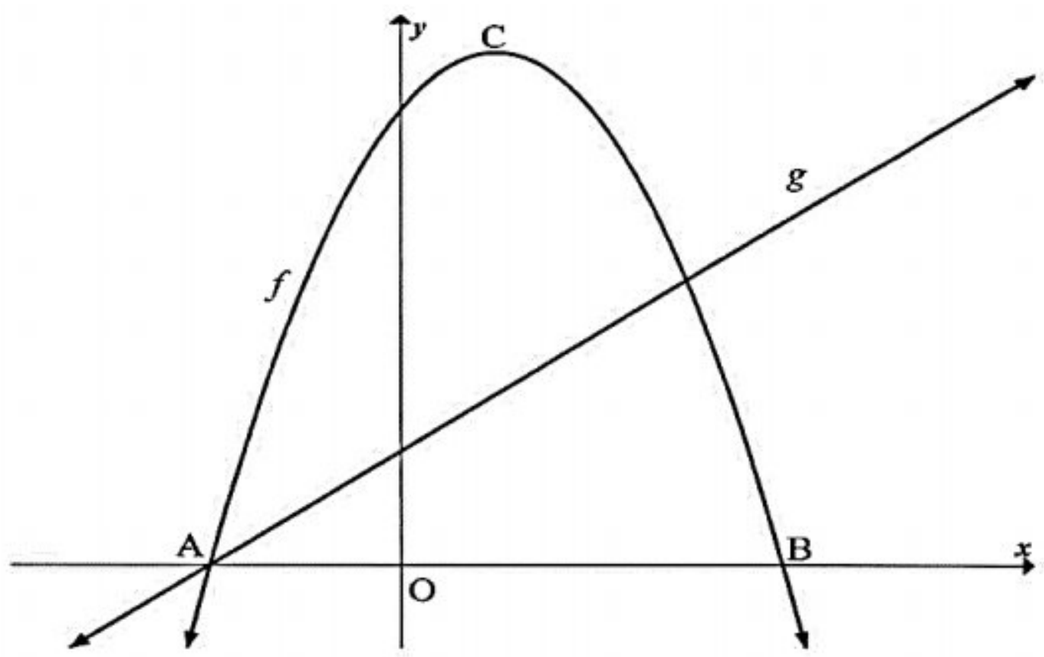
5 Sketched below are the graphs of $f(x)=-2x^2+4x+16$ and $g(x)=2x+4$. A and B are the x-intercepts of f. C is the point on f.



5.1 Calculate the coordinates of A and B. (3) **L2**

5.2 Determine the coordinates of C the turning point of f. (2) **L2**

5.3 Write down the range of f. (1) **L2**

5.4 The graph of $h(x)=f(x+p)+q$ has a maximum value of 15 at $x=2$. Determine the values of p and q. (3) **L3**

5.5 Determine the equation of g^{-1}, the inverse of g, in the form $y=...$ (2) **L2**

5.6 For which value(s) of x will $g^{-1}(x).g(x)=0$? (2) **L3**

5.7 If $p(x)=f(x)+k$, determine the values of k for which p and g will not intersect. (5) **L4**

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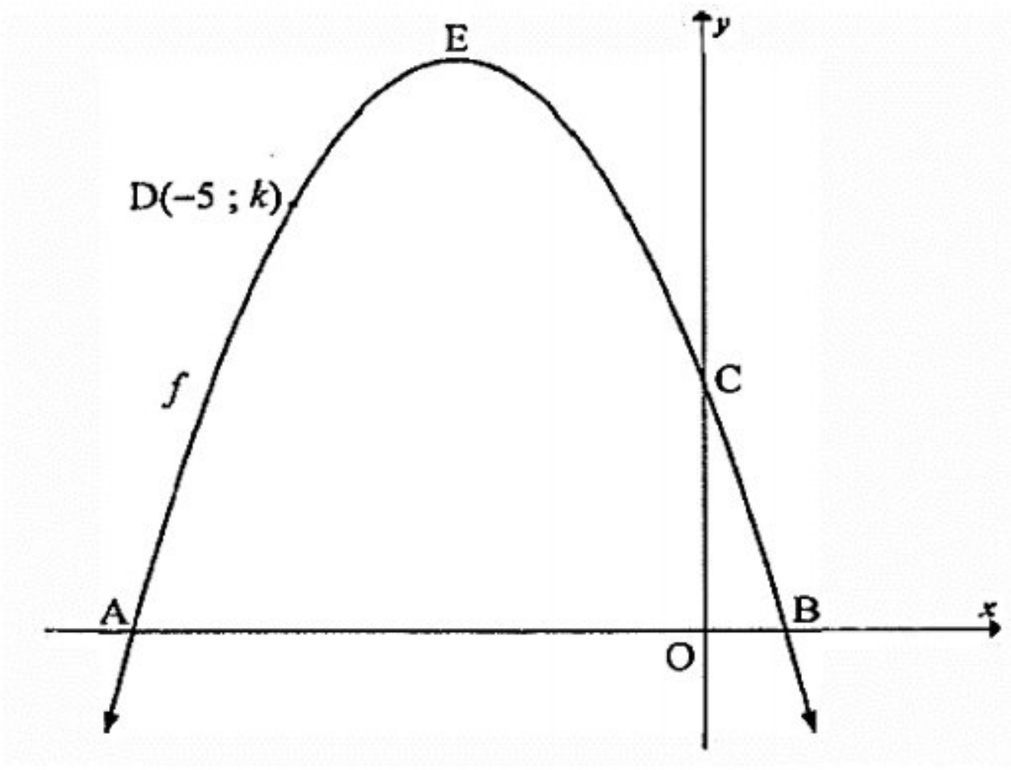
6. The sketch below shows the graph of $f(x)=-x^2-6x+7$.
C is the $y-$intercept of f, A and B are the $x-$intercepts of f, D(-5; k) is a point on f.



6.1 Calculate the coordinates of E, the turning point of f. (3) **L2**

6.2 Write down the value of k (1) **L1**

6.3 Determine the equation of the straight line passing through C and D. (4) **L2**

6.4 A tangent parallel to CD, touches f at P. Determine the coordinates of P. (4) **L3**

6.5 For which values of x will $f(x)-12>0$? (2) **L3**

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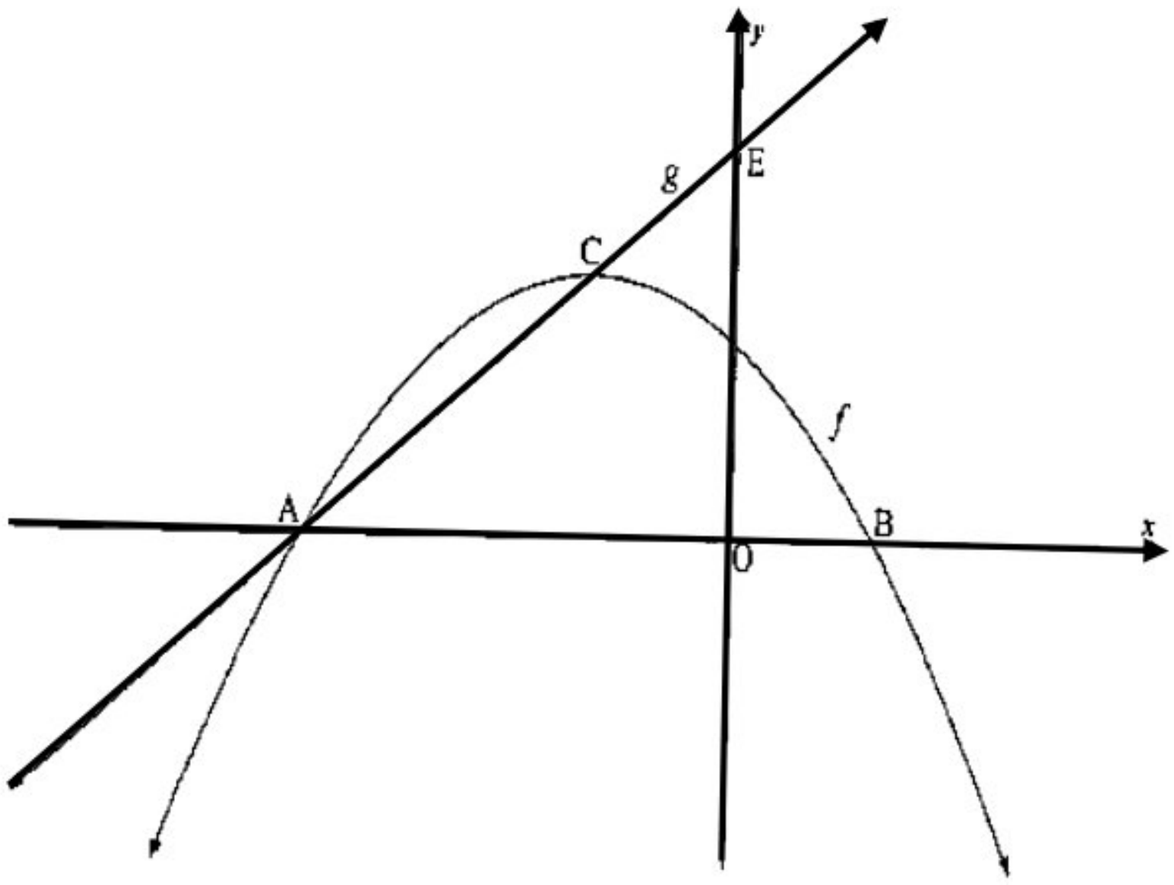
7 The sketch below shows the graphs of $f(x)=-x^2-2x+3$ and $g(x)=mx+q$.
The graph f has x-intercept at A and B(1;0) and a turning point at C.
The straight line g, passing through A and C, cuts the y-axis at E.



7.1 Write down the coordinates of the y-intercept of f. (1) **L1**

7.2 Show that the coordinates of C are $(-1;4)$. (3) **L2**

7.3 Write down the coordinates of A. (1) **L2**

7.4 Calculate the length of CE. (6) **L3**

7.5 Determine the values of k if $h(x)=2x+k$ is a tangent to the graph of f. (5) **L3**

7.6 Determine the equation of g^{-1}, the inverse of g, in the form $y=...$ (2) **L2**

7.7 For which values of x is $g(x)\geq g^{-1}(x)$? (3) **L3**

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8. Given: $f(x)=\dfrac{4}{x-1}+2$

8.1 Determine the equation of the asymptotes of f. (2) **L1**

8.2 Write down the y-intercept of f. (1) **L1**

8.3 Calculate the x-intercept of f. (2) **L2**

8.4 Sketch the graph of f, label all asymptotes and indicate the intercepts with the axes. (4) **L2**

8.5 Use your graph to determine the values of x for which $\dfrac{4}{x-1}\geq -2$ (2) **L4**

8.6 Determine the equation of the axis of symmetry of $f(x-2)$, that has a negative gradient. (3) **L3**

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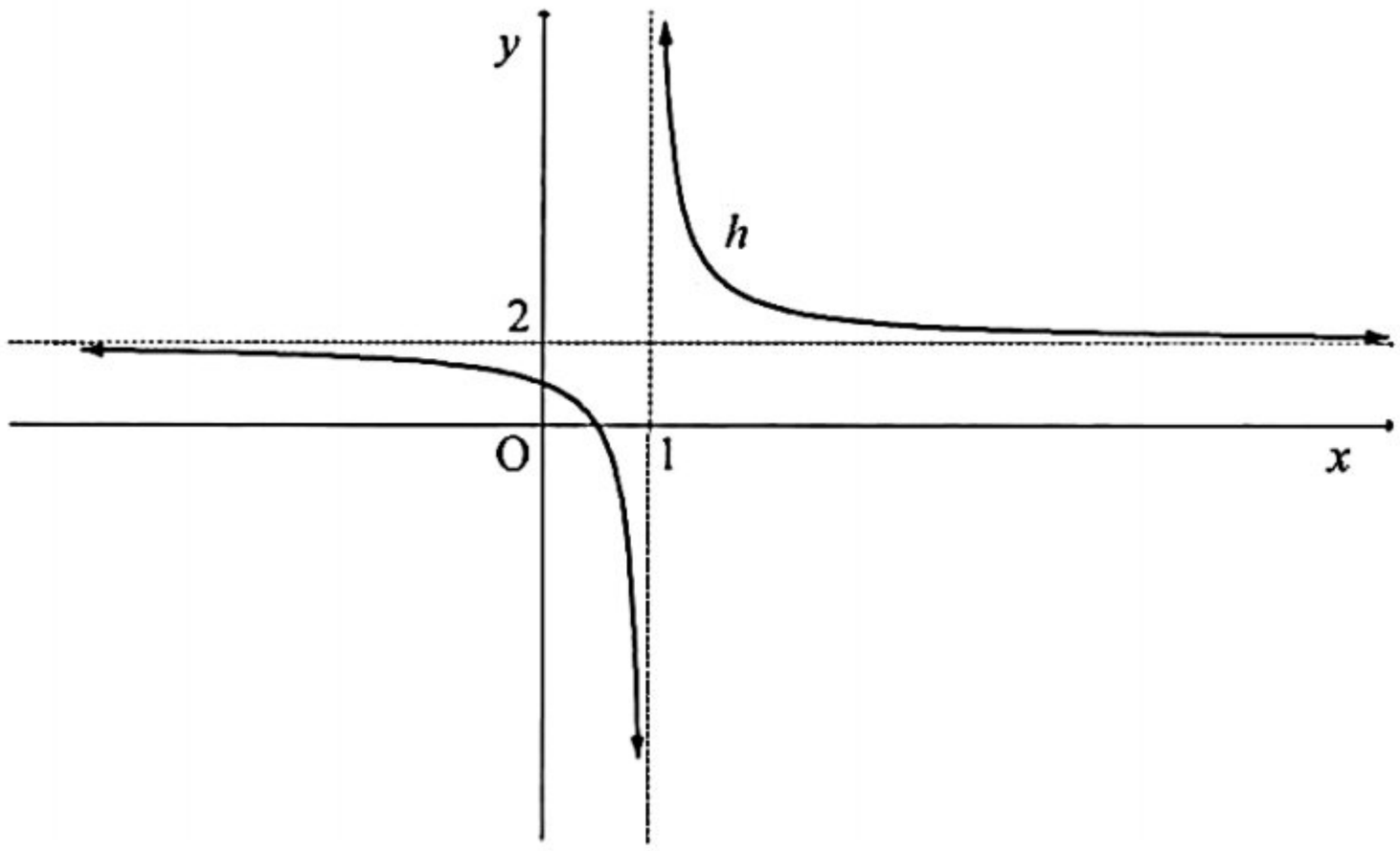
9 9.1 Determine the equation of asymptotes of: $f(x)=\dfrac{7-x}{x-1}$ (3) **L2**

9.2 Sketched below is the graph of $h(x)=\dfrac{1}{x+p}+q$. The asymptotes of h intersect at $(1;2)$.



9.2.1 Write down the values of p and q. (2) **L1**

9.2.2 Calculate the coordinates of the x-intercept of h. (2) **L2**

9.2.3 Write down the x-coordinate of the x-intercept of g if $g(x)=h(x+3)$. (2) **L2**

9.2.4 The equation of an axis of symmetry of h is $y=x+t$. Determine the value of t. (2) **L1**

9.2.5 Determine the values of x for which $-2\le\dfrac{1}{x-1}$ (2) **L3**

10 Given: $f(x)=\dfrac{4}{2-x}+1$

10.1 Write down the equation of the asymptotes. (2) **L1**

10.2 Sketch the graph of $g(x)$ if $g(x)=-f(x)$ (4) **L3**

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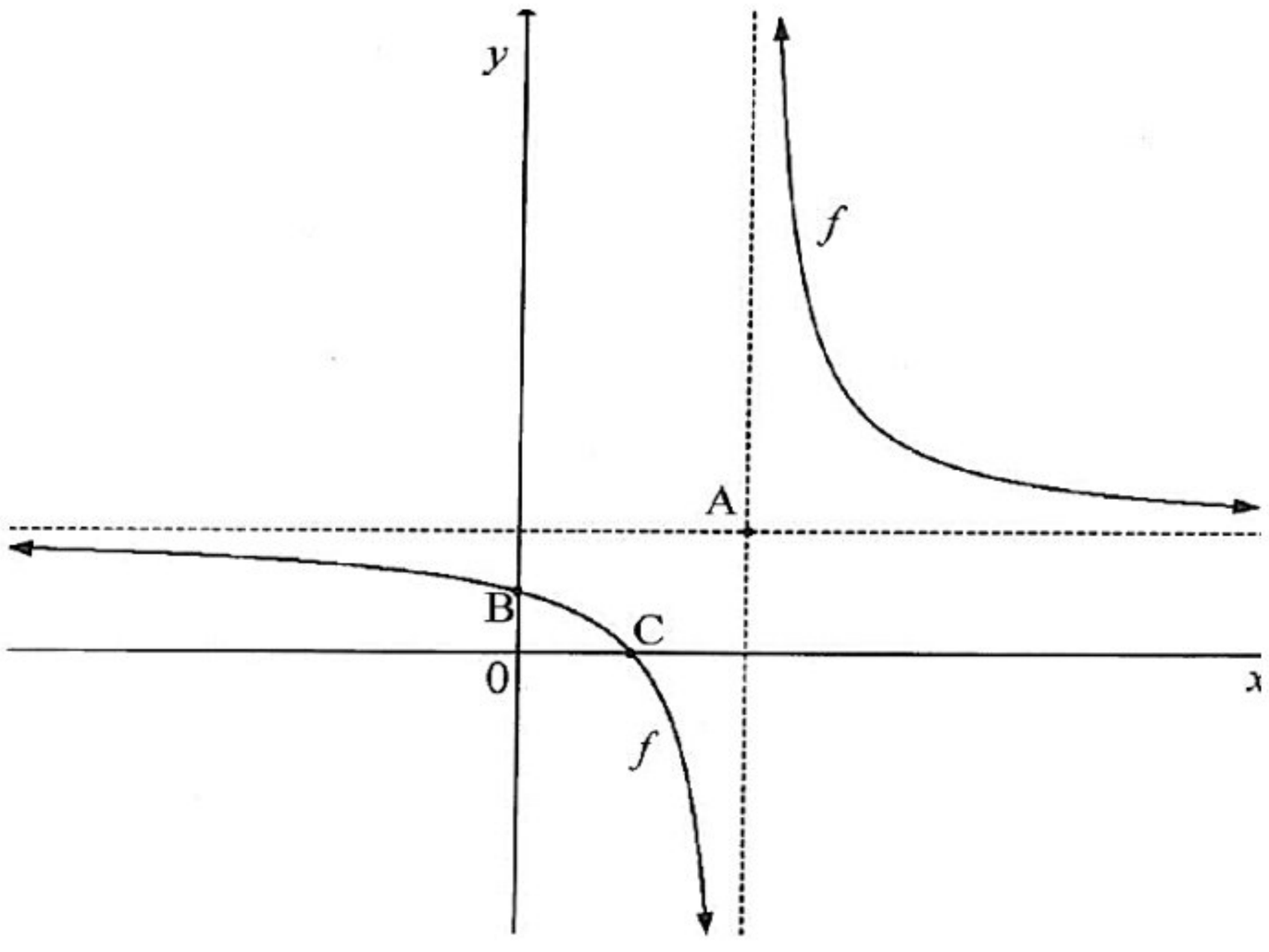
11. The sketch alongside shows the graph of $f(x)=\dfrac{6}{x-4}+3$. The asymptotes of f intersect at A. The graph f intersects the x-axis and y - axis at C and B respectively.



11.1 Write down the coordinates of A. (1) **L1**

11.2 Calculate the coordinates of B. (2) **L1**

11.3 Calculate the coordinates of C. (2) **L2**

11.4 Calculate the average gradient of f between B and C. (2) **L1**

11.5 Determine the equation of a line of symmetry of f which has a positive y - intercept. (2) **L2**

DBE/FEB-MARCH 2018

12 The function f, defined by $f(x)=\frac{a}{x+p}+q$, has the following properties:

- The range of f is $y\in R, y\neq 1$.
- The graph f passes through the origin.
- $\mathrm{P}\left(\sqrt{2}+2;\sqrt{2}+1\right)$ lies on the graph of f.

12.1 Write down the value of q. (1) **L1**

12.2 Calculate the values of a and p. (5) **L3**

12.3 Sketch a neat graph of this function. Your graph must include the asymptotes, if any. (4) **L2**

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13 Consider: $g(x)=\frac{a}{x+p}+q$

The following information of g is given:

- Domain: $x\in R; x\neq -2$
- x-intercept at K (1; 0)
- y-intercept at $\mathrm{N}\left(0;-\frac{1}{2}\right)$

13.1 Show that the equation of g is given by: $g(x)=\frac{-3}{x+2}+1$ (6) **L3**

13.2 Write down the range of g. (1) **L1**

13.3 Determine the equation of h, the axis of symmetry of g, in the form $y=mx+c$, where $m>0$ (3) **L2**

13.4 Write down the coordinates of K', the image of K reflected over h. (2) **L3**

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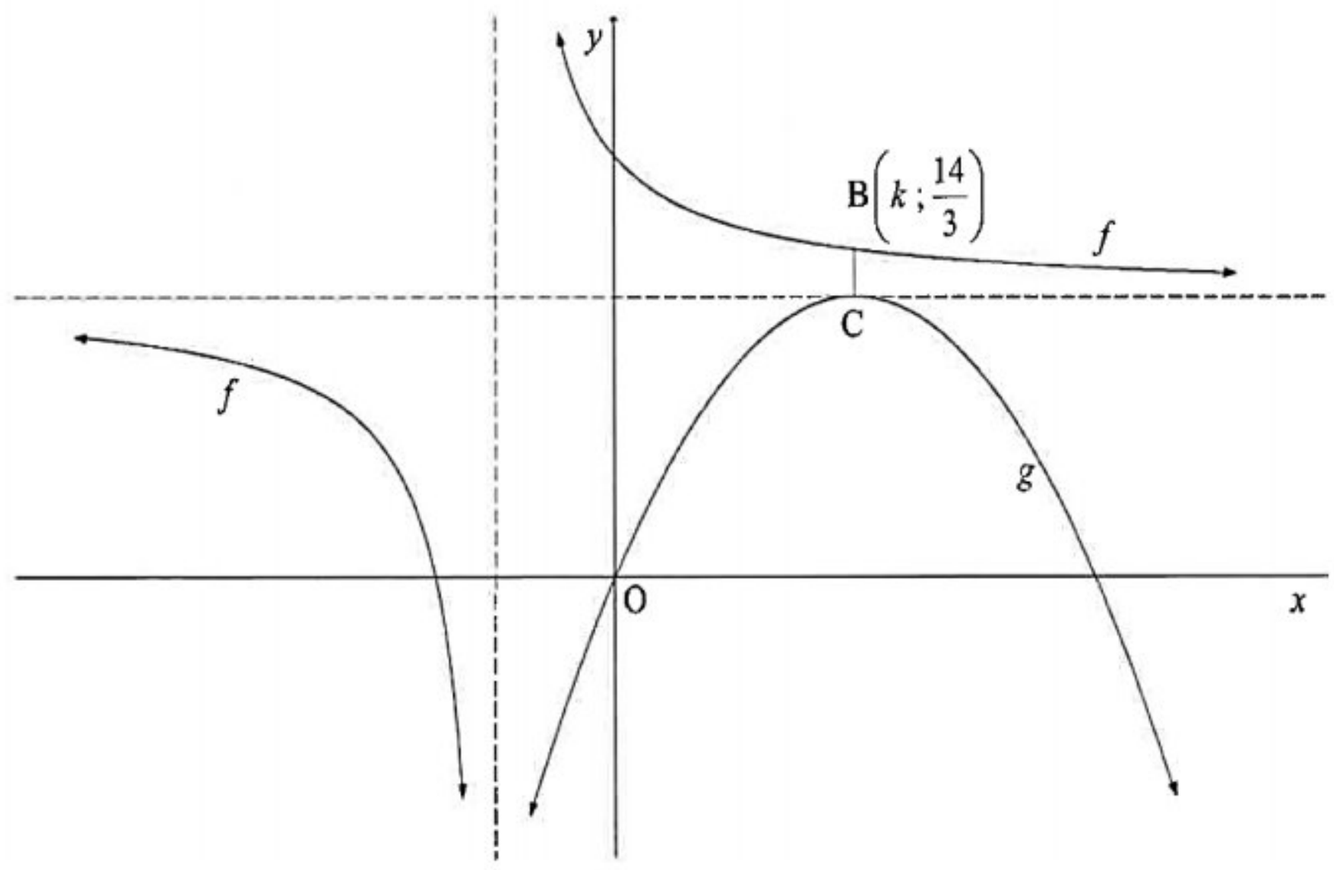
14 The graphs of $f(x)=\frac{2}{x+1}+4$ and parabola g are drawn below.

- C, the turning point of g, lies on the horizontal asymptotes of f.
- The graph of g passes through the origin.
- $B\left(k;\frac{14}{3}\right)$ is a point on f such that BC is parallel to the y -axis.



14.1 Write down the domain of f. (2) **L1**

20

14.2 Determine the x-intercept of f. (2) **L2**

14.3 Calculate the value of k. (3) **L2**

14.4 Write down the coordinates of C. (2) **L1**

14.5 Determine the equation of g in the form $y = a(x+p)^2 + q$. (3) **L2**

14.6 For which value(s) of x will $\frac{f(x)}{g(x)} \leq 0$? (4) **L3**

14.7 Use the graphs of f and g to determine the number of real roots of $\frac{2}{x} - 5 = -(x-3)^2 - 5$. Give reasons for your answer (4) **L4**

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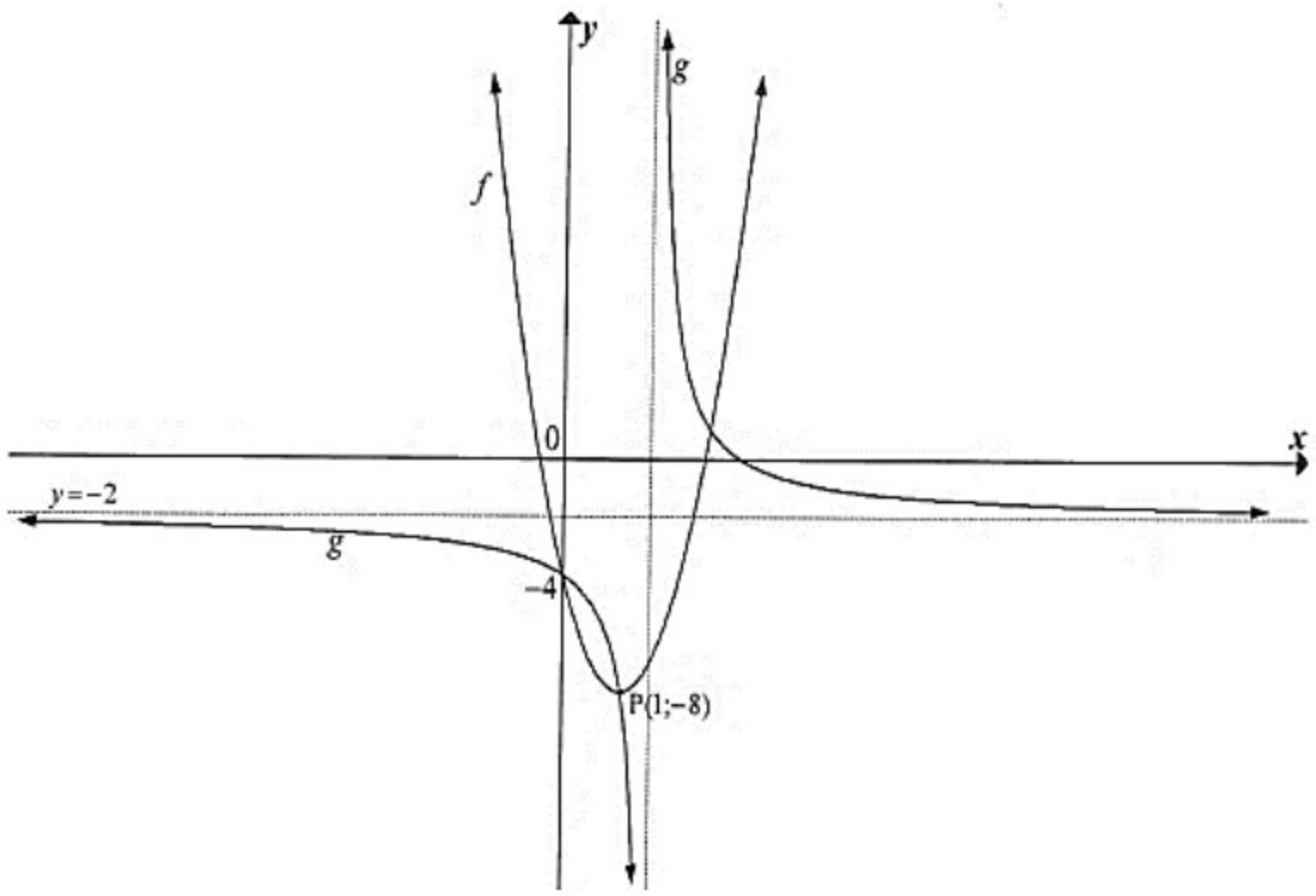
15 The graphs of the functions $f(x) = a(x+p)^2 + q$ and $g(x) = \frac{k}{x+r} + d$ are sketched below. Both graphs cut the $y-$axis at -4. One of the points of intersection of the graphs is P$(1;-8)$, which is also the turning point of f. The horizontal asymptote of g is $y = -2$.



15.1 Calculate the values of a, p and q. (4) **L1**

15.2 Calculate the values of k, r and d. (6) **L3**

15.3 Determine the value(s) of x in the interval $x \leq 1$ for which $g(x) \geq f(x)$. (2) **L2**

15.4 Determine the value(s) of k for which $f(x) = k$ has two, unequal positive roots. (2) **L2**

15.5 Write down the equation for the axis of symmetry of g that has a negative gradient. (3) **L2**

15.6 The point P is reflected in the line determined in QUESTION 15.5 to give the point Q. Write down the coordinates of Q. (2) **L3**

DBE/FEB-MARCH 2016

16. Given: $f(x) = 2^{-x} + 1$

16.1 Determine the coordinates of the $y-$intercept of f. (1) **L1**

16.2 Sketch the graph of f, clearly indicating ALL intercepts with the axes as well as any asymptotes. (3) **L2**

16.3 Calculate the average gradient of f between the points on the graph where $x = -2$ and $x = 1$. (3) **L2**

16.4 If $h(x) = 3f(x)$, write down an equation of the asymptote of h. (1) **L3**

DBE/MAY-JUNE 2023

17. Given the function: $p(x) = \left(\frac{1}{3}\right)^x$

17.1 Is p an increasing or decreasing function? (1) **L1**

17.2 Determine p^{-1}, the inverse of p in the form $y = ...$ (2) **L2**

17.3 Write down the domain of p^{-1} (1) **L1**

17.4 Write down the equation of the asymptote of $p(x) - 5$ (1) **L2**

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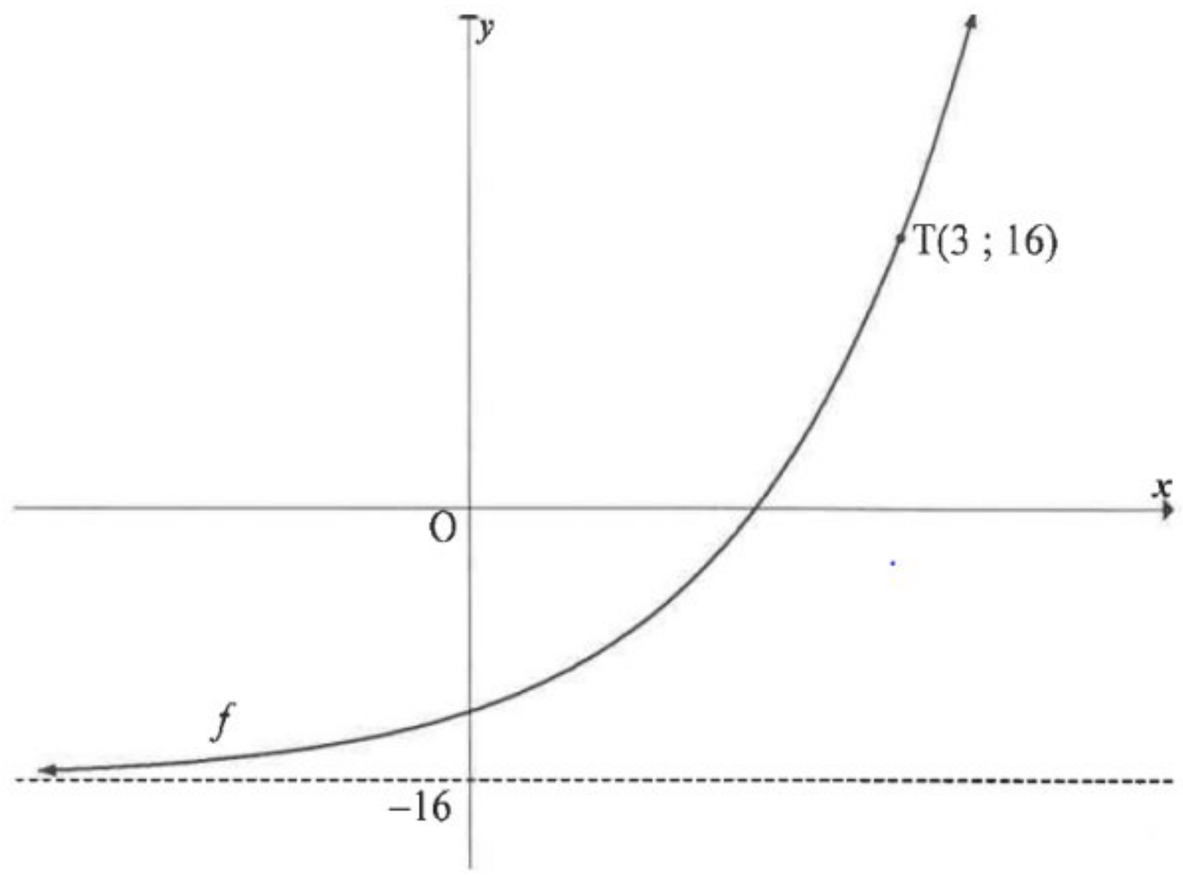
18 18.1 Given: $g(x) = 3^x$

18.1.1 Write down the equation g^{-1} in the form $y = ...$ (2) **L2**

18.1.2 Point P(6;11) lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to form g. Write down the coordinate of the image of P on g. (2) **L2**

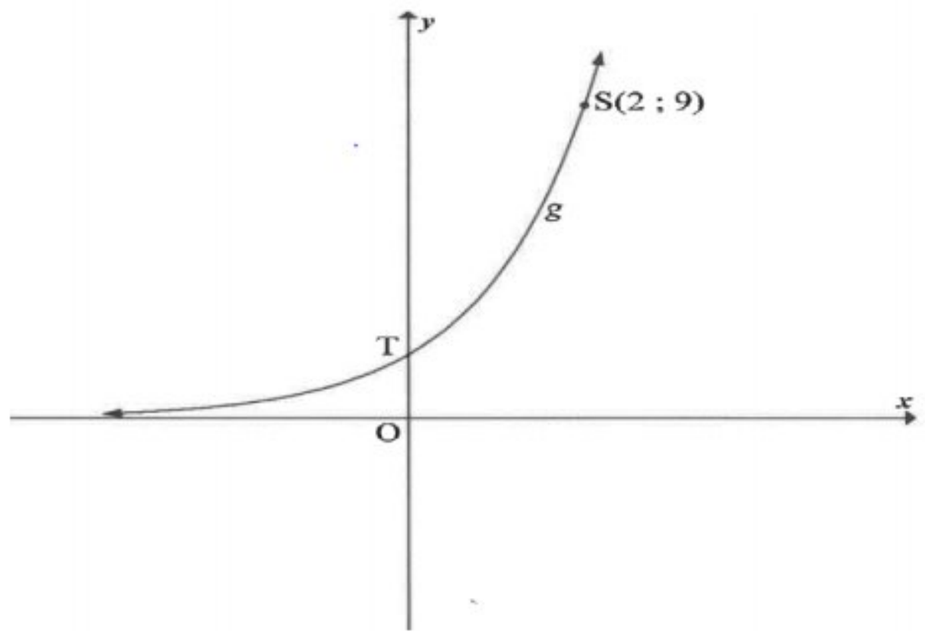
18.2 Sketched is the graph of $f(x) = 2^{x+p} + q$. T(3;16) is a point on f and the asymptote of f is $y = -16$.



Determine the values of p and q (4) **L2**

19 **DBE/FEB-MARCH 2018**

The graph of $g(x) = a^x$ is drawn in the sketch below. The point S(2;9) lies on g. T is the y-intercept of g.



19.1 Write down the coordinates of T. (2) **L1**

19.2 Calculate the value of a. (2) **L2**

19.3 The graph h is obtained by reflecting g in the y-axis. Write down the equation of h. (2) **L2**

19.4 Write down the values of x for which $0 < \log_3 x < 1$. (2) **L3**

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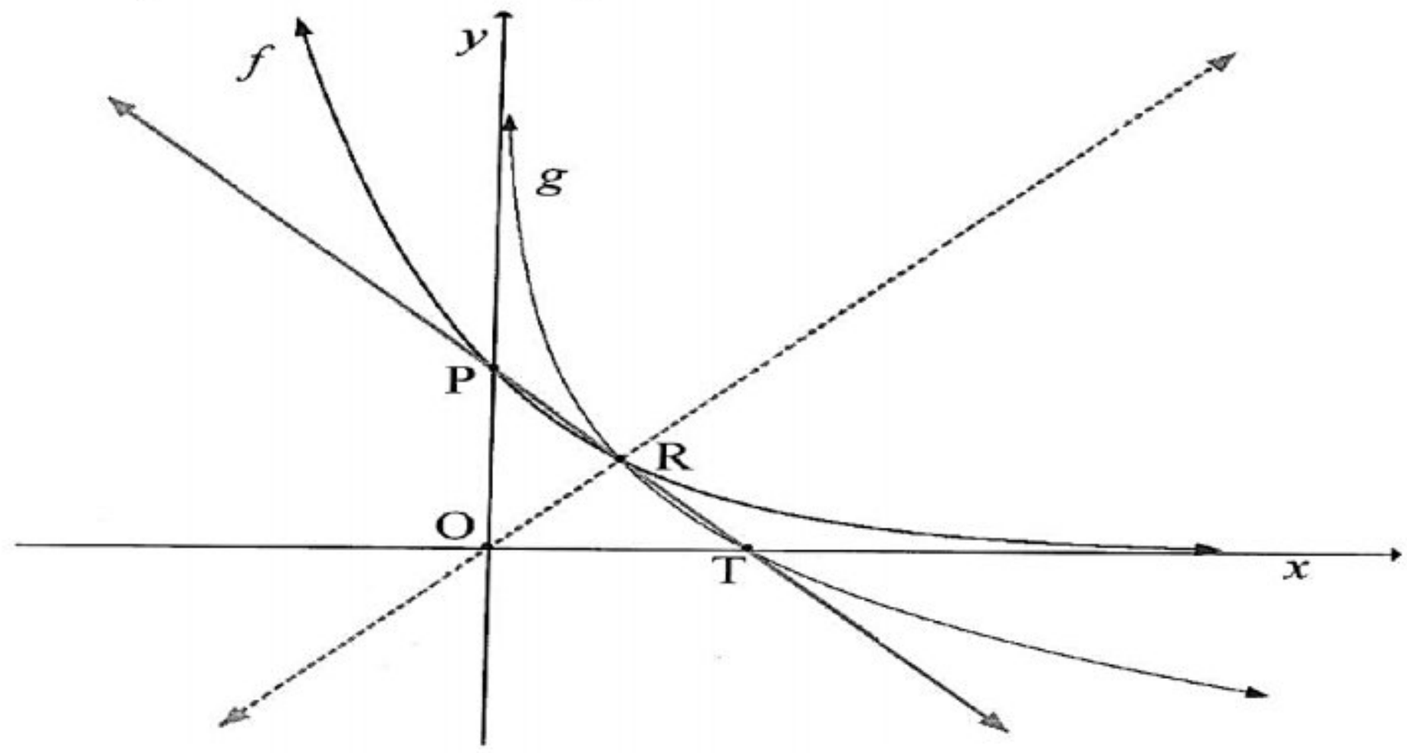
20 In the sketch below, P is the y-intercept of the graph of $f(x) = b^x$. T is the x-intercept of graph g, the inverse of f. R is the point of intersection of f and g. Straight lines are drawn through O and R and through P and T.



20.1 Determine the equation of g (in terms of b) in the form $y = ...$ (2) **L2**

20.2 Write down the equation of the line passing through O and R. (1) **L1**

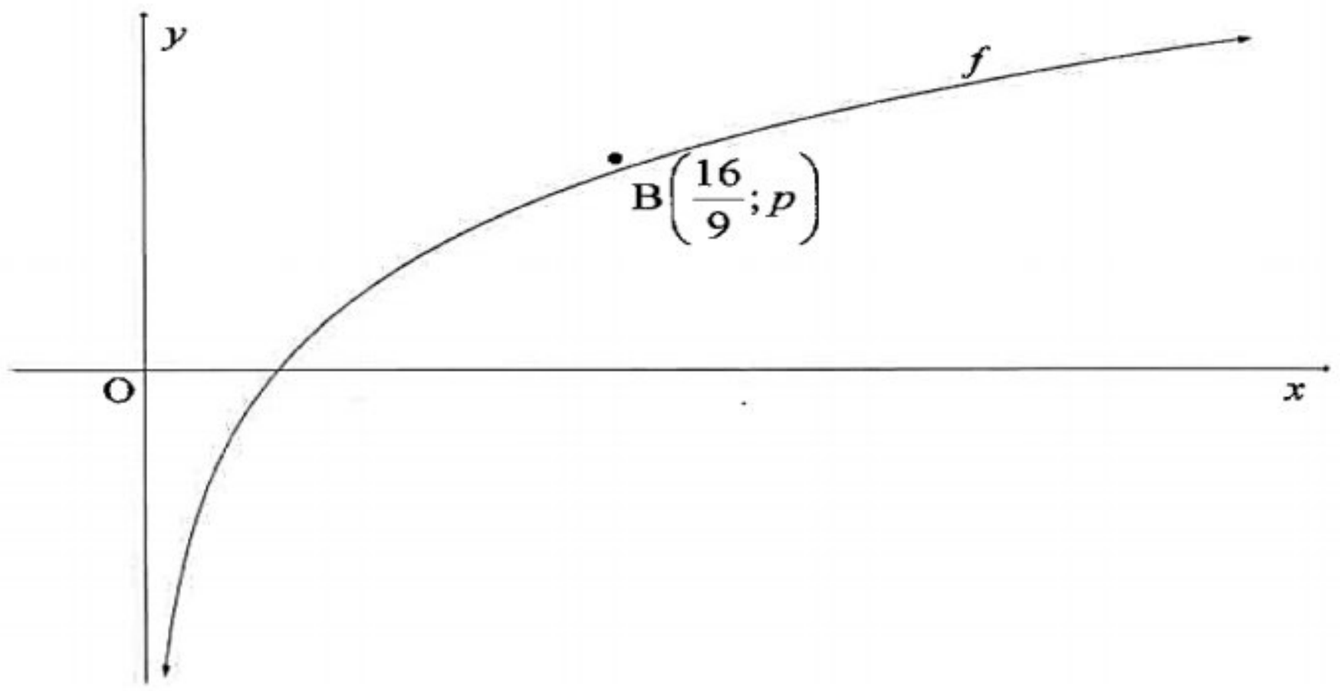
20.3 Write down the coordinates of point P. (1) **L1**

20.4 Determine the equation of the line passing through P and T. (2) **L2**

20.5 Calculate the value of b. (5) **L3**

DBE/MAY-JUNE 2018

21 The graph of $f(x) = \log_{\frac{4}{3}} x$ is drawn below. $B\left(\frac{16}{9}; p\right)$ is a point on f.



21.1 For which value(s) of x is $\log_{\frac{4}{3}} x < 0$? (2) **L2**

21.2 Determine the value of p ,without using a calculator (3) **L2**

21.3 Write down the equation of the inverse of f in the form $y = ...$ (2) **L2**

21.4 Write down the range of $y = f^{-1}(x)$ (2) **L1**

21.5 The function $h(x)=\left(\frac{3}{4}\right)^x$ is obtained after applying two reflections on f. Write down the coordinates of B'', the image of B an h. (2) **L3**

DBE/ MAY-JUNE 2022

22 The graph of $g(x)=a\left(\frac{1}{3}\right)^x+7$ passes through point $E(-2;10)$

22.1 Calculate the value of a. (3) **L2**

22.2 Calculate the coordinates of the y-intercept of g. (2) **L2**

22.3 Consider: $h(x)=\left(\frac{1}{3}\right)^x$

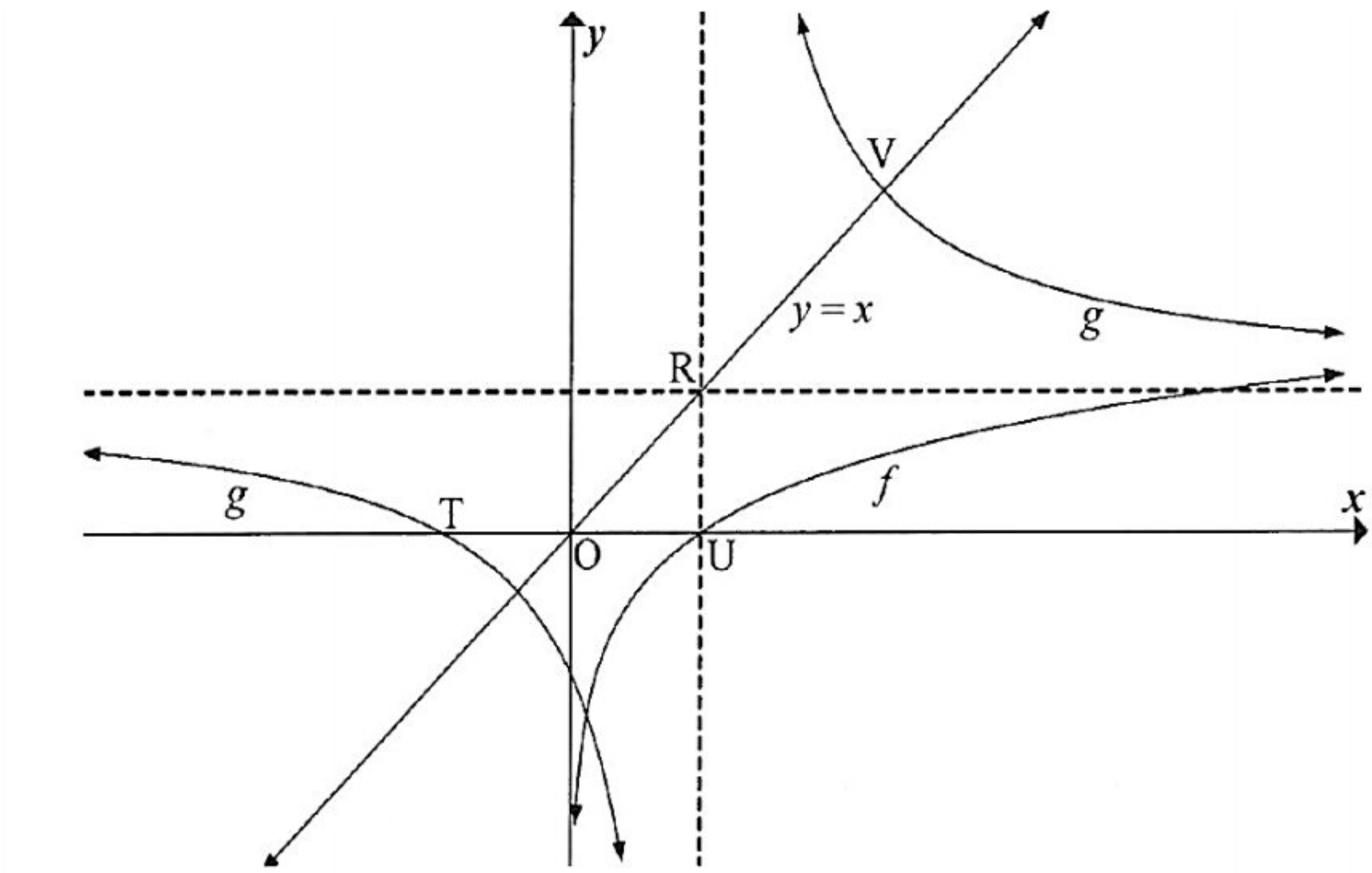
22.3.1 Describe the translation from g to h. (2) **L3**

22.3.2 Determine the equation of the inverse of h, in the form $y=....$ (2) **L2**

DBE/FEB-MARCH 2017

23 The sketch below shows the graphs of $f(x)=\log_5 x$ and $g(x)=\frac{2}{x-1}+1$.

- T and U are the x-intercepts of g and f respectively.
- The line $y=x$ intersects the asymptotes of g at R, and the graph of g at V.



23.1 Write down the coordinates of U. (1) **L1**

23.2 Write down the equation of the asymptotes of g. (2) **L2**

23.3 Determine the coordinates of T. (2) **L2**

23.4 Write down the equation of h, the reflection of f in the line $y=x$, in the form $y=...$ (2) **L2**

23.5 Write down the equation of the asymptotes of $h(x-3)$. (1) **L3**

23.6 Calculate the coordinates of V. (4) **L2**

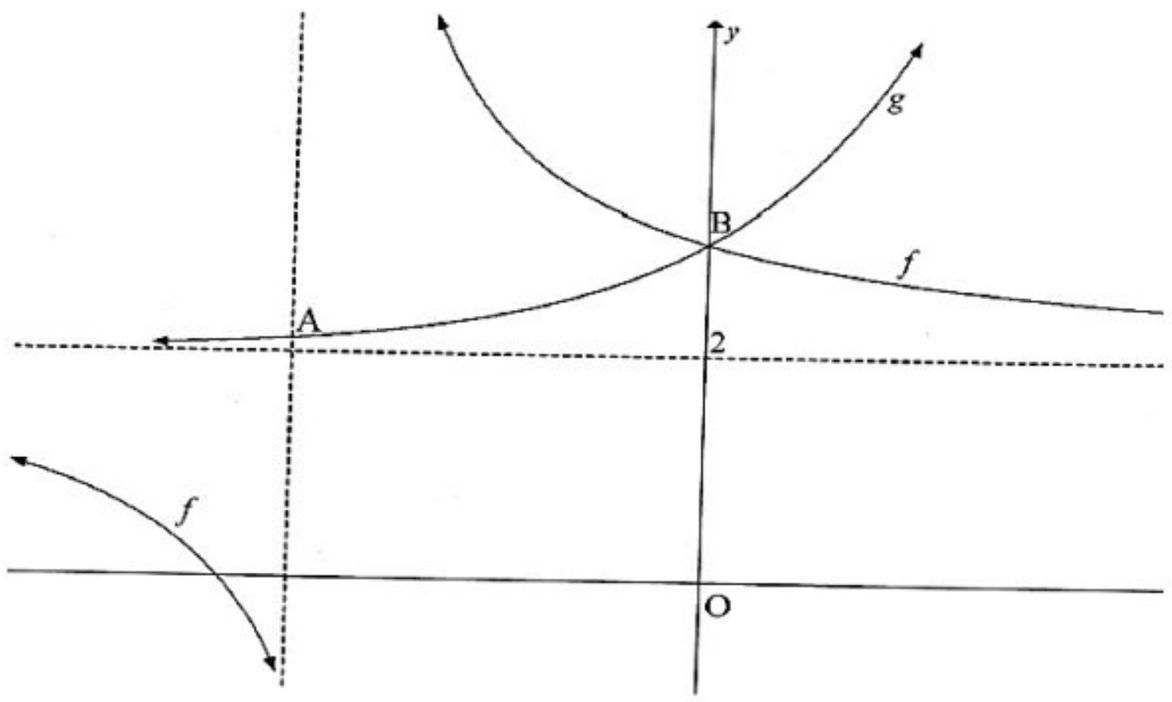
23.7 Determine the coordinates of T' the point which is symmetrical to T about the point R. (2) **L2**

DBE/ MAY-JUNE 2016

24 The sketch below shows the graphs of

$f(x)=\frac{3}{x-p}+q$ and $g(x)=2^x+r$

- g intersects the vertical asymptotes of f at A.
- B is the common y-intercept of f and g.
- $y=2$ is the common horizontal asymptote of f and g



24.1 Write down the value of r. (1) **L1**

24.2 Determine the value of p. (4) **L2**

24.3 Determine the coordinates of A. (3) **L2**

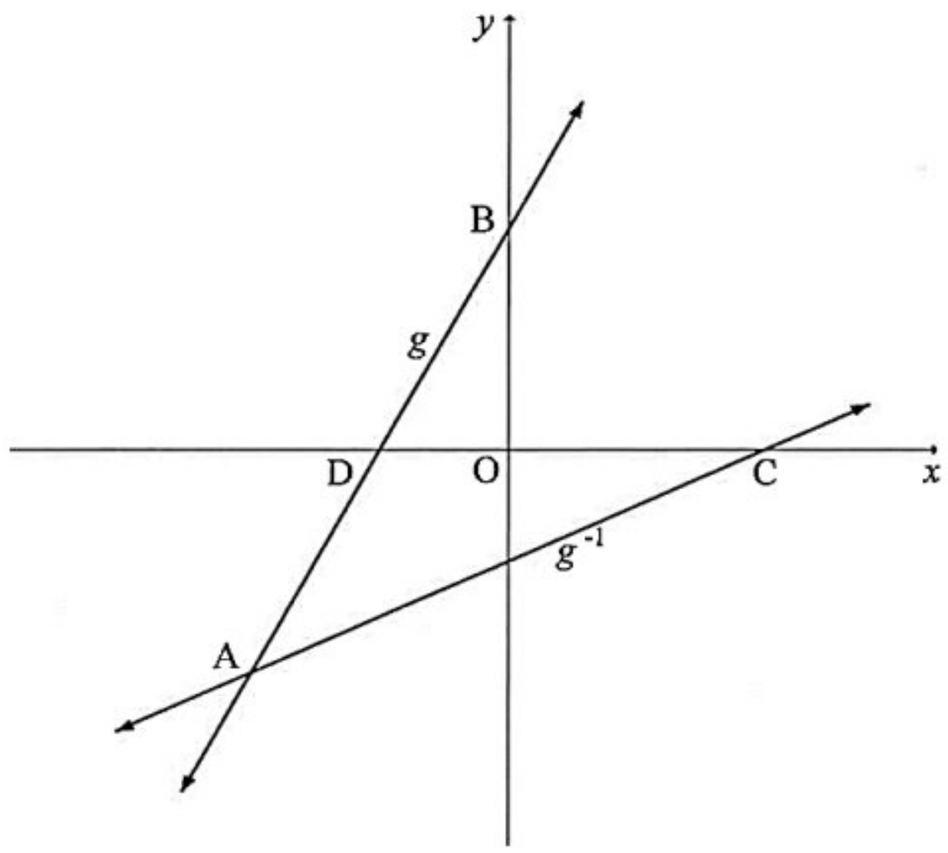
24.4 For which value(s) of x is $f(x)-g(x)\geq 0$? (2) **L2**

24.5 If $h(x)=f(x-2)$, write down the equation of h. (2) **L2**

DBE/ NOV 2022

25 The graphs of $g(x)=2x+6$ and g^{-1}, the inverse of g, are shown in the diagram below.

- D and B are the x and y-intercepts respectively of g.
- C is the x-intercept of g^{-1}.
- The graphs of g and g^{-1} intersect at A.



25.1 Write down the y-coordinate of B. (1) **L1**

25.2 Determine the equation of g^{-1} in the form $g^{-1}(x)=mx+n$. (2) **L2**

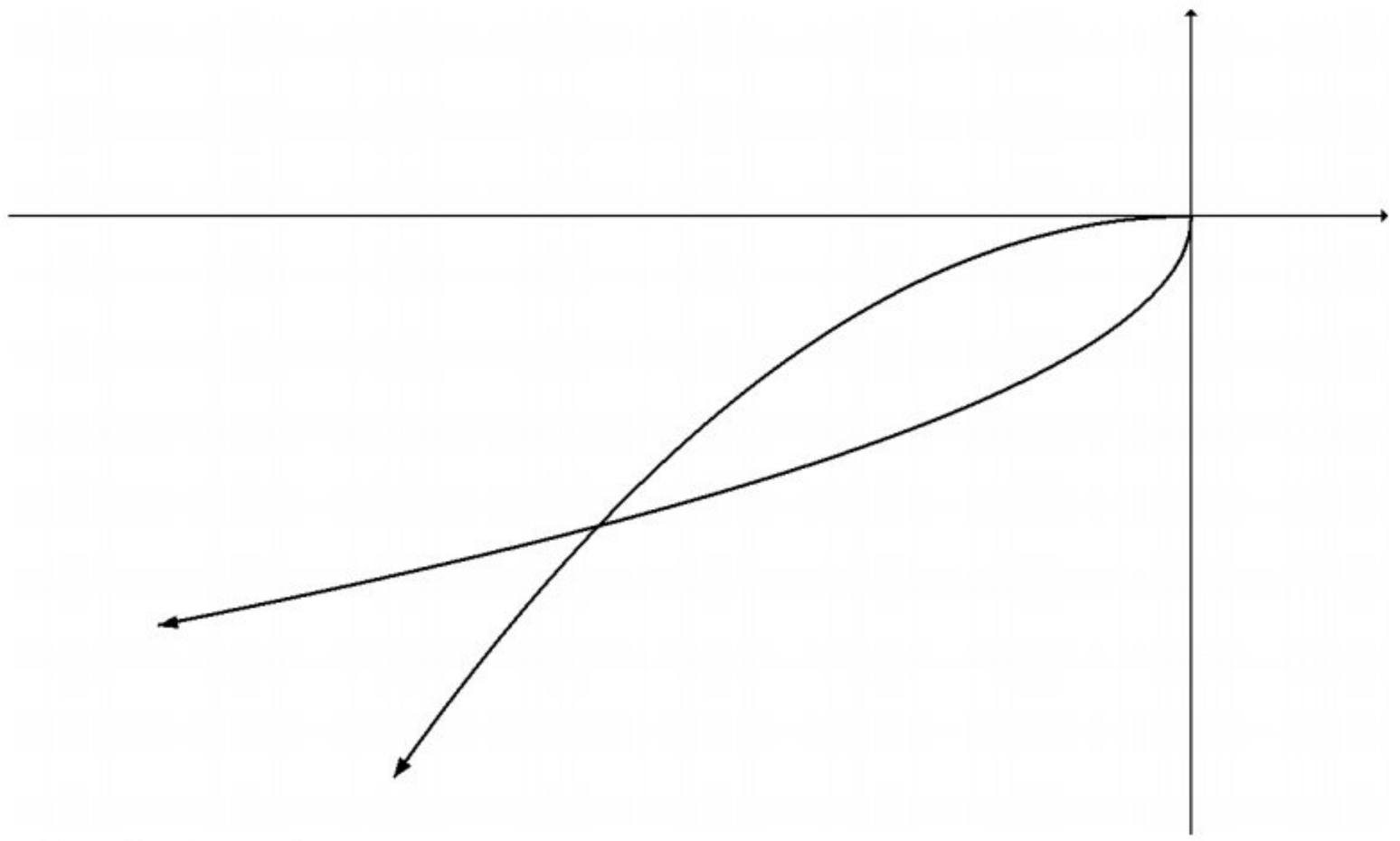
25.3 Determine the coordinates of A. (3) **L2**

25.4 Calculate the length of AB (2) **L2**

25.5 Calculate the area of ΔABC. (5) **L3**

DBE/ NOV 2018

26 In the diagram below, the graph of $f(x)=ax^2$ is drawn in the interval $x\leq 0$. The graph of f^{-1} is also drawn. P (–6; –12) is a point on f and R is a point on f^{-1}.



26.1 Is f^{-1} a function? Motivate your answer. (2) **L1**

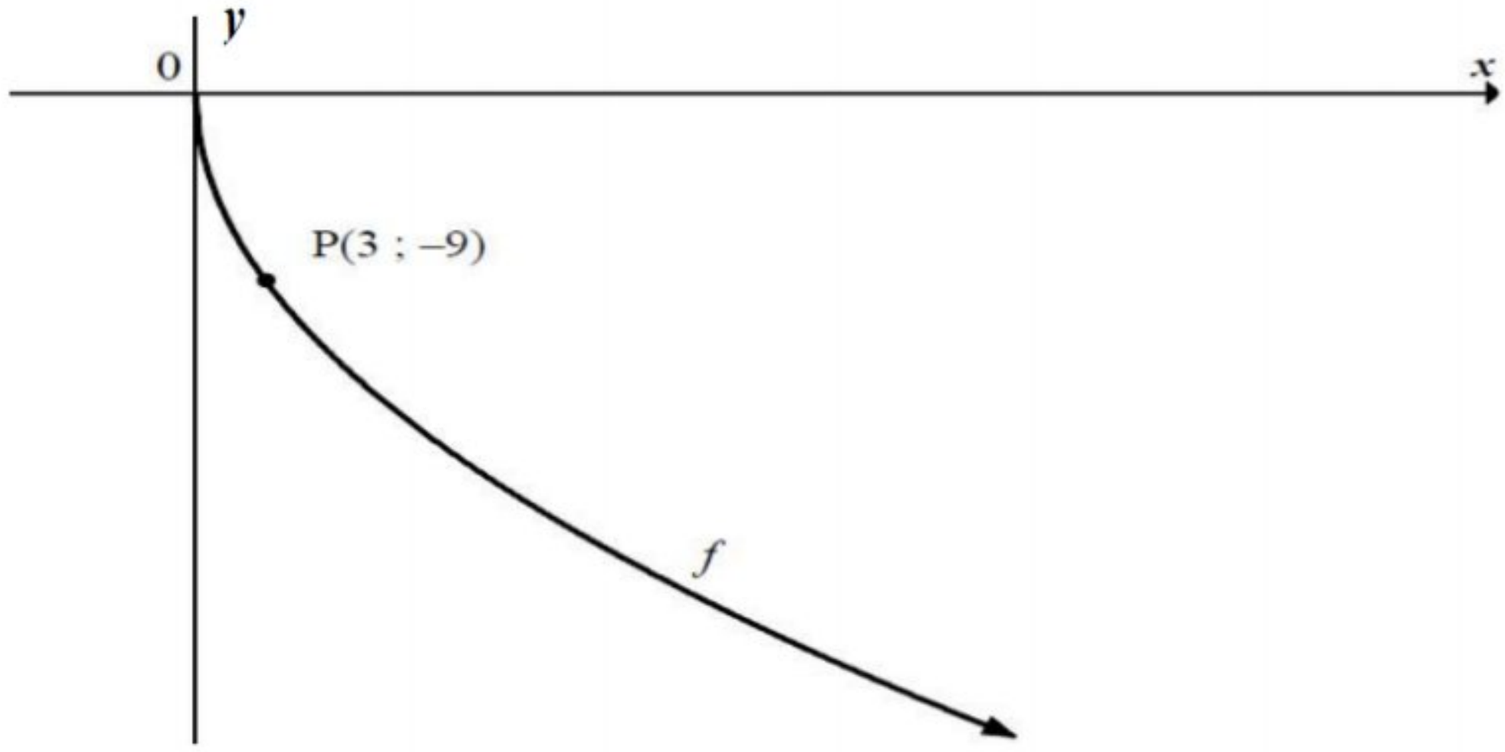
26.2 If R is the reflection of P in the line $y = x$, write down the coordinates of R. (1) **L1**

26.3 Calculate the value of a. (2) **L1**

26.4 Write down the equation of f^{-1} in the form $y = \ldots$ (3) **L2**

DBE/ NOV 2012

27 The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below. The point P(3;–9) lies on the graph of f.



27.1 Use your graph to determine the values of x for which $f(x) \geq -9$ (2) **L1**

27.2 Write down the equation of f^{-1} in the form $y = \ldots$include all restrictions. (3) **L2**

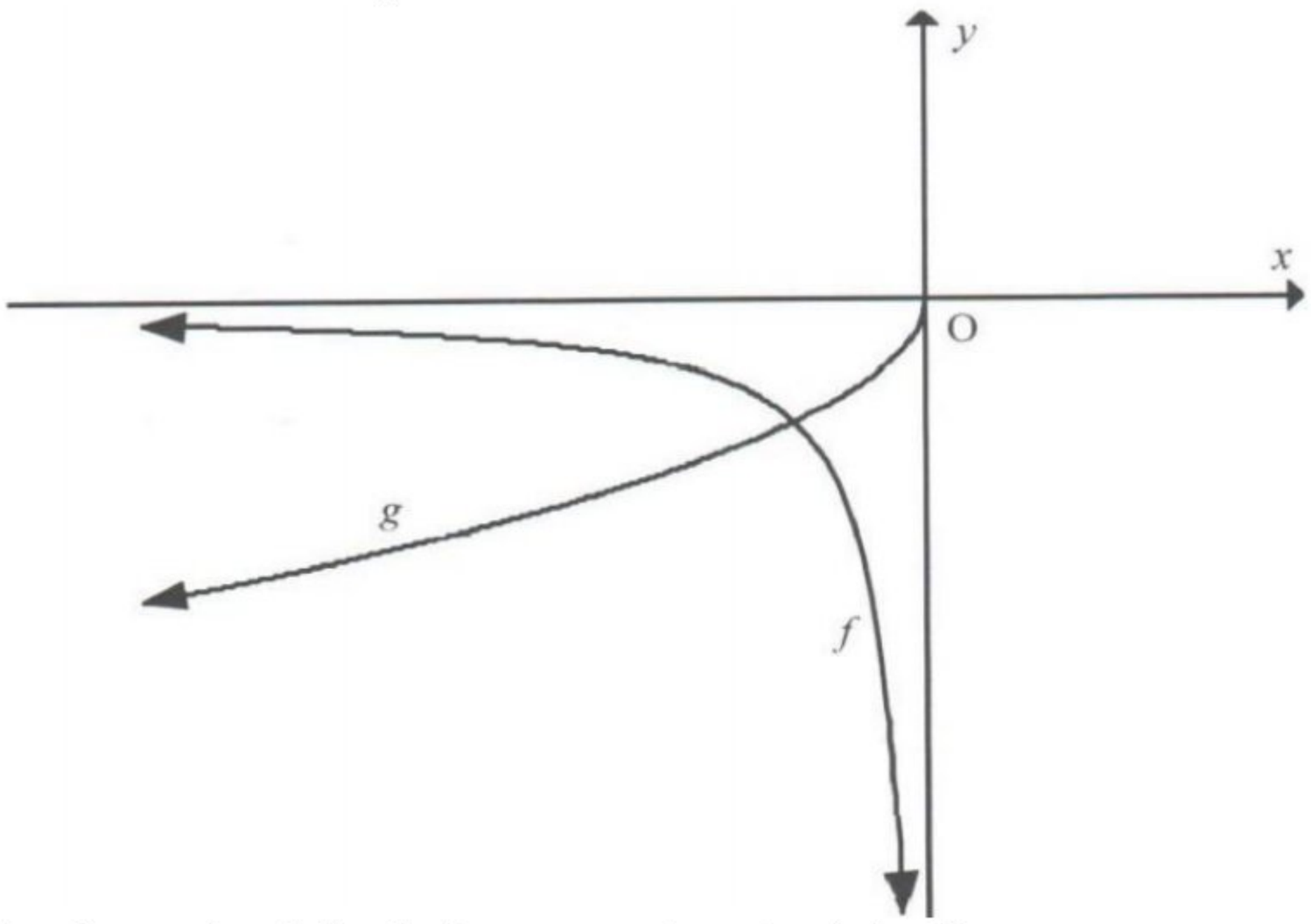
27.3 Sketch f^{-1}, the inverse of f, in your answer book. Indicate the intercept(s) with the axes and the coordinates of ONE other point. (3) **L2**

27.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$. (1) **L2**

28

DBE JUNE 2015

Given the graphs of $f(x)=\frac{1}{x}$ for $x<0$ and $g(x)=-\sqrt{-x}$ for $x\leq 0$.\



28.1 Prove that the graphs of f and g intersect at the point $(-1;-1)$. (4) **L2**

28.2 Determine the equation of g^{-1} in the form $y=...$ (3) **L2**

DBE/ MARCH 2016

29

Given $f(x)=\frac{1}{4}x^2,\ x\leq 0$

29.1 Determine the equation of f^{-1} in the form $f^{-1}(x)=...$ (3) **L2**

29.2 On the same system of axes, sketch the graphs of f and f^{-1}. Indicate clearly the intercepts with the axes, as well as another point on the graph of each of f and f^{-1}. (3) **L2**

29.3 Is f^{-1} a function? Give a reason for your answer. (2) **L1**

TOPIC	DIFFERENTIAL CALCULUS

GUIDELINES, SUMMARY NOTES, & STRATEGIES

TEACHING APPROACHES (CALCULUS)

1. FIRST PRINCIPLES:

The learners:

- ✓ Need to understand what is meant by determining the gradient from first principles and know the first principles formula.
- ✓ must be able to copy the first principle formula from the formula sheet correctly.
- ✓ Be able to simplify the first principles expression (It seems as if learners handled this question better when they determine $f(x+h)$ separately and then bring it back to the formula).
- ✓ Need to be mindful of the notation and apply it correctly when they simplify the first principle expression.
- ✓ At this stage, learners can also determine the equation of the tangent at a point.

2. RULES FOR DIFFERENTIATION

- ✓ The learners:
  - i. need to revise how to simplify surds, rational, irrational exponents.
  - ii. Must know how to simplify expressions before differentiation.
  - iii. Must know how to tell which variable they are required to differentiating with respect to.
- ✓ Must expose themselves to variety of questions having different notations including where a variable is given as constant.
- ✓ Following instructions is once more important, on how the answer should be provided whether with a + ve or - ve.
- ✓ Must always use of correct notation.

3. CUBIC FUNCTIONS $f(x) = ax^3 + bx^2 + cx + d$

The learners need to know and follow these steps when sketching a cubic function:

- ✓ Before learners can sketch a cubic function, they at least need to know the shape of their graph as guided by value of a where a could be $a > 0$ and $a < 0$.
- ✓ The learners must be able to Factories a third-degree polynomial using any other method to determine the ***x*-intercepts** (the x-intercepts are known as the: zero, roots, $f(x) = 0$. It would be an advantage if they can be able factories using a calculator.
- ✓ They must also be able to find the ***y*- intercept**, which is when $x = 0$, or given by the value of d.
- ✓ Learners must be able to use the first derivative to find the coordinates of the turning points, which are also known as the Stationery points or local minima and local maxima. In simple terms, this is finding $f'(x) = 0$, solve for x, and then find the corresponding y-values to give the coordinate of the turning point.
- ✓ Examiners often require learners to write the intercepts with the axes, stationary points and points of inflection in coordinate form $(a\ ;\ b)$. Make sure that the learners are aware of this.

4. INTERPRETATION OF A CUBIC FUNCTION:

The learners must be able to:

- ✓ Tell what the domain is, that $x \in R$
- ✓ Understand the relationship between the graph of a function and the graph of its derivative is important in that it explains to the learners why the second derivative is zero at a point of inflection.
- ✓ Understand that the point of inflection is determined by equating the second derivative to zero and solving for x. An alternative method is to add up the x-coordinates of the turning points and divide by 2 (i.e. determining the midpoint of the two turning points).

- ✓ Tell for which values of x will $f(x)$ be concave up: $f''(x) > 0$ & Concave down: $f''(x) < 0$
- ✓ Tell where f is increasing or decreasing: increasing ($f'(x) > 0$, decrease $f'(x) < 0$).
- ✓ Determine the values of x, for which: $x.f(x) > 0$, $f'(x) > 0$, $f'(x).f(x) < 0$
- ✓ when will f have three real roots, two real roots or one real root?

5. OPTIMIZATION

The learners need to develop the conceptual understanding on Optimization

- **Calculus of motion**

✓ In this regard, the equation will be given.
✓ The learners need to know that, Velocity is the derivative of displacement, and
✓ Acceleration (2^{nd} derivative) is the derivative of velocity

- **Rates of change**

✓ Knowledge of formulae for the surface area and volume of right prisms is required from learners.
✓ A list of relevant formulae will only be provided for the surface area and volume of cones, spheres and pyramids. Learners must select the correct one to use.

REVISION QUESTIONS

1.

1.1 Determine the derivative of f, using the first principle, if $f(x) = x - 2$. (5) **L1**

1.2 Given $f(x) = x^2 + 2x$, determine $f'(x)$ from first principles. (5) **L1**

1.3 Given $f(x) = -\frac{2}{x}$, determine $f'(x)$ from first principles. (5) **L2**

1.4 Determine the derivative of f, using the first principle, if $f(x) = -2x^3$. (5) **L2**

1.5 Determine $f'(x)$ from first principles if it is given that $f(x) = ax^2 + bx + c$. (5) **L2**

1.6 Determine $f'(x)$ from first principles if it is given that $f(x) = -3$. (3) **L1**

1.7 Determine, from the first principles, the equation of the tangent to $f(x) = 1 - 3x^2$ at $x = 5$. (8) **L2**

1.8 Determine $f'(x)$ from first principles if it is given that $f(x) = \frac{4}{3}x^3$ and hence find $f'(-1)$. (6) **L2**

2 **Rules: Determine:**

2.1 $D_t\left(\frac{t^2 - 3t^{-2}}{2\sqrt{t^3}}\right)$ (4) **L3**

2.2 $\frac{dy}{dx}$ **if** $\left(\frac{2x^2 - 7x + 3}{2x - 1}\right)$ (2) **L2**

2.3 the derivative of f if ; $f(x) = \sum_{r=0}^{3} rx^{3-r}$ (4) **L3**

2.4 $\frac{dy}{ds}$ **if** $y = -\frac{\sqrt{s}}{2} - \frac{1}{s^2}$ (4) **L2**

2.5 $p'(x)$ **if** $p(x) = \frac{-6 - 7x + 3x^2}{-3 + x}$ (3) **L2**

2.6 $\frac{dy}{dx}$ **if** $y = 3x^3\beta^4 - \beta^5 x$ (2) **L2**

2.7 $D_x\left[\frac{-6\sqrt[3]{x}+2}{x^4}\right]$ (4) **L3**

2.8 Determine $\frac{dy}{dx}$ if $yx-y+2x=2x^2;\quad x\neq 1$ (4) **L3**

2.9 Determine if $y=6-5x$

2.9.1 $\frac{dy}{dx}$ (1) **L1**

2.9.2 $\frac{dx}{dy}$ (3) **L1**

2.9.3 Prove that: $\frac{dy}{dx}=1\div\frac{dx}{dy}$ (2) **L2**

3 3.1 Determine the equation of the tangent to the curve $g(x)=\sqrt{x^3}$ at $x=4$ (4) **L2**

3.2 Determine the point on the curve of $y=4x^2+3x$ where the gradient is -1 (3) **L2**

3.3 Prove that $x+y=0$ is a tangent to the curve

$y=x^3-10x^2+24x$ (5) **L4**

3.4 The line $y=2x+3$ is a tangent to the curve , $y=x^2+ax+b$ at the point $(2;7)$.

Calculate the values of a and b. (5) **L3**

3.5 If g is a linear function with $g(1)=5$ and $g'(3)=2$, determine the equation of g in the form $y=\ldots..$ (3) **L2**

3.6 The function $f(x)=x^3+bx^2+cx-4$ has a point of inflection at (2 ; 4). Calculate the values of b and c. (7) **L3**

3.7 The curve with equation $y=x+\frac{12}{x}$ passes through the point A (2;b) . Determine the equation of the line perpendicular to the tangent to the curve at A . (4) **L3**

3.8 Given $h(x)=-x^3+\frac{3}{2}x^2+6x$ and $g(x)=-12x$. P and Q (2;10) are the turning points of h. The graph of h passes through the origin.

3.8.1 Calculate the average gradient of h between P and Q, if it is given that $x=-1$ at P. (4) **L2**

3.8.2 Show that the concavity of h changes at $x=\frac{1}{2}$. (3) **L2**

3.8.3 Explain the significance of change in question 3.8.2 with respect to h. (1) **L1**

3.8.4 Determine the value of x, given $x<0$,at which the tangent to h is parallel to g. (4) **L2**

4 Given the polynomial: $f(x)=(x+2)(x-3)(x-3)$

4.1 Calculate the coordinates of the turning points of the graph of f (5) **L2**

4.2 Sketch the graph of *cl*early indicating the axes and the turning points. (4) **L2**

4.3 For which value(s) of x will $x.f(x)<0$. (2) **L2**

5 For a certain function, f, the first derivative is given as $f'(x)=3x^2+8x-3$

5.1 Calculate the x – coordinates of the turning points of f. (3) **L2**

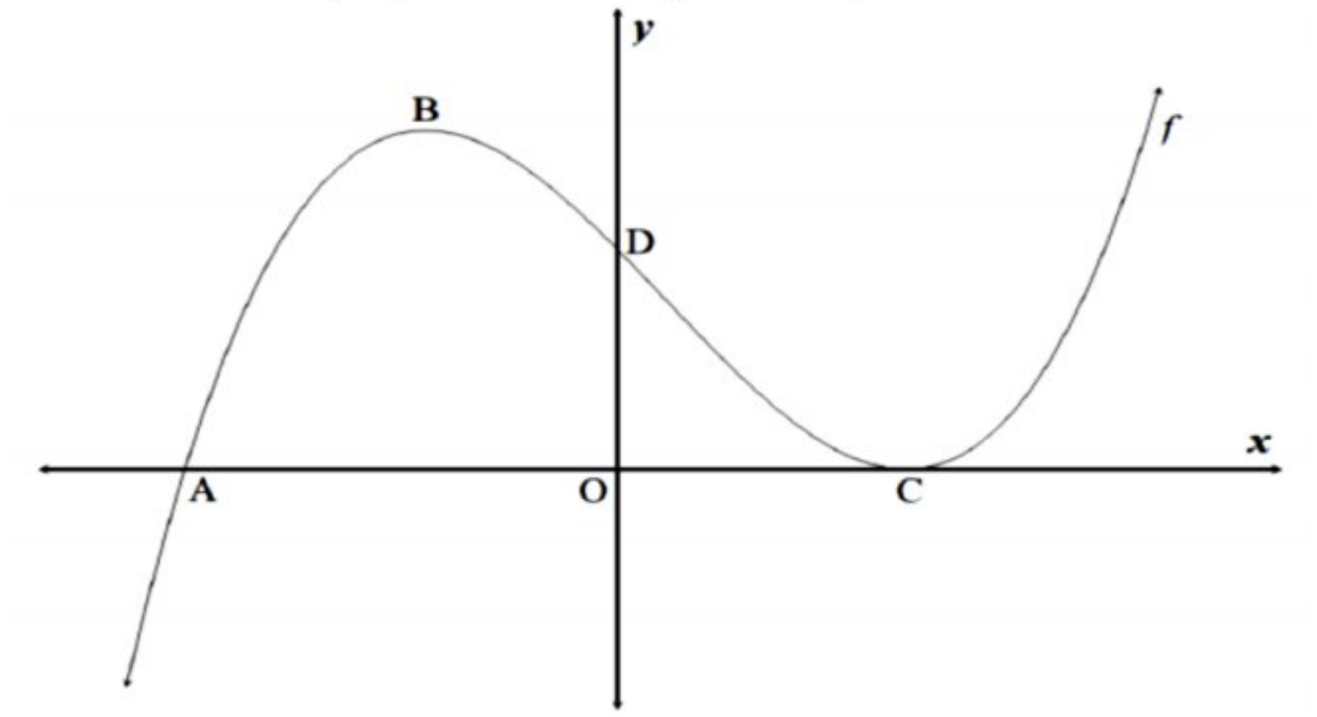
5.2 Draw a rough sketch of f. (2) **L2**

5.3 Determine the values of x for which f is increasing. (2) **L2**

5.4 If it is further given that $f(x)=ax^3+bx^2+cx+d$ and $f(0)=-18$,determine the equation of f. (5) **L3**

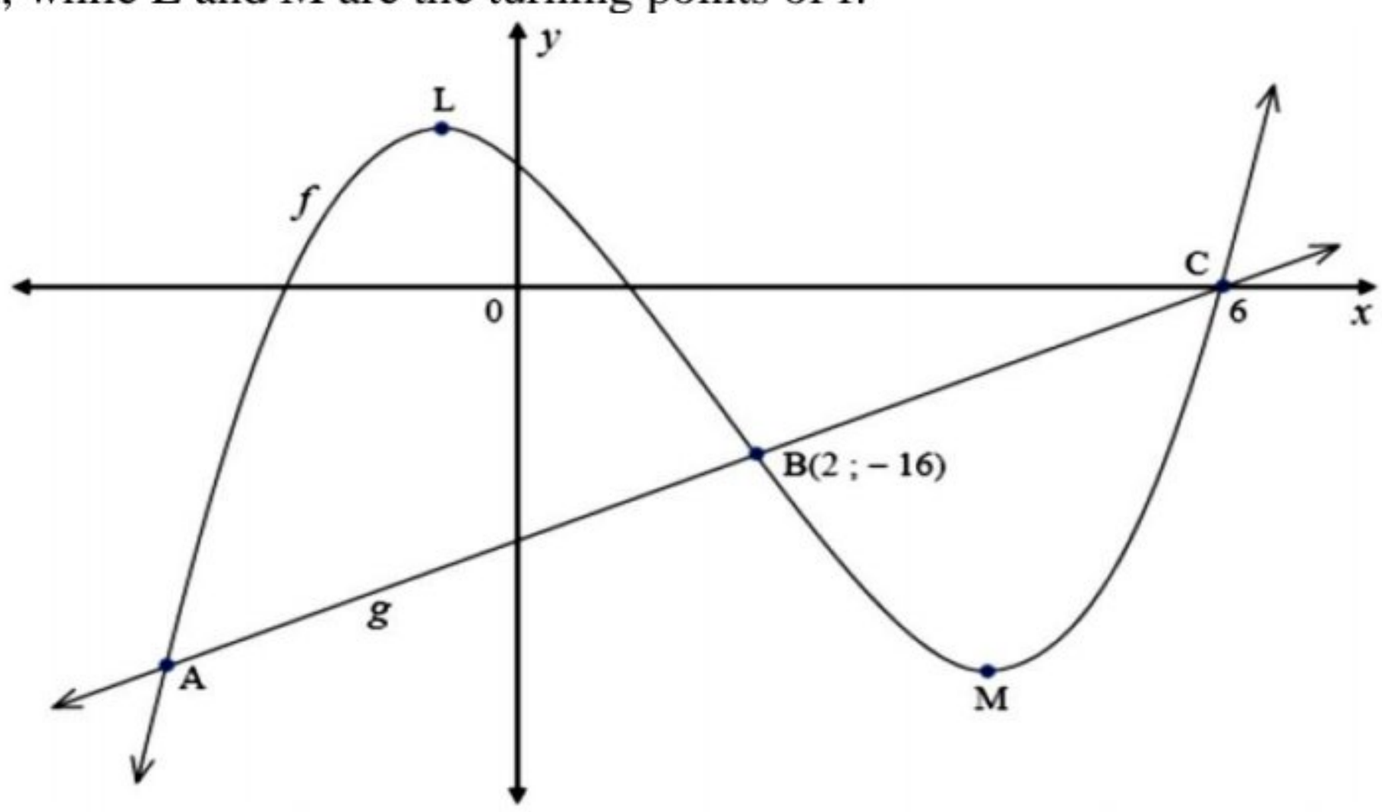
30

6 The graph of $f(x) = x^3 - x^2 - 8x + 12$ is sketched below. B and C are turning points, A and C are the x-intercepts, and D is the y-intercept.



6.1	Write down the coordinates of D	(2)	**L1**
6.2	Determine the coordinates of the turning points of f	(5)	**L2**
6.3	Show that $f(x)$ has a point of inflection at $x = \frac{1}{3}$.	(4)	**L2**
6.4	If $g(x) = f(-x) + 1$ write down the coordinates of C', the image of C.	(2)	**L3**
6.5	Write down the value(s) of k for which $f(x) = k$ will have		
	6.5.1 two unequal real roots	(2)	**L2**
	6.5.2 one of the roots equal to 0	(2)	**L2**

7 The sketch below represents the functions $f(x) = x^3 + bx^2 + cx + d$ and $g(x) = ax + q$. The points A, B(2;-16) and C, are points where the two graphs intersect. C(6;0) is an x-intercept of f, while L and M are the turning points of f.



Show that $b = -5$, $c = -8$ *and* $d = 12$ if it is given that

7.1	$f'(x) = 3x^2 - 10x - 8$	(4)	**L3**
7.2	Determine the coordinates of the turning points, L and M, of f.	(5)	**L2**
7.3	Determine the equation of g	(3)	**L2**
	If it is further given that the coordinates of point A are $(x;-36)$, determine the length of		
7.4	AM.	(3)	**L2**
7.5	For which value(s) of x….		
	7.5.1 is the graph of f increasing?	(2)	**L1**
	7.5.2 is the graph of f concave down?	(2)	**L2**

8 The following information is about a cubic polynomial $y = f(x)$

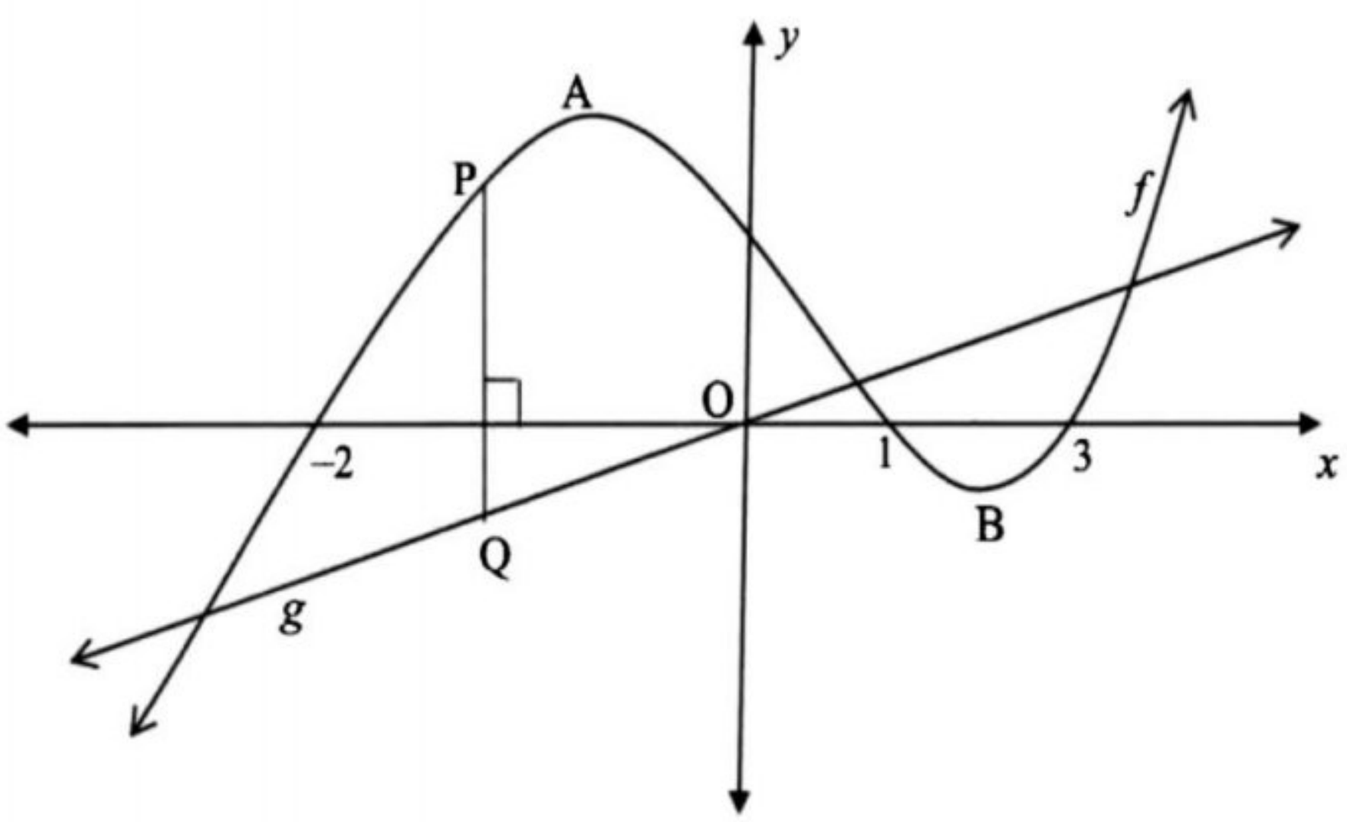
- $f(-1)=0$
- $f(5)=0$
- $f(0)=-2$
- $f'(-1)=f'(3)=0$
- $f(3)=6$
- If $x<-1$ then $f'(x)>0$
- If $x>3$ then $f'(x)>0$

8.1 Sketch a neat graph of $f(x)$ showing all intercepts and turning points. (5) **L3**

8.2 Use the graph to find the x-value of the point of inflection (2) **L2**

8.3 For which values of x is the graph decreasing? (2) **L1**

9 In the diagram, the graphs of $f(x)=x^3-bx^2-cx+d$ and $g(x)=2x$ are drawn. The graph passes through the x-axis at $x=-2, x=1$ and $x=3$. A and B are the turning points of f. P is a point on f and Q is a point on g such that PQ is perpendicular to the x-axis. $x_p<0$



9.1 Show that $f(x)=x^3-2x^2-5x+6$ (2) **L2**

9.2 Calculate the x-coordinate of B. (3) **L2**

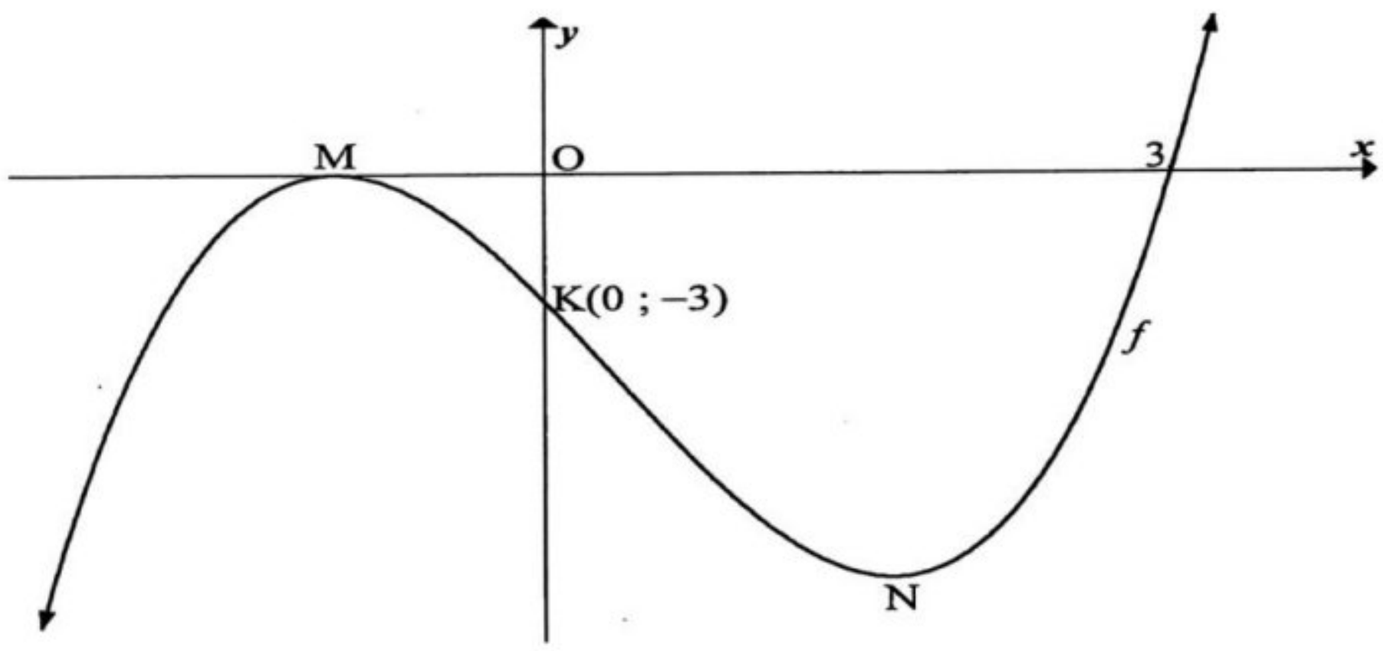
9.3 A tangent to f has gradient of -1. Explain why the point of contact of the tangent and the graph of f lies between A and B. (1) **L1**

9.4 For which values of x will f be concave up? (2) **L2**

9.5 Determine the maximum length of the line PQ. (5) **L3**

10

Sketched alongside is the graph of $f(x)=x^3+ax^2+bx+c$. The x-intercepts of f are at $(3;0)$ and M, where M lies on the negative x-axis.
$K(0;-3)$ is the y-intercept of f.
M and N are the turning point of f



10.1 Show that the equation of f $f(x)=x^3-x^2-5x-3$ (5) **L3**

10.2 Calculate the coordinates on N. (5) **L2**

10.3 For which values of x will:

10.3.1 $f(x) \leq 0$ (2) **L1**

10.3.2 f is increasing (2) **L1**

10.3.3 f is concave up (3) **L2**

10.4 Determine the maximum vertical distance between the graphs of f and f' in the interval $-1 < x < 0$. (5) **L4**

11 A cubic function $h(x) = -2x^3 + bx^2 + cx + d$ cuts the x-axis at (–3 ; 0); $\left(-\frac{3}{2} ; 0\right)$ and (1 ; 0).

11.1 Show that $h(x) = -2x^3 - 7x^2 + 9$. (3) **L2**

11.2 Calculate the x-coordinates of the turning points of h. (3) **L2**

11.3 Determine the value(s) of x for which h will be decreasing. (3) **L1**

11.4 For which value(s) of x will there be a tangent to the curve of h that is parallel to the line $y - 4x = 7$. (4) **L3**

12 **If** g is a cubic function with

- g(3)=g'(3)=0
- g(0)=27
- g"(x)>0 when x<3 and g"(x)<0 when x>3,

Draw a sketch of g indicating ALL relevant points. (3) **L3**

13 Given $f(x) = 3x^3$

13.1 **S**olve $f(x) = f'(x)$ (3) **L2**

13.2 The graphs f, f' and f'' all pass through the point $(0;0)$

13.2.1 For which of the graphs will $(0;0)$ be a stationary point? (1) **L1**

13.2.2 Explain the difference, if any, in the stationary points referred to in QUESTION13.2.1 (2) **L2**

13.3 Determine the vertical distance between the graphs of f' and f'' at $x = 1$ (2) **L2**

13.4 For which value(s) of x is $f(x) - f'(x) < 0$ (4) **L3**

14 The graph of $f(x) = ax^3 + bx^2 + cx + d$ has two turning points.

The following information about f is also given:

- $f(2) = 0$
- The x-axis is a tangent to the graph of f at $x = -1$
- $f'(1) = 0$
- $f'\left(\frac{1}{2}\right) > 0$

Without calculating the equation of f, use this information to draw a sketch graph of f, only indicating the x-coordinates of the x-intercepts and turning points. (4) **L3**

15 A rectangular box of cereal has a height of $(5 - x)$ units.

The expression for the volume (V) of the box is given by

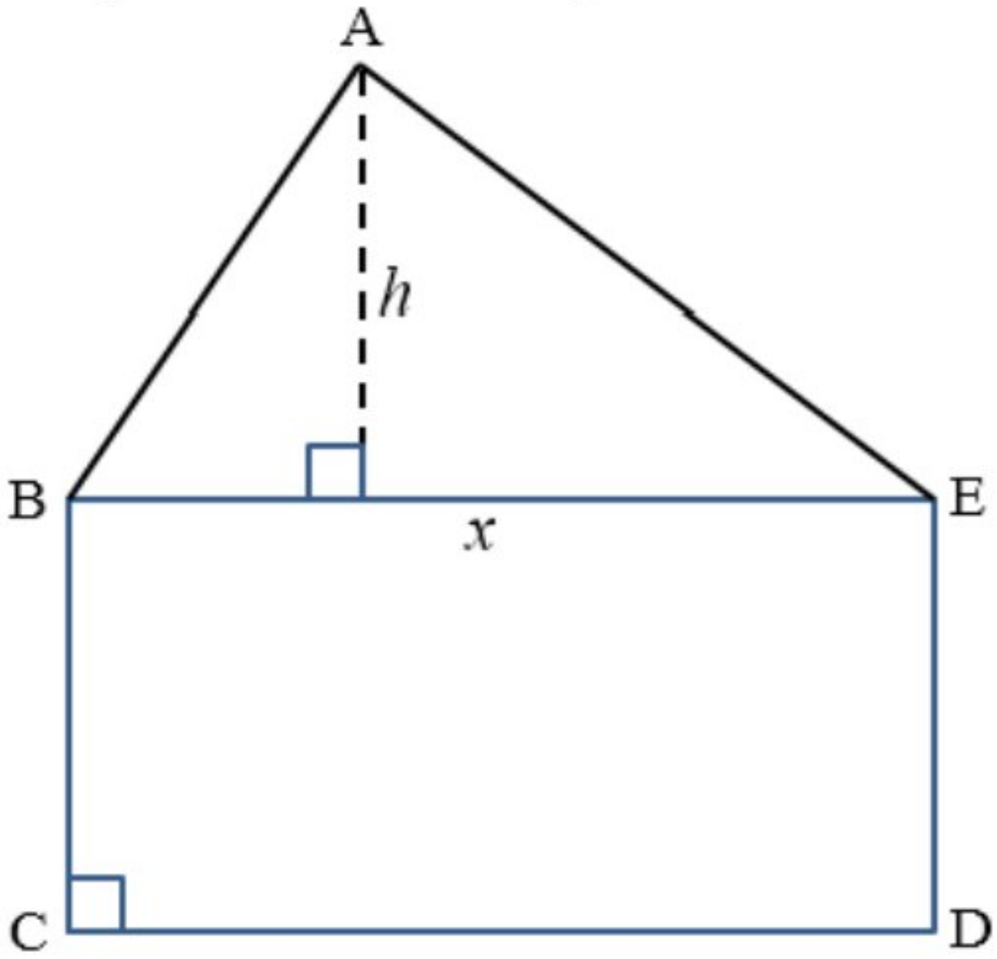
$V(x) = x^3 - 8x^2 + 5x + 50$

If the height of the cereal box is $(5 - x)$ units, determine the area of the base of the box

15.1 in terms of x. (3) **L3**

15.2 Calculate the value of x for which the volume of the box will be at maximum. (5) **L2**

16 It the diagram below, Triangle ABC has a base of x meters. The base and the perpendicular height off the triangle add up to 10 meters. The triangle is mounted on a rectangle BCDE which has a perimeter of 32 meters.

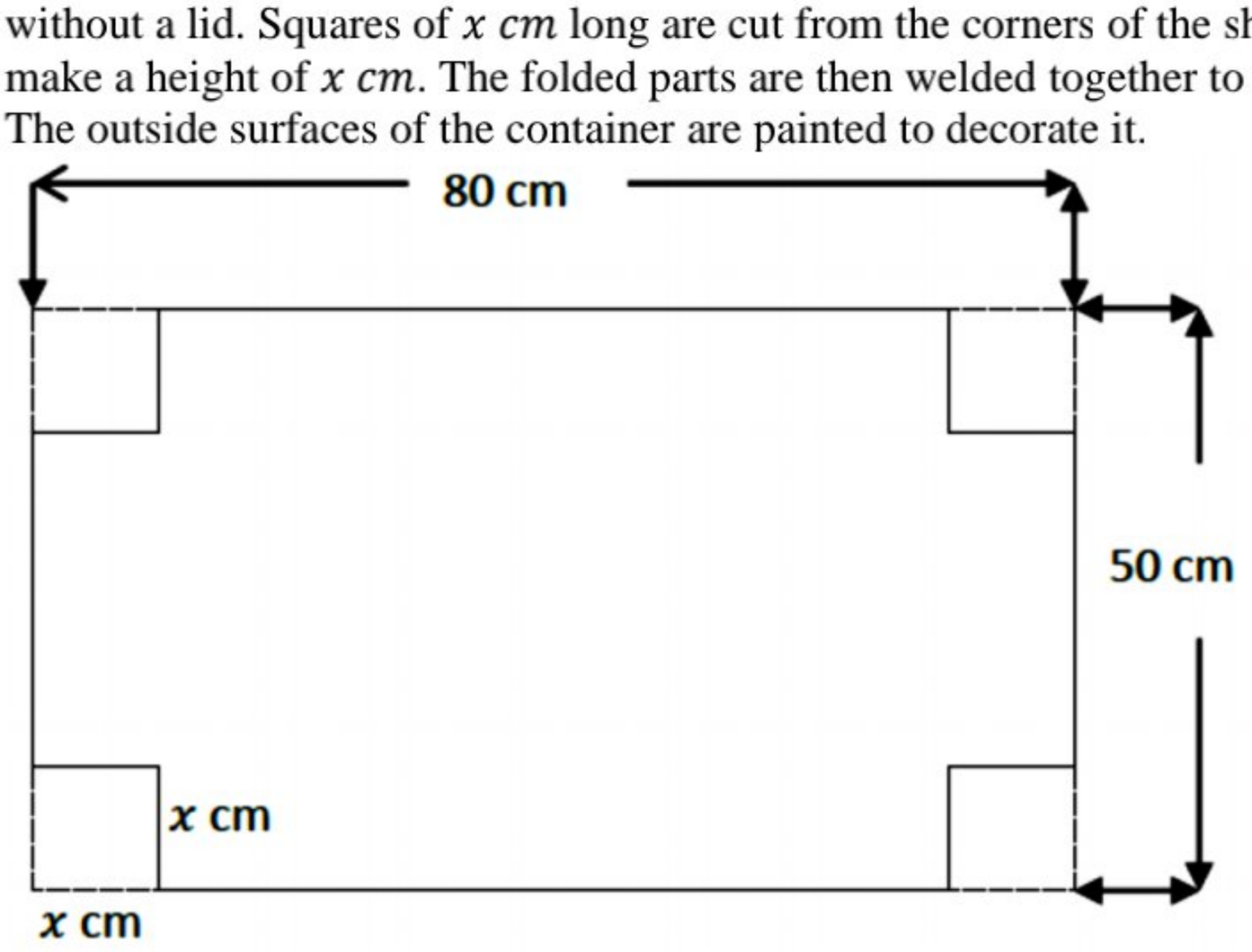


16.1 Show that the new area of the figure ABCDE is equal to $-\frac{3}{2}x^2 + 21x$ (5) **L2**

16.2 Determine the value of x for which ABCDE has a maximum area. (3) **L2**

16.3 Hence, determine the maximum area of ABCDE. (2) **L2**

17 A piece of metal sheet, 80cm long and 50cm wide, is used to make a rectangular container without a lid. Squares of $x\ cm$ long are cut from the corners of the sheet for proper folding to make a height of $x\ cm$. The folded parts are then welded together to close the corners properly. The outside surfaces of the container are painted to decorate it.



17.1 Prove that the volume of the container is given by $V(x) = 4x^3 - 260x^2 + 4000x$ (3) **L4**

17.2 For which values of x will the volume be a maximum. (2) **L2**

17.3 Calculate the surface area that is to be painted. (3) **L2**

18 After flying a short distance an insect came to rest on a wall. Thereafter the insect started crawling on the wall. The path that the insect crawled **can be** described by

$h(t) = (t-6)(-2t^2+3t-6)$, where h is the height (in cm) above the floor and t is the time (in minute) since the insect started crawling .

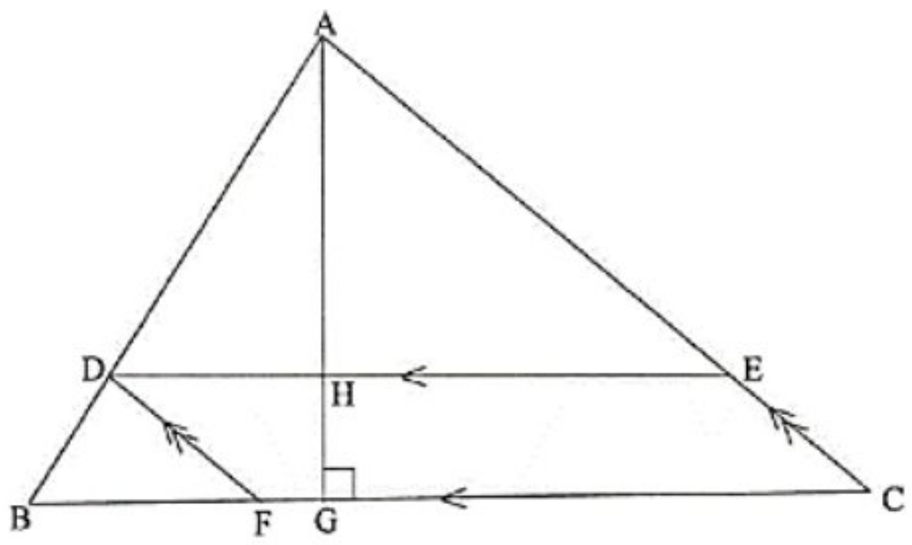
18.1 At what height above the floor did the insect start to crawl? (1) **L1**

18.2 How many times did the insect reach the floor? (3) **L2**

18.3 Determine the maximum height that the insect reached above the floor (4) **L2**

19 In ΔABC:

- D is a point on AB , E is a point on AC and F is a point on BC such that DECF is a parallelogram.
- BF:FC = 2:3
- The perpendicular height AG is drawn intersecting DE at H.
- AG =t units
- $BC = (5-t)$ units.



19.1 Write down AH: HG (1) **L1**

19.2 Calculate t if the area of the parallelogram is a maximum.
Note: (Area of a parallelogram = base × perpendicular height) (5) **L3**

Study and Master Grade12

20 Prove that $y = x^3+2x^2+9x+8$ has no maximum or minimum values. (5) **L3**

21 A metal ball is thrown vertically upwards at a velocity of $25\,m/s$, and its height, h meters above the ground after t seconds, is given by the formula $h = 25t-5t^2$.

21.1 Calculate $\frac{dh}{dt}$. (1) **L1**

21.2 Now calculate the velocity of the metal ball at $t = 1{,}5$. (2) **L1**

21.3 What is the average velocity between the points $(1;20)$ and $(2{,}5;31{,}25)$? (2) **L2**

21.4 After how many seconds does the metal ball reach its maximum height, and what is its maximum height? How fast is the metal ball travelling at this point? (4) **L3**

22 The relationship between the vertical displacement (y) and horizontal displacement (x) of a rocket that is launched is $y = 8x^2 \sin 60^\circ - x\cos 60^\circ$. Both vertical and horizontal displacements are measured in kilometers, Calculate:

22.1 The maximum height the projectile can reach, correct to the nearest meter. (5) **L3**

22.2 The horizontal displacement the projectile has undergone the moment it strike the earth, correct to the nearest km. (4) **L3**

22.3 The size of the angle at which the projectile is launched. (3) **L3**

TOPIC | **FINANCIAL MATHEMATICS**

GUIDELINES, SUMMARY NOTES, & STRATEGIES

SIMPLE INTEREST AND COMPOUND INTEREST (A>P)

- When **simple interest** is used, interest is calculated as a percentage of the original amount invested or borrowed.

$$A = P(1+in)$$

- When **compound interest** is used, interest is added after every period and the interest for the next period is calculated as a percentage of the new total.

$$A = P(1+i)^n$$

DEPRECIATION (A<P)

- The formulae that are used to do depreciation calculations are:

$A = P(1-in)$ Straight line depreciation

$A = P(1-i)^n$ Reducing balance depreciation

DIFFERENT COMPOUND PERIODS

- Interest is not always calculated annually (yearly), but often based on a different period such as:
- **Monthly** (Divide interest rate by 12 and multiply years by 12)
- **Quarterly** (Divide the interest by 4 and multiply years by 4)
- **Half-year or Semi-annually or bi-annually** (Divide interest rate by 2 and multiply years by 2)

EFFECTIVE AND NOMINAL INTEREST RATES

- To determine the annual effective rate, we use the formula:

$$1+i_{eff} = \left(1+\frac{i_{nom}}{m}\right)^m$$

- When working with different compounding periods use the formula

$$(1+i_{new})^m = (1+i_{nom})^n$$

ANNUITIES

- An annuity is a series of equal payments made at regular time intervals
- The annuity formulae are used under the following conditions:
  - ✓ All payments are equal
  - ✓ The payments are made at regular intervals
  - ✓ The interest rate remains fixed and
  - ✓ the compounding period for interest is the same as the payment intervals

THE FUTURE VALUE FORMULA

- We can use the following formula to calculate the future value of an annuity:

$$F = \frac{x\left[(1+i)^n - 1\right]}{i}$$

F is the future value.
x is the payment.
i is the interest rate per interval.
n is the number of payments.

THE PRESENT VALUE FORMULA

- We can use the following formula to calculate the present value of an annuity:

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

P is the present value.
x is the payment.
i is the interest rate per interval.
n is the number of payments.

SINKING FUND

(a) $A = P(1-i)^n$ (scrap value of old asset)

(b) $A = P(1+i)^n$ (cost of new asset)

(c) Sinking fund = new – old

(d) Calculate x

(e) Withdrawals (calculate x_{new}) – treat it separately and add it back

THE OUTSTANDING BALANCE ON A LOAN

Outstanding Balance = Loan with interest to date – Repayments with interest to date

$OB = P(1+i)^n - \frac{x\left[(1+i)^n - 1\right]}{i}$ **Or** $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

Note:

- When using the P formula, use the remaining number of payments.
- When using OB=A-F, use n as number of payments made.

DELAYED/ DEFERRED ANNUITIES

- When the first payment of a loan is made more than one period after the loan was received, this payment is referred to as a ***deferred annuity.***
- Apply the compound interest to the loan to move it to the same point on the timeline as the present value of the annuity

THE LAST/FINAL PAYMENT

Last payment = Outstanding balance after the last full payment multiplied by $(1+i)^1$

MISSED PAYMENTS

To calculate the new payment:

- We calculate the outstanding balance immediately after the last payment made.

We then apply the compound interest to this outstanding balance, till one period before payments resume. The result is the present value of the new annuity consisting of all the remaining payments.

REVISION QUESTION

1 A new cell phone was purchased for R7 200. Determine the depreciation value after 3 years if the cell phone depreciates at 25% p.a. on a reducing balance method. (3) **L2**

2 **Outstanding Balance**

Sipho negotiates a loan of R300 000 with a bank which has to be repaid by means of monthly payments of R5 000 and a final payment which is less than R5 000. The repayments start one month after the granting of the loan. Interest is fixed at 18% p.a., compounded monthly.

2.1 Determine the number of payments required to settle the loan. (6) **L3**
2.2 Calculate the balance outstanding after Sipho has paid the last R5 000. (5) **L2**
2.3 Calculate the value of the final payment made by Sipho to settle the loan. (2) **L2**
2.4 Calculate the total amount that Sipho paid to the bank. (2) **L2**

3 **Outstanding Balance**

James buys a house and takes out a loan of R2 million. He repays the loan over fifteen years. The interest charged on the outstanding balance of the loan is 8,5% p.a., compounded monthly.

3.1 Calculate his monthly payment of the loan. (4) **L2**
3.2 What is the outstanding balance on the loan at the end of five years. (3) **L2**
3.3 Determine the amount of money paid on the loan at the end of the first five years. (2) **L1**
3.4 What is the interest paid on the loan during the first five years. (4) **L3**

4 R1 430,77 was invested in a fund paying i% p.a. compounded monthly. After 18 months the fund had a value of R1 711,41. Calculate i. (4) **L2**

5 **Delayed/Deferred Annuities**

A father decided to buy a house for his family for R800 000. He agreed to pay monthly instalments of R10 000 on a loan which incurred interest at a rate of 14% p.a. compounded monthly. The first payment was made at the end of the first month.

5.1 Show that the loan would be paid off in 234 months. (4) **L3**

5.2 Suppose the father encountered unexpected expenses and was unable to pay any instalments at the end of the 120^{th}, 121^{st}, 122^{nd} and 123^{rd} months. At the end of the 124^{th} month he increased his payment so as to still pay off the loan in 234 months by 111 equal monthly payments. Calculate the value of this new instalment. (7) **L3**

6 **Sinking Fund**

A farmer buys a tractor for R2,2 million.

6.1 Determine the book value of a tractor at the end of 5 years if the depreciation is calculated at 14% p.a. on a reducing balance method. (3) **L1**

6.2 Determine the expected cost of buying a new tractor in five years' time if the average rate of inflation is expected to be 6% p.a. (3) **L1**

6.3 The farmer decides to replace the old tractor in five years' time. He will trade in the old tractor. Calculate the sinking fund. (3) **L1**

6.4 Calculate the monthly payment into the sinking fund if the payments commenced one month after he bought the tractor if the interest rate is 7% per annum compounded monthly. (4) **L2**

6.5 Suppose that at the end of each year he withdraws R5 000 from his account to pay for the maintenance of the tractor. Determine the new monthly deposit. (4) **L3**

7 **Sinking Fund**

A business buys a machine that costs R120 000. The value of the machine depreciates at 9% per annum according to the reducing balance method.

7.1 Determine the scrap value of the machine at the end of 5 years. (3) **L1**

7.2 After 5 years the machine needs to be replaced. During this time the inflation remained constant at 7% per annum. Determine the cost of the new machine at the end of 5 years. (3) **L1**

7.3 The business estimates that it will need R90 000 by the end of five years. A sinking fund for R90 000 into which equal monthly instalments must be paid, is set up. Interest on this is 8,5% p.a. compounded monthly. The first payment will be made immediately, and the last payment will be made at the end of the 5-year period. Calculate the monthly deposits. (5) **L2**

8 **Investment**

At the beginning of October 2018, Lungile opened a savings account with a single deposit of R10 000. She then made 24 monthly deposits of R1600 at the end of every month starting at the end of October 2018. She earns 15% p.a. interest compounded monthly in her account. Calculate the amount that should be in her savings account immediately after she makes the last deposit. (5) **L3**

9 How many years will it take for an investment to double in value, if it earns interest at a rate of 8,5% p.a., compounded quarterly? (5) **L3**

DBE-MAY/ JUNE 2022 QUESTION 7

10 **Sinking Fund**

A company purchased machinery for R500 000. After 5 years, the machinery was sold for R180 000 and new machinery was bought.

10.1 Calculate the rate of depreciation of the old machinery over the 5 years, using the reducing balance method. (4) **L2**

10.2 The rate of inflation for the cost of the new machinery is 6,3% p.a. over the 5 years. What will the new machinery cost at the end of 5 years. (2) **L1**

10.3 The company set up a sinking fund and made the first payment into this fund on the day the old machinery was bought. The last payment was made three months before the new machinery was purchased at the at the end of the 5 years. The interest earned on the sinking fund was 10,25% p.a., compounded monthly. The money from the sinking fund and the R180 000 from the sale of the old machinery was used to pay for the new machinery.
Calculate the monthly payment into the sinking fund. (5) **L3**

DBEMARCH 2010 QUESTION 9

11 **Investment**

Lindiwe receives a bursary of R80 000 for her studies at university. She invests the money at a rate of 13,75% p.a. compounded yearly. She decides to withdraw R25 000 at the end of each year for her studies, starting at the end of the first year.
Determine for how many full years will this investment finance her studies. (4) **L3**

DBE NOVEMBER 2012 QUESTION 7

12 **Investment**

Lorraine receives an amount of R900 000 upon her retirement. She invests this amount immediately at an interest rate of 10,5% per annum, compounded monthly. She needs an amount of R18 000 per month to maintain her current lifestyle. She plans to withdraw the first amount at the end of the first month.
For how many months will she be able to live from her investment? (6) **L3**

March 2015 Question 7

13 For each of the three years from 2010 to 2012 the population of town X decreased by 8% per year and the population of town Y increased by 12% per year. At the end of 2012, the populations of these two towns were equal.
Determine the ratio of the population of town X (call it P_x) to the population of town Y (call it P_Y) at the beginning of 2010. (4) **L4**

DBE -MARCH 2017 QUESTION 6

14 Lerato wishes to apply for a home loan. The bank charges interest at 11% per annum, compounded monthly. She can afford a monthly instalment of R9 000 and wants to repay the loan over a period of 15 years. She will make the first monthly repayment one month after the loan is granted. Calculate, to the nearest thousand rand, the maximum amount that Lerato can borrow from the bank. (5) **L2**

DGC SEPTEMBER 2021 QUESTION 5

15 **Outstanding Balance**

Khwezi is planning to buy her first home. The bank will allow her to use a maximum of 30% of her monthly salary to repay the bond. She earns R18 480 per month. Suppose, at the end of each month, Khwezi repays the maximum amount allowed by the bank. The first instalmet is made one month after the loan is granted.

15.1 How much money does Khwezi borrow if she takes 25 years to repay the loan at a rate of 8% p.a. compounded monthly? (4) **L2**

15.2 Calculate the outstanding balance after 20 years of paying back the loan. (3) **L2**

RUSTENBURG GIRLS' HIGH SCHOOL QUESTION3

16 **Delayed/Deferred Annuities**

Lynne purchases a new car for R350 000. They take out a 6-year loan on 1 January 2019. The monthly instalments are paid at the end of every month. Interest is fixed at 18% p.a. compounded monthly.

16.1 Calculate the monthly repayment. (4) **L2**

16.2 Due to financial difficulty, Lynne misses the 40^{th}, 41^{st} and 42^{nd} payments. Determine the balance outstanding at the end of the 42^{nd} month. (4) **L3**

16.3 If Lynne's monthly repayment is R10 000. How many months will it take her to pay back the rest of the loan. (4) **L3**

IEB - MAY 2022, QUESTION 3

17 **Outstanding Balance**

If Khanya took a loan of R1 800 000 to purchase a house at the start of 2008 and the bank charged her interest at 9,25% per annum compounded monthly over a 30-year contract period:

17.1 Calculate her monthly repayments. (Assume that she received the loan immediately and that the first payment was made at the end of the first month). (4) **L2**

17.2 Calculate her outstanding balance at the end of 12 years. (5) **L2**

IEB - NOVEMBER 2022, QUESTION 6

18. You start a business and buy a delivery truck for R450 000.

18.1 **Sinking Fund**

If the inflation rate is 6% per annum, how much will a new delivery truck cost in five years' time? (2) **L1**

18.2 If you calculate depreciation at 20% per annum using the reducing-balance method, what will the value of your delivery truck be in five years' time? (2) **L1**

18.3 You set up a sinking fund for the next 5 years so that you can trade in your delivery truck and buy a new one by financing the difference with the sinking fund. The bank offers an interest rate of 9% per annum compounded monthly. How much should you put away at the end of each month so that you have enough to finance the difference between the new vehicle and trade-in value of your delivery truck at the end of 5 years? The last payment into the sinking fund is made at the end of the 5 years. (5) **L3**

IEB - NOVEMBER 2021, QUESTION 6

19. **Investment**

The details of an investment opportunity with a company called Phantom Investment House are given below:

- Monthly payments of R5 000 need to be made at the end of each month.
- The interest rate offered is 15% per annum compounded monthly.

You decide to invest with Phantom Investment House with the terms given above.

19.1 What will the value of your investment be at the end of 3 years? (4) **L2**

19.2 What are the minimum number of months required for your investment to reach a value greater than R2 500 000? (5) **L3**

IEB - NOVEMBER 2021, QUESTION 7

20. **Mixed Payments**

You approach a bank for a home loan to the value of R850 000. This needs to be paid off monthly over a period of twenty years. The interest rate offered on the home loan is 9% per annum compounded monthly.

20.1 What are the minimum monthly payments required to pay off the loan over the 20 year-period? (4) **L2**

20.2 If the monthly payment is changed to R9 000 per month, then what will the balance outstanding on your loan be at the end of 12 years?

(This is directly after the payment is made at the end of 12 years.) (5) **L2**

IEB - MAY 2021, QUESTION 4

21. Simon wants to buy a car that costs **R345 000**

21.1 He opens a savings account and six months after opening the account, he makes his first deposit of R12 895 and continues depositing R12 895 at the end of every six-month period.
Interest is paid at 13% per annum compounded half-yearly.

21.1.1 How much money will be in Simon's account three years after opening the account? (3) **L2**

21.1.2 Ignoring the effect of inflation on the price of the car, determine how long it will take Simon to save the money needed to buy the car. (5) **L2**

21.1.3 If the effect of inflation is considered, determine the cost of the car eight years after opening the bank account. Inflation for this period is calculated at 3,5% per annum. (3) **L1**

21.2 Instead of the savings plan, he considers a second plan which is getting a loan for R345 000 under the following agreement:
• Interest is charged at 13% per annum compounded half-yearly.
• The loan must be settled in eight years.
Determine his minimum monthly repayment. (3) **L2**

IEB - MARCH 2019, QUESTION 8

22. (a) **Mixed Payments**

Greta purchased a car for R325 000, 9 years ago. The car depreciated on a reducing balance at a rate of 7% per annum over the 9 years. Determine what percentage this depreciation value would be of the car's original value. (3) **L2**

(b) Greta was granted a loan of R1 825 000 to purchase a house which was repayable over 25 years. The bank offered her an interest rate of 9,5% per annum compounded monthly. After 9 years of repaying her loan, she decided to purchase a second car. Instead of applying for vehicle finance she decided to access money from her home loan.

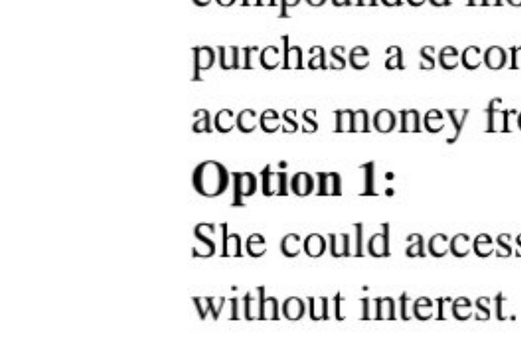
Option 1:
She could access 75% of each of her monthly repayments over the 9 years without interest.

Option 2:
She could access five(5) times the monetary **difference** between the original loan and the balance on the loan after 9 years, immediately after the 108th payment.
Which option would yield the highest amount available to purchase a new car?
Show all working. (10) **L3**

IEB - NOVEMBER 2019, QUESTION 2

23. **Outstanding Balance**

Busi opens a new credit card account that charges compound interest at 12,3% p.a. compounded weekly.



[<https://www.postoffice.co.uk/credit-card/platinum>

Interest:
• 12,3% p.a. compounded weekly

Note: For the calculations in this question, assume that the relevant years have 52 weeks each.

She purchases a computer for an amount of R12 349,00 immediately after activating her credit card.

23.1 Show that the balance owing on the credit card one week after the purchase will be R12 378,21 (to the nearest cent) (3) **L1**

23.2 Determine how long it will take Busi to pay off the money that she owes on her credit card if she repays R94,75 per week and she does not make any other purchases using this card. (4) **L2**

23.3 If the depreciation rate of her computer is 20% per annum on a straight-line basis, determine what its value will be after two years. (3) **L1**

23.4 She wants to sell the computer after two years. Will the depreciated value of the computer be sufficient to pay off the outstanding balance immediately after the 104th payment? Show all working. (6) **L3**

CRAWFORD- SEPTEMBER 2022, QUESTION 8

24 **Outstanding Balance**

Mr. Govender and Mrs. Williams have both taken loans from ABSA Bank. Repayments are made consistently after one month of granting of the loan. The details are tabulated below.

	Mr Govender	Mrs Williams
Loan Amount	R75 000	P
Monthly payment	R1 200	R 1 180,42
Interest rate	15% p.a. compounded monthly	13% p.a. compounded monthly
Balance outstanding after 62 months	K	K

If the interest rates and monthly repayments remain constant, determine:

24.1 K, the balance outstanding on Mr. Govender's loan after 62 months. (5) **L2**

24.2 Immediately after 62 months, Mrs. Williams received the following summary from ABSA Bank.

Outstanding Balance	R50 636
Total interest paid	R44 764

Using this information, calculate P, the initial amount that Mrs. Williams borrowed from ABSA Bank. (5) **L3**

REDHILL HIGH- SEPTEMBER 2022, QUESTION 2

25. **Investment**

Buhle decided to start saving before retirement. She makes payments of R10 000 monthly into an account yielding 7,72% p.a. compounded monthly, starting on 1 November 2016 with a final payment on 1 April 2026.

25.1 Calculate how much will be in the savings account immediately after the last deposit is made. (4) **L2**

25.2 At the end of the investment period Buhle re-invested the full amount for her to be able to draw a monthly pension from the fund. She re invested the money at an interest rate of 10% p.a. compounded monthly. If she draws an amount of R30 000 per month from the investment, for how many full months will she be able to receive R30 000. (4) **L3**

25.3 After withdrawing R30 000 for 20 months Buhle requires R 1 500 000. Determine whether she can access this amount of money from the annuity. (3) **L3**

TOPIC **PROBABILITY**

GUIDELINES, SUMMARY NOTES, & STRATEGIES

The probability scale: $0 \le P \le 1$. If P (an event) = 0, the event is impossible; If P (an event) = 1, the event is certain to happen.

The **definition of probability**: $P(E) = \frac{n(E)}{n(S)}$

Addition Rule for any 2 events A and B: **P (A or B) = P(A) + P(B)−P (A and B)**
Mutually exclusive events A and B: **P (A or B) = P(A) + P(B)**
NOTE: Since P (A and B) = 0
Independent events A and B: **P (A and B) = P(A) × P(B)**
The complementary rule: **P (not A) = 1−P(A)**
Venn-Diagram, Tree diagram and Contingency Table

The fundamental counting principle: If one operation can be done in m ways and a second operation can be done in n ways then the total possible number of different ways in which both operations can be done is $m \times n$.

- *Pin codes and Passwords*
- *Arrangements [(a) Different/Selection (b) Identical)]*
- *Re-arrangements*

REVISION QUESTIONS

1. **DBE NOVEMBER 2008 QUESTION 4**

A smoke detector system in a large warehouse uses two devices, A and B. If smoke is present, the probability that it will be detected by device A is 0,95. The probability that it will be detected by device B is 0,98 and the probability that it will be detected by both devices simultaneously is 0,94.

1.1 If smoke is present, what is the probability that it will be detected by device A or device B or both devices? (3) **L2**

1.2 What is the probability that the smoke will not be detected? (1) **L1**

2. **DBE MARCH 2009 QUESTION 4**

In a company there are three vacancies. The company had identified candidates to fill each post.

POST	CANDIDATES
Clerk	Craig, Luke and Tom
Sales representative	Ann, Sandile, Sizwe and Devon
Sales manager	John and Debby

2.1 In how many different ways can these three posts be filled? (3) **L1**

2.2 If it is certain that Craig will get the job as clerk, in how many different ways can the three posts be filled? (2) **L2**

3. **DBE MARCH 2012 QUESTION 7**

Three items from four different departments of a major chain store will be featured in a one-page newspaper advertisement. The page layout for the advertisement is shown in the diagram below where one item will be placed in each block.

A	B	C
D	E	F
G	H	I
J	K	L

3.1 In how many different ways can all these items be arranged in the advertisement? (2) **L1**

3.2 In how many different ways can these items be arranged if specific items are to be placed in blocks A, F and J? (2) **L2**

3.3 In how many different ways can these items be arranged in the advertisement if items from the same department are grouped together in the same row? (3) **L2**

4. Consider the word: CONDUCT

4.1 How many different arrangements are possible if all the letters are used? (2) **L1**

4.2 How many different arrangements can be made if the first letter is T and the fifth letter is C? (2) **L1**

4.3 How many different arrangements can be made if the letters O, N and D must follow each other, in any order? (3) **L2**

5. **NORTH WEST TRIAL 2021**

A certain school want to paint their computer room. A survey was conducted among the boys and girls who take CAT and IT as a subject to establish their preference of colour.

The results were recorded in the table below:

	Blue	**Green**	**Total**
Boys	19	31	50
Girls	41	35	76
Total	60	66	126

5.1 If a learner from this group is selected at random, what ie the probability that he/she choose green? (2) **L1**

5.2 Are the events of being a boy and choosing blue as a preferred colour independent? Use calculations, correct to two decimal places, to motivate your answer (4) L3

5.3 Are the events 'being a male' and 'staying at home' independent events. Motivate your answer with relevant calculations. (4) **L2**

6. Consider the word LONDI.

6.1 How many different 5-letter arrangements can be made using all the above letters? (2) **L1**

6.2 Determine the probability that the letters I and N will always be the first two letters of the arrangements in question 6.1. (3) **L2**

7. The letters of the word SENZIWA are randomly arranged into a new 'word', also consisting of seven letters. How many different arrangements are possible if:

7.1 Letters may be repeated (2) **L1**

7.2 Letters may not be repeated (2) **L1**

7.3 The arrangements must start with a vowel and end in a consonant and no repetition of letters is allowed. (4) **L2**

DBE NOVEMBER 2015 QUESTION 11.3

8. There are ***t*** orange balls and 2 yellow balls in the bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

Calculate how many orange balls are in the bag. (6) **L4**

9. **DBE NOVEMBER 2016 QUESTION 12**

The digits 1 to 7 are used to create a four-digit code to enter a locked room. How many different codes are possible if the digits may not be repeated and the code must be an even number bigger than 5000? (5) **L4**

10. **MARCH 2016 QUESTION 10**

Each passenger on a certain Banana Airways flight chose exactly one beverage from tea, coffee or fruit juice. The results are shown in the table below.

	MALE	**FEMALE**	**TOTAL**
Tea	20	40	60
Coffee	***b***	***c***	80
Fruit juice	***d***	***e***	20
TOTAL	60	100	***a***

10.1 Write down the value of ***a***. (1) **L1**

10.2 What is the probability that a randomly selected passenger is male? (2) **L1**

10.3 Given that the event of a passenger choosing coffee is independent of being a male, calculate the value of ***b***. (4) **L2**

11. **DBE JUNE 2016 QUESTION 11**

Five boys and four girls go to the movies. They are all seated next to each other in the same row.

11.1 One boy and girl are a couple and want to sit next to each other at any end of the row of friends. In how many different ways can the entire group be seated? (3) **L3**

11.2 If all the friends are seated randomly, calculate the probability that all the girls are seated next to each other. (3) **L3**

12. **DBE NOVEMBER 2016 QUESTION 11**

A survey was conducted among 100 boys and 60 girls to determine how many of them watched TV in the period during which examinations were written. Their responses are shown in the partially completed table below.

	WATCHED TV DURING EXAMINATIONS	**DID NOT WATCH TV DURING EXAMINATIONS**	**TOTALS**
Male	**80**	***a***	
Female	**48**	**12**	
Totals	***b***	**32**	**160**

12.1 Calculate the values of a and b. (2) **L1**

12.2 Are the events 'being a male' and 'did not watch TV during examinations' mutually exclusive? Give a reason for your answer. (2) **L2**

12.3 If a learner who participated in this survey is chosen at random, what is the probability that the learner:

12.3.1 Watched TV in the period during which the examinations were written? (2) **L2**

12.3.2 Is not a male and did not watch TV in the period during which examinations were written? (2) **L2**

13. **DBE JUNE 2018 QUESTION 10**

Ben, Nhlanhla, Owen, Derick and 6 other athletes take part in a 100 m race. Each athlete will be allocated a lane in which to run. The athletic track has 10 lanes.

13.1 In how many different ways can all the athletes be allocated a lane? (2) **L1**

13.2 Four athletes taking part in the event insist on being placed in lanes next to each other. In how many different ways can the lanes be allocated to the athletes now? (3) **L2**

13.3 If lanes are randomly allocated to athletes, determine the probability that Ben will be placed in lane 1, Nhlanhla in lane 3, Owen in lane 5 and Derick in lane 7. (2) **L2**

14. **DBE NOVEMBER 2018 QUESTION 12**

Given: $P(A) = 0,45$; $P(B) = y$ and $P(A \text{ or } B) = 0,74$.

Determine the value(s) of y if A and B are mutually exclusive. (3) **L2**

DBE JUNE 2017 QUESTION 11.3

15. Grade 12 learners in a certain town may choose to attend any one of three high schools. The table below shows the number of Grade 12 learners (as a percentage) attending the different schools in 2016 and the matric pass rate in that school (as a percentage) in 2016.

SCHOOLS	NUMBER OF LEARNERS ATTENDING (%)	MATRIC PASS RATE (%)
A	**20**	**35**
B	**30**	**65**
C	**50**	**90**

If a learner from this town, who was in Grade 12 in 2016, is selected at random, determine the probability that the learner:

15.1 Did not attend School A (2) **L1**

15.2 Attended School B and failed Grade 12 in 2016 (3) **L2**

15.3 Passed Grade 12 in 2016 (4) **L3**

DBE MARCH 2018 QUESTION 11

16 Veli and Bongi are learners at the same school. Some days they arrive late at school. The probability that neither Veli nor Bongi will arrive late on any day is 0,7.

16.1 Calculate the probability that at least one of the two learners will arrive late on a randomly selected day. (1) **L2**

16.2 The probability that Veli arrives late for school on a randomly selected day is 0,25, while the probability that both of them arrive late for school on that day is 0,15. Calculate the probability that Bongi will arrive late for school on that day. (3) **L2**

16.3 The principal suspects that the late coming of the two learners is linked. The principal asks you to determine whether the events of Veli arriving late for school and Bongi arriving late for school are statistically independent or not. What will be your response to him? Show ALL calculations. (3) **L4**

DBE NOVEMBER 2018 QUESTION 12

17. An organisation decided to distribute gift bags of sweets to a Grade R class at a certain school. There is a mystery gift in exactly $\frac{1}{4}$ of the total number of bags.

Each learner in the class may randomly select two gift bags of sweets, one after the other. The probability that a learner selects two bags of sweets with a mystery gift is.

Calculate the number of gift bags of sweets with a mystery gift inside. (6) **L4**

DBE JUNE 2019 QUESTION 11

18. Two learners from each grade at a high school (Grades 8, 9, 10, 11 and 12) are elected to form a sports committee.

18.1 In how many different ways can the chairperson and the deputy chairperson of the sports committee be elected if there is no restriction on who may be elected? (2) **L1**

18.2 A photographer wants to take a photograph of the sports committee. In how many different ways can the members be arranged in a straight line if:

18.2.1 Any member may stand in any position? (1) L1

18.2.2 Members from the same grade must stand next to each other and the Grade 12 members must be in the centre? (3) L3

19. Events S and T are independent. P(S) = 0,4 and P(T) = 0,25.

19.1 Represent the given information on a Venn diagram. Indicate on the Venn diagram the probabilities associated with each region. (3) **L3**

19.2 Determine P[S or (not T)]. (2) **L2**

DBE NOVEMBER 2020 QUESTION 11

20. Harry shoots arrows at a target board. He has a 50% chance of hitting the bull's eye on each shot.

20.1 Calculate the probability that Harry will hit the bull's eye in his first shot and his second shot. (2) **L2**

20.2 Calculate the probability that Harry will hit the bull's eye at least twice in his first three shots. (3) **L3**

20.3 Glenda also has a 50% chance of hitting the bull's eye on each shot. Harry and Glenda will take turns to shoot an arrow and the first person to hit the bull's eye will be the winner. Calculate the probability that the person who shoots first will be the winner of the challenge. (3) **L4**

DBE JUNE 2021 QUESTION 11.3

21. A three-digit number is made up by using three randomly selected digits from 0 to 9. No digit may be repeated.

21.1 Determine the total number of possible three-digit numbers, greater than 100, that can be formed. (2) **L2**

21.2 Determine the total number of possible three-digit numbers, both even and greater than 600, that can be formed. (4) **L3**

DBE JUNE 2022 QUESTION 10.3

22. There are 120 passengers on board an aeroplane. Passengers have a choice between a meat sandwich or a cheese sandwich, but more passengers will choose a meat sandwich. There are only 120 sandwiches available to choose from. The probability that the first passenger chooses a meat sandwich and the second passenger chooses a cheese sandwich is $\frac{18}{85}$.

Calculate the probability that the first passenger will choose a cheese sandwich. (5) **L4**

23. In Gauteng number plates are designed with 3 alphabetical letters, excluding the 5 vowels, next to one another and then any 3 digits, from 0 to 9, next to one another.

The GP is constant in all Gauteng number plates, for example BBB 012 GP.

Letters and digits may be repeated on a number plate.

23.1 How many unique number plates are available? (3) **L2**

23.2 What is the probability that a car's number plate will start with a N? (3) **L2**

23.3 What is the probability that a car's number plate will contain only one 5? (3) **L4**

23.4 How many unique number plates will be available if the letters and numbers are not repeated? (3) **L2**

24. There are 15 girls in a mixed class. If two learners from the class are selected at random to represent the class on the RCL, the probability that both will be girls is 0,35. How many boys are there in the class? (5) **L4**

25. DBE GR. 11 NOVEMBER 2017 QUESTION 9

25. A survey was done among 80 learners on their favourite sport. The results are shown below:

52 learners like rugby (R)
42 learners like volleyball (V)
5 learners like chess only (C)
14 learners like rugby and volleyball, but not chess
12 learners like rugby and chess, but not volleyball
15 learners like volleyball and chess, but not rugby
x like all three types of sport

3 learners do not like any sport

25.1 Draw a Venn diagram to represent the information above. (5) **L2**

25.2 Show that $x = 8$. (2) **L2**

25.3 How many learners like only rugby? (1) **L1**

25.4 Calculate the probability that a learner, chosen randomly, likes at least two different types of sport. (3) **L3**

DBEMARCH 2015 QUESTION 10

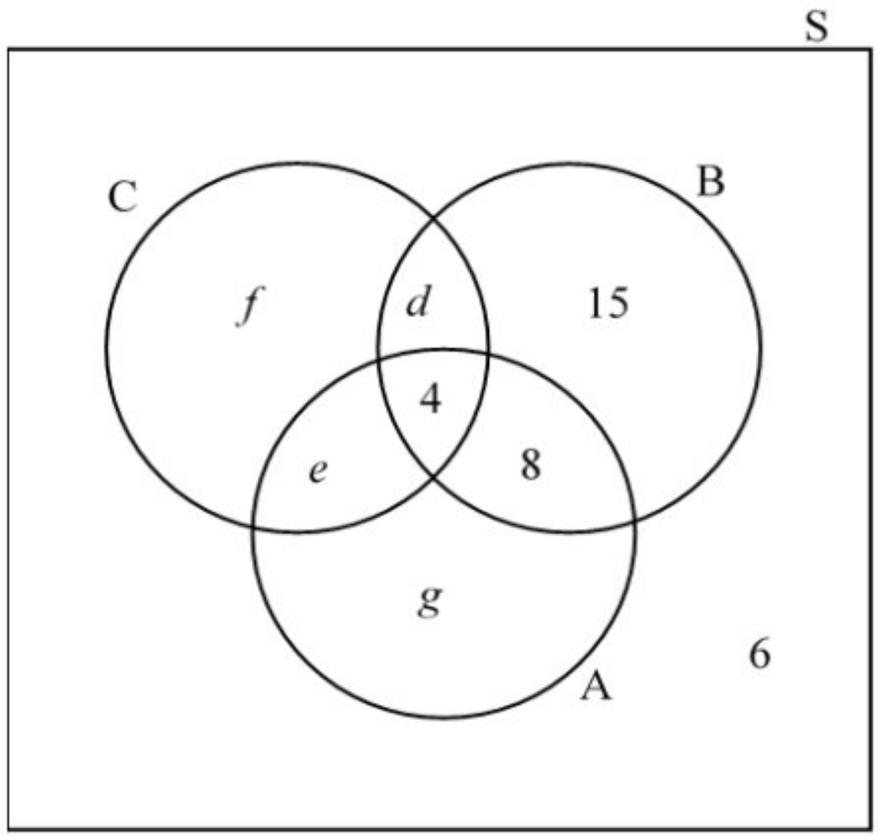
26. Research was conducted about driving under the influence of alcohol. Information obtained from traffic authorities in 54 countries about the methods used to measure alcohol levels in a person are summarised below:

4 countries use all three methods (A, B and C.)
12 countries use the alcohol content of breath (A) and blood-alcohol concentration (B).
9 countries use blood-alcohol concentration (B) and certificates issued by doctors (C)
8 countries use A and C

21 countries use A
32 countries use B
20 countries use C

6 countries use none of these methods.

The partially completed Venn diagram below represents this information:



26.1 Use the given information and the Venn diagram to determine the values of d, e, f and g. (4) **L2**

26.2 For a randomly selected country, calculate:

26.2.1 P(A and B and C) (1) **L1**

26.2.2 P(A or B or C) (1) **L1**

26.2.3 P(only C) (1) **L1**

26.2.4 P(that a country uses exactly two methods) (1) **L2**

26.3 Determine the probablity of getting at least one six when rolling a six-sided dice three times. (4) **L4**

27. In a certain country, 10-digit telephone numbers with the following format were introduced:

Format	**Area code**	**Exchange code**	**Number**
Number of digits	3 digits	3 digits	4 digits
Example	321	755	3578

Digits may be repeated.

27.1 How many possible 10-digit telephone numbers could be formed? (2) **L1**

27.2 Certain restrictions were placed in the group of digits:

Area code: must be 3 digits and first and second letter MUST be 0 or 1

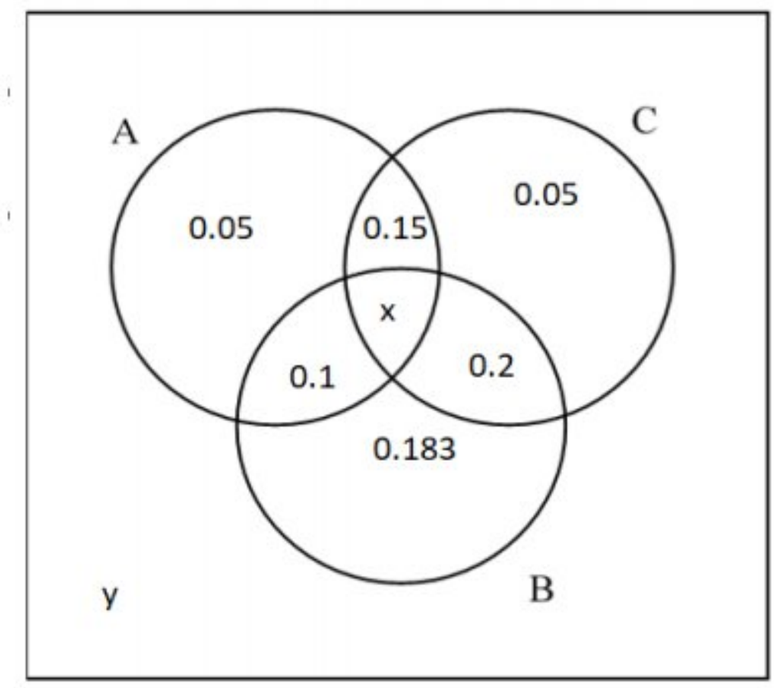
Exchange code: must be 3 digits and the first and second digits can NOT be 0 or 1
Number: must be 4 digits and the first digit MUST be an even number

27.2.1 How many valid 10-digit telephone number could be formed by applying the given restriction? (3) **L4**

27.2.2 Determine the probability that any randomly chosen 10-digit telephone number would be a valid phone number. (2) **L2**

28. DBE **NOVEMBER 2022 QUESTION 10**

28.1 A, B and C are three events. The probabilities of these events (or any combination of them) occurring is given in the Venn-diagram below



28.1.1 If it is given that the probability that at least one of the events will occur is 0,893. Calculate the value of:

a) y, the probability that none of the events of occur. (1) **L1**

b) x, the probability that all three events will occur. (1) **L1**

28.1.2 Determine the probability that at least two of the events will take place. (2) **L2**

28.1.3 Are events B and C independent? Justify you answer. (5) **L3**

28.2 A four-digits code is required to open a combinations lock, the code must be an even number and may not contain the digits 0 or 1. Digits may not be repeated.

28.2.1 How many possible 4 digit combinations are there to open the lock (3) **L2**

28.2.2 Calculate the probability that you will open the lock at the first attempt if is given that the code is greater than 5000 and the third digit is 2. (5) **L4**

29 **DBE MAY/JUNE 2023**

29. A group of people participated in a trial test for a new headache pill

- 50% of the participants received headache pill
- 50% of participants received a sugar pill
- $\frac{2}{5}$ of the group receiving the headache pill were not cured
- $\frac{3}{10}$ of the group receiving sugar pill were cured

29.1 Represent the given information on a tree diagram. Indicate on your diagram the probability associated with each branch as well as the outcomes (3) **L2**

26.2 Determine the probability that a person chosen at random from the group will NOT be cured (2) **L2**

29.3 Three events, A, B and Care considered:

$$P(A)=\frac{2}{5},\ P(B)=\frac{1}{4}\ and\ P(AorB)=\frac{13}{20}$$

29.2.1 Are the events A and B mutually exclusive? Support your answer with necessary calculations (2) **L3**

29.2.2 Determine the P(Conly), if it is further given that

$$P(AorC)=\frac{7}{10},\ P(AandC)=\frac{2}{5}\ and\ 2P(BorC)=P(AandC)$$ (3) **L4**

29.2.3 Determine the probability that events A, B or C do NOT take place (2) **L3**

29.4 Seven friends (4 boys and 3 girls want to stand in a straight line next to each other to take a photo.

29.4.1 In how many ways can the 3 girls stand next to each other in the photo? (2) **L2**

29.4.2 In the next photo, determine the probability that Selwyn (a boy) and Lindiwe (a girl) will NOT stand next to each other in the photo (3) **L3**

STATISTICS/DATA HANDLING
GUIDELINES, SUMMARY NOTES, & STRATEGIES

Definition:
Data Handling is a process during which data (information) is collected, recorded, and presented.
Terminology:

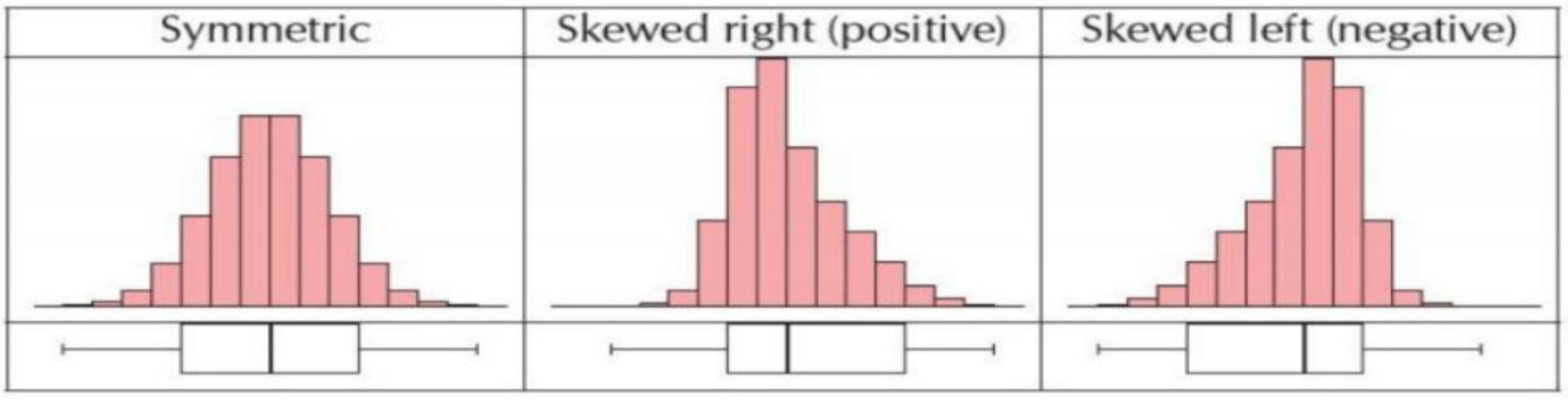
- Data – information that is being analysed.
  - ❖ **Population** – data is collected on the entire group of elements.
  - ❖ **Sample** – data is collected on a specified set from a larger group of elements.
  - ❖ **Ungrouped data** – a set of random data elements gathered for analysis.
  - ❖ **Grouped data** – data elements aggregated into different classes, groups, or intervals.
  - ❖ **Univariate data** – single set of data that is distinguished by specific characteristics.
  - ❖ **Bivariate data** – data set that compares two related variables.
- Measures of central tendency – single numbers around which all data items seem to be spread.
  - ❖ The **Mean**, also known as the average, is the sum of all the data values in a set, divided by number of all elements in the set i.e $\bar{x} = \frac{\sum f x}{n}$ or $\bar{x}_{est} = \frac{\sum f m}{\sum f}$; where f is the frequency and m is the midpoint of a class interval.
  - ❖ The **Median,** (Q_2) is the middle most data item in an ordered data set.

    Position of median $= \frac{1}{2}(n+1)$
  - ❖ The **Mode** is the most frequent data item in a set. In grouped data, the modal group will have the highest frequency. Data sets may have no mode, two modes (bimodal), three modes (trimodal), etc.
- Measures of dispersion – numbers that describe the spread of the data.
  - ❖ The **Range** is the difference between the maximum and the minimum data values in a given data set.
  - ❖ The **Inter-Quartile-Range** (IQR) is the difference between the third and first quartiles, i.e. $IQR = Q_3 - Q_1$
  - ❖ **Standard Deviation** (σ) is a measure of how dispersed data is around the mean. The square of the standard deviation is the **variance** (σ^2).
- Quartiles – numbers that divide data into quarters in an ordered data set.
  - ❖ **Lower quartile,** (Q_1), is a data item below which a quarter of the data lies in an ordered data set.

    Position of lower quartile $\frac{1}{4}(n+1)$
  - ❖ **Upper quartile**, (Q_3) is a data item above which a quarter of the data lies in an ordered data set.

    Position of upper quartile $= \frac{3}{4}(n+1)$
- Percentiles – numbers below which a certain percentage of data item lies in an ordered data set.
  - ❖ Position of percentile $= \frac{percentile}{100} \times number \quad of \quad data \quad items \quad in \quad a \quad set$
- Five Number Summary – five numbers that separate a data set into quarters.
  - ❖ Minimum value
  - ❖ Lower quartile (Q_1)
  - ❖ Median (Q_2)
  - ❖ Upper quartile (Q_3)
  - ❖ Maximum value
- Box – and – Whisker Diagram (drawn using the five number summary)

- ❖ It is important in analysing the distribution of data in a given set.
- ❖ If mean – median = 0, then the distribution is symmetric.
- ❖ If mean – median > 0, then the distribution is positively skewed.
- ❖ If mean – median < 0, then the distribution is negatively skewed.



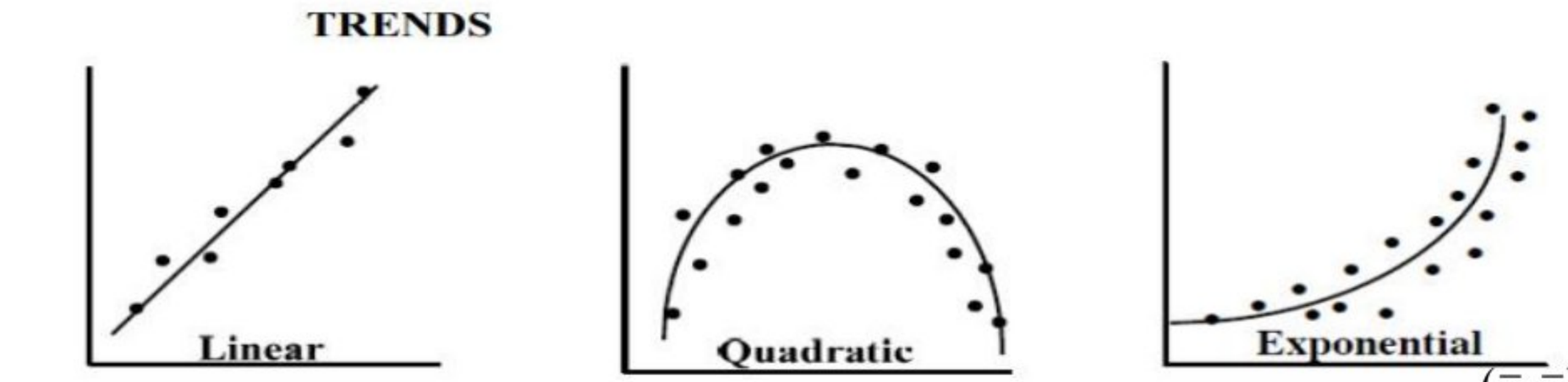
- ❖ In a symmetrical data set approximately 68% of the data will fall within one standard deviation of the mean $[\overline{x}-\sigma;\overline{x}+\sigma]$ and approximately 96% of the data will lie within two standard
- ❖ deviations of the mean $[\overline{x}-2\sigma;\overline{x}+2\sigma]$

- Outliers – data items that are a lot bigger or smaller than the rest of the elements in the data set.

They are determined as follows:

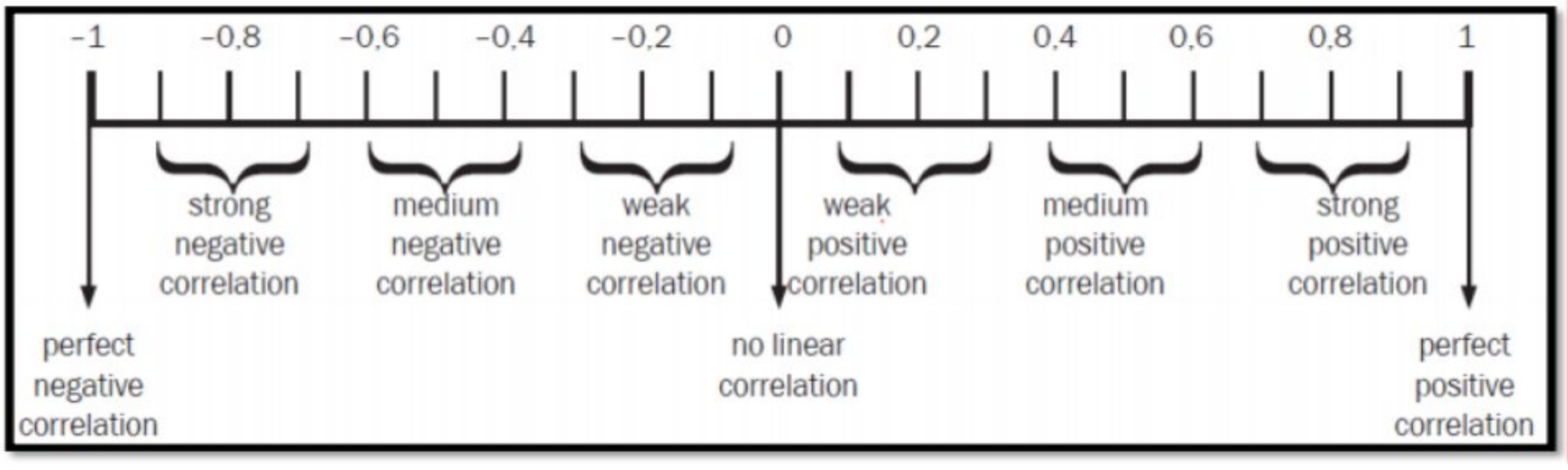
- ❖ Lower outliers are numbers $< Q_1 - 1.5 \times IQR$
- ❖ Upper outliers are numbers $> Q_1 + 1.5 \times IQR$

- Graphical representations
  - ❖ **Histogram** – represents grouped data as condensed bars whose widths and lengths represent class intervals and frequency respectively.
  - ❖ **Ogive (Cumulative Frequency Curve)** – an *s*-shaped smooth curve drawn by plotting upper limits of class intervals of a grouped data against cumulative frequency of a set.
  - ❖ **Scatter plot** – representation of bivariate data as discrete data points.
- Bivariate data summaries
  - ❖ **Regression line (line of best fit) -** a line drawn on the scatter plot that shows a general trend that bivariate data seems to follow.



- ❖ **Least squares regression line** – is a straight line that passes through the mean point $(\overline{x};\overline{y})$ relating bivariate data

- ❖ **Corelation Coefficient ®** – indicates the strength of the relationship between the variables in bivariate data. It lies between -1 and 1.



REVISION QUESTIONS

DBE FEBRUARY/MARCH 2014 QUESTION 1

1 The tuck shop at Great Future High School sells cans of soft drinks. The Environmental Club at the school decided to have a can-collection project for three weeks to make learners aware of the effects of litter on the environment.

The data below shows the number of cans collected on each school day of the three-week project.

58 83 85 89 94 97 98 100 105 109 112 113 114 120 145

1.1 Calculate the mean number of cans collected over the three-week period. (2) **L1**

1.2 Calculate the standard deviation. (2) **L2**

1.3 Determine the lower and upper quartiles of the data. (2) **L1**

1.4 Draw a box and whisker diagram to represent the data. (3) **L2**

1.5 On how many days did the number of cans collected lie outside ONE standard deviation of the mean? (3) **L2**

DBE MAY/JUNE 2022 QUESTION 1

2 The table below shows the mass (in kg) of the school bags of 80 learners.

MASS (kg)	FREQUENCY	CUMULATIVE FREQUENCY
$5 < m \leq 7$	6	
$7 < m \leq 9$	18	
$9 < m \leq 11$	21	
$11 < m \leq 13$	19	
$13 < m \leq 15$	11	
$15 < m \leq 17$	4	
$17 < m \leq 19$	1	

2.1 Write down the modal class of the data. (1) **L1**

2.2 Complete the cumulative frequency column in the table (2) **L1**

2.3 Draw a cumulative frequency graph (ogive) for the given data. (3) **L2**

2.4 Use the graph to determine the median mass for this data. (2) **L2**

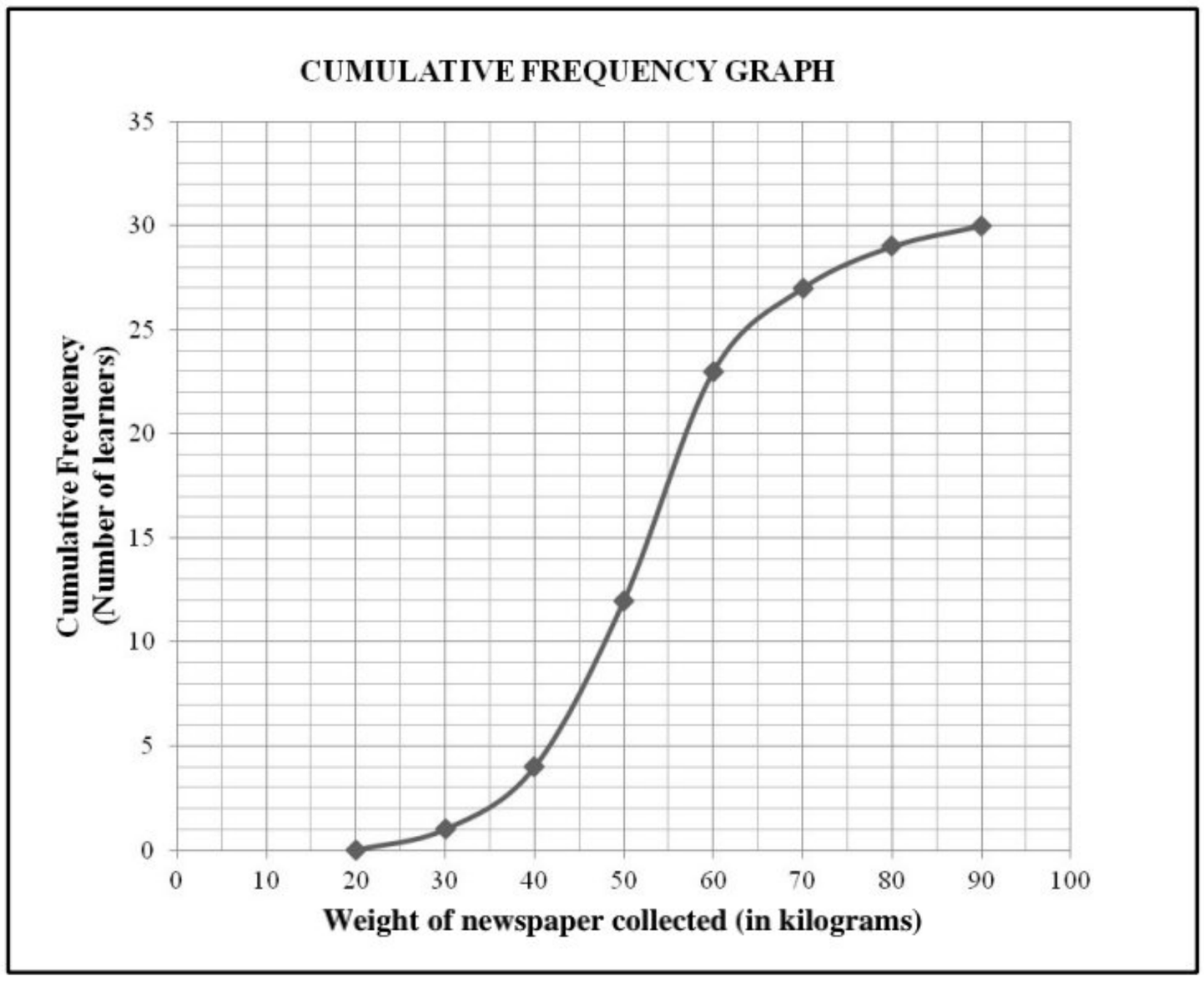
2.5 The international guideline for the mass of a school bag is that it should not exceed 10% of a learner's body mass.

2.5.1 Calculate the estimated mean mass of the school bag (2) **L2**

2.5.2 The mean mass of this group of learners was found to be 80kg. On average, are these school bags satisfying the international guideline with regards to mass? Motivate your answer. (2) **L3**

DBE NOVEMBER 2012 QUESTION 4

3 As part of an environmental awareness initiative, learners of Greenside High School were requested to collect newspapers for recycling. The cumulative frequency graph (ogive) below shows the total weight of the newspapers (in kilograms) collected over a period of 6 months by 30 learners.



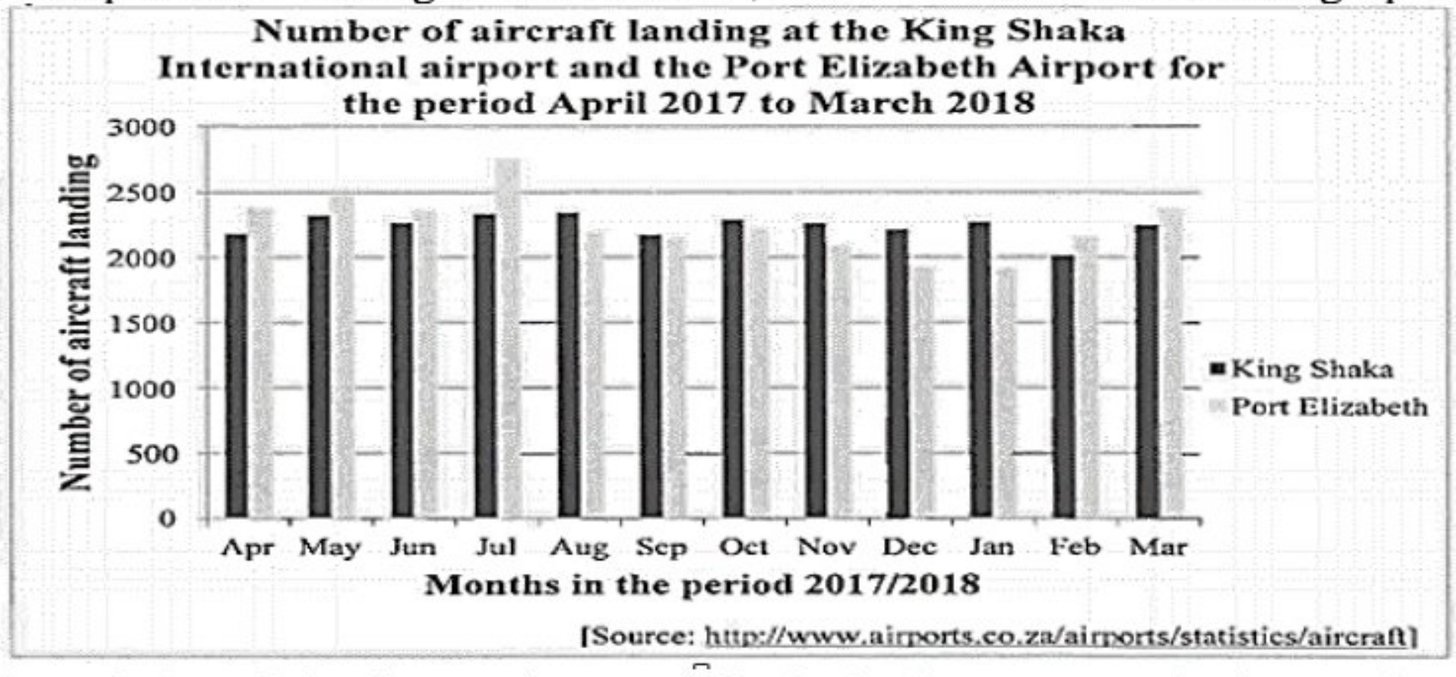
3.1 Determine the modal class of the weight of the newspapers collected (1) **L1**

3.2 Determine the median weight of the newspapers collected by this group of learners. (1) **L2**

3.3 How many learners collected more than 60 kilograms of newspaper? (2) **L2**

DBE NOVEMBER 2020 QUESTION 2

4 The number of aircraft landing at the King Shaka International and the Port Elizabeth Airport for the period starting in April 2017 ending in March 2018, is shown in the double bar graph below.



4.1 The number of aircraft landing at the Port Elizabeth Airport exceeds the number of aircraft landing at the King Shaka International Airport during some months of the given period. During which month is this difference the greatest? (1) **L1**

4.2 The number of aircraft landing at the King Shaka International Airport during these months are given below. Calculate the mean for the data.

2 182	2 323	2 267	2 334	2 346	2 175
2 293	2 263	2 215	2 271	2018	2 254

(2) **L1**

4.3 Calculate the standard deviation for the number of aircraft landing at the King Shaka International Airport for the given period. (2) **L2**

4.4 Determine the number of months in which the number of aircraft landing at the King Shaka International Airport were within one standard deviation of the mean. (3) **L2**

4.5 Which one of the following statements is CORRECT?

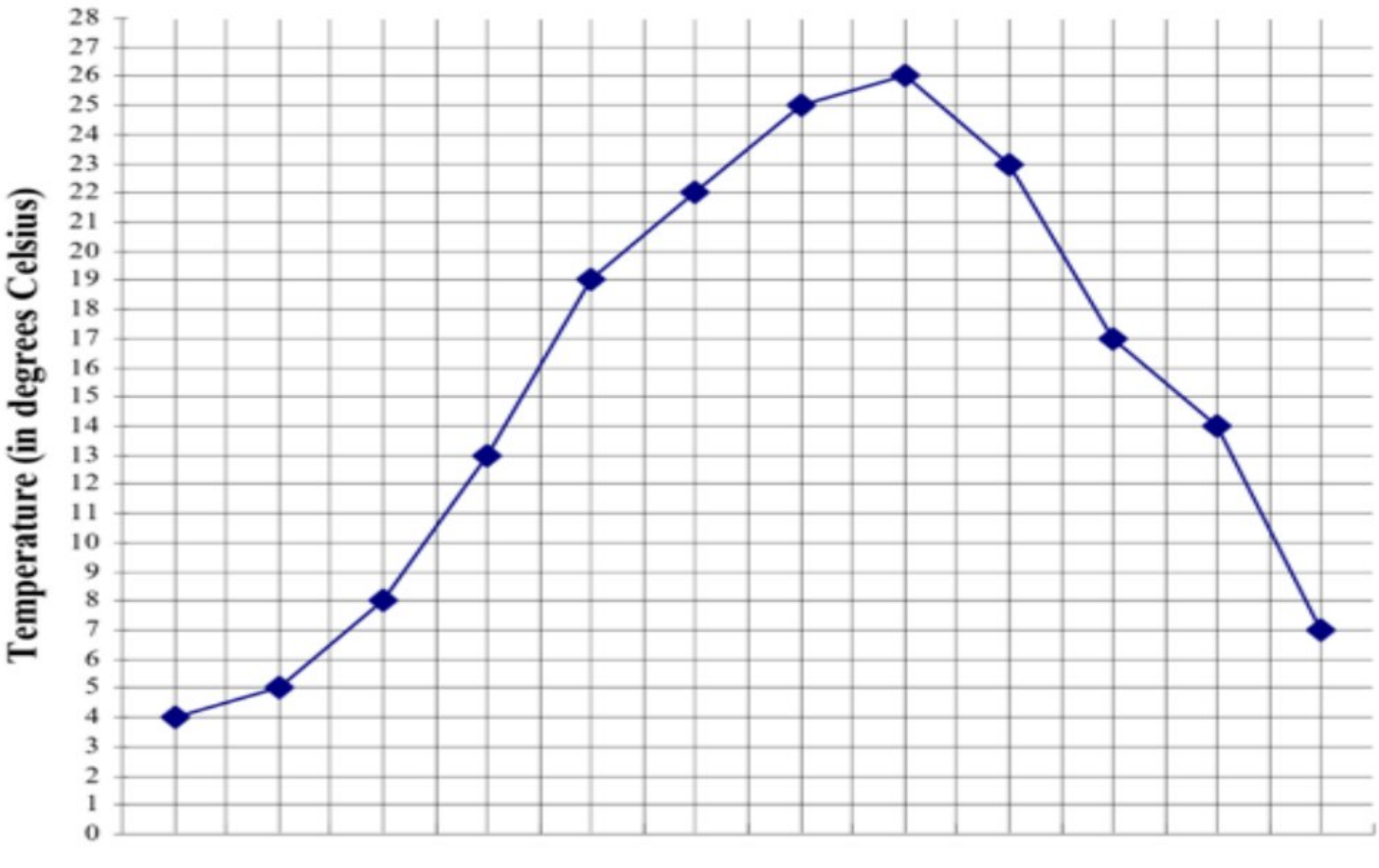
a) During December and January, there were more landings at the Port Elizabeth Airport than at the King Shaka International Airport.

b) There was a greater variation in the number of aircraft landings at the King Shaka International than at the Port Elizabeth for the given period.

c) The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of landings at the King Shaka International Airport (1) **L3**

DBE FEBRUARY/MARCH 2010 QUESTION 1

5 The graph below shows the monthly maximum temperatures in a certain city.



5.1 What is the range of the monthly maximum temperatures? (2) **L1**

5.2 Calculate the mean monthly maximum temperature. (3) **L1**

5.3 Calculate the standard deviation of the monthly maximum temperature. (2) **L2**

5.4 It is predicted that one hundred years from now, global warming is likely to increase the city's monthly maximum temperature by 5° C in December, January and February. It will also result in an increase of 1° C in the other months of the year.

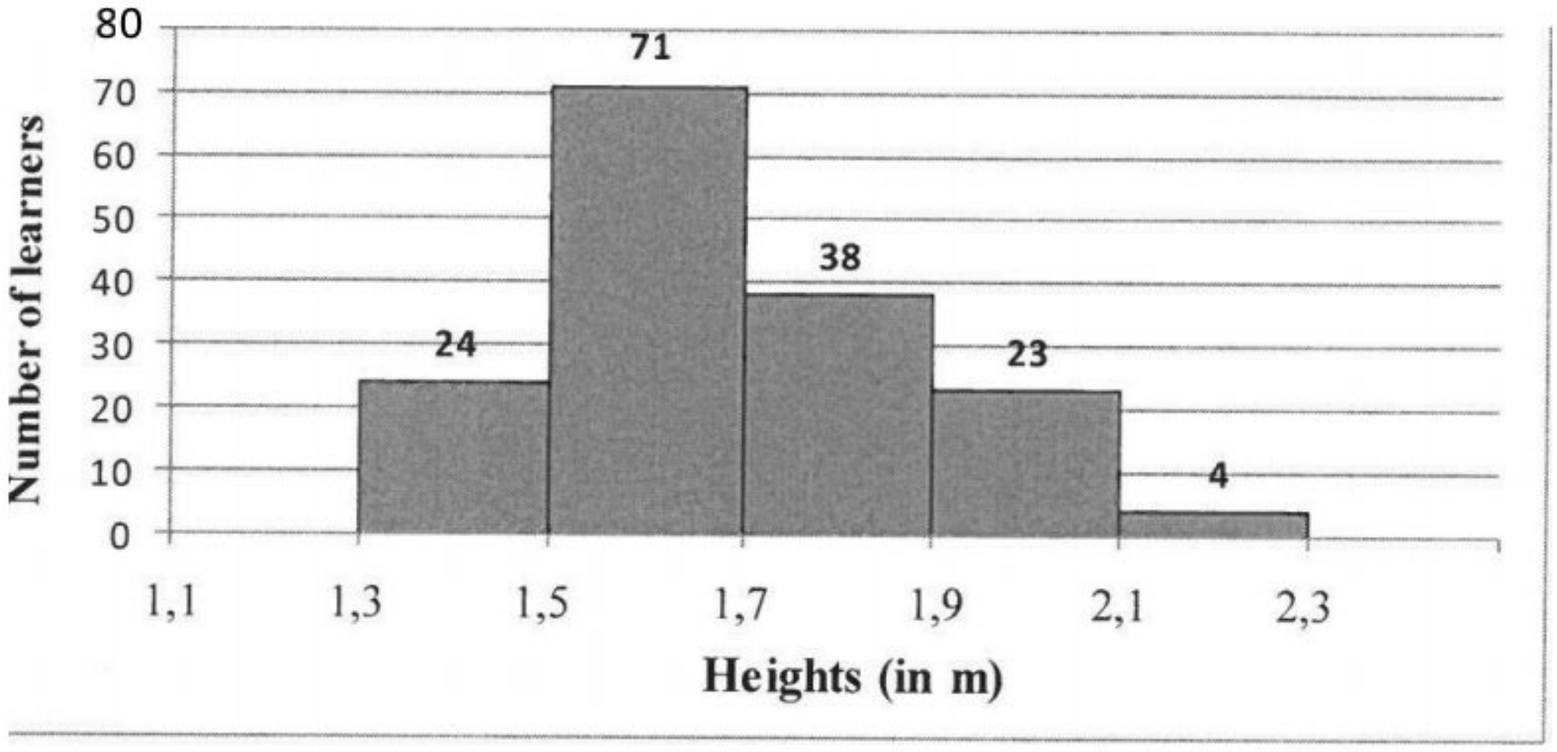
5.4.1 By how much does the mean increase? (2) **L3**

5.4.2 Describe the effect that the predicted increases in temperature will have on the standard deviation. Justify your answer. (2) **L4**

DBE NOVEMBER 2016 QUESTION 2

6 The heights of 160 learners in a school are measured. The height of the shortest learner is 1,39 m and the height of the tallest learner is 2,21 m. The heights are represented in the histogram below.

Histogram



6.1 Describe the skewness of the data. (1) **L1**

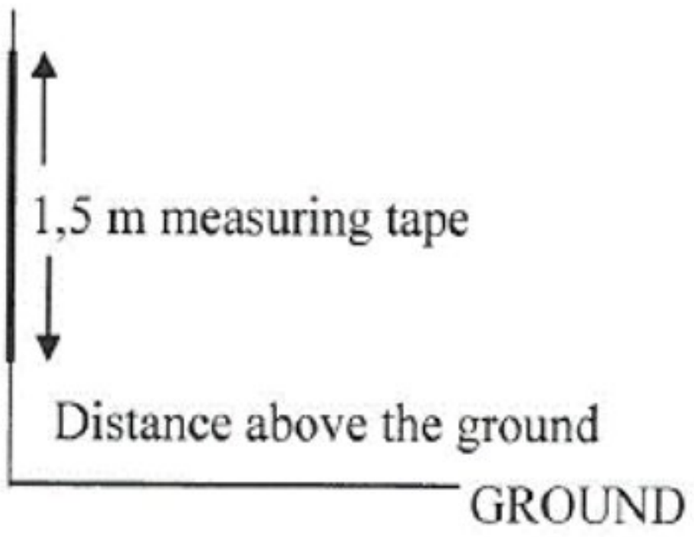
6.2 Calculate the range of the heights. (2) **L1**

6.3 Draw and complete a cumulative frequency table. (2) **L2**

6.4 Draw an ogive (cumulative frequency curve) to represent the data. (4) **L2**

6.5 Eighty learners are less than x metres in height. Estimate x. (2) **L2**

6.6 The person taking the measurements only had 1,5 m measuring tape available. In order to compensate for the short measuring tape, he decided to mount the tape on a wall at a height of 1m above the ground. After recording the measurements, he discovered that the tape was mounted at 1,1 m above the ground instead of 1 m. How does this error influence the following?



6.6.1 Mean of the data set. (1) **L3**

6.6.2 Standard deviation of the data set. (1) **L4**

DBE NOVEMBER 2015 QUESTION 2

7 A group of 30 learners each randomly rolled two dice once and the sum of the values on the uppermost faces of the dice was recorded. The data is shown in the frequency table below.

Sum of the values on uppermost faces	Frequency
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

7.1 Calculate the mean of the data. (2) **L2**

7.2 Determine the median of the data. (2) **L2**

7.3 Determine the standard deviation of the data. (2) **L2**

7.4 Determine the number of times that the sum of the recorded values of the dice is within ONE standard deviation from the mean. Show your calculations. (3) **L2**

FS/SEPTEMBER 2020 QUESTION 1

8 The table below gives the average exchange rate and the average monthly oil price for the year 2010.

	Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sept**	**Oct**	**Nov**	**Dec**
Exchange rate in R/S	**7.5**	**7.7**	**7.2**	**7.4**	**7.7**	**7.7**	**7.6**	**7.3**	**7.1**	**7.0**	**6.9**	**6.8**
Oil price in $	**69.9**	**68.0**	**72.9**	**70.3**	**66.3**	**67.1**	**67.9**	**68.3**	**71.3**	**73.6**	**76.0**	**81.0**

8.1 Draw a scatterplot to represent the exchange rate (in R/S) versus the oil price (in $). (3) **L2**

8.2 Determine the equation of the least square regression line. (3) **L2**

8.3 Calculate the value of the correlation coefficient. (1) **L1**

8.4 Comment on the strength of the relationship between the exchange rate (in R/S) and the oil price (in $). (2) **L1**

8.5 Determine the mean oil price. (1) **L1**

8.6 Determine the standard deviation of the oil price. (1) **L2**

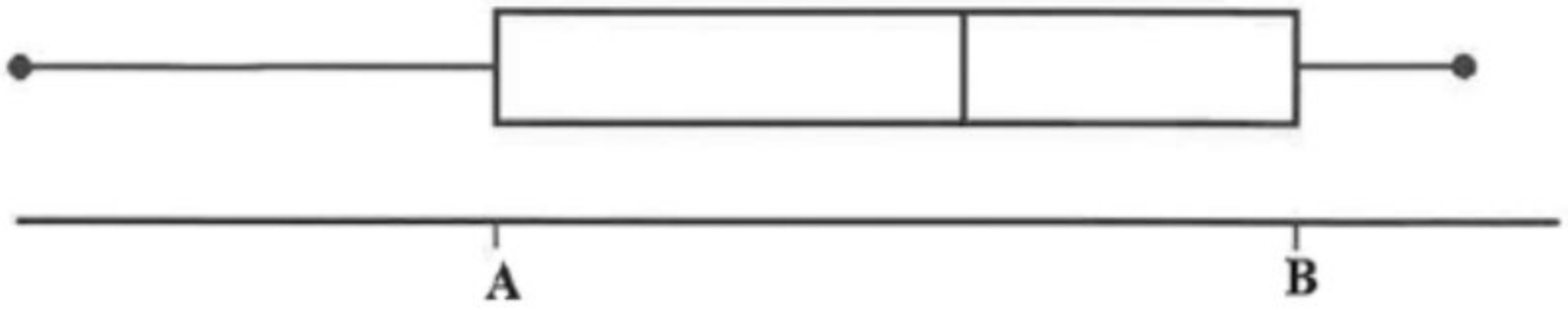
8.7 Generally, there is a concern from the public when the oil price is higher than two standard deviations from the mean. In which months would the public have been concerned? (2) **L3**

DBE FEBRUARY/MARCH 2018 QUESTION 1.2 – 1.3

9 An organisation decided that it would set up blood donor clinics at various colleges. Students would donate blood over a period of 10 days. The number of units of blood donated per day by students of college X is shown in the table below.

DAYS	1	2	3	4	5	6	7	8	9	10
UNITS OF BLOOD	45	59	65	73	79	82	91	99	101	106

The number of units of blood donated by students of college X is represented in the box and whisker diagram below.



9.1 Describe the skewness of the data. (1) **L1**

9.2 Write down the values of **A** and **B**, the lower quartile and the upper quartile of the data, respectively. (2) **L1**

9.3 It was discovered that there was an error in counting the number of units of blood donated by college X each day. The correct mean of the data is 95 units of blood. How many units of blood were NOT counted over the ten days? (1) **L4**

10 **FEBRUARY/MARCH 2012 P3 QUESTION 2**

10 A large company employs several people. The table below shows the number of people employed in each position and the monthly salary paid to each person in that position.

POSITION	NUMBER EMPLOYED IN POSITION	MONTHLY SALARY PER PERSON (IN RAND)
Managing director	1	150 000
Director	2	100 000
Manager	2	75 000
Foreman	5	15 000
Skilled workers	30	10 000
Semi-skilled workers	40	7 500
Unskilled workers	65	6 000
Administration	5	5 000

10.1 Calculate the total number of people employed at this company. (1) **L1**

10.2 Calculate the total amount needed to pay salaries for ONE month. (2) **L1**

10.3 Determine the mean monthly salary for an employee in this company. (2) **L2**

10.4 Is the mean monthly salary calculated in QUESTION 10.3 a good indicator of an employee's monthly salary? Motivate your answer. (2) **L3**

11 A group of learners from Mr Smith's class wrote a Mathematics test which was scored out of 75 marks. The results were represented in the table below.

MARKS	FREQUENCY	CUMULATIVE FREQUENCY
$5 < x \leq 15$	3	
$15 < x \leq 25$	6	
$25 < x \leq 35$	m	21
$35 < x \leq 45$	4	
$45 < x \leq 55$	7	
$55 < x \leq 65$	9	
$65 < x \leq 75$	n	
Total	**51**	

11.1 How many learners wrote the test? (1) **L1**

11.2 Determine the value of m and n. (2) **L2**

11.3 Complete the given table on the diagram sheet. (2) **L1**

11.4 Draw a cumulative frequency curve (ogive) to represent above data (3) **L2**

11.5 Hence, or otherwise estimate the value of the median for the above data (2) **L2**

DBE FEB/MARCH 2012 QUESTION 4

12 In the grid below a, b, c, d, e, f and g represent values in a data set written in an increasing order. No value in the data set is repeated.

a	*b*	*c*	*d*	*e*	*f*	*g*

Determine the value of a, b, c, d, e, f and g if:

- The maximum value is 42
- The range is 35
- The median is 23
- The difference between the median and the upper quartile is 14
- The interquartile range is 22
- $e = 2c$
- The mean is 25

DBE FEB./MARCH 2009 P2 QUESTION 12

13 A motor company did research on how the speed of a car affects the fuel consumption of the vehicle. The following data was obtained:

Speed in km/h	60	75	115	85	110	95	120	100	70
Fuel consumption in ℓ/100 km	11,5	10	8,4	9,2	7,8	8,9	8,8	8,6	10,2

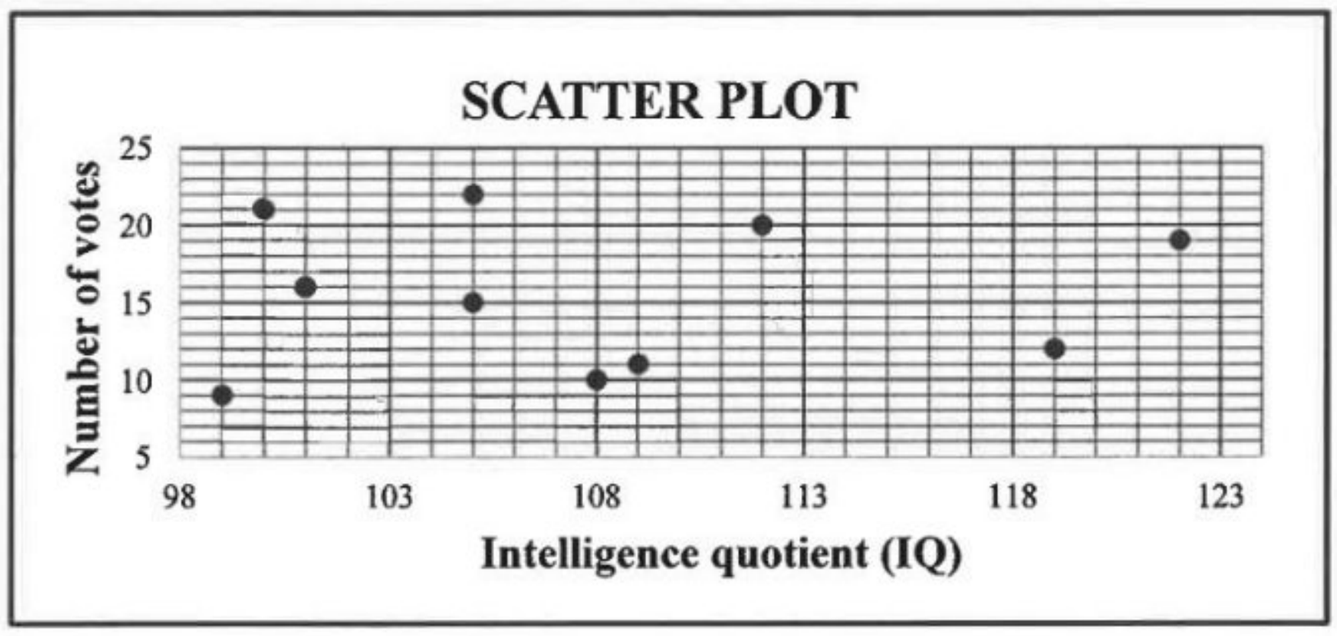
13.1 Represent the data as a scatter plot. (3) **L2**

13.2 Suggest whether a linear, quadratic or exponential function would best fit the data. (1) **L1**

13.3 What advice can the company give about the driving speed in order to keep the cost of fuel to a minimum? (2) **L2**

DBE NOVEMBER 2022 QUESTION 1

14 The matric class of a certain high school had to vote for the chairperson of the RCL (representative council of learners). The scatter plot below shows the IQ (intelligent quotient) of the 10 learners who received the most votes and the number of votes that they received.



Before the election, the popularity of each of these ten learners was established and a popularity score (out of a 100) was assigned to each. The popularity scores and the number of votes of the same 10 leamers who received the most votes are shown in the table below:

Popularity score (x)	32	89	35	82	50	59	81	40	79	65
Number of votes (y)	9	22	10	21	11	15	20	12	19	16

14.1 **Calculate**

14.1.1 Mean number of votes that these 10 learners received (1) **L1**

14.1.2 Standard deviation of the number of votes that these 10 learners received (2) **L2**

14.2 The learners who received fewer votes than one standard deviation below the mean were not invited for an interview. How many learners were invited? (2) **L3**

14.3 Determine the equation of the least squares regression line for the data given in the table. (2) **L2**

14.4 Predict the number of votes that a learner with a popularity score of 72 will receive. (3) **L2**

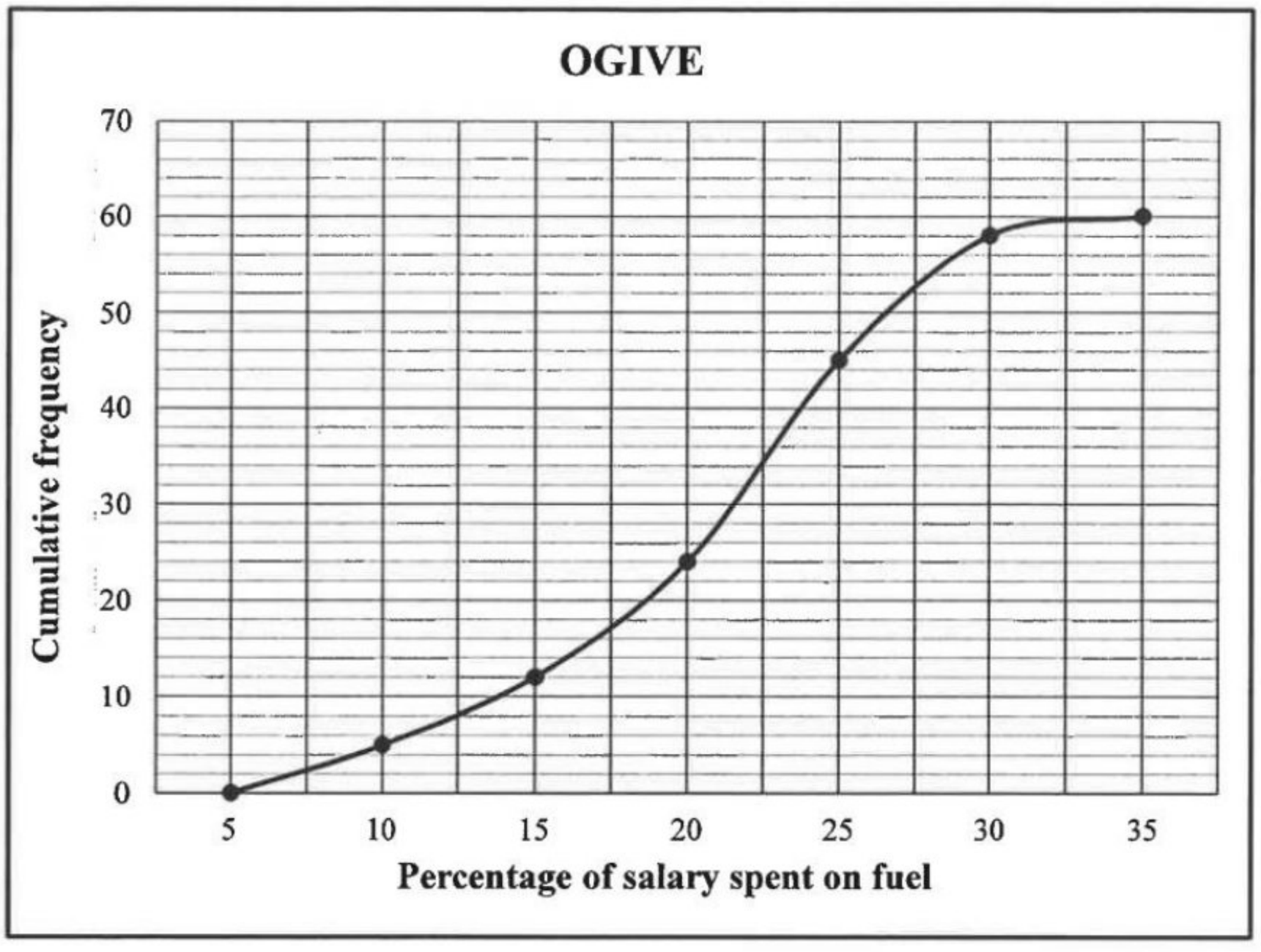
14.5 Using the scatter plot and table above, provide a reason why:

14.5.1 IQ is not a good indicator of the number of votes that a learner could receive (1) **L2**

14.5.2 The prediction in QUESTION 14.4 is reliable (1) **L3**

15 **DBE NOVEMBER 2022 P2 QUESTION 2**

A company conducted research among all its employees on what percentage of their monthly salary was spent on fuel in a particular month. The data is represented in the ogive (cumulative frequency graph) below



15.1	How many people are employed at this company?	(1)	**L1**
15.2	Write down the modal class of the data.	(1)	**L1**
15.3	How many employees spent more than 22,5% of their monthly salary on fuel?	(2)	**L2**
15.4	An employee spent R2 400 of his salary on fuel in that particular month. Determine the monthly salary of this employee if he spends 7% of his salary on fuel.	(2)	**L2**
15.5	The monthly salaries of these employees remain constant and the number of litres of fuel used in each month also remains constant. If the fuel price increases from R21,43 per litre to R22,79 per litre at the beginning of the next month, how will the above ogive change?	(2)	**L4**

16 **MPUMALANGA MAY/ JUNE 2023 MATHS P2 QUESTION 1**

An estate agent did a survey on the salaries of the people renting homes in a complex. She selected 12 homes where two salary earners live for her survey. The data collected is recorded in the table below.

Salary of person 1	3000	2100	5100	3560	6250	7400	4210	3200	2600	1000	4100	8000
Salary of person 2	4500	8320	6500	3500	1500	4200	6420	3520	10500	11000	7800	19350

16.1	Determine the median salary for person 1 in this data.	(2)	**L1**
16.2	Determine the mean salary for person 2 in this data.	(2)	**L1**
16.3	Determine the number of salaries for person 2 that are above ONE standard deviation from the mean.	(3)	**L2**
16.4	Determine the equation of the least squares regression line for this data in the table.	(3)	**L2**
16.5	Draw a scatter plot and the least squares regression line for the given data.	(4)	**L3**
16.6	Explain what will happen to the least squares regression line if the income of the person receiving an income of R19 350 decreases by 50%	(2)	**L3**

TOPIC	ANALYTICAL GEOMETRY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

DISTANCE BETWEEN TWO POINTS

The formula to calculate the length of a line segment between two points $A(x_A; y_A)$ and $B(x_B; y_B)$ is given by the formula:

$$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2 \text{ or}$$

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

MIDPOINT OF A LINE SEGMENT

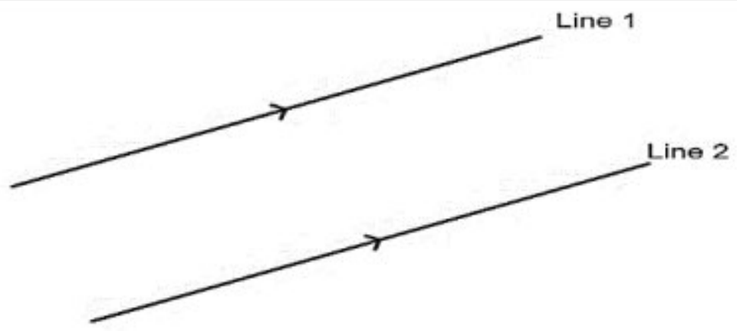
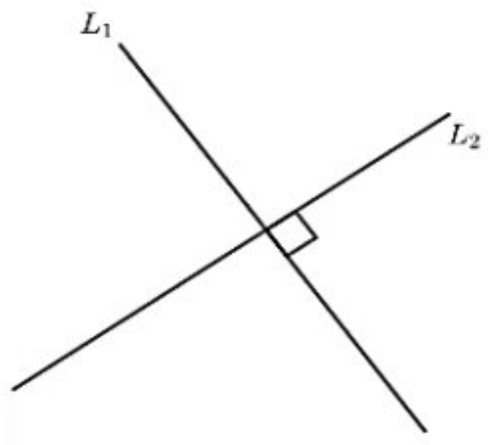
The formula for point M, the midpoint of a line segment AB joining the points $A(x_A; y_A)$ and $B(x_B; y_B)$ is given by the formula:

$$M(x_M; y_M) = M\left(\frac{x_A + x_B}{2}; \frac{y_A + y_B}{2}\right)$$

GRADIENT OF A LINE

A formula to calculate the gradient of a line joining two points $A(x_A; y_A)$ and $B(x_B; y_B)$ is given by the formula:

The gradient of line AB: $M_{AB} = \frac{y_B - y_A}{x_B - x_A}$

PARALLEL LINES	PERPENDICULAR LINES
	
If $L_1 \parallel L_2$ then $m_1 = m_2$	If $L_1 \perp L_2$ then $m_1 \times m_2 = -1$

VERTICAL LINES

The vertical line always cuts through the x-axis. It is parallel to the y- axis and perpendicular to the x-axis.The equation of a line cutting the x axis at a: $x = a$

HORIZONTAL LINES

The horizontal line cuts through the y- axis. It is parallel to the x-axis and perpendicular to the y- axis. The equation of a line cutting through the y axis at b: $x = b$

COLLINEAR POINTS

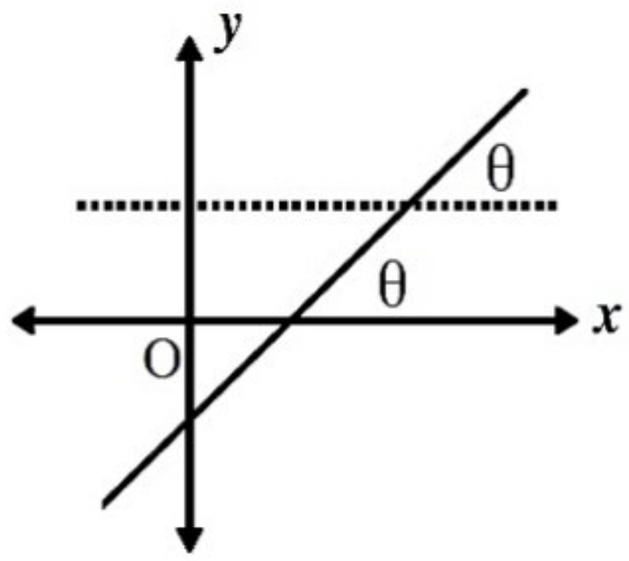
Points that are collinear lie on the same line. The gradient between each pair of points is the same. For example, if the points A, B and C are collinear, then:

$\text{Gradient}_{AB} = \text{Gradient}_{BC} = \text{Gradient}_{AC}$

EQUATION OF A LINE: $y = mx + c$ or $y - y_1 = m(x - x_1)$

INCLINATION OF A LINE

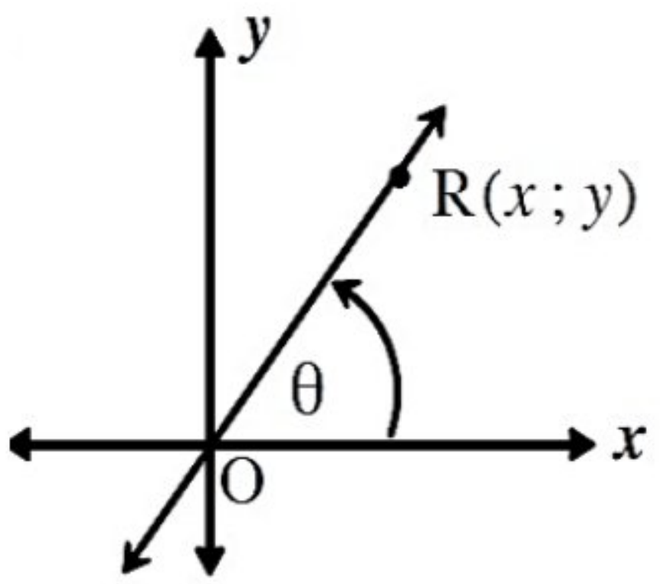
The inclination of a line is the angle formed with the horizontal in an anti-clockwise direction. On the Cartesian plane, the inclination of a line is calculated by finding the angle formed at the x-axis measured in anticlockwise direction. θ is the angle of inclination of line AB



Formula for finding the angle of inclination of a line

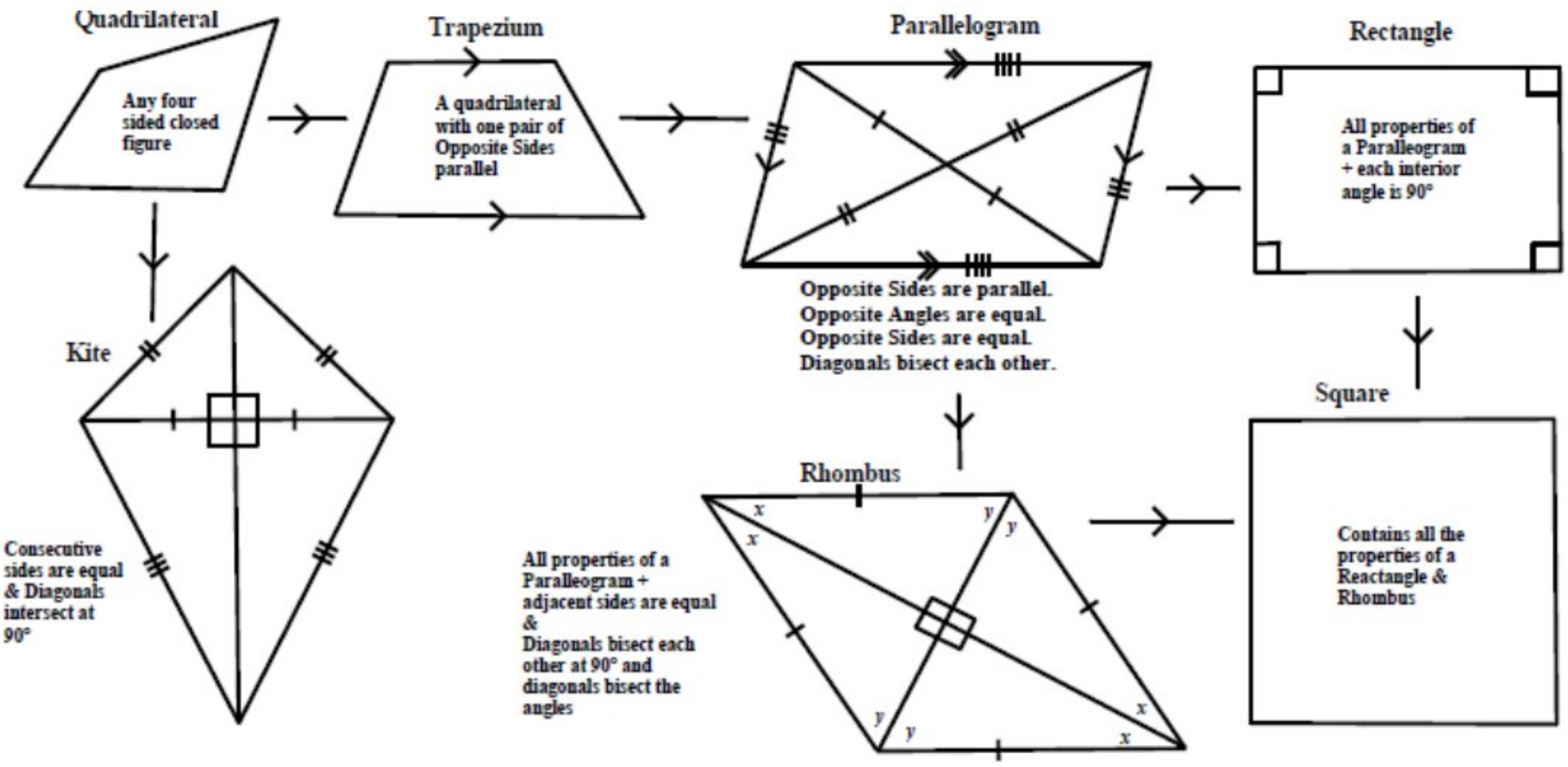
If $R(x; y)$ is a point on the terminal arm of θ, then by defination, $\tan\theta = \frac{y}{x}$,

But with $O(0;0)$, the gradient of line $OR = \frac{y-0}{x-0} = \frac{y}{x}$.



$\tan\theta = \text{Gradient}_{OR}$, where θ is the angle of inclination of line OR.

PROPERTIES OF QUADRILATERALS



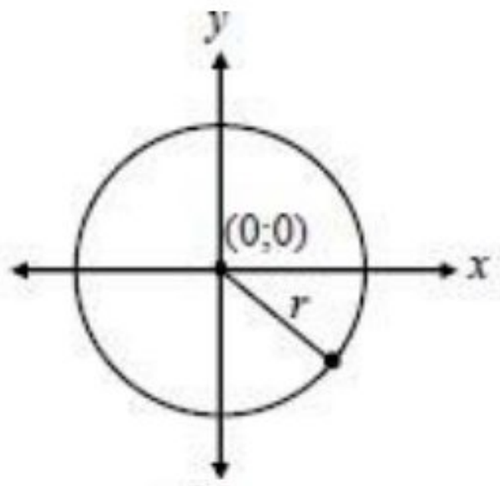
A CIRCLE

- Radius is the line from the centre to the circle.
- Radii of the circle have the same length.
- The diameter is a straight line from the circle through the centre to the circle (twice the radius)
- A chord touches the circle twice internally and divides the circle into segments
- A circumference is the distance around the circle.

CIRCLE WITH CENTRE AT THE ORIGIN

Equation: $r^2 = x^2 + y^2$

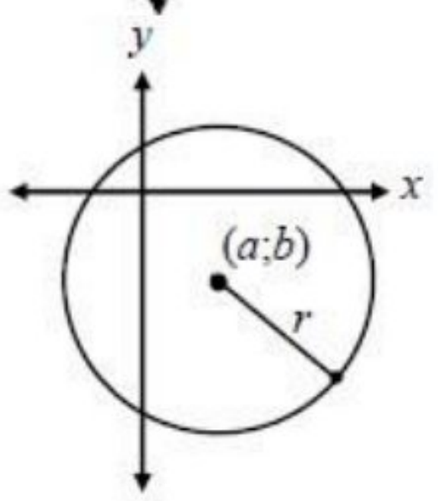
- This formula should remind you of Pythagoras.
- r is the radius and $(x; y)$ is a point on the circle.



CIRCLE WITH ANY CENTRE

Equation: $r^2 = (x-a)^2 + (y-b)^2$

- A circle with any centre is simply a circle with a centre at the origin that has been shifted left or right and up or down.
- r is the radius
- a is the x-coordinate of the centre
- b is the y-coordinate of the centre.



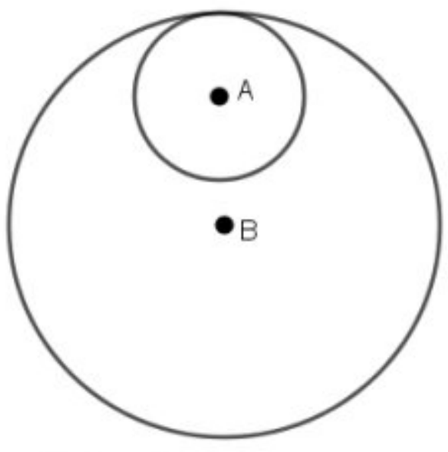
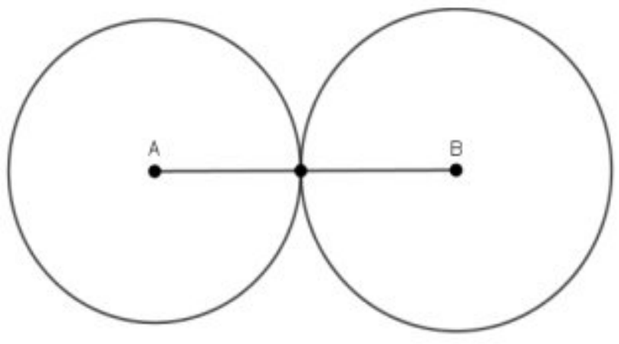
If the equation of the circle is given in the form: $x^2 + ax + y^2 + by + c = 0$, To determine the co-ordinates of the centre of the circle and the radius, be able to complete the square

EQUATION OF THE TANGENT TO THE CIRCLE

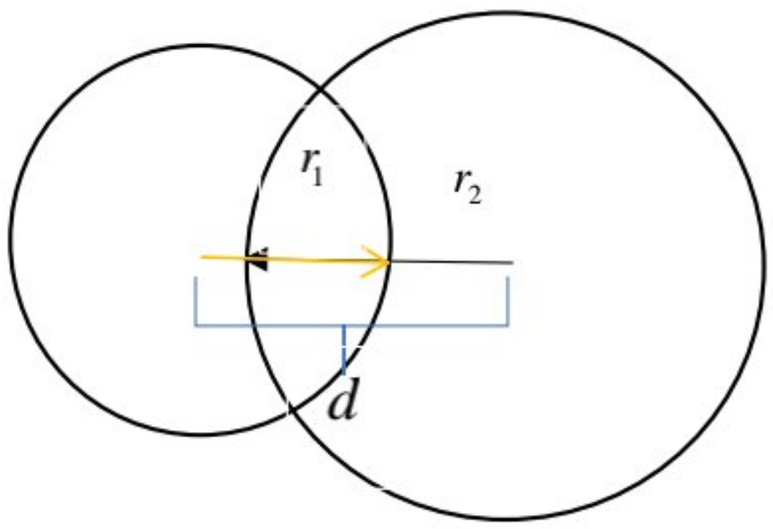
- ✓ A tangent is a straight line that touches the circle at only one point $(x; y)$
- ✓ A tangent is perpendicular to the radius (**this means the radius and the tangent form of a 90° angle at the point of** contact. Therefore, $m_{\tan gent} \times m_{radius} = -1$
- ✓ The equation of the tangent is determined by using the formula: $y = mx + c$, where m is the gradient and c is the y-intercept.

CIRCLES THAT TOUCH INTERNALLY AND CIRCLES THAT TOUCH EXTERNALLY

Let the centre of the one circle be A and the other B. Calculate the distance AB using the distance formula. Then add R (the radius of the one circle) to *r* the radius of the other

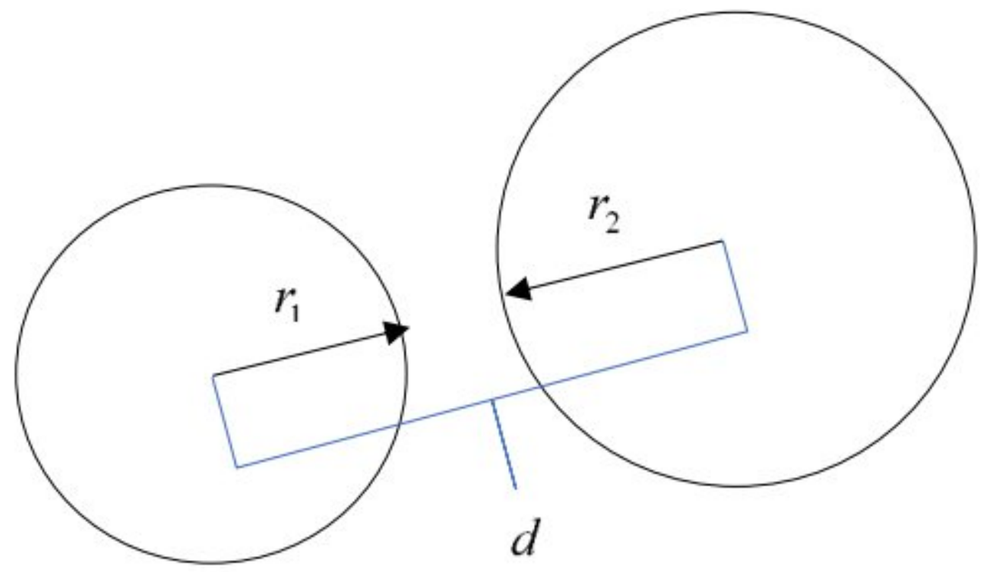
1. Two circles touch **internally**	2. . Two circles touch **externally**
 **AB = R − *r*.**	 **AB = R + r**

3. Circles intersecting at two points



$d < r_1 + r_2$

4. Circles that do not intersect

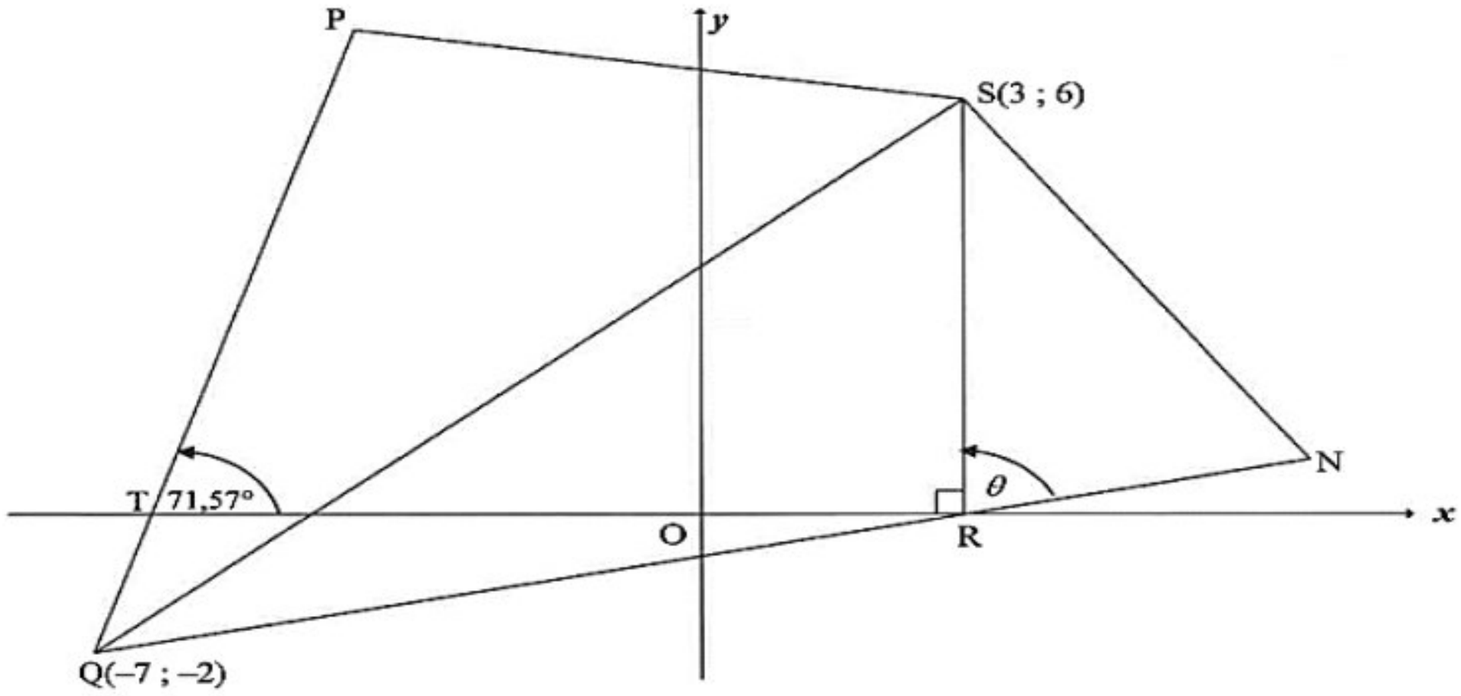


$d > r_1 + r_2$

DBE FEB-MARCH 2018 Question3

1 In the diagram, P, Q (-7; -2), R and S (3; 6) are vertices of a quadrilateral. R is a point on the x-axis.

QR is produced to N such that QR = 2RN. SN is drawn. $\hat{PTO} = 71,57°$ and $\hat{SRN} = \theta$



Determine:

1.1 The equation of SR. (1) **L1**

1.2 The gradient of QP to the nearest integer. (2) **L2**

1.3 The equation of QP in the form $y = mx + c$ (2) **L2**

1.4 The length of QR. Leave your answer in surd form. (2) **L2**

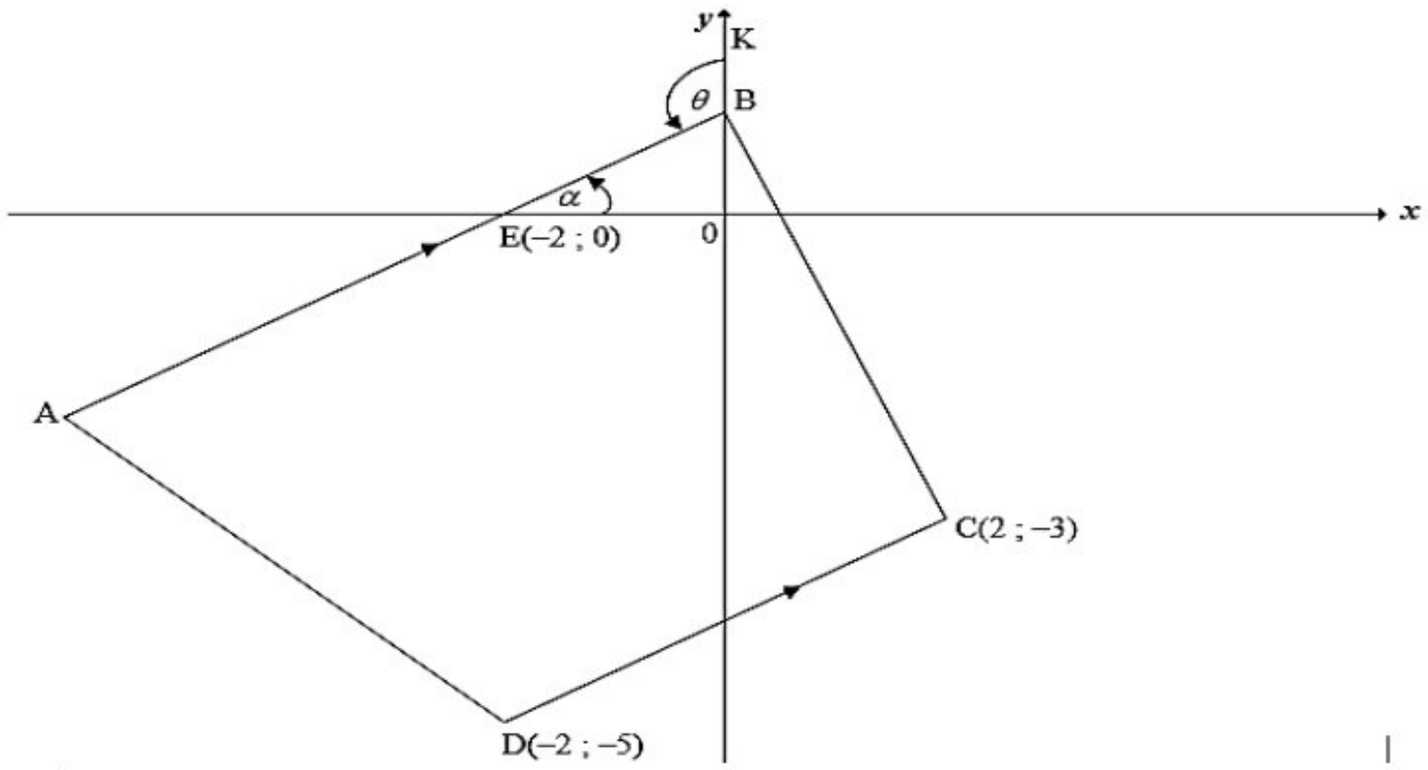
1.5 . $\tan(90° - \theta)$. (3) **L2**

1.6 The area of ΔRSN without using a calculator. (6) **L3**

[16]

DBE MAY-JUNE 2019 QUESTION 3

2. In the diagram, A, B, C (2; a – 3) and D(–2 ; -5) are vertices of a trapezium with AB || DC.
E (–2; 0) is the x-intercept of AB. The inclination of AB is α. K lies on the y-axis and $\hat{KBE} = \theta$



2.1 Determine:

2.1.1 The midpoint of EC. (2) **L1**

2.1.2 The gradient of DC. (2) **L1**

2.1.3 The equation of AB in the form $y = mx + c$. (3) **L2**

2.1.4 The size of θ. (3) **L3**

2.1.5 Prove that AB $\perp$ BC. (3) **L2**

2.2 The points E, B and C lie on the circumference of a circle. Determine:

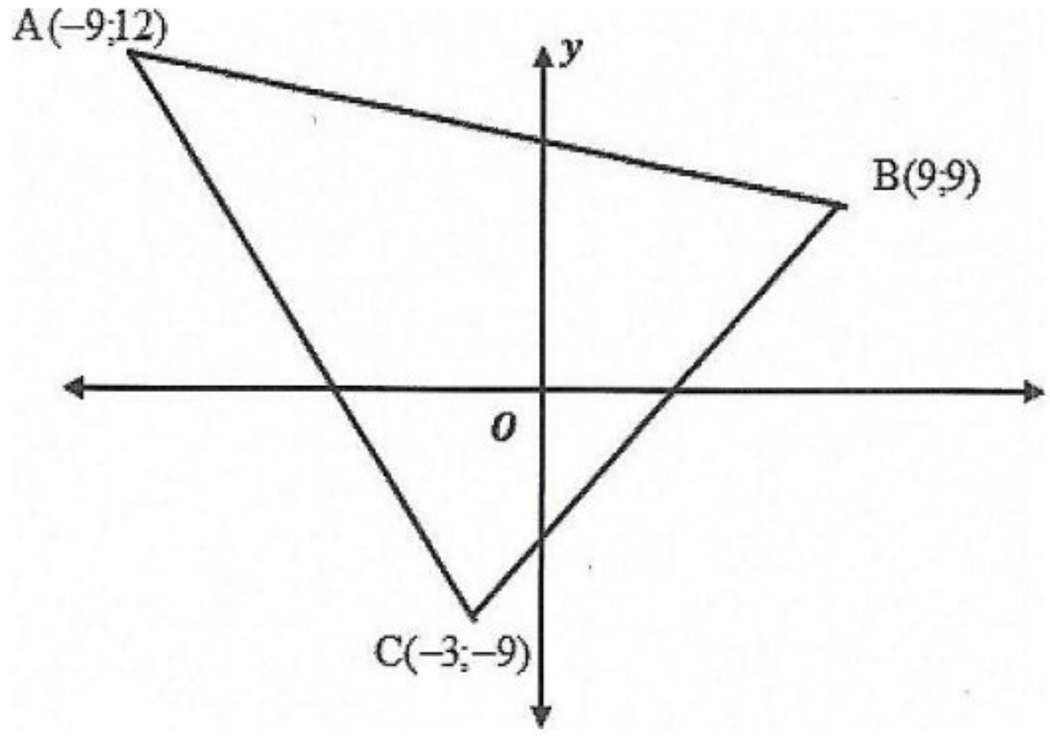
2.2.1 The centre of the circle. (1) **L1**

2.2.2 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (4) **L2**

[18]

LIMPOPO TRIAL 2019 QUESTION3

3. In the diagram A$(-9;12)$, B$(9;9)$ and C$(3;-9)$ are the vertices of ΔABC. K (m; n) is a point in the second quadrant.



3.1 Calculate the gradient of AB. (2) **L1**

3.2 Calculate the size of $\hat{B}$, rounded off to two decimal digits. (5) **L3**

3.3 Determine the coordinates of M, the midpoint of BC. (2) **L1**

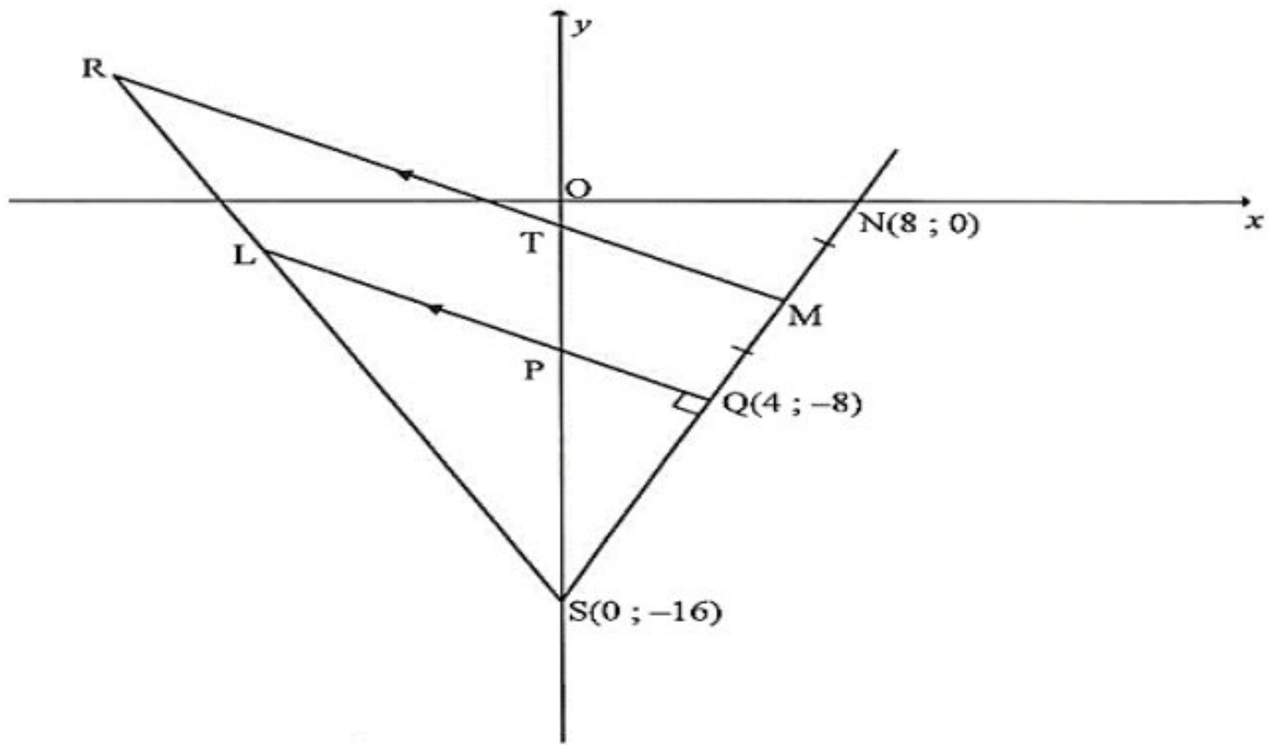
3.4 Determine the equation of AM. (3) **L2**

3.5 Determine the coordinates of K, if A, K and M are collinear and $BK = 5\sqrt{5}$ units (8) **L3**

[20]

DBE MAY-JUNE 2021 QUESTION 3

4 In the diagram, S (0; −16), L and Q(4;-8) are the vertices of ΔSLQ having LQ perpendicular to SQ. SL and SQ are produced to points R and M respectively such that RM || LQ. SM produced cuts the *x*-axis at N(8;0). QM = MN T and P are the y-intercepts of RM and LQ respectively.



4.1 Calculate the coordinates of M. (2) **L1**

4.2 Calculate the gradient of NS. (2) **L1**

4.3 Show that the equation of line LQ is $y = -\frac{1}{2}x - 6$ (3) **L2**

4.4 Determine the equation of a circle having centre at O, the origin, and passing through S. (2) **L2**

4.5 Calculate the coordinates of T. (3) **L3**

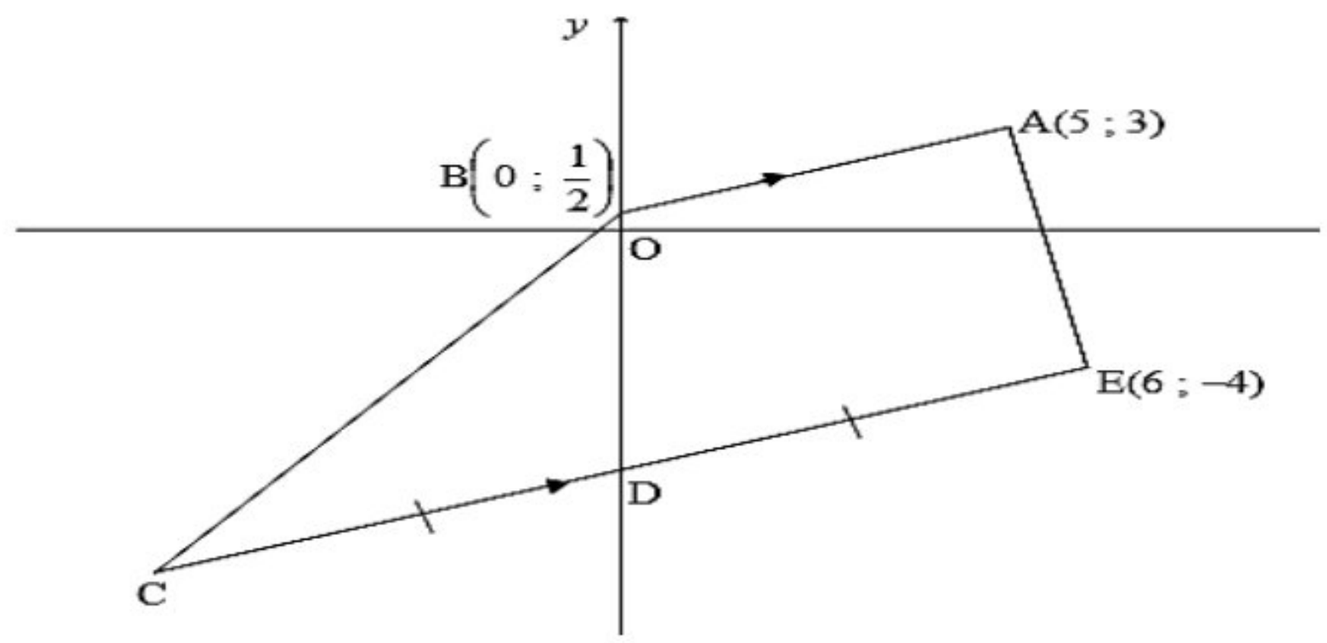
4.6 Determine $\frac{LS}{RS}$. (3) **L3**

4.7 Calculate the area of PTMQ. (4) **L3**

[19]

DBE MAY-JUNE 2022 QUESTION 3

5. In the diagram, A (5; 3). $B\left(0\ ;\ \frac{1}{2}\right)$. C and $E(6;-4)$ are the vertices of a trapezium having BA || CE. D is the *y*-intercept of CE and CD = DE.



5.1 Calculate the gradient of AB. (2) **L1**

5.2 Determine the equation of CE in the form $y = mx + c$. (3) **L2**

5.3 Calculate the Coordinates of C. (3) **L2**

5.2 Calculate the area of quadrilateral ABCD. (4) **L3**

5.3 If point K is the reflection of E in the *y*-axis:

5.3.1 Write down the coordinates of K (2) **L2**

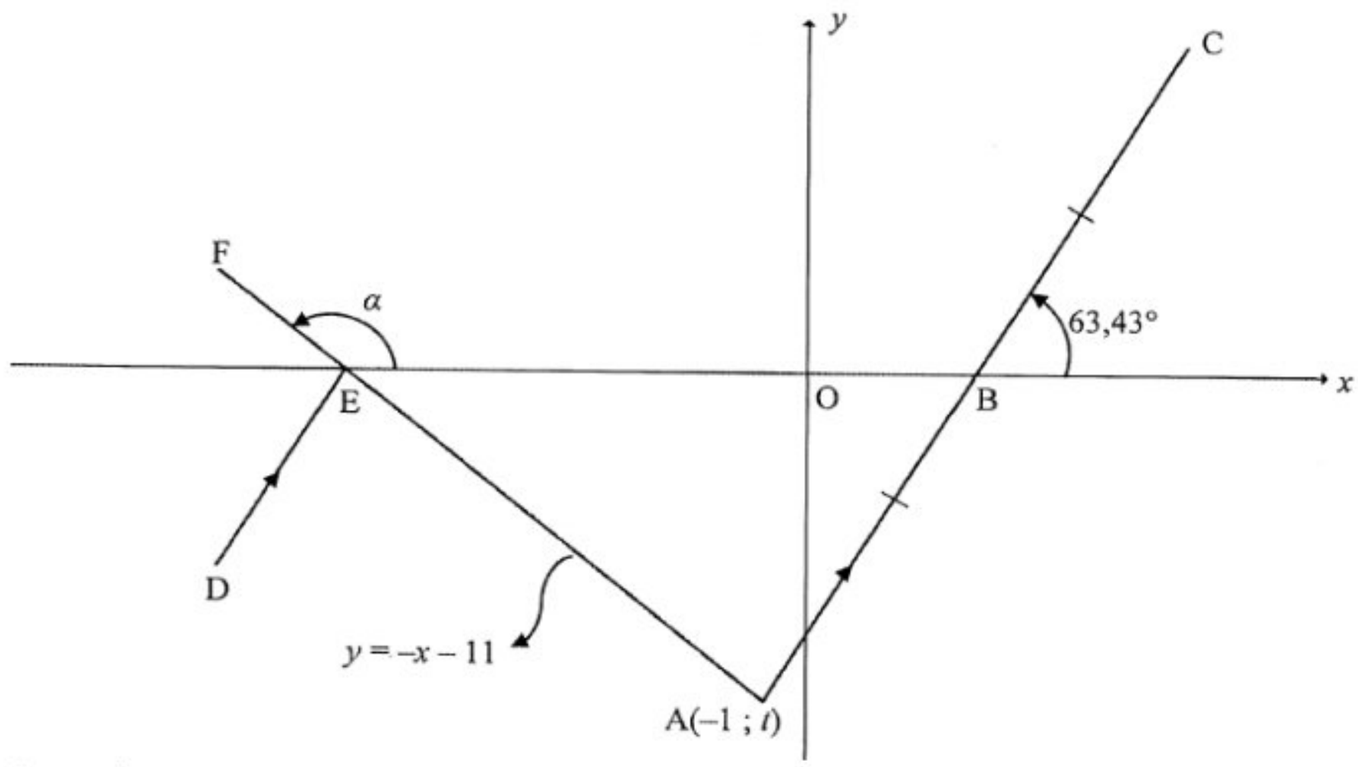
5.3.2 Calculate the perimeter of ΔKEC (4) **L2**

5.2.3 Calculate the size of $\hat{KCE}$ (3) **L2**

[21]

DBE MAY- JUNE 2023 QUESTION3

6 In the diagram, the equation of line AF is $y=-x-11$. B, a point on the x-axis, is the midpoint of the straight line joining A$(-1;\ t)$ and C.The angles of inclination of AF and AC are a and $63,43°$ respectively. AF cuts the x-axis in E. D is a point such that DE || AC.



6.1. Calculate the:

6.1.1 Value of t (2) **L1**

6.1.2 Size of α (2) **L2**

6.1.3 Gradient of AC, to the nearest whole number (2) **L2**

6.2 Determine the equation of AC in the form $y=mx+k$. (2) **L1**

6.3 Calculate the:

6.3.1 Coordinates of C (3) **L2**

6.3.2 Size of $F\hat{E}D$ (3) **L3**

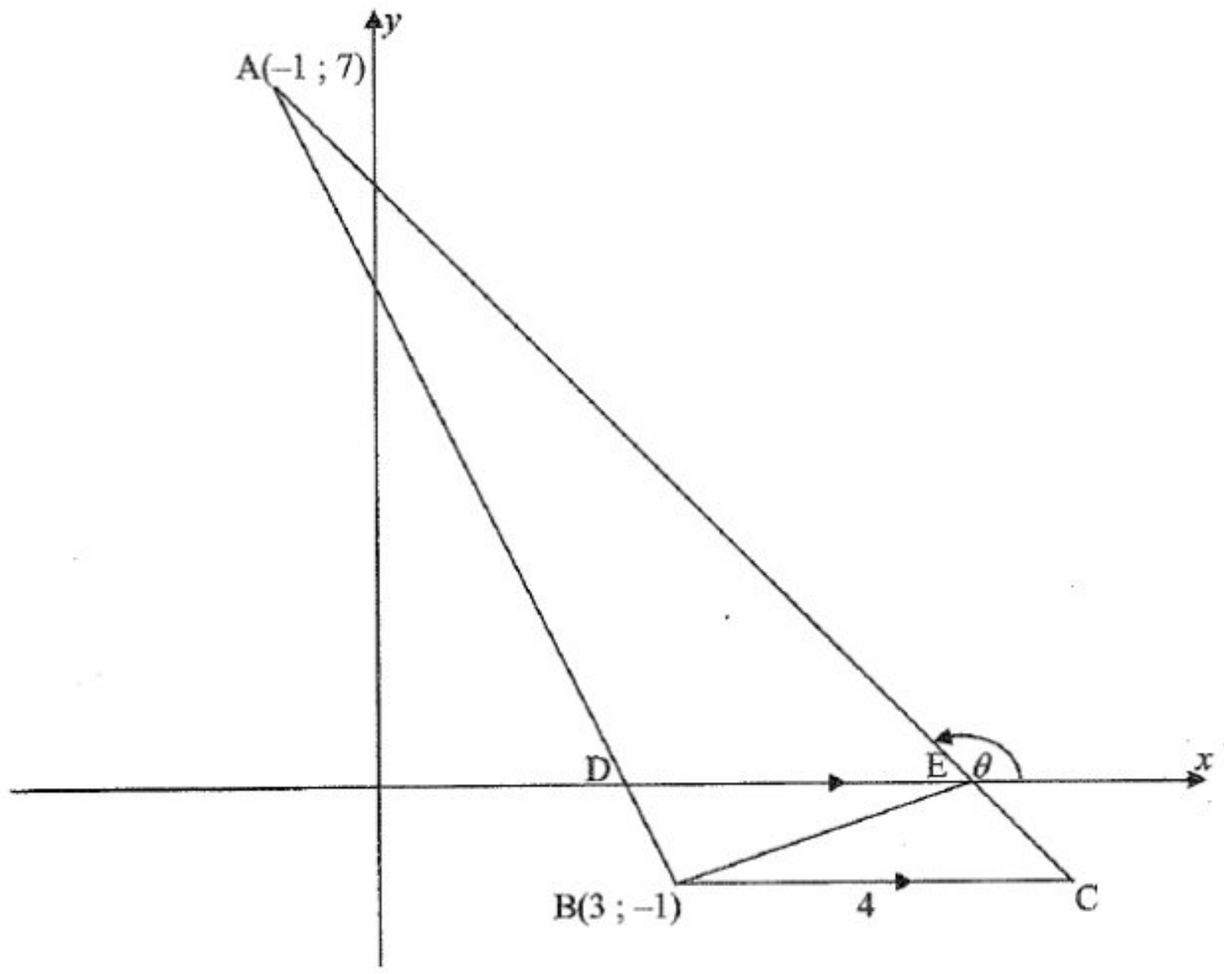
6.4 G is a point such that EAGC, in that order, is a parallelogram.

Determine the equation of a circle centred at G and passing through the point B. Write your answer in the form $(x-a)^2+(y-b)^2=r^2$. (4) **L3**

[18]

NORTH WEST SEPT 2022 QUESTION 3

7. In the diagram below, A$(-1;7)$, B$(3;-1)$ and C are vertices of ΔABC. The angle of inclination of AC is θ. BC || x - axis and BC=4 units. D and E are the x-intercepts of the lines AB and AC respectively. B and E are joined.



7.1 Write down the equation of the line BC. (1) **L1**

7.2 Write down the x-coordinate of C. (1) **L1**

7.3 Determine the equation of the line AC. (4) **L2**

7.4 Calculate the size of $A\hat{C}B$. (4) **L2**

7.5 If point K is the reflection of E in the line BC:

7.5.1 Write down the coordinates of K. (3) **L2**

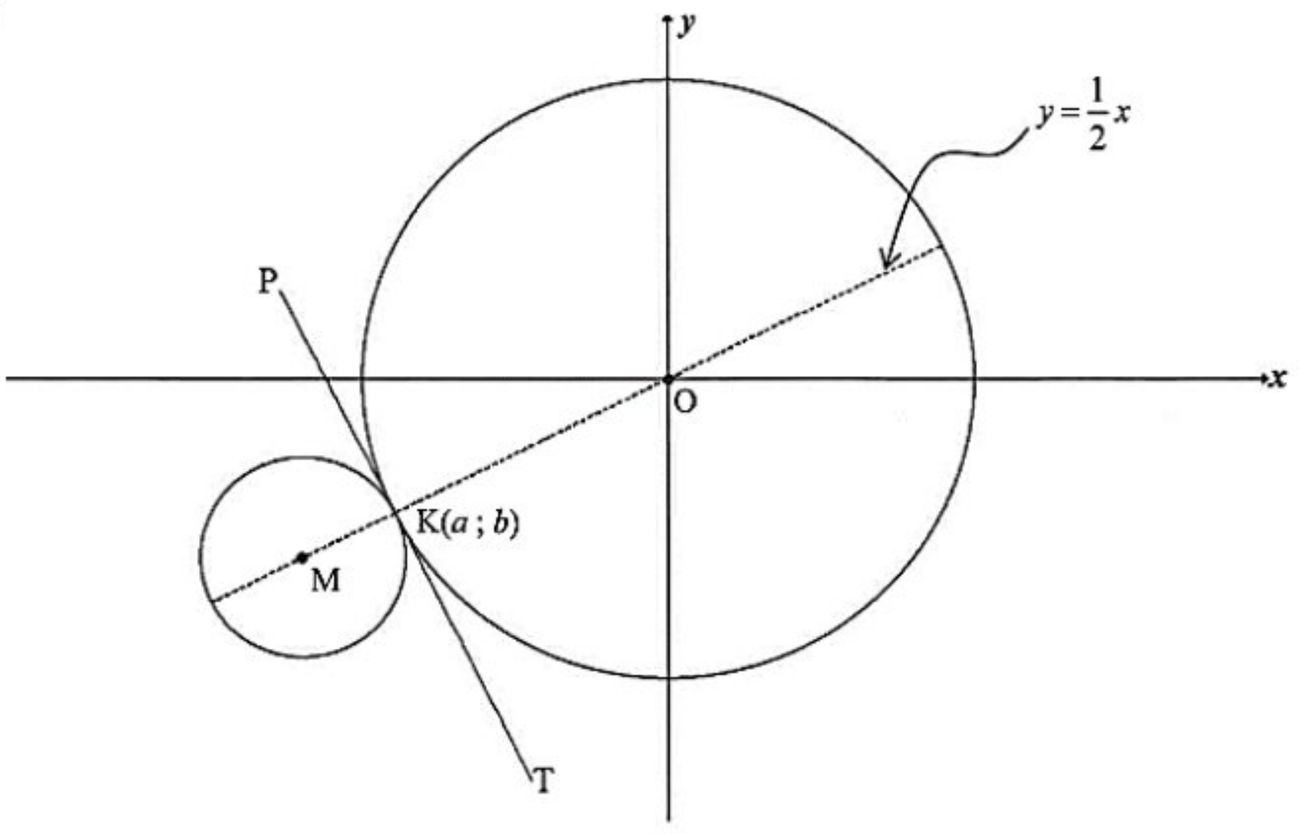
7.5.2 Calculate the area of quadrilateral BECK. (3) **L3**

7.6 F is a point in the first quadrant, such that it forms an equilateral ΔBFC. Calculate the coordinates of F. (5) **L4**

[21]

DBE FEB - MARCH 2018 QUESTION 4

8 In the diagram, PKT is a common tangent to both circles at K (a; b). The centres of both circles lie on the line $y = \frac{1}{2}x$. The equation of the circle centred at O is $x^2 + y^2 = 180$. The radius of the circle is three times that of the circle centred at M.



8.1 Write down the length of OK **in surd form.** (1) **L1**

8.2 Show that K is the point $(-12;-6)$ (4) **L2**

8.3 Determine:

8.3.1 The equation of the common tangent, PKT, in the form $y = mx + c$ (3) **L2**

8.3.2 The coordinates of M (6) **L2**

8.3.3 The equation of the smaller circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2) **L2**

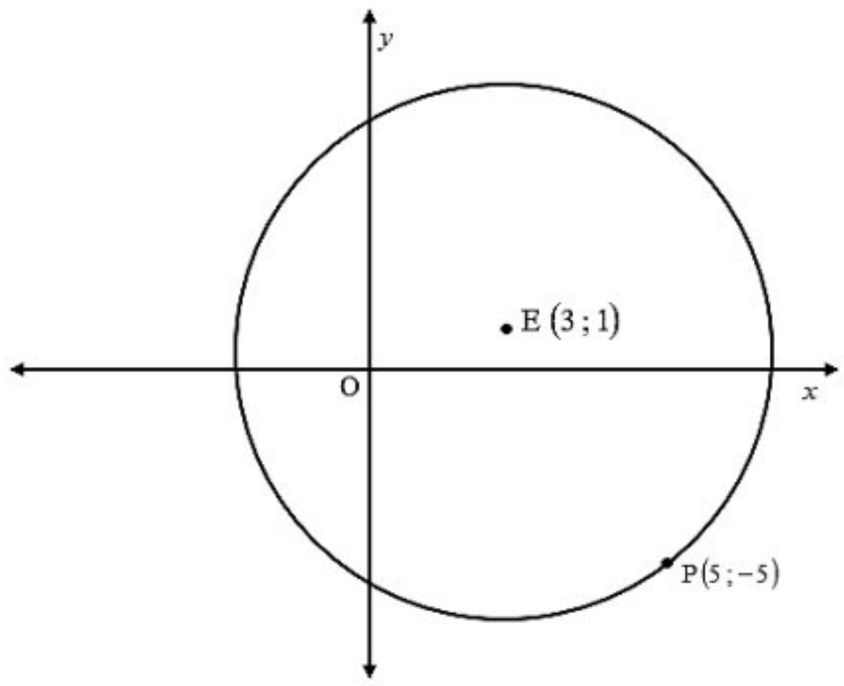
8.4 For which value(s) of r will another circle, with equation $x^2 + y^2 = r^2$, intersect the circle centred at M at two distinct points? (3) **L3**

8.5 Another circle, $x^2 + y^2 + 32x + 16y + 240 = 0$ is drawn. Prove by calculation that this circle does NOT cut the circle with centre $M(-16;-8)$ (5) **L4**

[24]

FS PREP EXAM 2019 QUESTION4

9 In the diagram below, the circle centred at E $(3\,;1)$ passes through point P(5; -5).



9.1 Determine the equation of:

9.1.1 The circle in the form $x^2 + y^2 + Ax + By + C = 0$. (4) **L2**

9.1.2 The tangent to the circle at $P(5\,;-5)$ in the form $y = mx + c$ (5) **L2**

9.2 A smaller circle is drawn inside the circle. Line EP is a diameter of the small circle. Determine the:

9.2.1 Coordinates of the centre of the smaller circle. (3) **L2**

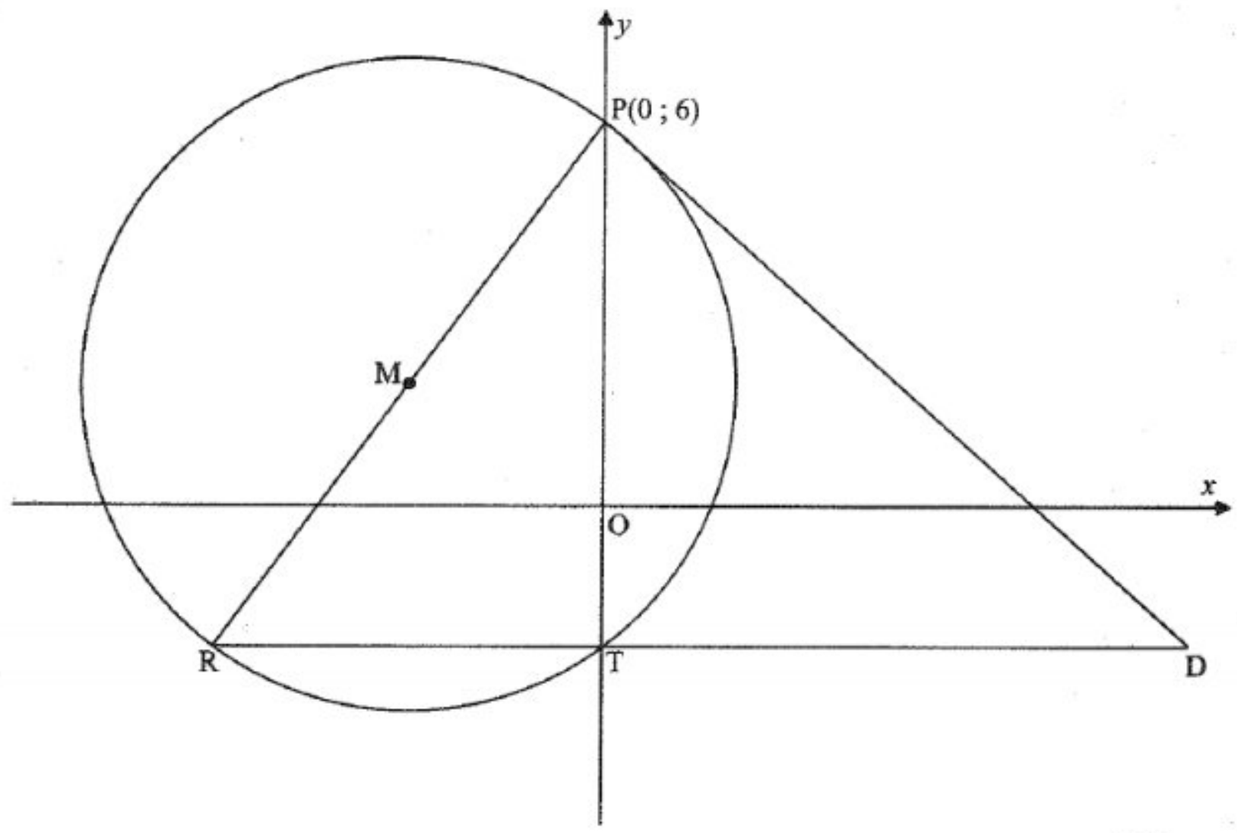
9.2.2 Length of the radius. (3) **L2**

9.3 Hence, or otherwise, determine whether point $C(9\,;3)$ lies inside or outside the circle centre at E. (3) **L3**

[18]

NORTH WEST SEPT 2022 QUESTION 4

10 In the diagram below, a circle with centre M and a tangent PD to the circle at P (0 ; 6) are drawn. The equation of the circle is given as $x^2 + 6x + y^2 - 4y - 12 = 0$. PR is a diameter, T is a y-intercept and chord is produced to D. RTD $\parallel$ x-axis.



10.1 Determine the coordinates of M. (3) **L2**

10.2 Determine the length of PR. (2) **L2**

10.3 Determine the equation of the tangent PD. (3) **L3**

10.4 Determine the coordinates of D. (4) **L3**

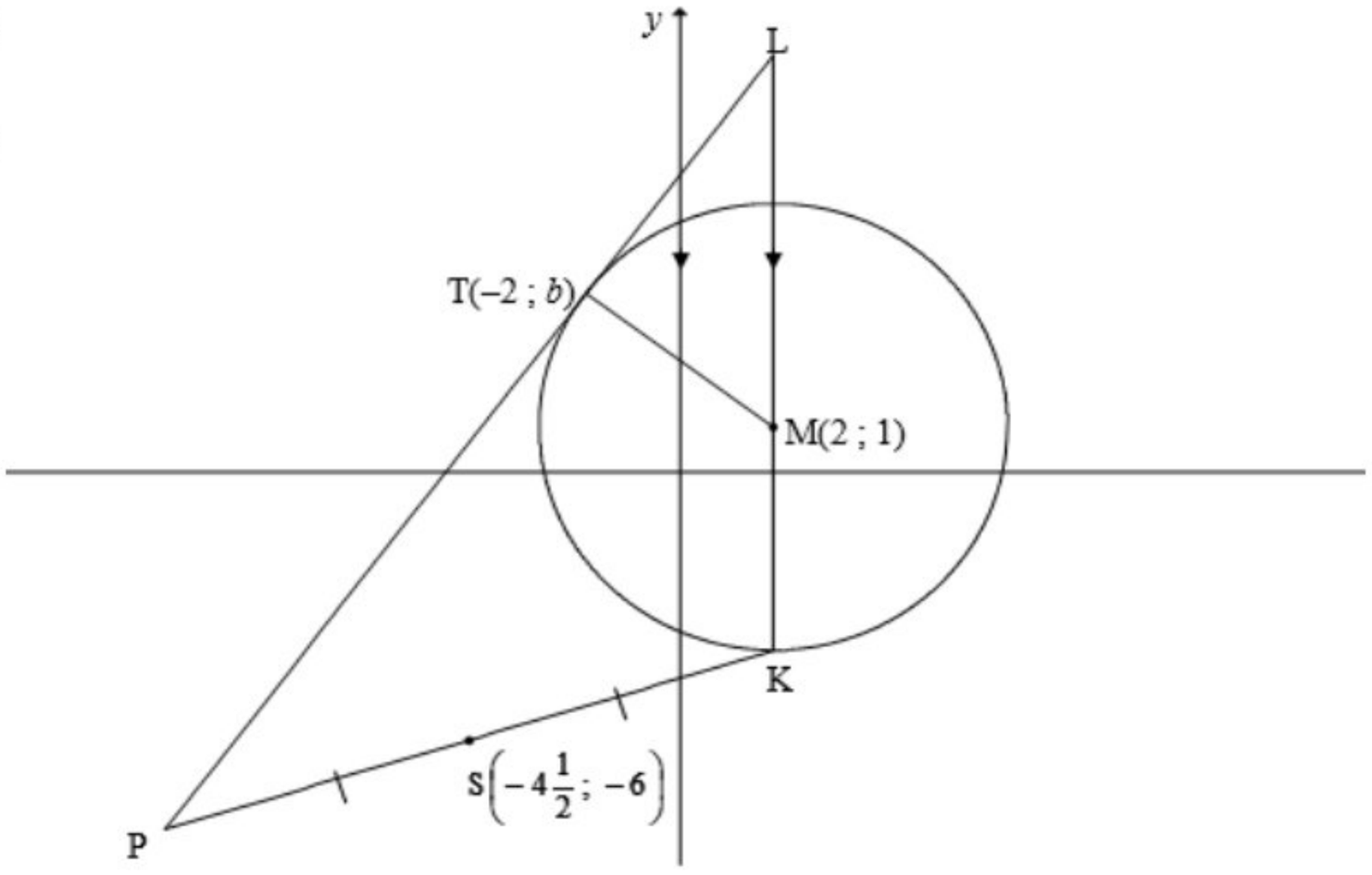
10.5 The circle, having centre M, is translated to the right such that the point D lies on the translated circle. Determine the coordinates of the possible centres of the translated circle. (4) **L4**

[16]

DBE MAY- JUNE 2019 QUESTION4

11 In the diagram, the circle is centred at M (2; 1). Radius KM is produced to L, a point outside the circle, such that KML || *y*-axis. LTP is a tangent to the circle at T (–2; *b*).

$S\left(-4\frac{1}{2};\ -6\right)$ is the midpoint of PK.



11.1 Given that the radius of the circle is 5 units, show that $b = 4$. (4) **L2**

11.2 Determine:

11.2.1 The coordinates of K (2) **L2**

11.2.2 The equation of the tangent LTP in the form $y = mx + c$ (4) **L2**

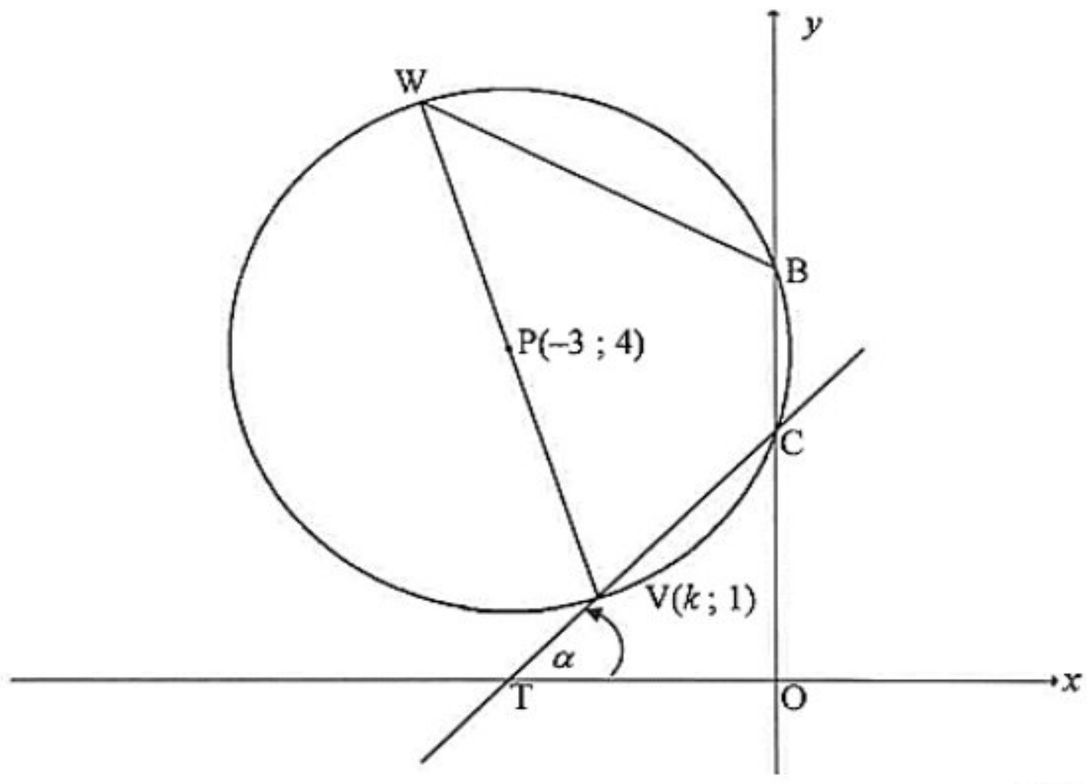
11.2.3 The area of ΔLPK (7) **L3**

11.3 Another circle with equation $(x-2)^2 + (y-n)^2 = 25$ is drawn. Determine, with an explanation, the value(s) of *n* for which the two circles will touch each other externally. (4) **L3**

[21]

DBE MAY- JUNE 2021 QUESTION4

12 In the diagram, P (-3; 4) is the centre of the circle. $V(k;1)$ and W are the endpoints of a diameter. The circle intersects the y-axis at B and C. BCVW is a cyclic quadrilateral. CV is produced to intersect the *x*- axis at T. $O\hat{T}C = \alpha$



12.1 The radius of the circle is $\sqrt{10}$. Calculate the value of k if point V is to the right of point P. Clearly show ALL calculations. (5) **L3**

12.2 The equation of the circle is given as $x^2+6x+y^2-8y+15=0$ Calculate the length of BC. (4) **L2**

12.3 If $k=-2$, calculate the size of:

12.3.1 α (3) **L2**

12.3.2 $\hat{VWB}$ (2) **L3**

12.4 A new circle is obtained when the given circle is reflected about the line $y=1$.Determine the:

12.4.1 Coordinates of Q, the centre of the new circle (2) **L2**

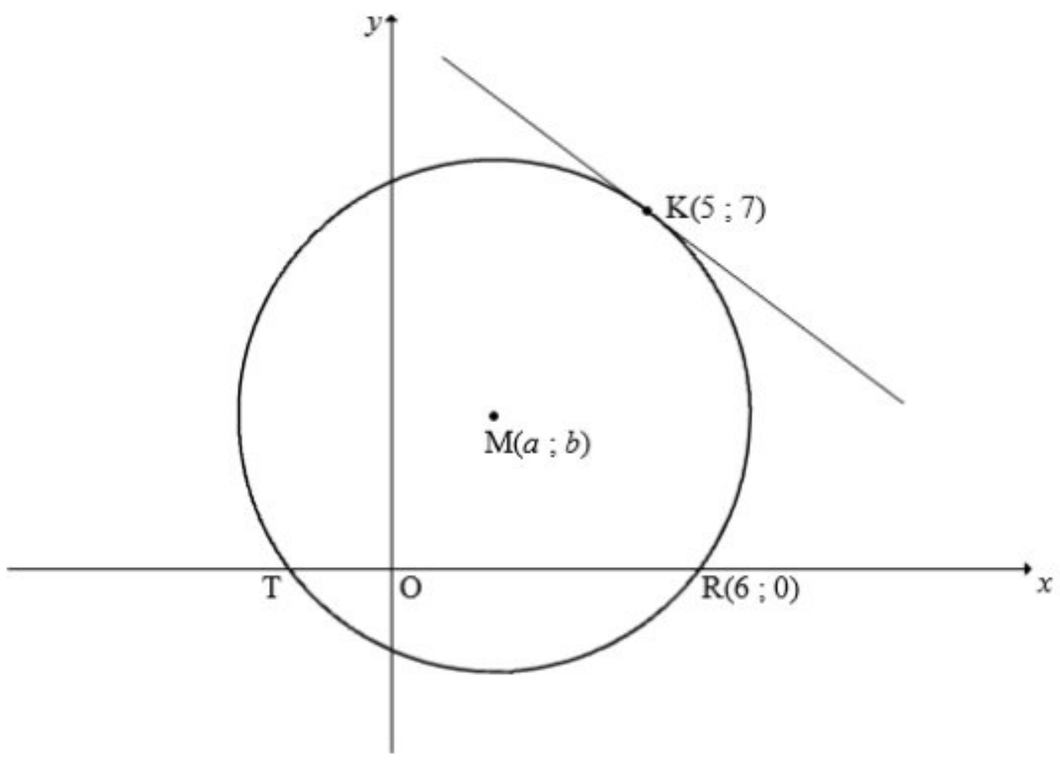
12.4.2 Equation of the new circle in the form $(x-a)^2+(y-b)^2=r^2$ (2) **L2**

12.4.3 Equations of the lines drawn parallel to the y-axis and passing through the points of intersection of the two circles. (2) **L3**

[20]

DBE MAY- JUNE 2022 QUESTION4

13 In the diagram, the circle centred at M (a; b) is drawn. T and R (6; 0) are the x-intercepts of the circle. A tangent is drawn to the circle at K (5 ; 7).



13.1. M is a point on the line $y=x+1$.

13.1.1 Write b in terms of a. (1) **L1**

13.1.2 Calculate the coordinates of M. (5) **L3**

13.2 If the coordinates of M are (2; 3), calculate the length of:

13.2.1 The radius of the circle (2) **L2**

13.2.2 TR (2) **L2**

13.3 Determine the equation of the tangent to the circle at K. Write your answer in the form $y=mx+c$. (5) **L2**

13.4. A horizontal line is drawn as a tangent to the circle M at the point N (c ; d), where $d<0$.

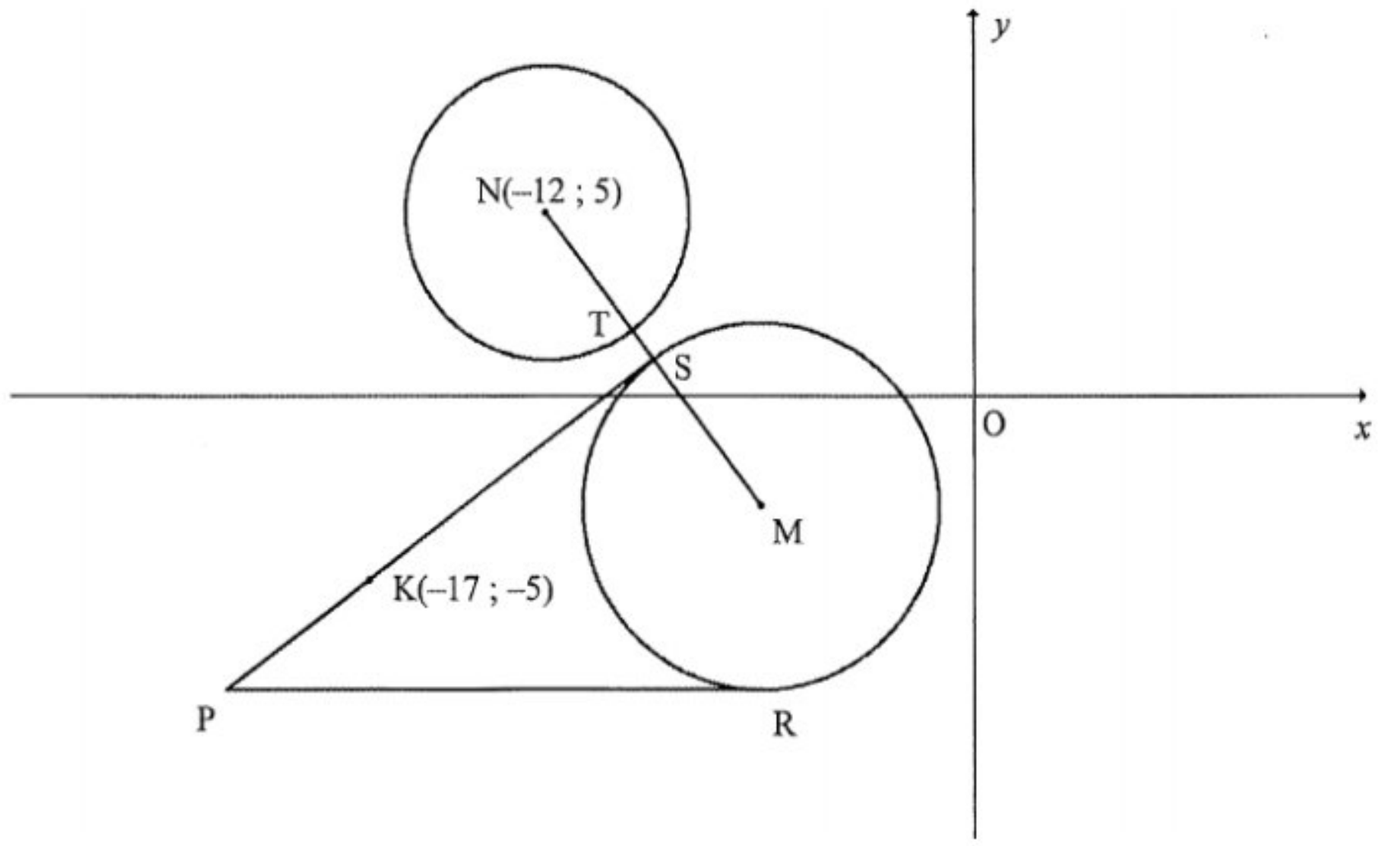
13.4.1 Write down the coordinates of N. (2) **L2**

13.4.2 Determine the equation of the circle centred at N and passing through T. Write your answer in the form $(x-a)^2+(y-b)^2=r^2$. (3) **L2**

[20]

DBE MAY- JUNE 2023 QUESTION4

14 In the diagram, the equation of the circle centred at $(-12;5)$ is $x^2+y^2+24x-10y+153=0$. The equation of the circle centred at M is $(x+6)^2+(y+3)^2=25$. PS and PR are tangents to the circle centred at M at S and R respectively. PR is parallel to the x-axis. K$(-17;-5)$ is a point on PS. The straight line joining N and M cuts the smaller circle at T and the larger circle at S.



14.1 Write down the coordinate of M. (2) **L1**

14.2 Calculate the:

14.2.1 Length of the radius of the smaller circle (2) **L2**

14.2.2 Length of TS (4) **L2**

14.3 Determine the equation of the tangent:

14.3.1 PR (2) **L2**

14.3.2 PS, in the form $y = mx + c$ (5) **L3**

14.4 Quadrilateral PSMR is drawn. Calculate the:

14.4.1 Perimeter of PSMR (5) **L3**

14.4.2 Ratio of $\dfrac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$ (2) **L3**

[22]

TOPIC TRIGONOMETRY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

1. Definitions of trig ratios:

In a right angled triangle: $\sin\theta = \frac{opposite}{hypotenuse}$; $\cos\theta = \frac{adjacent}{hypotenuse}$ and $\tan\theta = \frac{opposite}{adjacent}$

SOH CAH TOA helps you to remember these definitions.

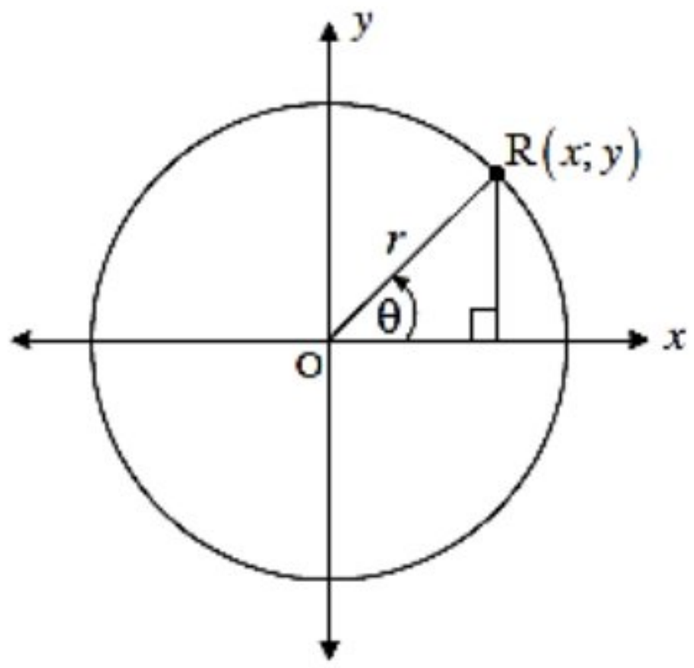
In a Cartesian plane:

$\sin\theta = \frac{y}{r}$;

$\cos\theta = \frac{x}{r}$;

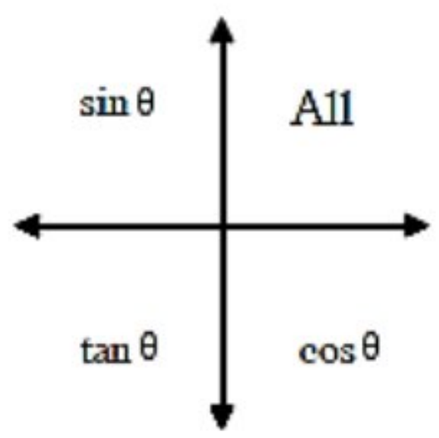
$\tan\theta = \frac{y}{x}$

and $r^2 = x^2 + y^2$



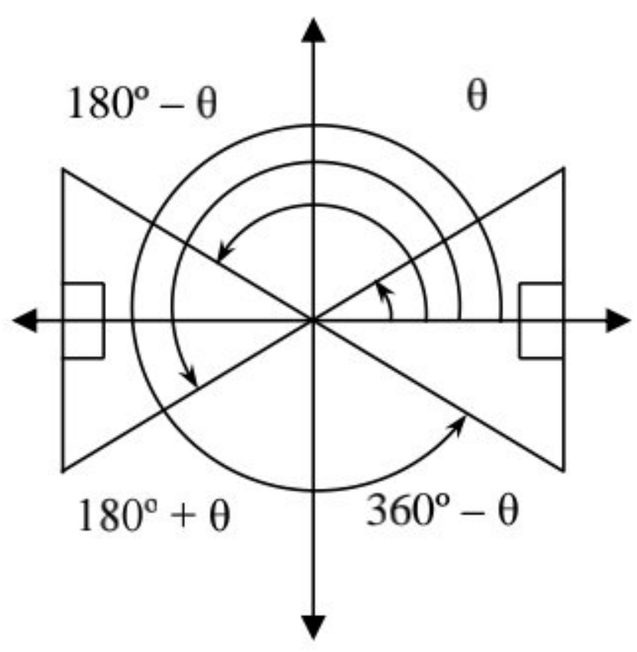
2. CAST Rule:

All trig ratios are positive in the 1st quadrant. **All**
Only $\sin\theta$ is positive in the 2nd quadrant. **Students**
Only $\tan\theta$ is positive in the 3rd quadrant. **Take**
Only $\cos\theta$ is positive in the 4th quadrant. **Care**



3. Reduction Formulae:

If θ is an acute angle, i.e. in the 1st quadrant,
$180° - \theta$ will lie in the 2nd quadrant,
$180° + \theta$ will lie in the 3rd quadrant,
and $360° - \theta$ will lie in the4th quadrant.



$\sin\theta = \sin(180^\circ - \theta) = -\sin(180^\circ + \theta) = -\sin(360^\circ - \theta)$

$\cos\theta = -\cos(180^\circ - \theta) = -\cos(180^\circ + \theta) = \cos(360^\circ - \theta)$

$\tan\theta = -\tan(180^\circ - \theta) = \tan(180^\circ + \theta) = -\tan(360^\circ - \theta)$

For $90° - \theta$ and $90° + \theta$ the ratio changes to its co-function.
The co-function of cos is sin and the co-function of sin is cos.

$\sin(90^\circ - \theta) = \cos\theta$; and $\cos(90^\circ - \theta) = \sin\theta$

$\sin(90^\circ + \theta) = \cos\theta$; and $\cos(90^\circ + \theta) = -\sin\theta$

Trigonometric identities:

Square identity: $\sin^2\theta + \cos^2\theta = 1$

Quotient identity: $\frac{\sin\theta}{\cos\theta} = \tan\theta$

Compound Angles:

$\sin(\theta \pm \beta) = \sin\theta\cos\beta \pm \cos\theta\sin\beta$

$\cos(\theta \pm \beta) = \cos\theta\cos\beta \mp \sin\theta\sin\beta$

Double Angles:

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

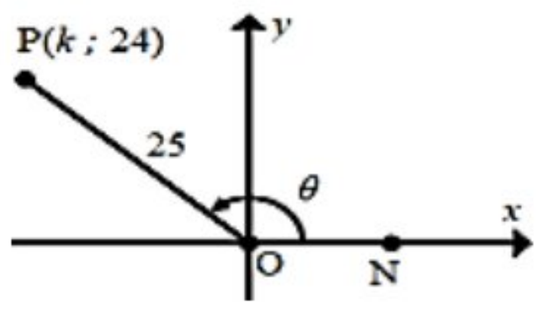
$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

REVISION QUESTIONS

1. DBE GR11 NOV 2013:

In the diagram alongside, P(k; 24) is a point in the second quadrant such that: $OP = 25$ units. N is a point on the positive x-axis and $P\hat{O}N = \theta$.



Without calculating the size of θ, determine the value of
The following:

1.1 k (1) **L1**

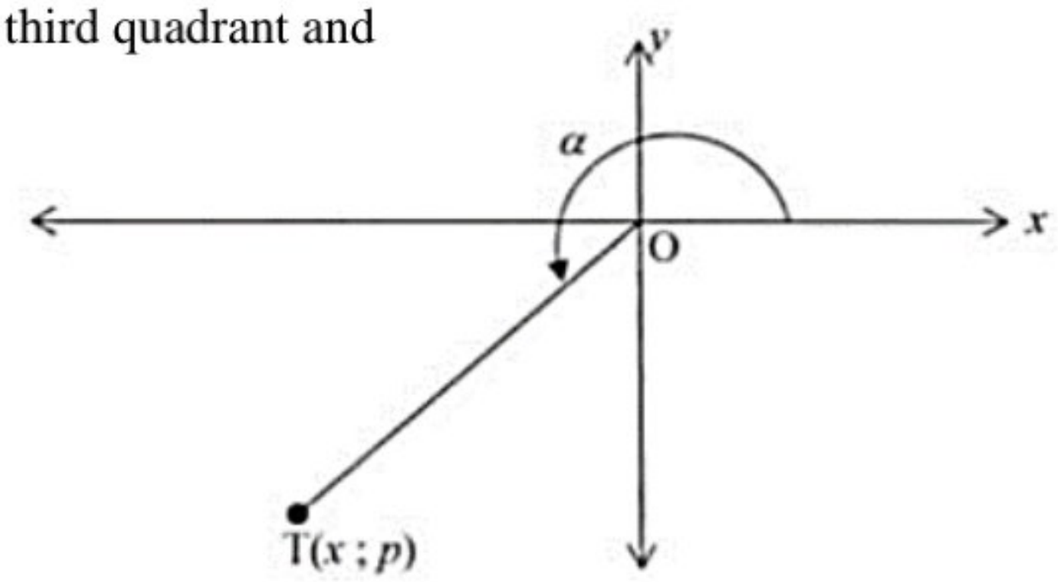
1.2 $\sin\alpha$ if $\theta+\alpha=360°$ (3) **L2**

1.3 $\tan\theta$ (1) **L1**

1.4 $\cos^2\theta-\sin^2\alpha$ (3) **L2**

2. **DBE FEB/ MARCH 2015**

In the diagram alongside, T(x ; p) is a point in the third quadrant and it is given that $\sin\alpha = \frac{p}{\sqrt{1+p^2}}$.



2.1 Show that $x=-1$ (2) **L2**

2.2 Write $\cos(180°+\alpha)$ in term of p in its simplest form (2) **L2**

2.3 Show that $\cos 2\alpha$ can be written as $\frac{1-p^2}{1+p^2}$ (3) **L3**

3. If $5\cos x = 4$ and $x\in[90°;360°]$, determine the value of $3\sin 2x$ without using a calculator. (5) **L3**

4. If $5\cos\theta-3=0$; $180°<\theta<360°$ and $17\sin\alpha=8$; $\cos\alpha<0$, determine without using a calculator:

4.1 $\cos(90°+\theta)$ (4) **L3**

4.2 $\tan\alpha+\tan\theta$ (4) **L2**

4.3 $\cos(\theta-\alpha)$ (3) **L2**

5. Given $\sin\hat{A}=\frac{6}{10}$ with $90°<\hat{A}<360°$ and $\cos 42° = p$, determine, without the use of a calculator, the value of $\cos(-A-42°)$ in terms of p. (6) **L3**

6. If $4\tan\theta=3$ and $180°<\theta<360°$, determine without using a calculator and with the aid of a diagram.

6.1 $\sin\theta+\cos\theta$ (4) **L2**

6.2 $\tan 2\theta$ (5) **L3**

7. If $\sin 2\theta=\frac{\sqrt{5}}{3}$ with $90°<2\theta<270°$, determine without the use of a calculator the value of:

7.1 $\sin\theta\cos\theta$ (2) **L2**

7.2 $\sin\theta$ (3) **L3**

8. If $\sin 36° = k$, determine the following in terms of k.

8.1 $\cos(-36°)$ (2) **L2**

8.2 $\cos 72°$ (3) **L2**

8.3 $\sin 72°$ (3) **L2**

8.4 $\cos 126°.\tan(-1116°)$ (3) **L2**

9. If $\sin 12° = p$, determine the following in terms of p:

9.1 $\tan 12°$ (2) **L1** 9.2 $\sin 24°$ (2) **L2**

9.3 $\sin 57°$ (4) **L3** 9.4 $\sin 6°$ (3) **L3**

10. **DBE JUNE 2023 QUESTION 5.2:**

Given that $\cos 20° = p$. Without using a calculator, write each of the following in terms of p: (4) **L3**

10.1 $\cos 200^\circ$ (2) **L1** 10.2 $\sin(-70^\circ)$ (2) **L2**

10.3 $\sin 10°$ (3) **L3**

11. Given $\cos 20° = p$ and $\sin 14° = q$. Without using a calculator, calculate the value of the following in terms of p or q.

11.1 $\sin 20°$ (2) **L1** 11.2 $\cos 6°$ (4) **L2**

12. If $\sin 38°\cos 10° = p$ and $\cos 38°\sin 10° = q$, determine in terms of p and q the value of:

12.1 $\sin 48°$ (3) **L2** 12.2 $\sin 28°$ (3) **L2**

13. **DBE JUNE 2023 QUESTION 5.3**

Determine, without using a calculator, the value of:

$\cos(A+55^\circ)\cos(A+10^\circ)+\sin(A+55^\circ)\sin(A+10^\circ)$ (3) **L2**

14. Evaluate the following trigonometric expressions without using a calculator

14.1 $\dfrac{\sin 140^\circ.\tan 315^\circ}{\cos 230^\circ \sin 420^\circ}$ (6) **L2**

14.2 $\dfrac{\sin 75^\circ.\cos 75^\circ}{\cos(45^\circ-2x).\cos 2x-\sin(45^\circ-2x).\sin 2x}$ (6) **L3**

14.3 $\dfrac{\tan 300^\circ+\cos(90^\circ+x)}{\sin(180^\circ-x)+2\cos(-30^\circ)}$ (6) **L3**

15. Simplify the following trigonometric expressions into a single trigonometric ratio.

15.1 $\dfrac{\sin(-x).\sin(x-180°).\sin 35°}{\cos(360°+x).\cos(90°-x).\cos 55°}$ (6) **L2**

15.2 $\sin x.\tan x+\cos x$ (3) **L2**

15.3 $\dfrac{\cos x}{1-\sin x}-\dfrac{\cos x}{1+\sin x}$ (4) **L2**

15.4 $\dfrac{\sin 2x+\sin x}{\cos 2x+\cos x+1}$ (5) **L3**

15.5 $\dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ (5) **L3**

16. **DBE JUNE 2023 QUESTION 5.5**

Consider the following trigonometric expression: $16\sin x.\cos^3 x-8\sin x.\cos x$

16.1 Rewrite this expression as a single trigonometric ratio. (4) **L3**

16.2 For which value of x in the interval $x\in[0^\circ;90^\circ]$ will $16\sin x.\cos^3 x-8\sin x.\cos x$ have its minimum value? (1) **L3**

17. Prove the following trigonometric identities.

17.1 $\dfrac{\sin^3 x+\sin x\cos^2 x}{\cos x}=\tan x$ (2) **L2**

17.2 $(\cos\theta-\sin\theta)^2=1-\sin 2\theta$ (3) **L2**

17.3 $\dfrac{1-\sin 2x}{\cos 2x}=\dfrac{\cos x-\sin x}{\cos x+\sin x}$ (2) **L2**

17.4 $\tan\alpha+\dfrac{1}{\tan\alpha}=\dfrac{2}{\sin 2\alpha}$ (5) **L3**

17.5 $\cos 4x=1-8\sin^2 x+8\sin^4 x$ (4) **L3**

17.6 $\cos 3\theta=4\cos^3\theta-3\cos\theta$ (4) **L3**

17. **DBE JUNE 2023 QUESTION 5.4**:

Consider: $\dfrac{\cos 2x+\sin 2x-\cos^2 x}{\sin x-2\cos x}=-\sin x$

17.1 Prove the given identity. (3) **L3**

17.2 Determine the value of $\dfrac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin^2 x+6\sin x\cos x}$. (2) **L3**

18. Given: $\dfrac{2\tan x-\sin 2x}{2\sin^2 x}=\tan x$

18.1 Prove the above identity. (6) **L3**

18.2 For which value(s) of x will the above identity be undefined in the interval $-180°\le x\le 180°$? (3) **L3**

19. Given that $\dfrac{\sin 2x}{1+\cos 2x}=\tan x$

19.1 Prove the above identity. (3) **L3**

19.2 Hence, determine the value of $\tan 22,5°$ without the use of a calculator. (3) **L2**

20. Determine the general solution of the following trigonometric equations.

20.1 $\cos\theta=0,4$ (2) **L2**

20.2 $\sin(2\theta+16°)=-0,67$ (3) **L2**

20.3 $\sqrt{3}\cos\theta-3\sin\theta=0$ (3) **L2**

20.4 $2\cos^2\theta=\cos\theta$ (4) **L2**

20.5 $2\sin^2\theta-\sin\theta=1$ (4) **L2**

20.6 $\cos(2\theta+45°)=\cos(20°-\theta)$ (3) **L2**

20.7 $\cos(\theta+30°)=\sin 2\theta$ (5) **L3**

20.8 $\cos 2\theta-5\cos\theta=1-\cos^2\theta$ (6) **L4**

20.9 $\cos(\theta+30°)=-\sin 2\theta$ (6) **L3**

20.10 $\tan x=2\sin 2x$, where $\cos x<0$ (7) **L3**

21. **DBE JUNE 2023 QUESTION 5.5**

Given: $3\tan 4x=-2\cos 4x$

21.1 Without using a calculator, show that $\sin 4x=-0,5$ is the only solution to the above equation. (4) **L3**

21.2 Hence, determine the general solution of x in the equation $3\tan 4x=-2\cos 4x$. (3) **L2**

TOPIC **TRIGONOMETRIC GRAPHS**

GUIDELINES, SUMMARY NOTES, & STRATEGIES

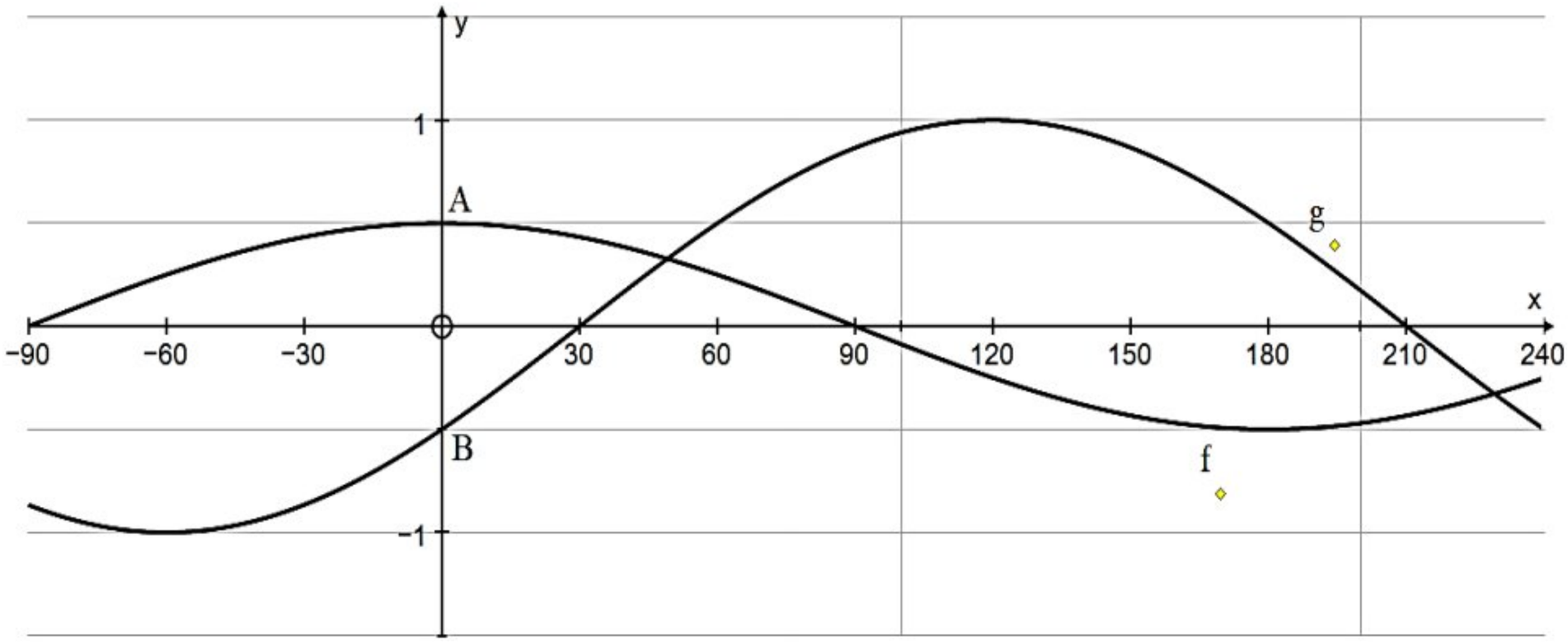
- The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.
- Candidates must be able to use and interpret functional notation. Learners must understand how $f(x)$ has been transformed to generate $f(-x)$, $-f(x)$, $f(x+a)$, $f(x)+a$ and $a.f(x)$ where $a \in \mathrm{R}$.

REVISION QUESTIONS

1. **DURBAN GIRLS' HIGH –SEPTEMBER 2019 QUESTION 7**

In the diagram below, the graphs of :

$f(x) = 2\sin x$ and $g(x) = \cos(x-30°)$, are drawn for $x \in [-180°;180°]$



1.1 Write down the range of g. (1) **L1**

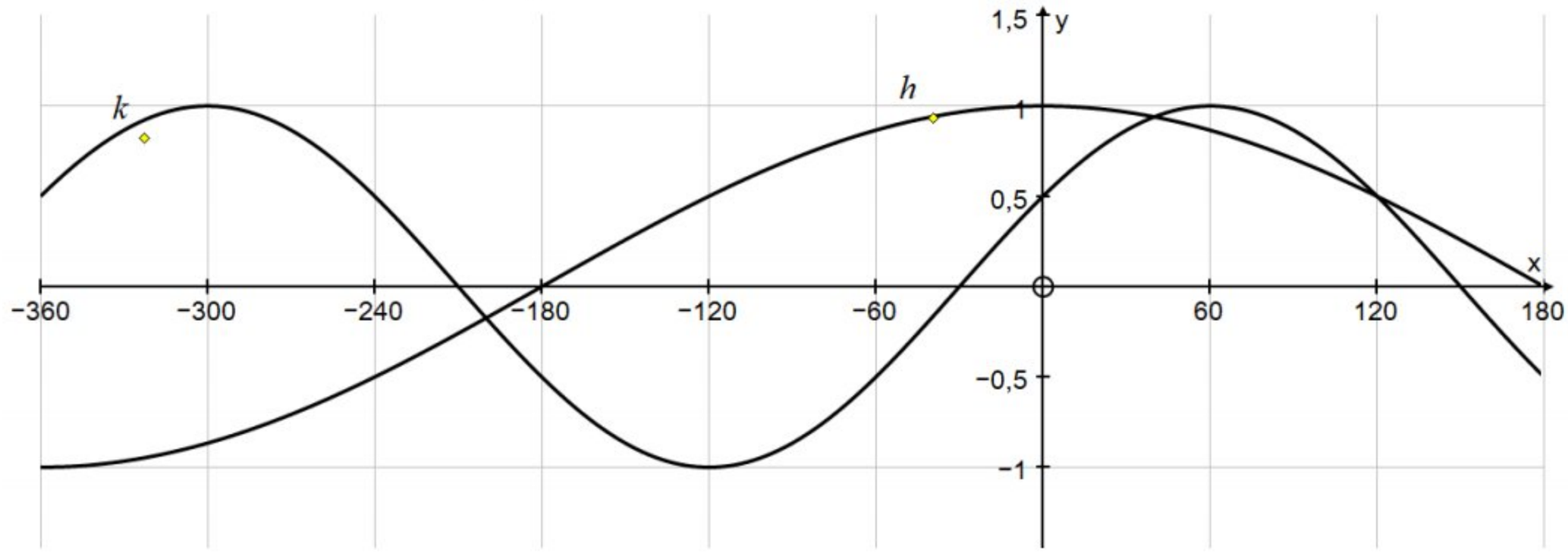
1.2 Write down the period of h if $h(x) = f\left(\frac{1}{2}x\right)$ (1) **L2**

1.3 Use your graph to write down the values of x in the interval $x \in [-180°;180°]$ for which :

1.3.1 $\frac{f(x)}{g(x)} = 1$ (2) **L3** 1.3.2 $f'(x).g(x) < 0$ (2) **L3**

2. **WESTVILLE BOYS HIGH –SEPTEMBER 2019 QUESTION 5**

The diagram below shows the graphs of $h(x) = \cos px$ and $k(x) = \sin(x+q)$ for $x \in [-360°;180°]$.



2.1 Write down the period of h. (1) **L1**

2.2 Determine the values of p and q (2) **L1**

2.3 For which values of x in the interval $x \in [-360^0; -60^0]$ is $\frac{h(x)}{k(x)} \leq 0$? (3) **L3**

3. **DBE MARCH 2018 QUESTION 5.5**

Consider $g(x) = -4\cos(x+30°)$

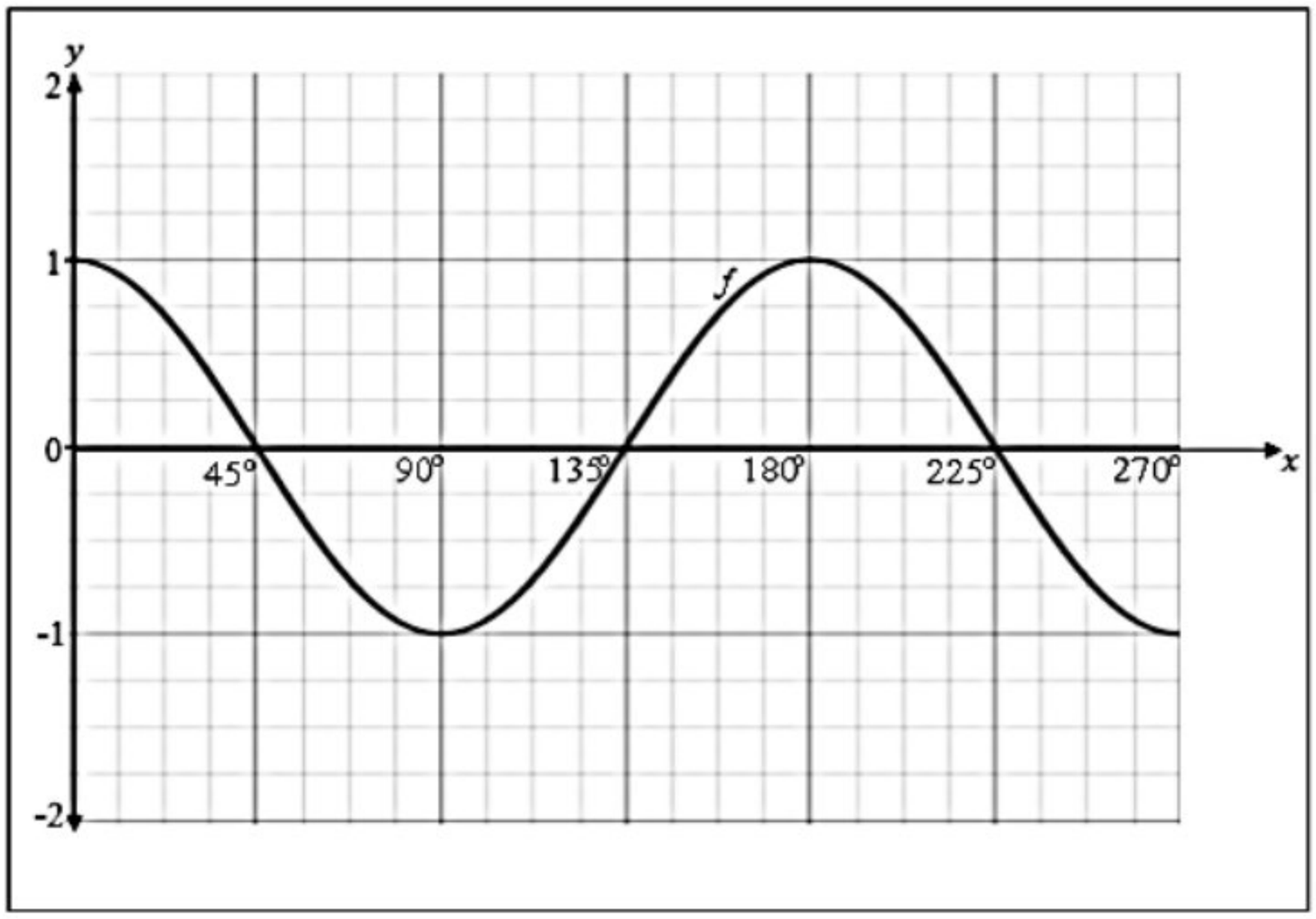
3.1 Write down the maximum value of $g(x)$. (1) **L1**

3.2 Determine the range of $g(x)+1$ (2) **L1**

3.3 The graph of g is shifted $60°$ to the left and reflected about x-axis to form a new graph h.

Determine the equation of h in its simplest form. (3) **L3**

4. In the diagram below, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [0°; 270°]$.



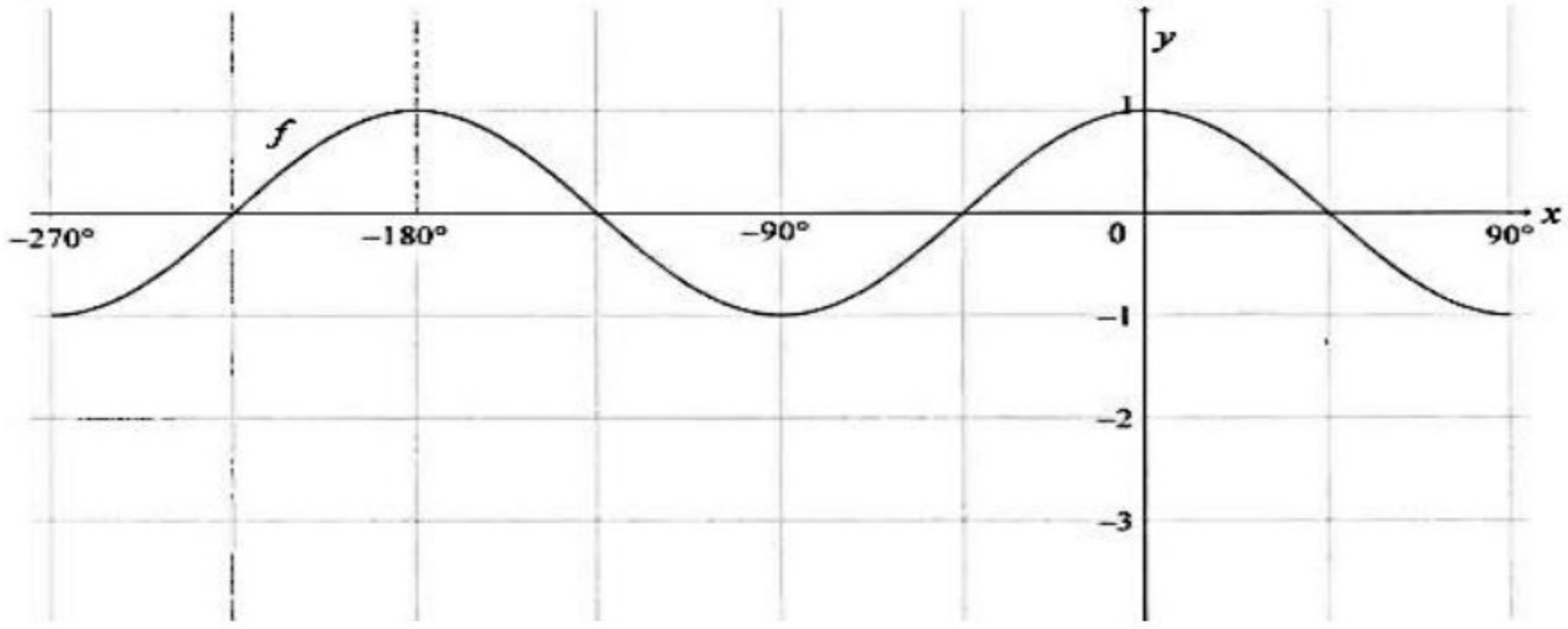
4.1 Draw the graph of $g(x) = -\frac{1}{2}\tan x$ for the interval $x \in [0°; 270°]$. Show all intercept with the axes and asymptotes. (4) **L2**

4.2 Write down the range of $h(x) = 3 - f(x)$. (1) **L2**

4.3 Use the graph to determine the value(s) of x in the interval $x \in [135°; 270°]$ for which $\frac{f(x)}{g(x)} \geq 0$. (2) **L2**

5. **DBE MAY/JUNE 2017 QUESTION 6**

In the diagram, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [-270°; 90°]$.



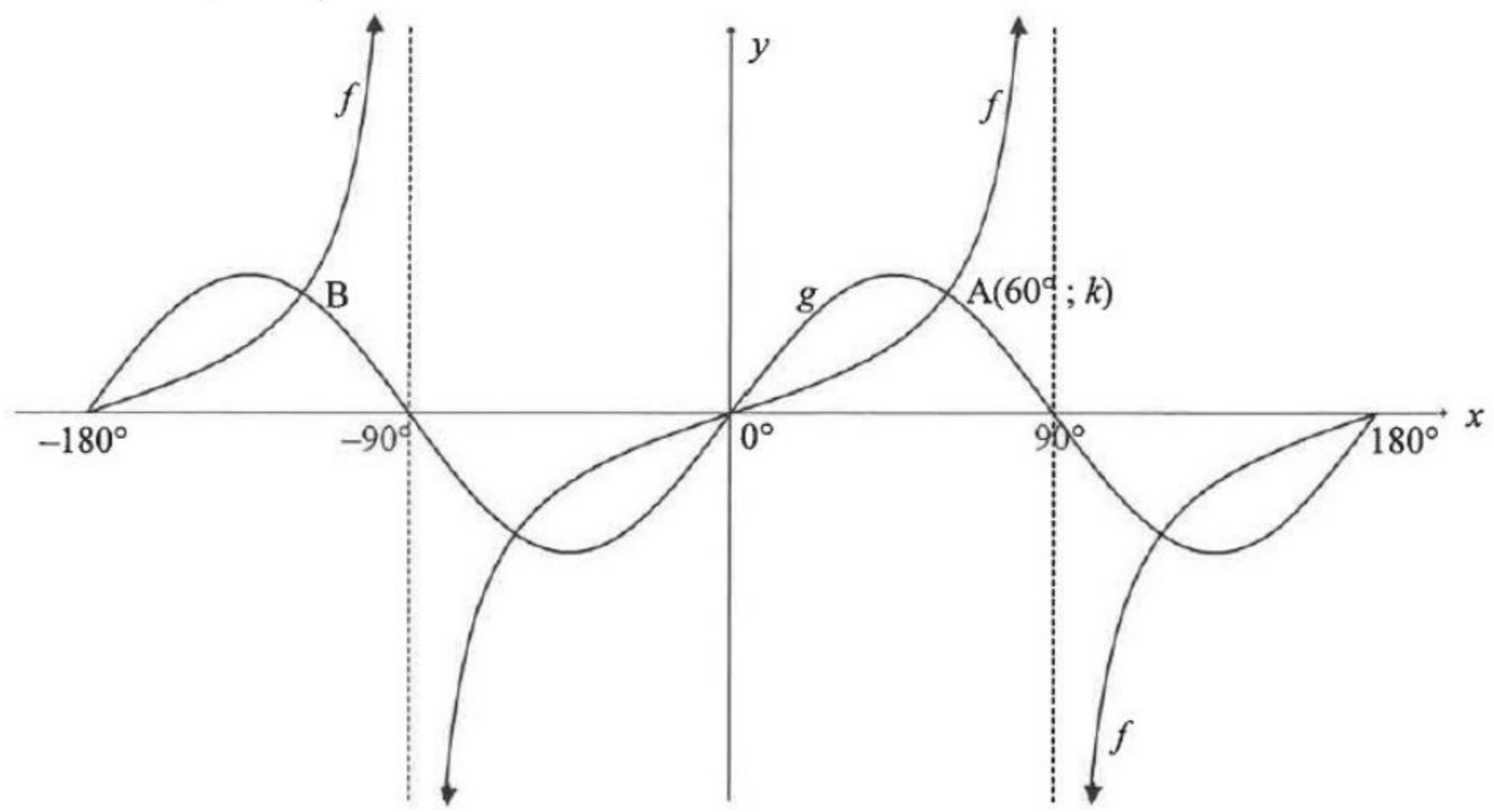
5.1 Draw the graph of $g(x) = 2\sin x - 1$ for the interval $x \in [-270°; 90°]$ on the grid given. Show all the intercepts with the axes as well as turning points. (4) **L2**

5.2 Let A be a point of intersection of the graphs of f and g. Show that the x-coordinate of A satisfies the equation $\sin x = \dfrac{-1+\sqrt{5}}{2}$. (4) **L3**

5.3 Hence, calculate the coordinates of the points of intersection of graphs of f and g on the interval $x \in [-270°; 90°]$. (4) **L3**

6. **DBE NOVEMBER 2022 QUESTION 6**

In the diagram below, the graphs of $f(x) = \tan x$ and $g(x) = 2\sin 2x$ are drawn for the interval $x \in [-180°; 180°]$. $A(60°; k)$ and B are two points of intersection of f and g.



6.1 Write down the period of g. (1) **L1**

6.2 Calculate the

6.2.1 Value of k (1) **L1** 6.2.2 Coordinates of B (1) **L2**

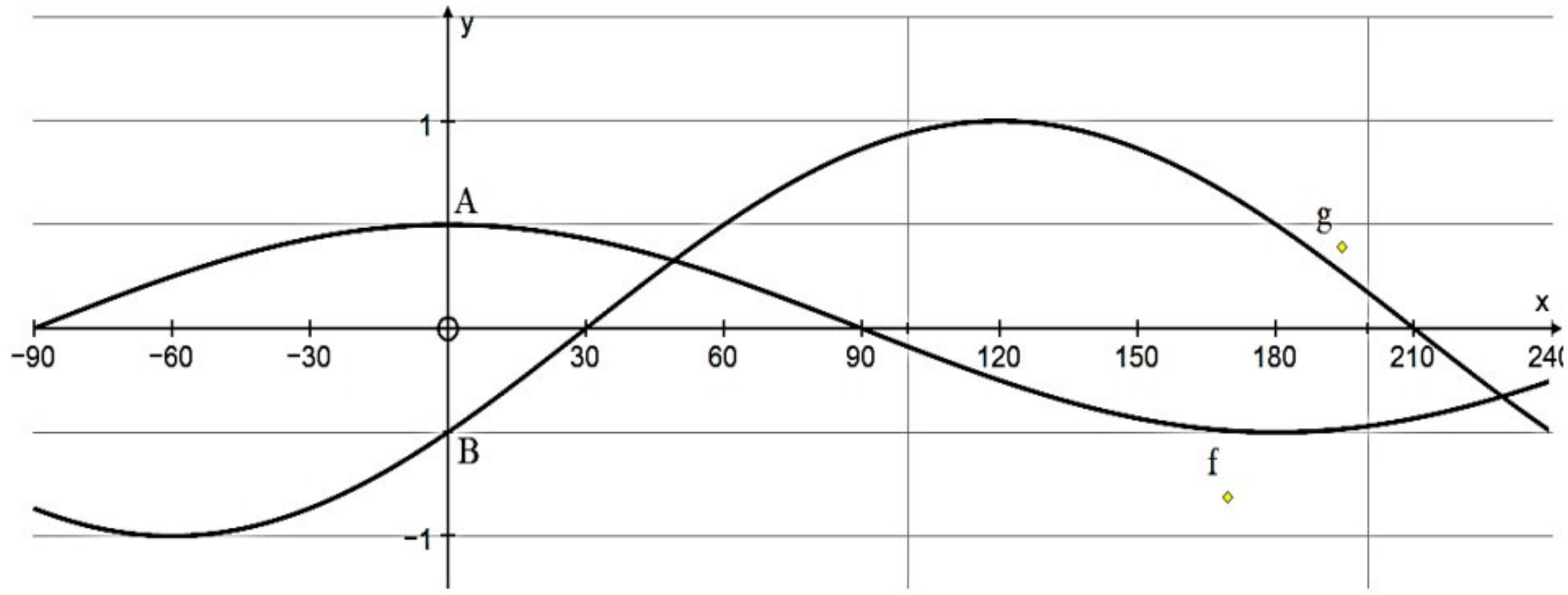
6.3 Write down the range of $2g(x)$. (2) **L2**

6.4 For which values of x will $g(x+5°) - f(x+5°) \leq 0$ in the interval $x \in [-90°; 0°]$? (2) **L4**

6.5 Determine the values of p for which $\sin x \cos x = p$ will have exactly two real roots in the interval $x \in [-180°; 180°]$. (3) **L4**

7. **DBE MAY/JUNE 2022 QUESTION 7**

In the diagram below, the graphs of $f(x) = \dfrac{1}{2}\cos x$ and $g(x) = \sin(x - 30°)$ are drawn for the interval $x \in [-90°; 240°]$. A and B are the y-intercepts of f and g respectively.



7.1 Determine the length of AB. (2) **L1**

7.2 Write down the range of $3f(x)+2$. (2) **L1**

7.3 Read off from the graphs a value of x for which $g(x)-f(x)=\frac{\sqrt{3}}{2}$. (2) **L2**

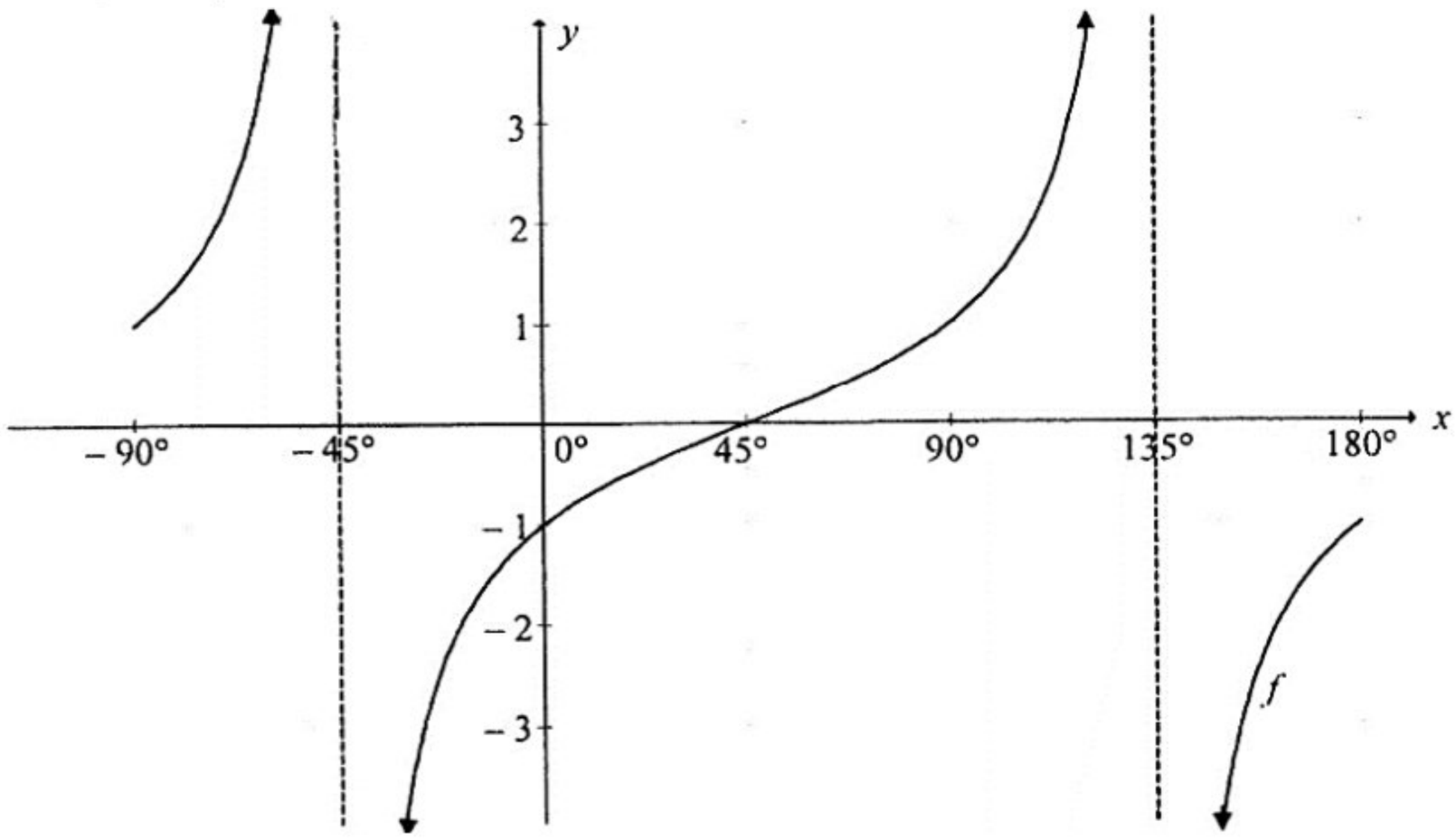
7.4 For which values of x, in the interval $x\in[-90°\ ;\ 240°]$, will:

7.4.1 $f(x).g(x)>0$ (2) **L2**

7.4.2 $g'(x-5°)>0$ (2) **L4**

8. **QUESTION 6 MAY/JUNE 2023**

In the diagram below, the graph of $f(x)=\tan\left(x-45^\circ\right)$ is drawn for the interval $x\in[-90°\ ;\ 180°]$.



8.1 Write down the period of f. (1) **L1**

8.2 Draw the graph of $g(x)=-\cos 2x$ for the interval $x\in[-90°\ ;\ 180°]$ on the given grid. Show all intercepts with the axes, as well as the minimum and maximum points of the graph. (3) **L2**

8.3 Write down the range of g. (1) **L1**

8.4 The graph of g is shifted 45° to the left to form the graph of h. Determine the equation of h in its simplest form. (2) **L3**

8.4 Use the graph(s) to determine the values of x in the interval $x\in[-90°\ ;\ 90°]$ for which:

8.5.1 $f(x)>1$ (2) **L2**

8.5.2 $2\cos 2x-1>0$ (4) **L3**

TOPIC TRIGONOMETRY: PROBLEMS IN TWO AND THREE DIMENSIONS

GUIDELINES, SUMMARY NOTES, & STRATEGIES

THE SINE RULE

In any Δ ABC it is true that:

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

Important: Use the Sine Rule when given **two angles and a side** in a triangle, also when **two sides and a non-included angle** are given.

It is advisable that when calculating **sides** have the **sides as numerators**: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ and when calculating **angles, have the angles as numerators:** $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$.

THE COSINE RULE

In any Δ ABC it is true that: $a^2=b^2+c^2-2bc.\cos A$, $b^2=a^2+c^2-2ac.\cos B$ and $c^2=a^2+b^2-2ab.\cos C$

Important: Use the Cosine Rule when given **two sides and an included angle,** also when you are given **all the three sides.**

THE AREA RULE

In any Δ ABC it is true that:

$$\text{Area of } \Delta ABC=\frac{1}{2}bc.\sin A=\frac{1}{2}ac.\sin B=\frac{1}{2}ab.\sin C$$

Important: To use the Area Rule, you need **two sides and an included angle** of the triangle**.**

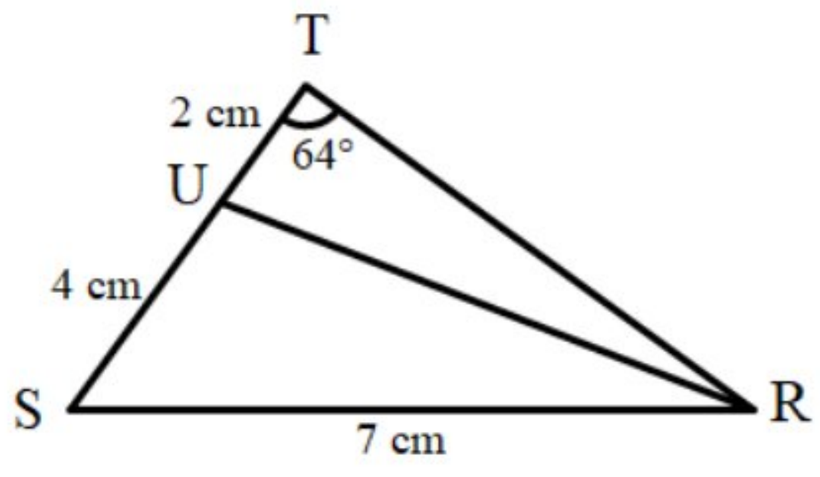
STRATEGIES

Note: When solving 3D problems separate all the triangle so that they will be 2D and easy to solve. It is also advisable that write all your findings back to the diagrams to help you with the next sub-question.

REVISION QUESTIONS

1. In the diagram below, Δ TSR is drawn with U on TS.

   US = 4 cm, UT = 2 cm, SR = 7 cm and $\hat{T}=64°$.



1.1 Calculate the value of $\hat{S}$ (*correct to ONE decimal place*). (4) **L3**

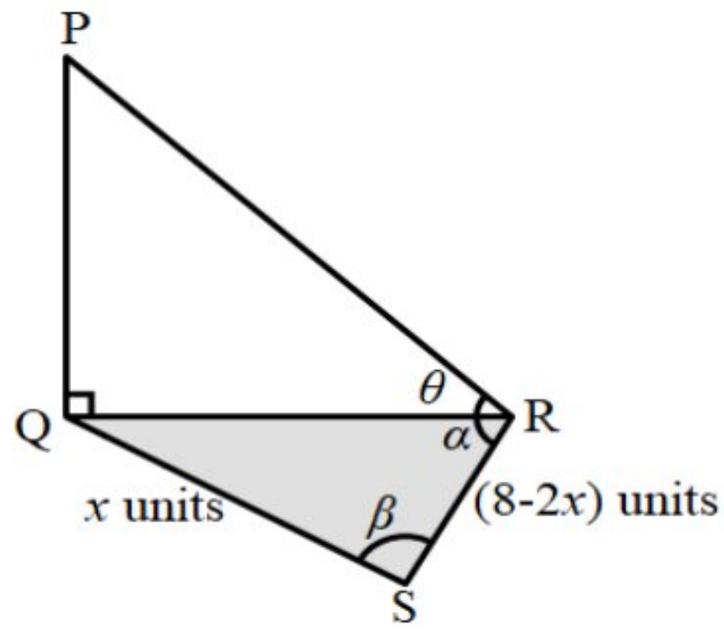
1.2 If $\hat{S}=65,6°$, calculate the following:

1.2.1 The area of Δ USR. (3) **L2**

1.2.2 The length of UR. (3) **L2**

2 **KZN STEP AHEAD DOCUMENT 2022**

In the diagram alongside, PQ is a vertical mast. R and S are two points on the same horizontal plane as Q, such that:
$Q\hat{R}S = \alpha,\ Q\hat{S}R = \beta,\ SR = 8-2x,\ QS = x.$



2.1 Show that: $PQ = \dfrac{x\sin\beta\tan\theta}{\sin\alpha}$ (5) **L3**

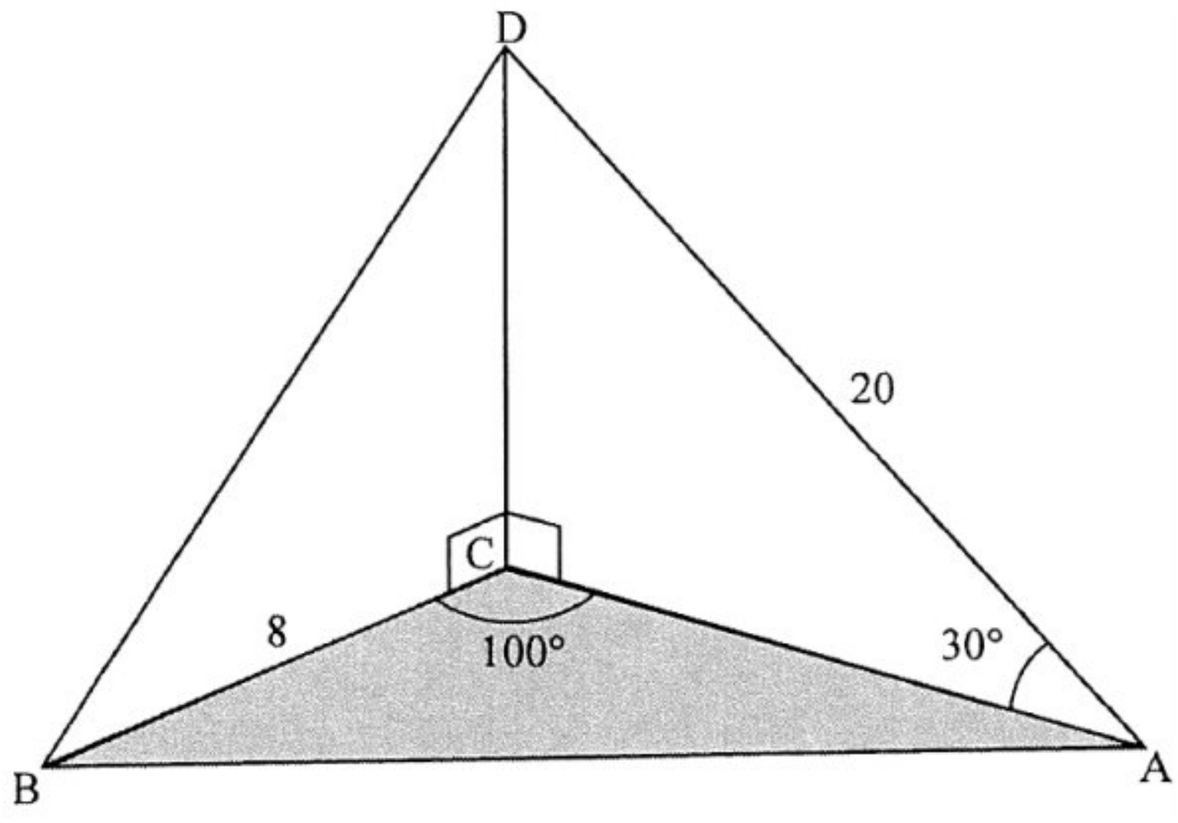
2.2 If $\beta = 60°$, show that the area of $\Delta QSR = 2\sqrt{3}.x - \dfrac{\sqrt{3}}{2}x^2$. (3) **L2**

2.3 Determine the value of x for which the area of Δ QSR will be maximum. (3) **L3**

2.4 Calculate the length of QR if the area of Δ QSR is maximum. (3) **L2**

3 **DBE MAY/JUNE 2023**

In the diagram, A, B and C are points in the same horizontal plane. D is a point directly above C, that is DC $\perp$ BC. It is given that $A\hat{C}B = 100°$, $C\hat{A}D = 30°$, AD = 20 units and BC = 8 units.



3.1 Calculate the length of

3.1.1 AC (2) **L2**

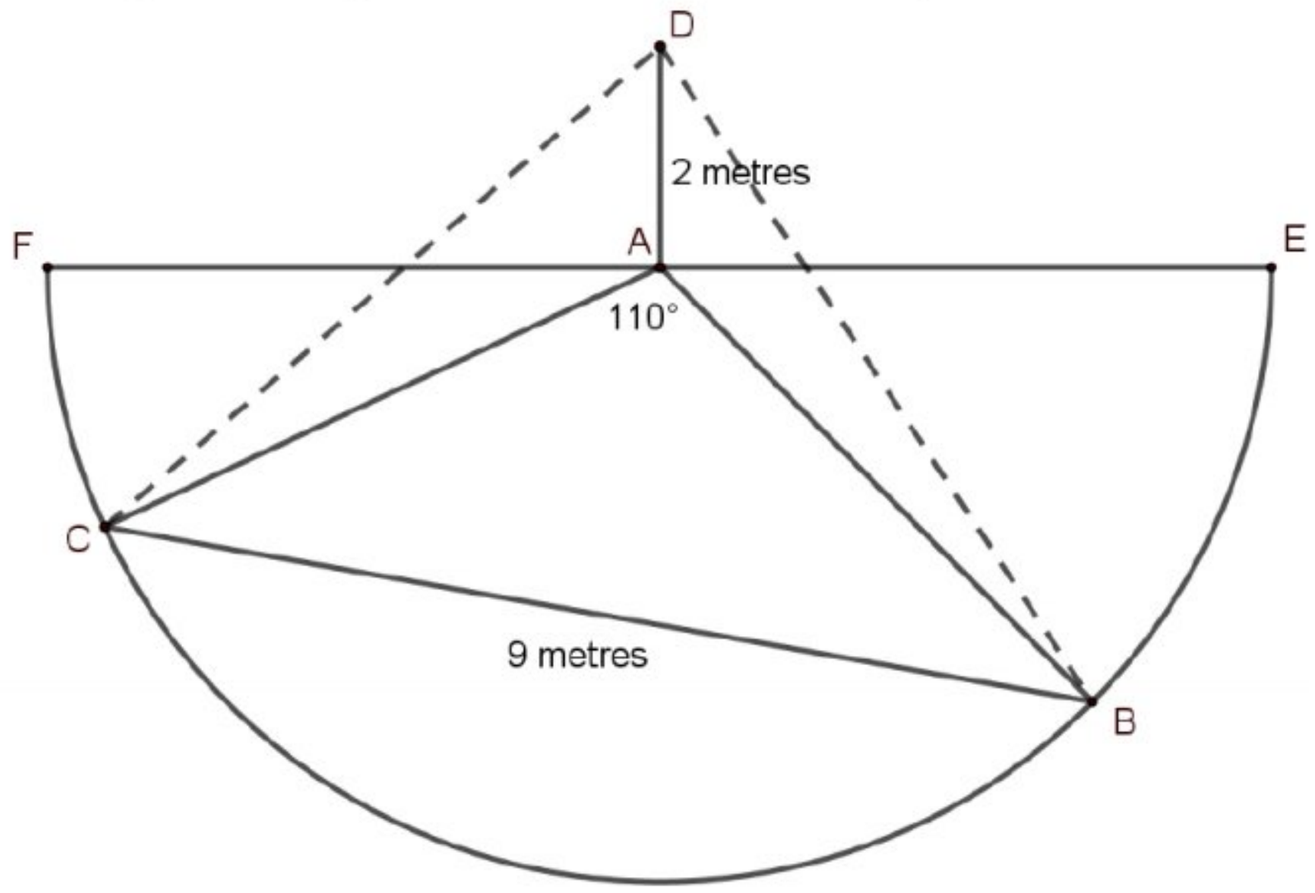
3.1.2 AB (3) **L2**

3.2 If it is further given that $A\hat{B}D = 73,4°$, calculate the size of $A\hat{D}B$. (3) **L2**

4. **IEB NOVEMBER 2020 QUESTION 8**

In the diagram below:

- The semi-circle with centre A and diameter FAE going through B and C lie on a horizontal plane.
- CB = 9 metres and $C\hat{A}B = 110^\circ$.
- AD is a vertical pole to the plane and is 2 metres in height.



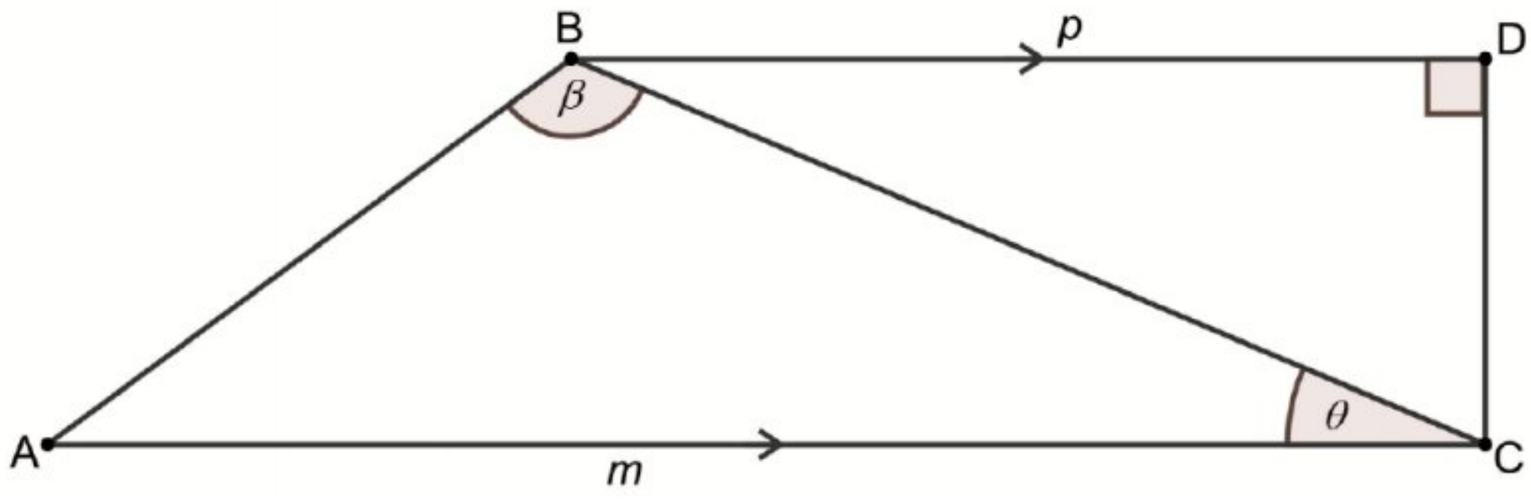
4.1 Calculate the length of DB. (3) **L3**

4.2 Calculate the size of $C\hat{D}B$. (3) **L3**

5. **IEB MAY/JUNE 2022 QUESTION 10 c)**

In the diagram below, quadrilateral CABD is drawn.

- AC || BD
- BD = p and $B\hat{C}A = \theta$
- $A\hat{B}C = \beta$ and $B\hat{C}A = \theta$.

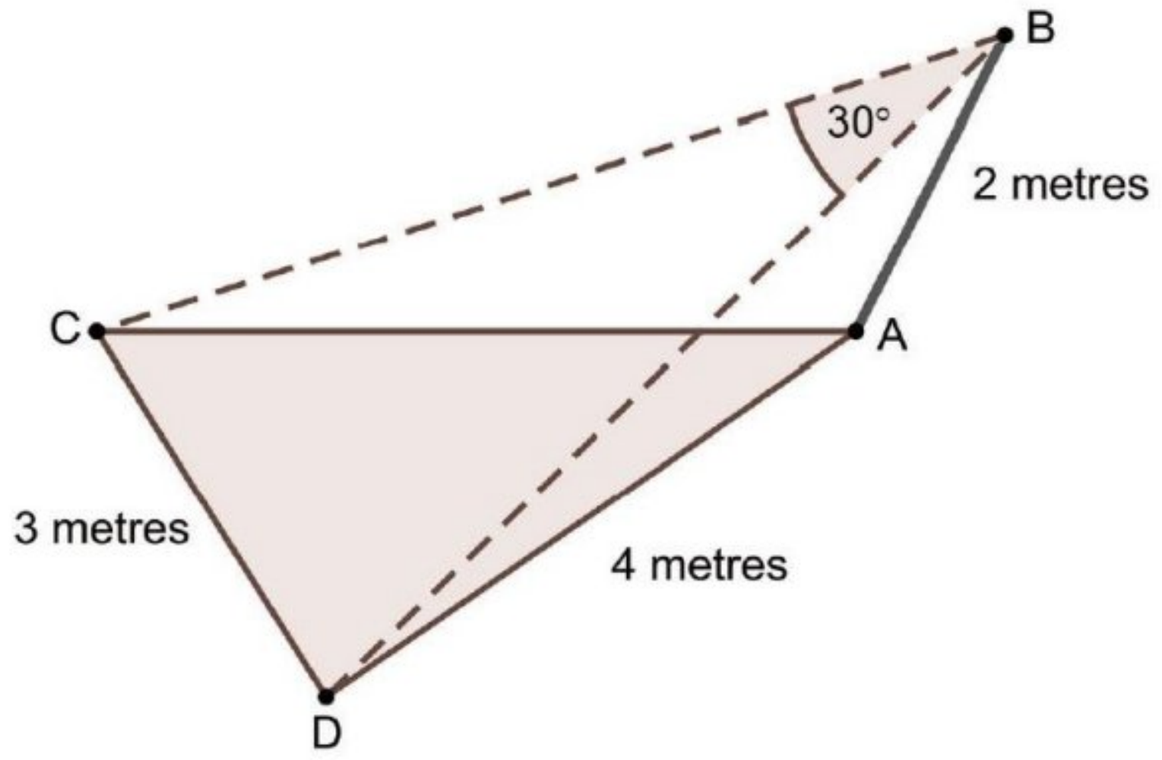


Prove that: $\dfrac{m(\sin\beta\cos\theta + \sin\theta\cos\beta)}{\sin\beta} = \dfrac{p}{\cos\theta}$ (5) **L3**

6. **IEB MAY/JUNE 2022 QUESTION 10 d)**

In the diagram below, A, C and D lie in the same horizontal plane.

- AB is a pole with its one end positioned at A.
- BC and BD are ropes used to hold the pole AB up, with $C\hat{B}D = 30^\circ$.
- The length of rope BC = the length of rope BD.
- CD = 3 metres and AD = 4 metres.



If the length of the pole AB is 2 metres, then how much shorter should each rope be so that the pole, AB, is perpendicular to the ground? (6) **L4**

TOPIC EUCLIDEAN GEOMETRY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

WAYS IN WHICH EUCLIDEAN GEOMETRY IS TESTED

1. Completing a statement of a theorem in words.
2. Determining the value of an angle in **two ways:** numerical and / or in terms of the variable(s)
3. **Proofs in riders:** Direct and indirect proofs
4. **Similarity** and **Proportionality** Theorems
   - Proportionality theorem: Question involving parallel lines in proportions, Areas (common angle vs. common vertex/same height)
   - Similarity theorem: AAA, ratios after similar triangles.
5. **Examinable proofs** to be known

1. COMPLETING A STATEMENT OF A THEOREM IN WORDS.

- Know by heart all the theorems and be able to complete the statement.

2. DETERMINING THE VALUE OF AN ANGLE

- Know all the theorems about **lines**, **triangles** and **circles** (**Centre** group, **non-centre** group, **tangent** group and **cyclic quad** group).
- Every statement must come with a reason and reasons must be stated according to the list of acceptable reasons from the exam guidelines

  E.g. base $\angle' s$ of an iso. Δ (unacceptable) the acceptable reason is: $\angle' s$ opp = sides

3. PROOFS IN RIDERS

Know how theorems and their converses are being formed in diagrams.

- When given 3 points on the circumference look out for a possibility of a triangle. If one side is produced then you may expect exterior angle of a triangle. If there is a tangent on the circle then there is a possibility of having a Tan Chord Theorem
- When given 4 or 5 points on the circumference then there is a possibility that 4 points may be joined and then there is a cyclic quad. In a case that one side is produced then you may expect exterior angles of a cyclic quad.
- Start with a given angle linking with what is required to prove
- Visualization: Mind picture of diagrams of theorems

DIRECT AND INDIRECT PROOFS IN RIDERS.

- In Geometry we mostly use angles to prove in questions.

1. Direct proof question: Prove A = B

2. Indirect proof question: Prove that a line // to another line.

Remember in Euclidean geometry- we mostly use angles to prove. This question is not asking about the angles directly. Here we need to prove sides but using angles i**ndirectly. Why indirectly?** Because we mostly use angles to prove.

∴ First, we need to change this question to be direct, and then prove. If we say it must be direct we mean that it must ask to prove angles 1st, then conclude by stating the sides that are parallel

4. SIMILARITY AND PROPORTIONALITY THEOREMS

PROPORTIONALITY THEOREM

- Identify parallel lines, and use ratios for proportion.
- **Useful strategies in solving problems involving ratio in areas of triangles:**

  CASE 1: If triangles share a **common angle** use area rule. Area = $\frac{1}{2}$a.bsinC

CASE 2: If triangles share a common vertex or height use $\text{Area} = \frac{1}{2}bh$

CASE 3: If none of the cases above apply then identify a common triangle and relate the two triangles in question to it, then use any of the two methods mentioned above. **OR**

Required Area = Area of big Δ − other known Area

SIMILARITY THEOREM

CASE 1: Prove that triangles are similar e.g. $\Delta ABC ||| \Delta DEF$

- Angles and / or sides in proportion can be used to prove that two triangles are similar.
- Always name the triangles you are referring to when proving similar triangles

CASE 2: Prove that $\frac{AB}{PQ} = \frac{AC}{PR}$. First prove: $\Delta ABC ||| \Delta PQR$ and then deduce the proportion of the sides.

CASE 3: Prove that: KN. PX = NR. YP. Find two triangles in which KN, PX, NR and Y, (or sides equal to these), and thus prove that: $\Delta KNR ||| \Delta YPX$, then deduce what you were asked to prove. Identify triangles. This method is used when proved similarity don't give asked ratios.

CASE 4: Prove: Proportion with square, with division, with + in between, there is a possibility that two similarities were used or Pythagoras theorem was used.

e.g. $\frac{CF^2}{EF^2} = \frac{BD}{DE}$

5. EXAMINABLE PROOFS

Five grade 11 proofs to be known for exam purposes:

5.1 Line from the centre ⊥ chord

5.2. **NEW**: line from centre to midpt of chord

5.3. Angle at the centre is 2× angle at the circumference.

5.4. Opposite angles of a cyclic quad are supplementary.

5.5. Tan chord theorem.

Two grade 12 proofs:

5.6. Line drawn parallel to one side of a triangle, divides the other two sides proportionally:
Proportionality theorem

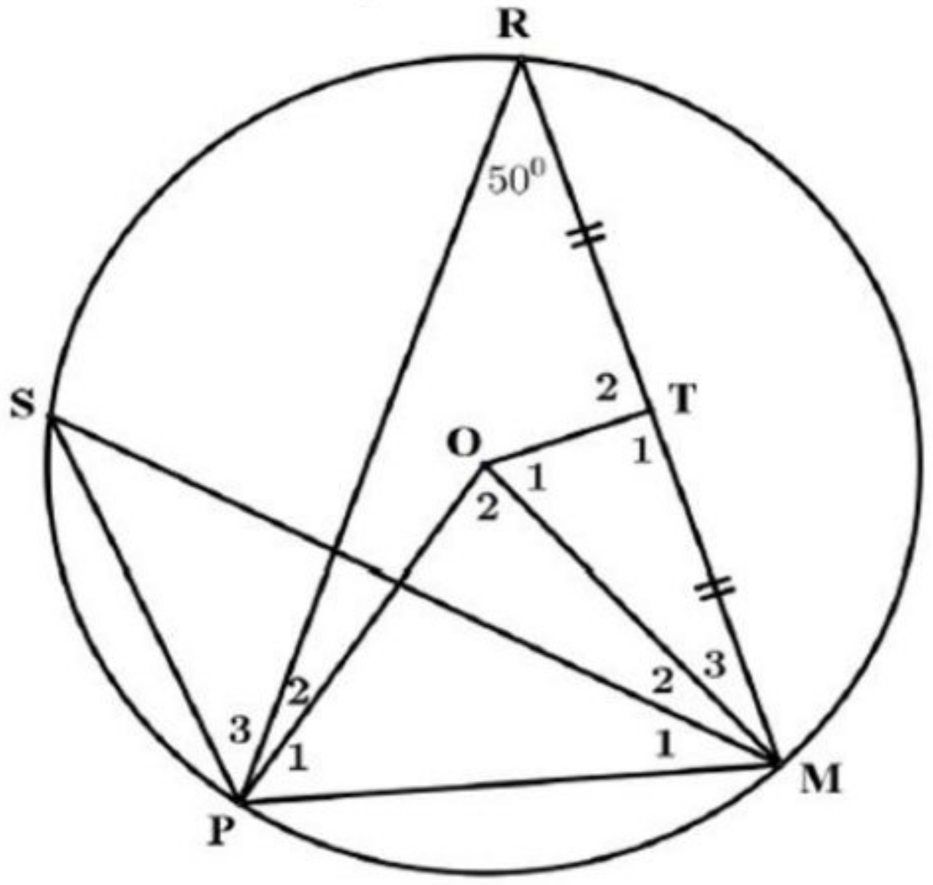
5.7 If two triangles are equiangular, then their corresponding sides are in proportion:
Similarity theorem

NB!!!!!

- Do not make any assumption e.g. do not assume that a line is a tangent or a diameter, unless you are told that it is.
- Look for key words in the statement such as centre, // lines, tangents, cyclic quads, bisects, etc.
- Continuously update the diagram as you read the statement and as you find the angles.
- When proving theorems, no construction no marks.
- You will not always be told that you have a cyclic quadrilateral. Therefore check lines joining four points on the circumference.
- For every statement there **must** be a reason.

REVISION QUESTIONS

1. In the figure, O is the centre of the circle RMPS. T is the midpoint of RM. and $\widehat{R} = 50°$. Calculate with reasons the size of the angles that follows.



.

1.1 $\hat{T}_1$ (1) **L1**

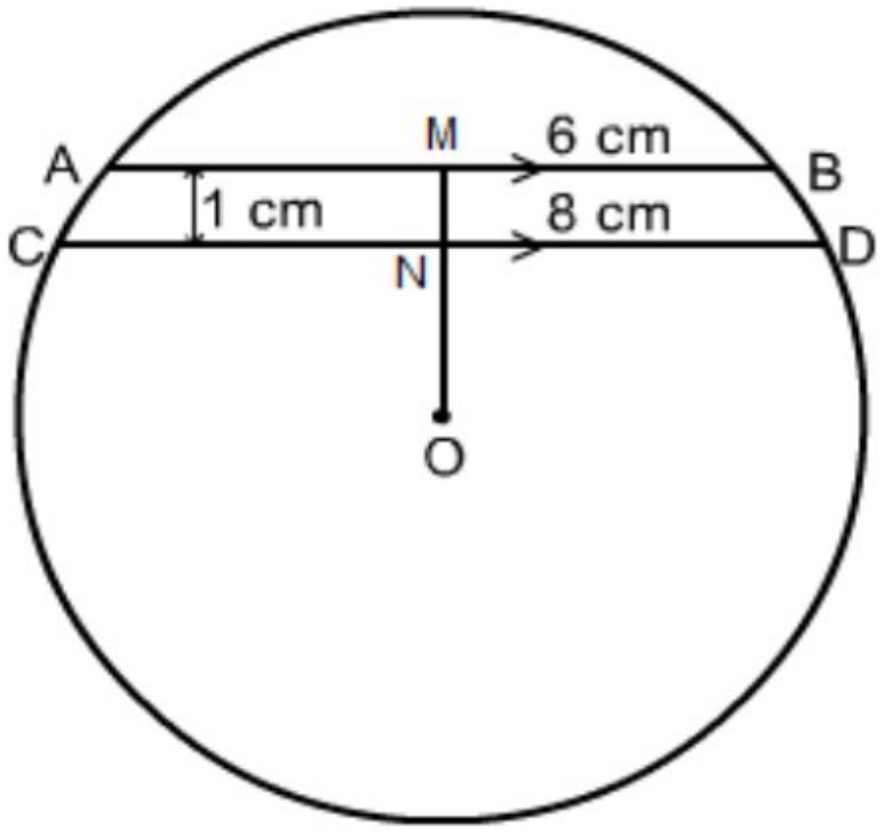
1.2 $\hat{O}_2$ (4) **L1**

1.3 $\hat{S}$ (3) **L1**

1.4 $\hat{P}_1$ (3) **L2**

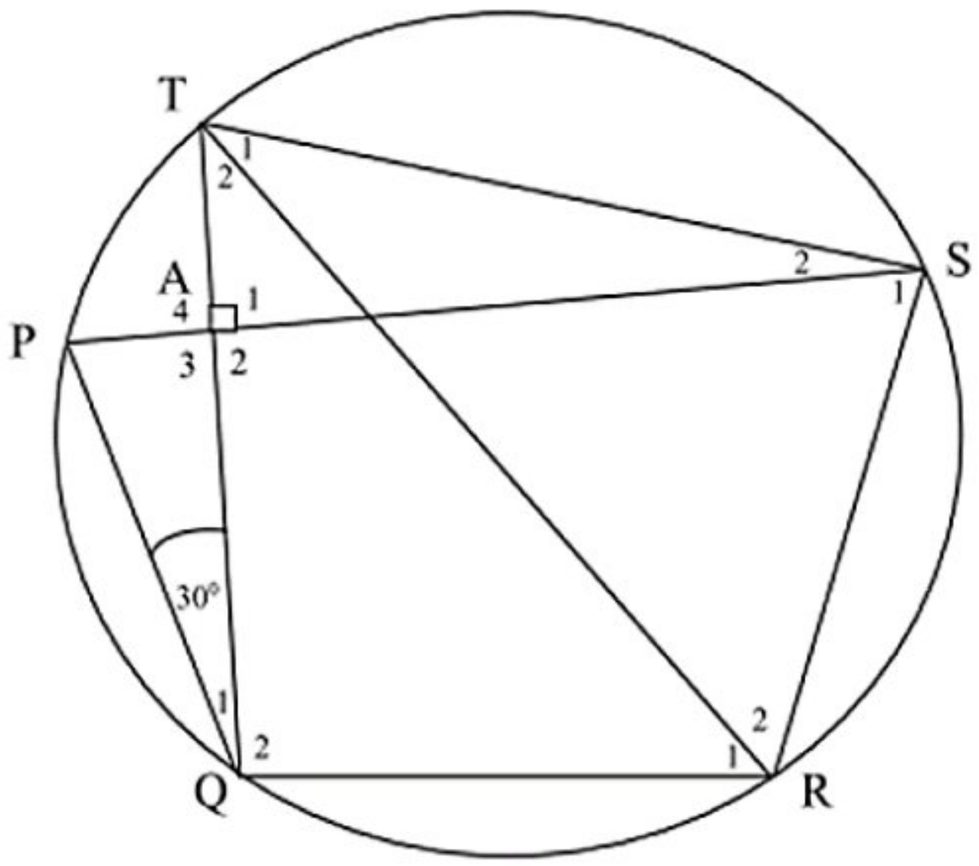
1.5 Is TOPM a cyclic quadrilateral? Give a reason for your solution. (2) **L2**

2. AB and CD are two parallel chords on the same side of the centre O of a circle. The shortest distance between AB and CD is 1 cm. The length of AB is 6 cm and of CD is 8 cm. M is the midpoint of AB and the line OM intersects CD at N which is the midpoint of CD. Calculate the length of the radius of the circle.



(4) **L3**

3. In the figure, TR is a chord of the circle PQRST. QAT ⊥ PAS. $\widehat{Q}_1 = 30°$ and $\widehat{P} = \widehat{S}_1$



3.1 Name 3 angles each equals to 60° (4) **L2**

3.2 Calculate the size of $Q\widehat{R}S$ (2) **L2**

3.3 Prove that PS || QR (2) **L3**

3.4 Prove that TR is a diameter of the circle (3) **L3**

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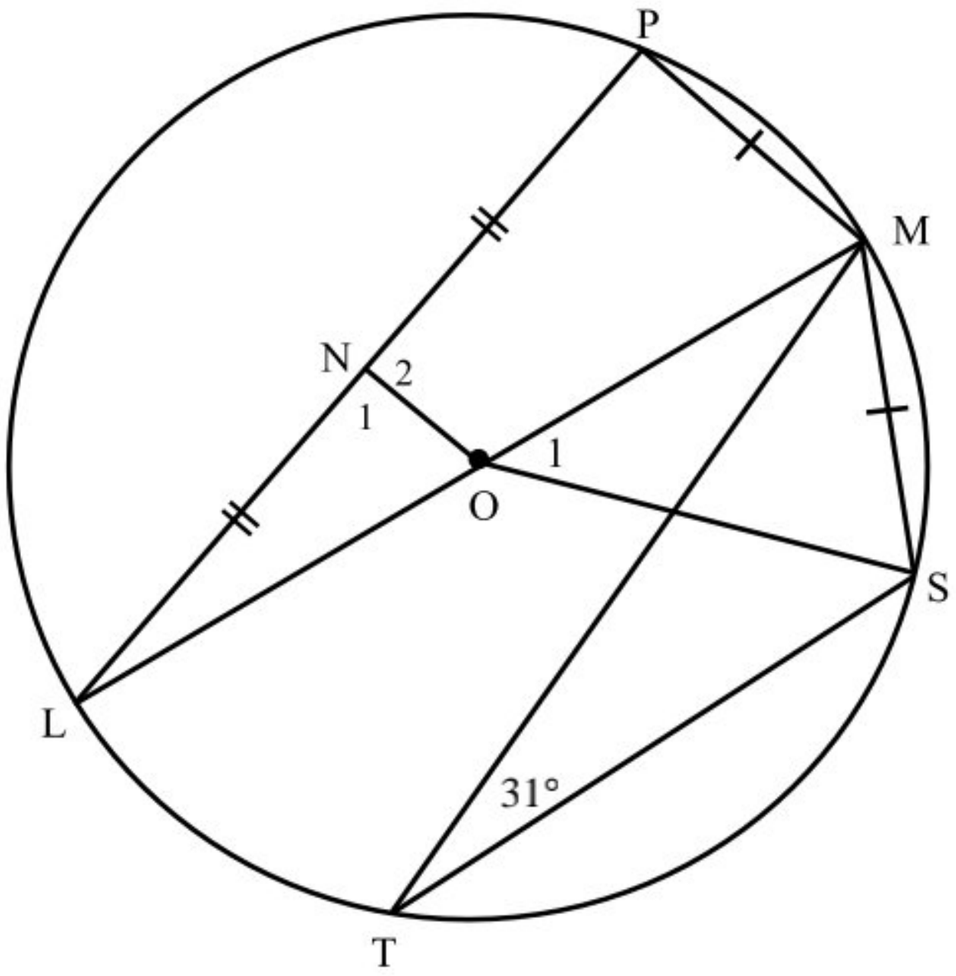
4. In the diagram below, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn. PM = MS and $M\hat{T}S = 31°$.



Calculate giving reasons the values of:

Determine, with reasons, the size of each of the following angles:

4.1 $M\hat{O}S$ (1) **L1**

4.2 $\hat{L}$ (1) **L1**

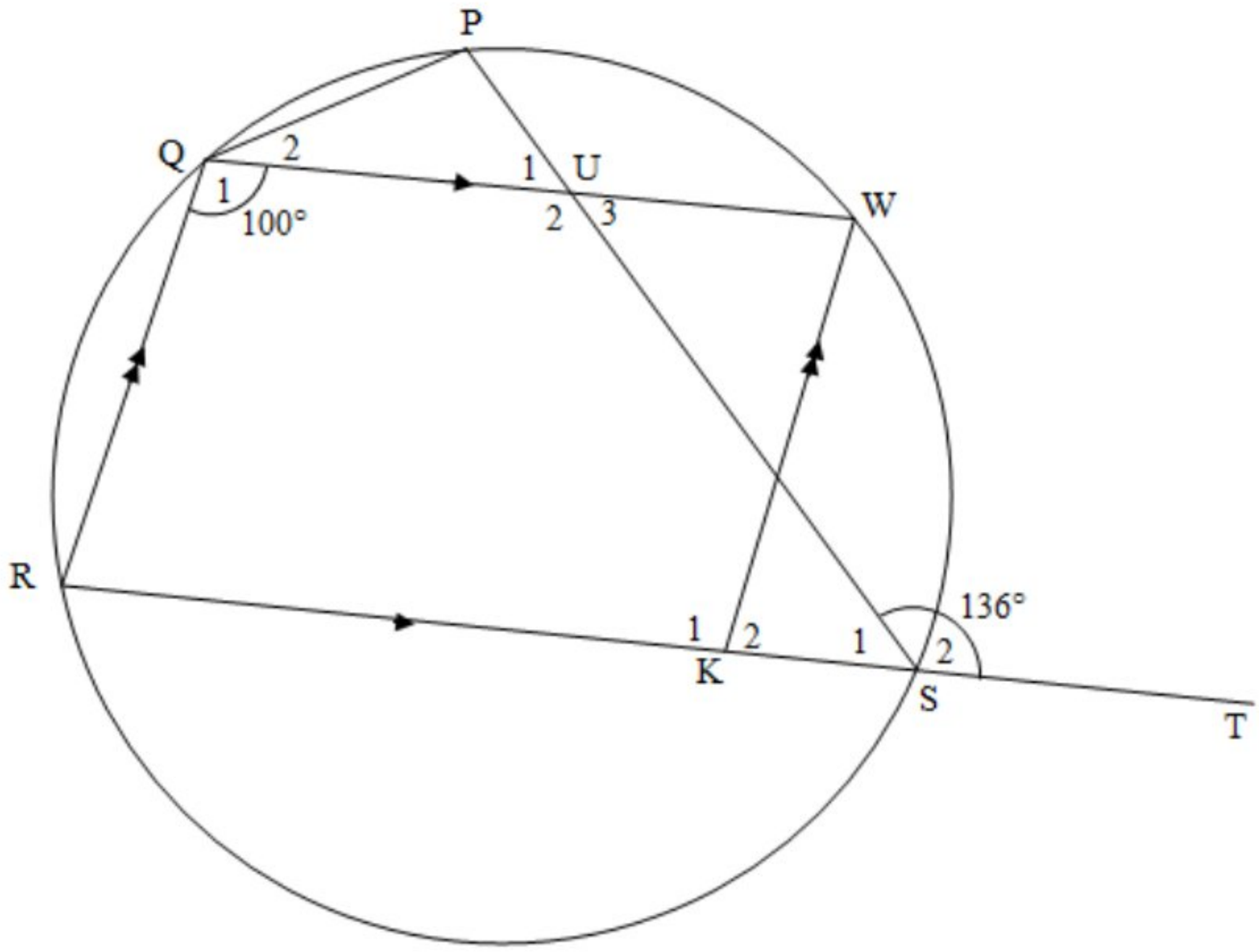
4.3 $\hat{P}$ (1) **L1**

4.4 $\hat{N}_2$ (1) **L1**

4.5 Prove that $ON = \frac{1}{2}MS$ (2) **L2**

DBE NOVEMBER 2019

5. In the diagram, PQRS is a cyclic quadrilateral. Chord RS is produced to T. K is a point on RS and W is a point on the circle such that QRKW is a parallelogram. PS and QW intersect at U. $P\hat{S}T = 136°$ and $\hat{Q}_1 = 100°$.



Determine, with reasons, the size of:

5.1 $\hat{R}$ (2) **L1**

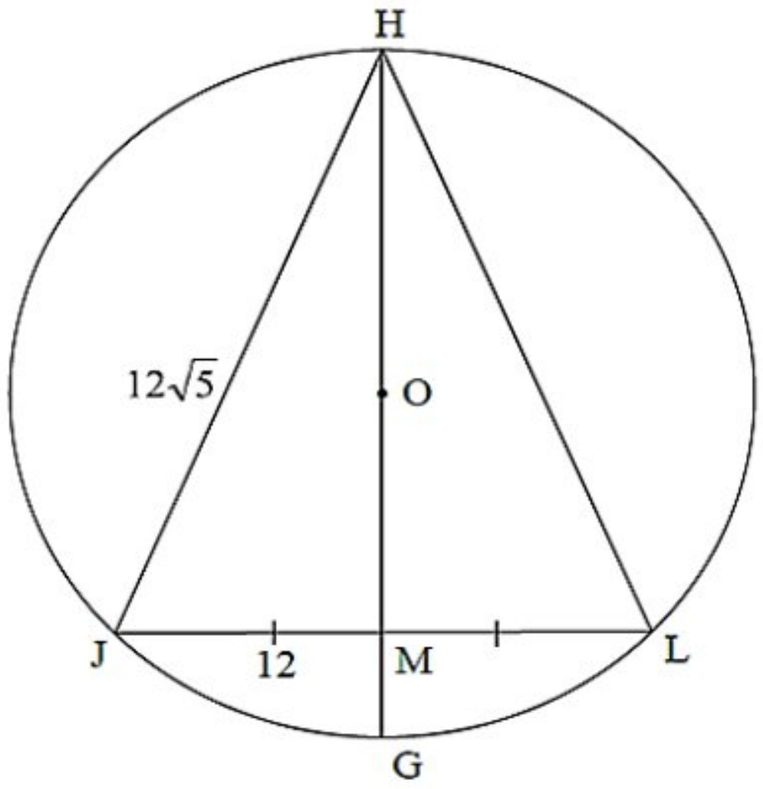
5.2 $\hat{P}$ (2) **L1**

5.3 $P\hat{Q}W$ (3) **L2**

5.4 $\hat{U}_2$ (2) **L2**

NOVEMBER 2020(2)

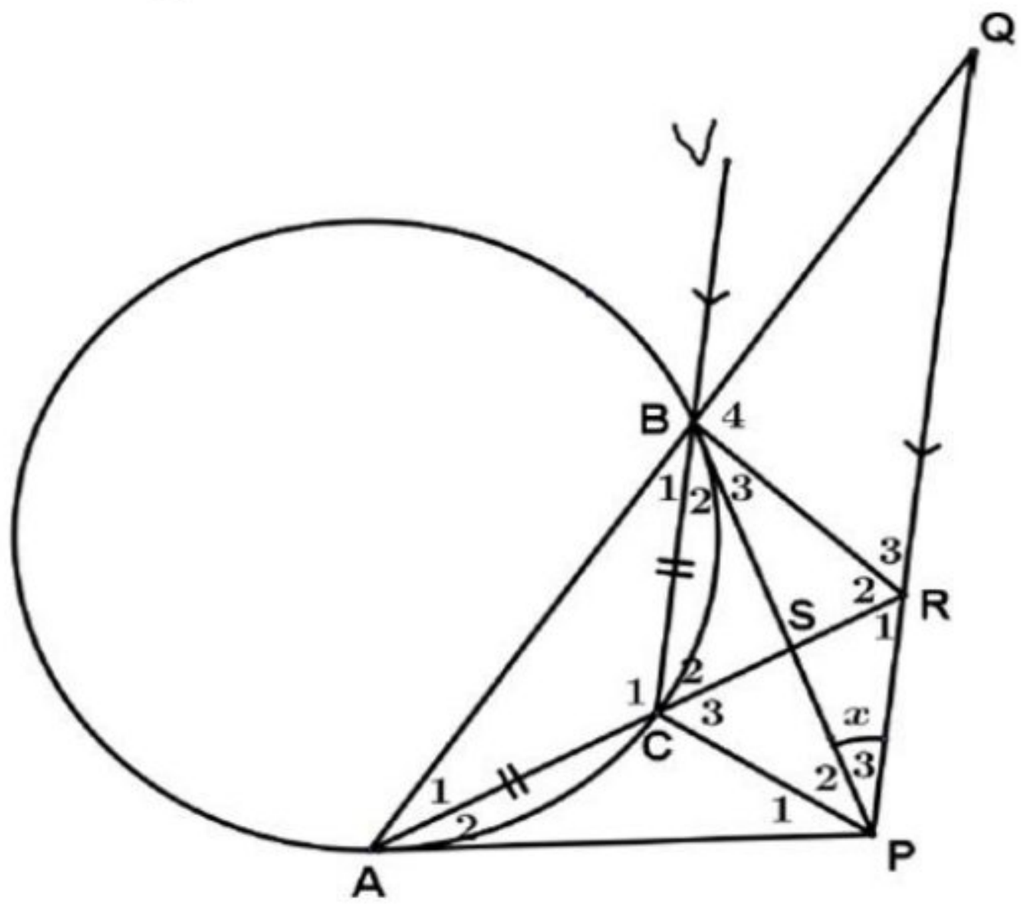
6. In the diagram along, a circle centred at O is drawn. H, J, G and L are points on the circle. ΔHJL is drawn. HOG bisects JL at M. $HJ = 12\sqrt{5}$ units and JM=12 units.



6.1 If MG = 6 units and OM = x, write HM in terms of x. (2) **L2**

6.2 Calculate, giving reasons, the length of the radius of the circle (5) **L3**

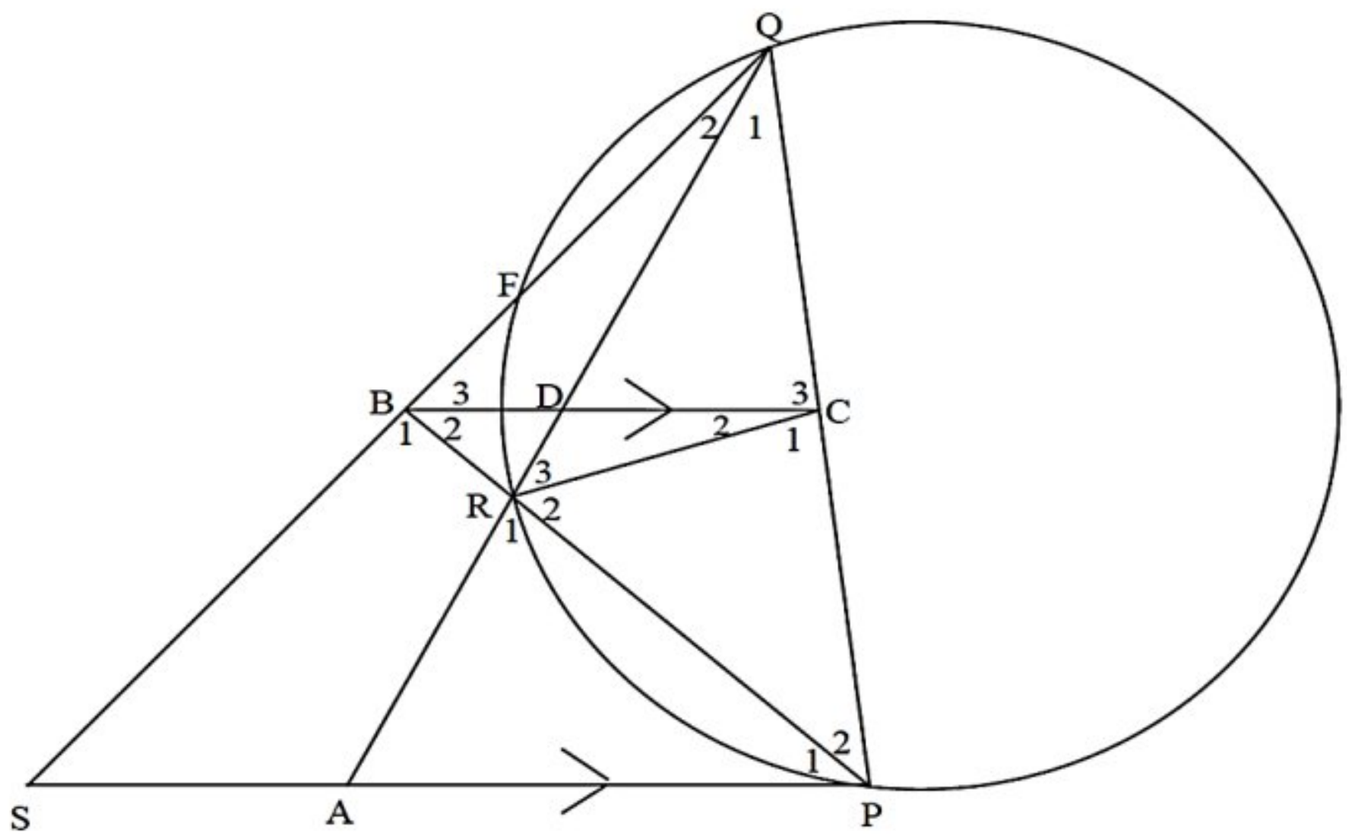
7. In the figure, AP and PB are tangents to the circle at A and B. C is a point on the circle such that AC = CB. AB is extended to Q such that PQ|| CB. AC is extended to meet PQ at R. BR is joined. Let $B\hat{P}R = x$



7.1 Write down with reasons 5 other angles each equal to x. (6) **L2**

7.2 Prove that ABRP is a cyclic quadrilateral (4) **L3**

8. In the diagram below, SP is a tangent to the circle at P and PQ is a chord. Chord QF produced meets SP at S and chord RP **bisects** $\hat{QPS}$. PR produced meets QS at B. BC|| SP and cuts the chord QR at D. QR produced meets SP at A. Let $B_2 = x$



8.1 Name, with reasons, 3 angles equal to x 6 **L2**

8.2 Prove that PC = BC 2 **L2**

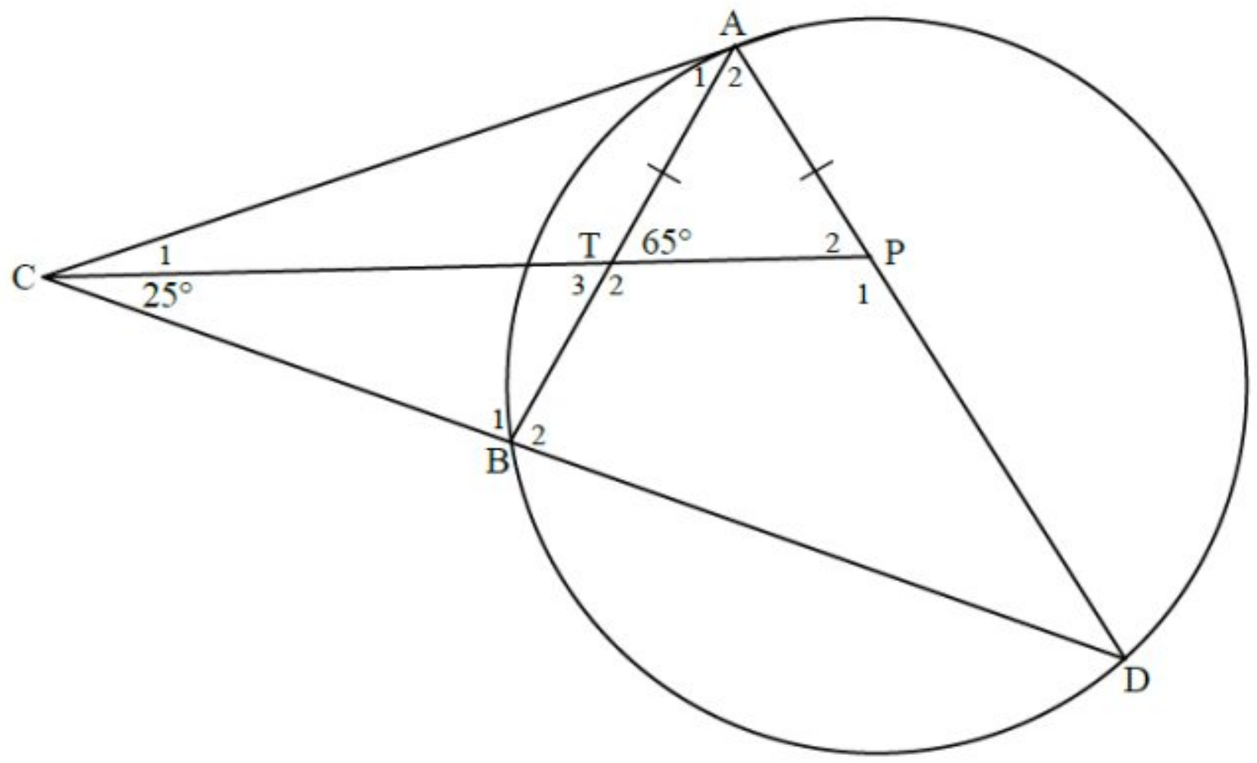
8.3 Prove that RCQB is a cyclic quadrilateral. 2 **L3**

8.4 Prove that $\Delta PBS \;|||\; \Delta QCR$ 5 **L3**

8.5 Show that PB.CR = QB.CP 4 **L4**

GAUTENG 2nd PUSH JUNE 2023

9. In the diagram ΔACD is drawn with points A and D on the circumference of a circle. CD cuts the circle at B. P is a point on AD with CP the bisector of $A\hat{C}D$. CP cuts the chord AB at T. AT = AP, $A\hat{T}P = 65°$ and $P\hat{C}D = 25°$



9.1 Determine the size of each of the following with reasons:

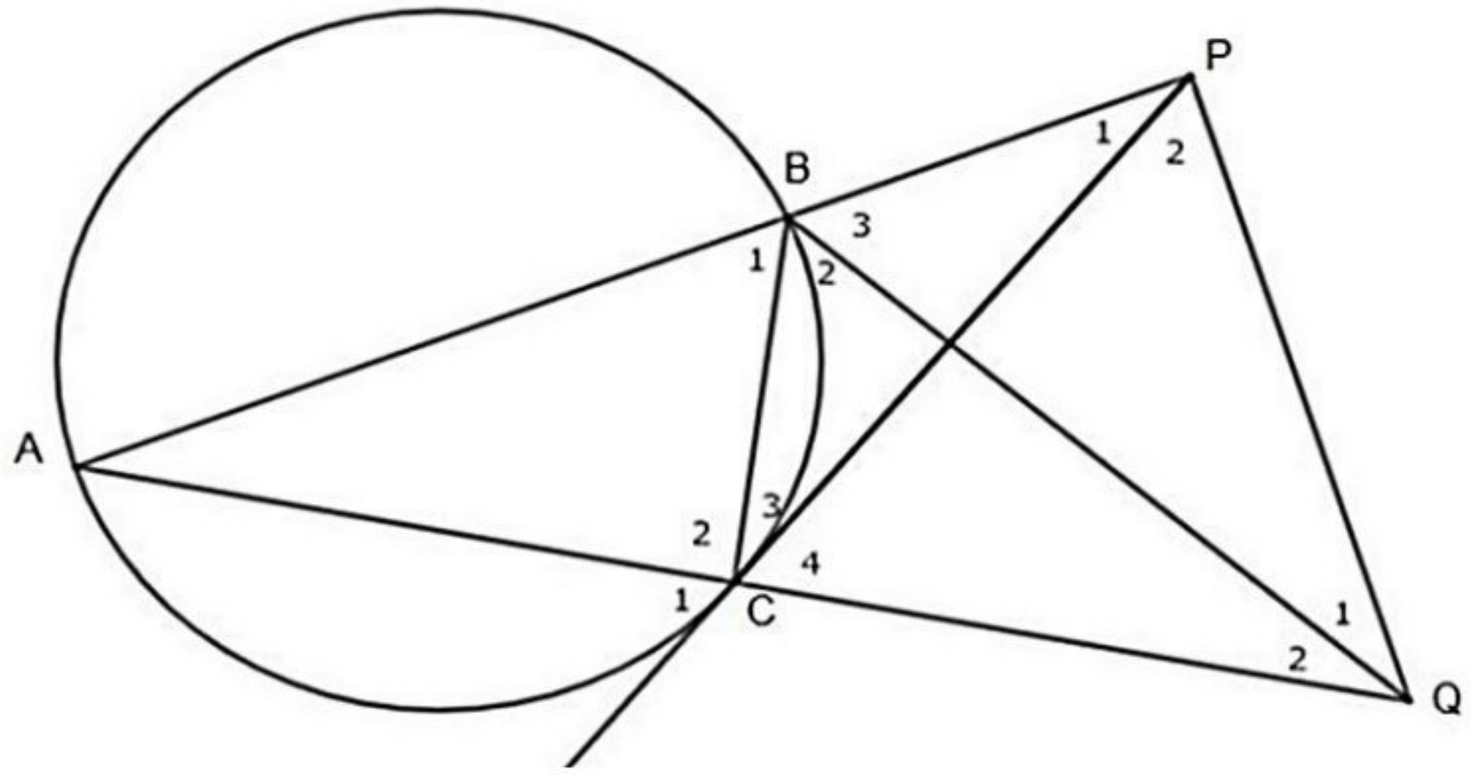
9.1.1 $\hat{P}_2$ (2) **L1**

9.1.2 $\hat{D}$ (2) **L1**

9.1.3 $\hat{A}$ (2) **L2**

9.2 Is CA a tangent to the circle ABD? Motivate your answer. (2) **L3**

10. A, B and C are concyclic. AB produced meets the tangent through C at P. AC is produced to Q so that PQ = PC.



10.1 If $\hat{C}_1 = x$, determine, with reasons THREE other angles each equal to x. (6) **L2**

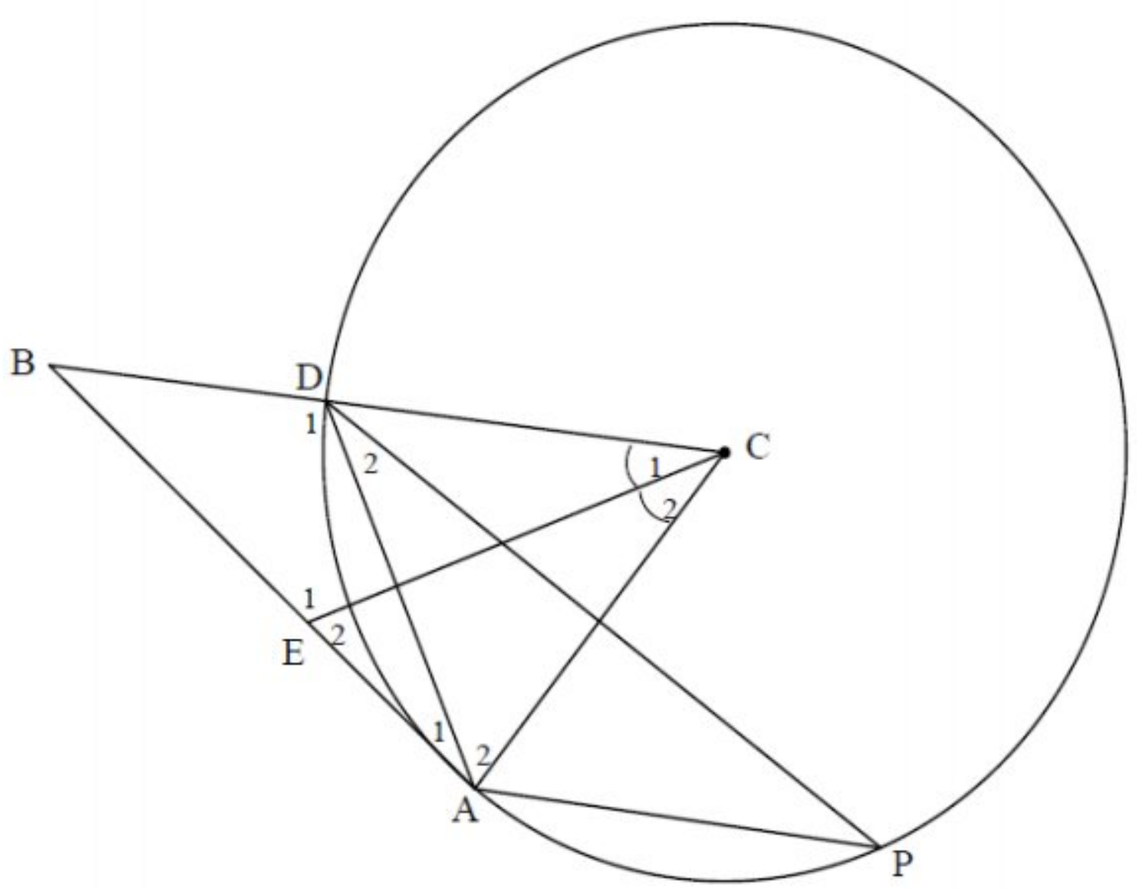
10.2 Prove that: (a) BCQP is a cyclic quadrilateral. (2) **L3**

10.3 (b) PQ is a tangent to circle ABQ. (4) **L3**

10.4 (c) $PQ^2 = PA.PB$ (5) **L3**

GAUTENG 2nd PUSH JUNE 2023

11 In the diagram C is the centre of the circle DAP. BA is a tangent to the circle at A. CD is produced to meet the tangent to the circle at B. DP and DA are drawn. E is a point on BA such that EC bisects $D\hat{C}A$.
Let $\hat{C}_1 = x$.



11.1 Prove that $\Delta BAD \mid\mid\mid \Delta BCE$. (7) **L4**

11.2 If it is also given that AB = 8 units and AC = 6 units, calculate:

11.2.1 The length of BD (5) **L3**

11.2.2 The length of BE (3) **L3**

11.2.3 The size of x (3) **L3**

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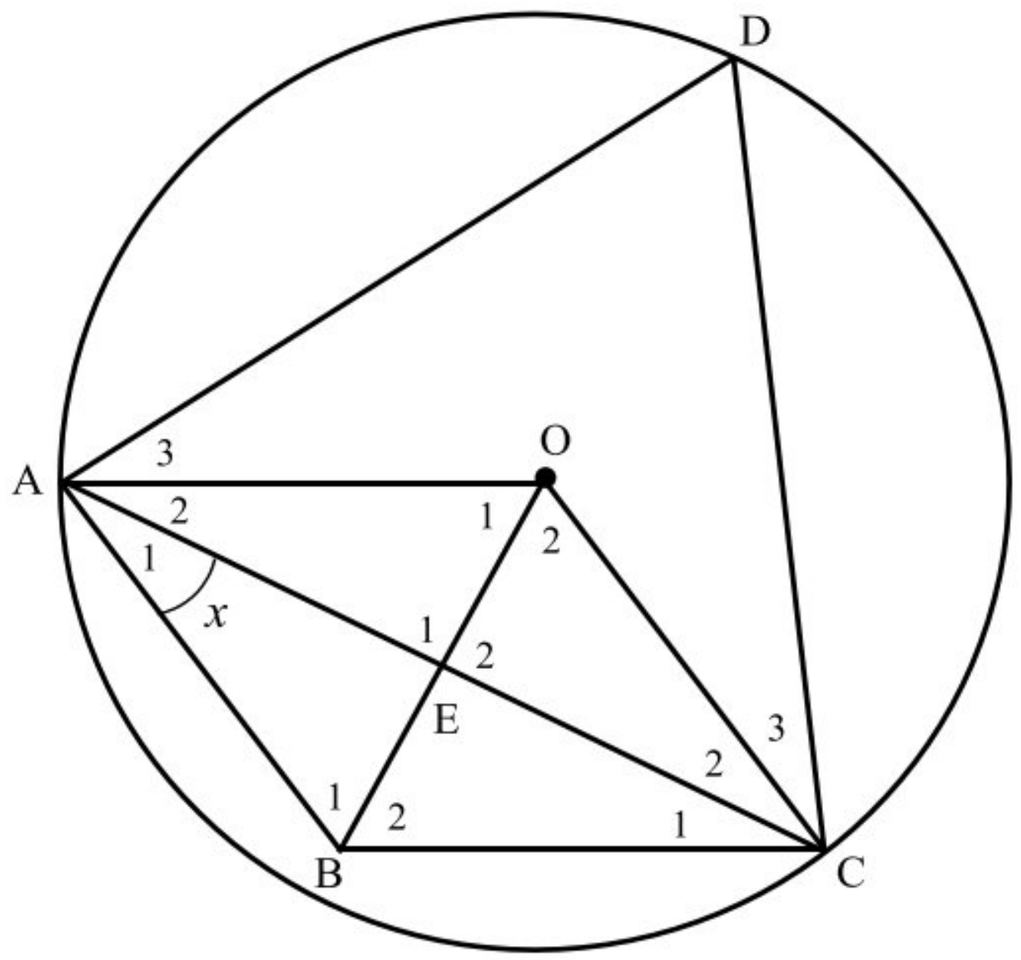
12. In the diagram below, the circle with centre O passes through the points A, C and D.
ABCO is a **rhombus** with diagonals intersecting at E.
ABCD is a **kite**. Let $\hat{A}_1 = x$.



12.1 Why is $\hat{A}_2 = x$? (1) **L1**

12.2 Write, with reasons, the following angles in terms of x:

12.2.1 $\hat{O}_1$ (2) **L2**

12.2.2 $A\hat{D}C$ (2) **L2**

12.2.3 $\hat{C}_3$ (4) **L3**

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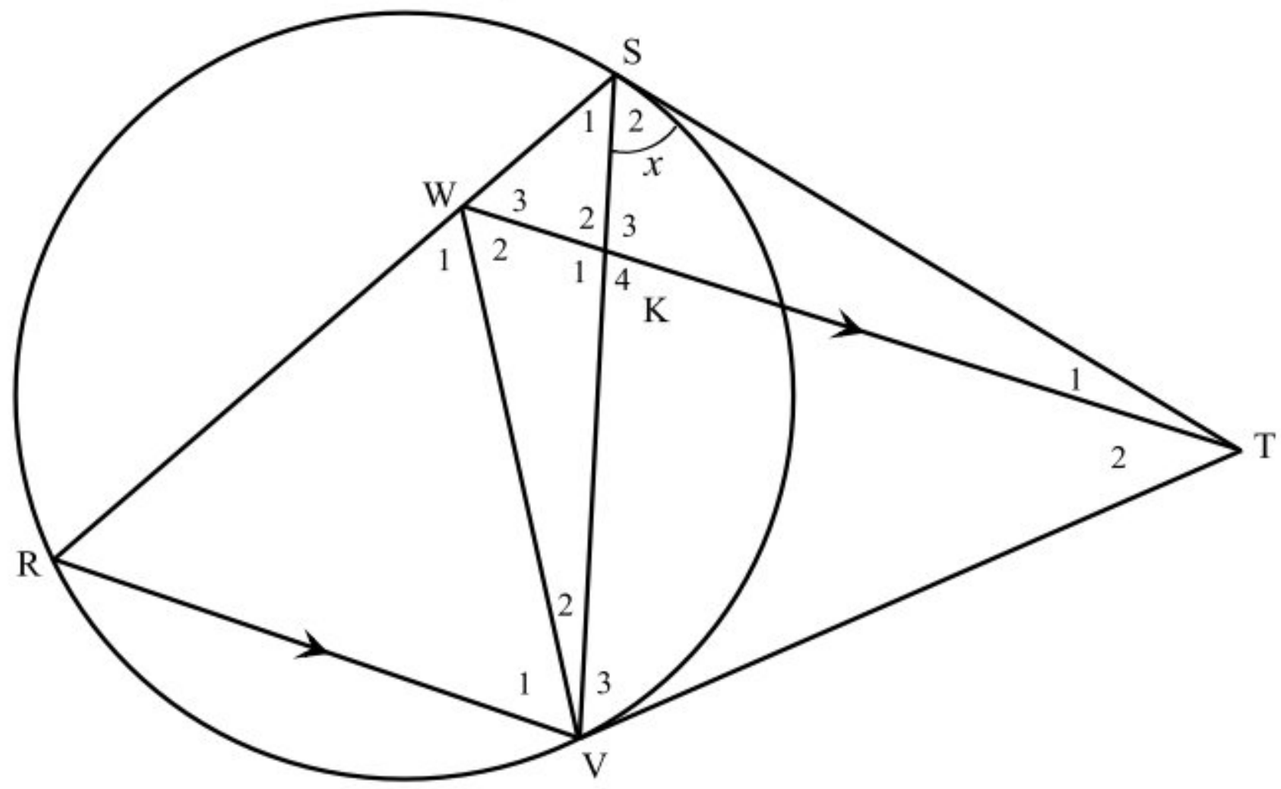
13. In the diagram below, ST and VT are tangents to the circle at S and V respectively. R is a point on the circle and W is a point on chord RS such that WT || RV. SV and WV are drawn. WT intersects SV at K. Let $\hat{S}_2 = x$.



13.1 Write down, with reasons, THREE other angles each equal to *x*. (3) **L2**

13.2 Prove, with reasons, that:

13.2.1 WSTV is a cyclic quadrilateral. (5) **L2**

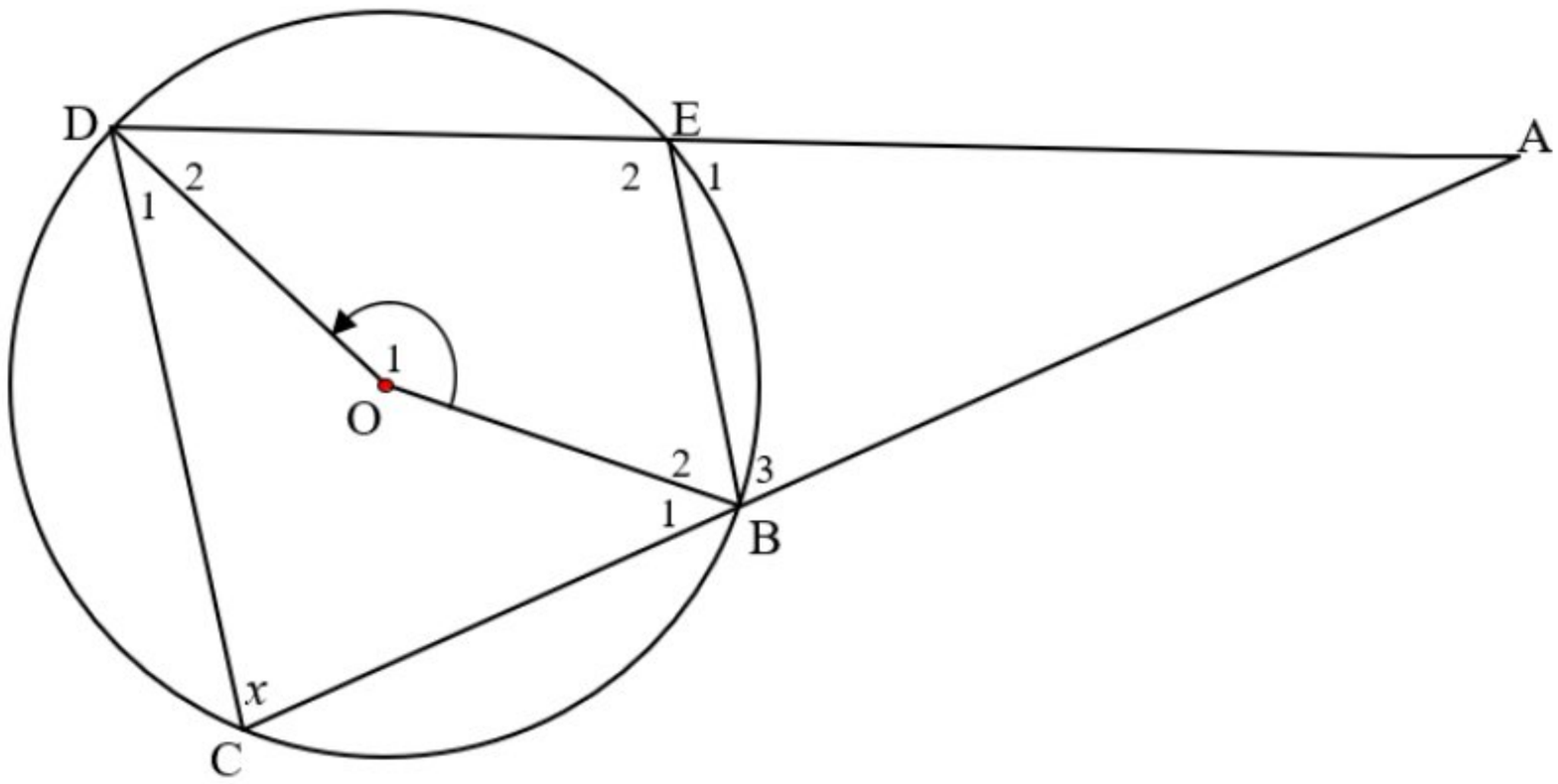
13.2.2 WR = WV (1) **L1**

13.2.3 $\Delta WRV \,|||\, \Delta TSV$ (5) **L3**

13.2.4 $\frac{RV}{SR} = \frac{KV}{TS}$ (5) **L4**

14. **GAUTENG 2nd PUSH JUNE 2023**

In the diagram O is the centre of the circle and BO and OD are drawn. Chords CB and DE are produced to meet in A. Chords BE and CD are drawn. $B\hat{C}D = x$.



14.1 Give a reason why the following statements are correct.

14.1.1 $\hat{E}_1 = x$ (1) **L1**

14.1.2 $\hat{O}_1 = 2x$ (1) **L1**

14.2 If it is given that BE | | CD, prove that:

14.2.1 AC = AD (4) **L3**

14.2.2 ABOD is a cyclic quadrilateral (3) **L3**

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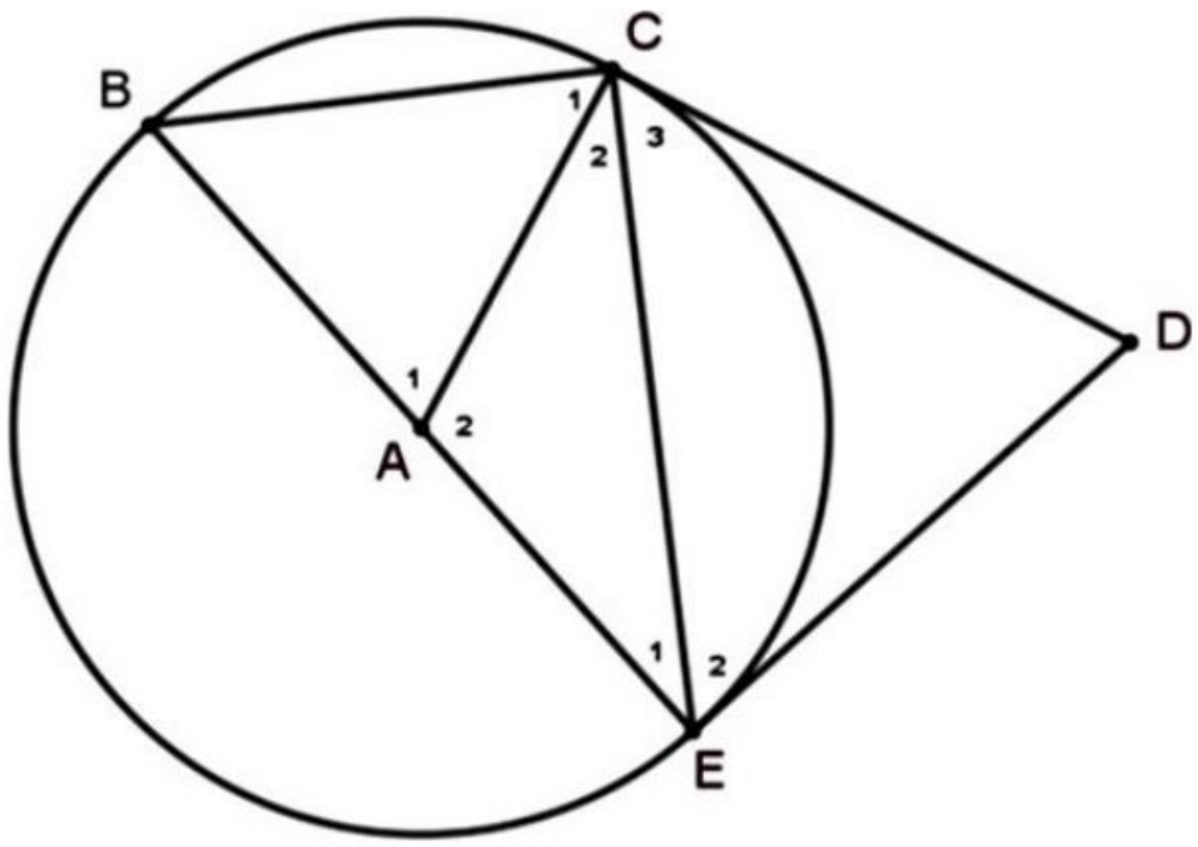
15. In the diagram below:
    - *DC* and *DE* are tangents to the circle at *C* and *E* respectively.
    - *A* is the centre of the circle.
    - *B* lies on the circle and *BAE* is a straight line.



15.1 Prove that $\Delta ABC \,|||\, \Delta DEC$. (5) **L3**

15.2 Hence, show that $AE.EC = BC.DE$. (3) **L2**

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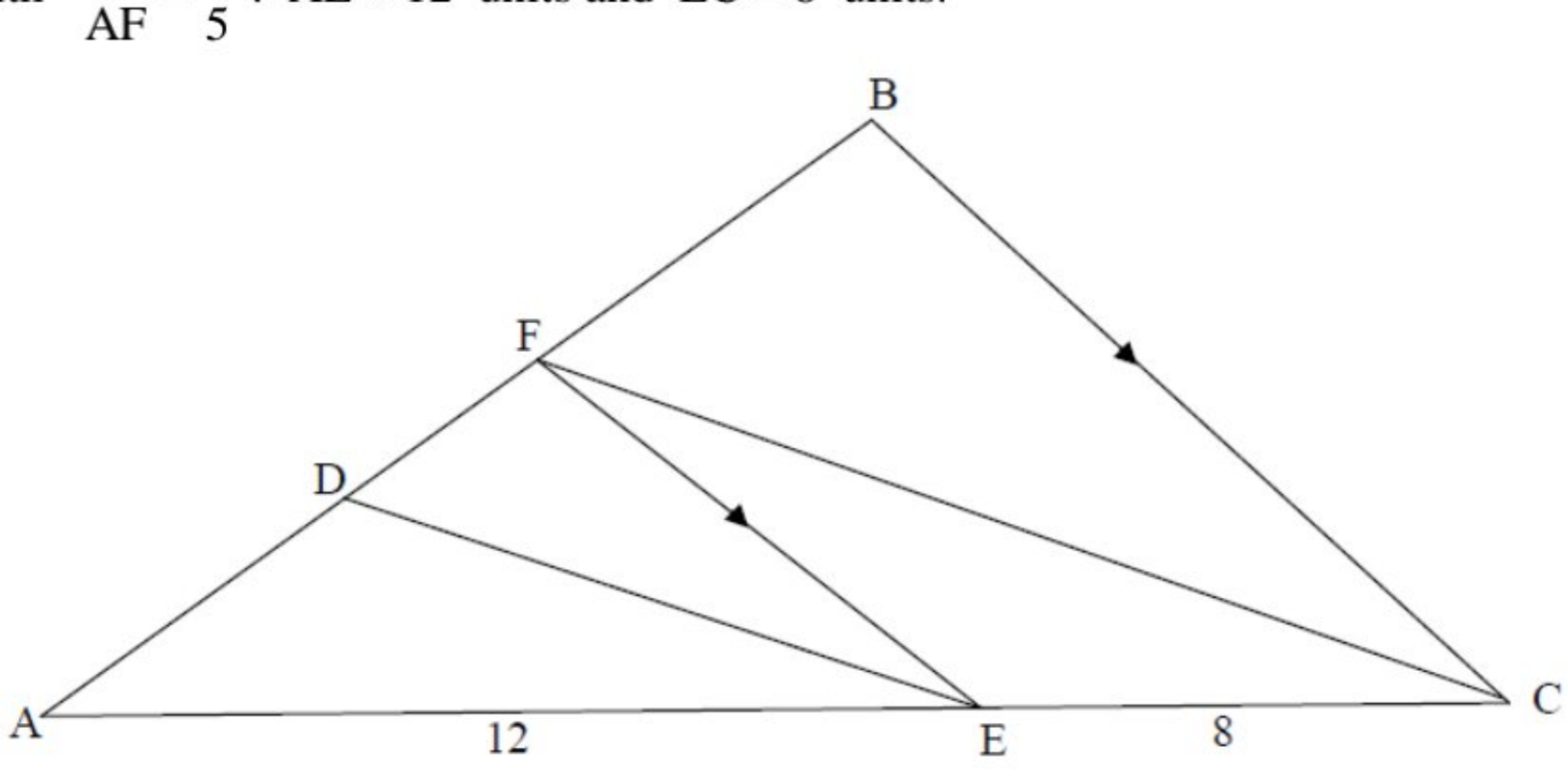
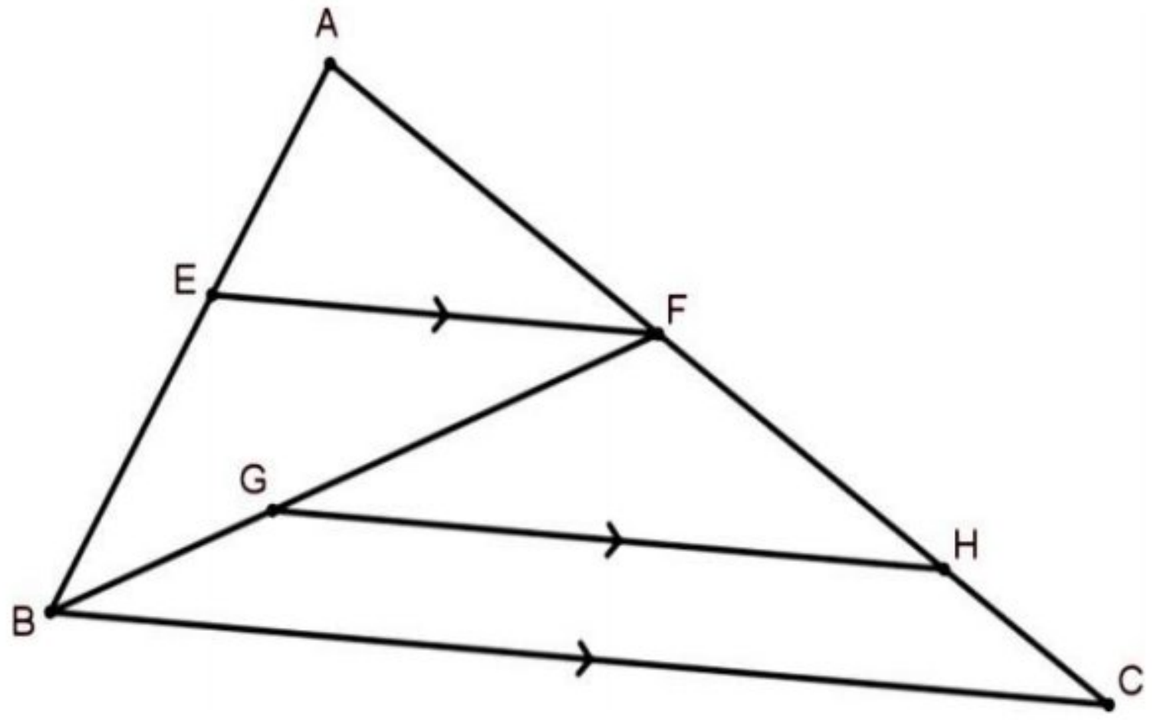
16. In the diagram ABC is a triangle with F on AB and E on AC. BC | | FE. D is on AF with $\frac{AD}{AF} = \frac{3}{5}$. AE = 12 units and EC = 8 units.



16.1 Prove that DE | | FC. (3) **L2**

16.2 If AB = 14 units, calculate the length of BF. (3) **L3**

17. In the diagram below: E lies on AB and F on AC in Δ APC with EF|| BC . G lies on FB and H on FC in Δ FBC with GH || BC. 5AE=4EB and $\frac{FG}{FB} = \frac{5}{8}$.



17.1 Calculate the value of $\frac{AF}{AC}$ (2) **L2**

17.2 Calculate the value of $\frac{HF}{AF}$ (4) **L3**

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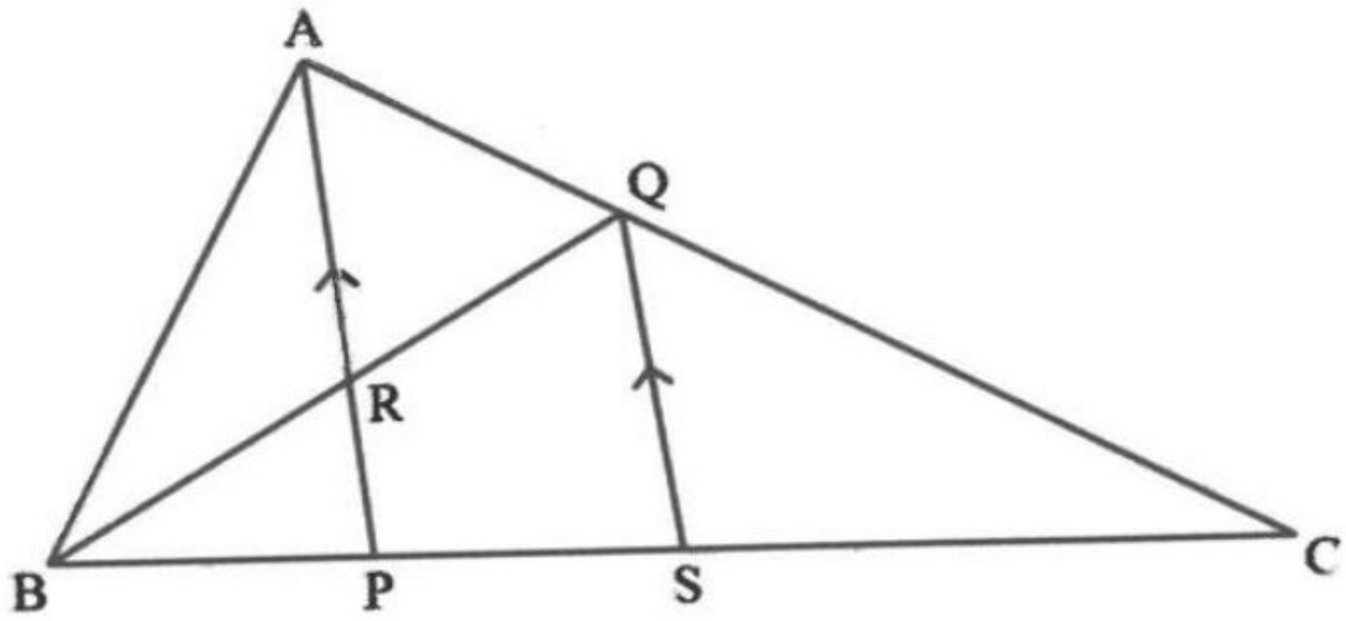
18. In ΔABC, AQ : QC = 1 : 3. AP//QS with P and S on BC and Q on AC. BQ intersects AP in R. $BP = \frac{1}{3}BC$.



Determine with reasons, the following:

18.1 $\frac{BP}{PS}$ (5) **L3**

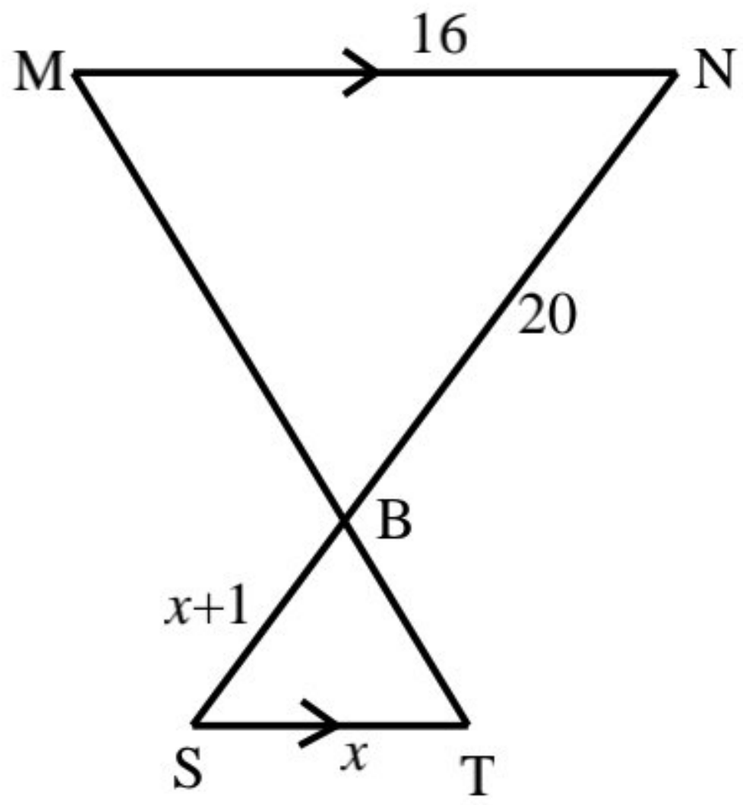
18.2 $\frac{BR}{QR}$ (2) **L2**

18.3 $\frac{\text{Area } \Delta \text{ APC}}{\text{Area } \Delta \text{ BQC}}$ (3) **L3**

18.4 $\frac{\text{Area } \Delta \text{ ABC}}{\text{Area } \Delta \text{ APC}}$ (3) **L3**

18.5 $\frac{\text{Area } \Delta \text{ QSC}}{\text{Area } \Delta \text{ ABP}}$ (4) **L4**

19. Refer to the diagram below where MN = 16 cm, NB = 20 cm, BS = $(x + 1)$ cm and ST = x cm. MN//ST.



19.1 Calculate the value of x. (4) **L3**

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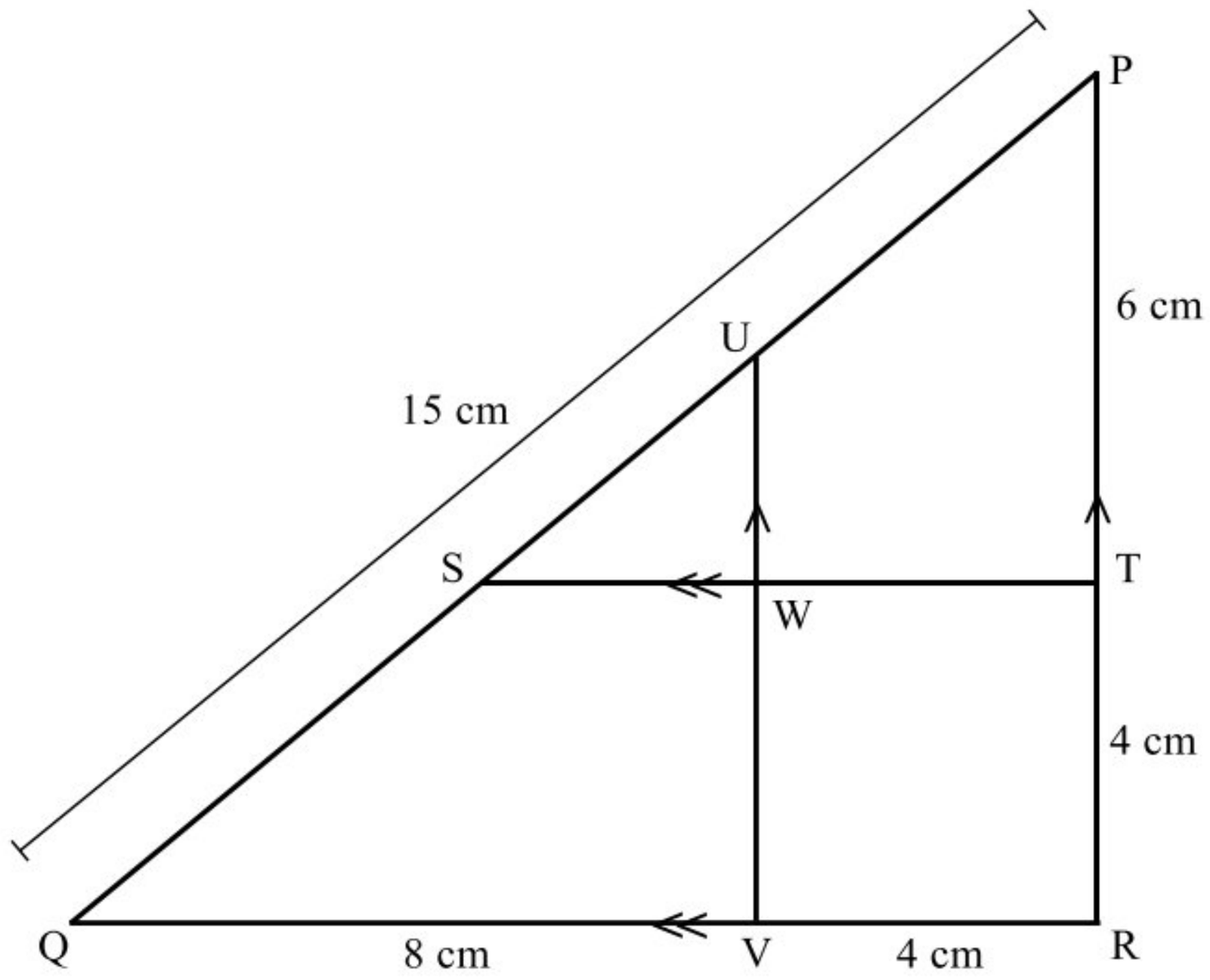
20. In ΔPQR, ST || QR. UV || PR, PT = 6 cm, VR = TR = 4 cm, PQ = 15 cm and QV = 8 cm. Straight lines ST and UV meet at W.



Calculate, with reasons:

20.1 the length of PS. (2) **L2**

20.2 the length of PU. (2) **L2**

20.3 Determine the numerical value of $\frac{VW}{UW}$ (1) **L1**

20.4 Calculate, with reasons, the length of UW. (2) **L2**

ANSWERS

SEQUENCES AND SERIES

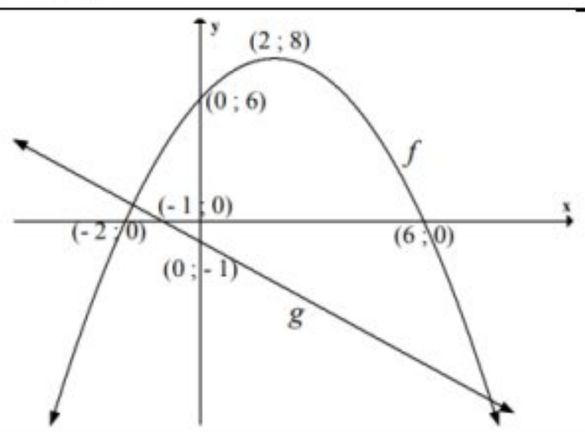
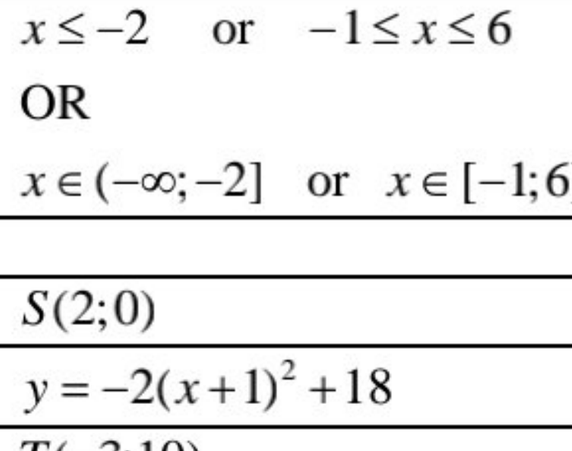
1	
1.1.1	$-1<\frac{1}{3}<1$ $\therefore$ *the series is convergent*
1.1.2	$\frac{3}{10}$
1.2.1	$4x;\frac{1}{81}$
1.2.2	xn
1.2.3	$\frac{1}{1594323}$
1.2.4	$\frac{1}{2}$
2.1	
2.2	T_8
3.1.1	$\frac{x}{2}$
3.1.2	$-2<x<2$
3.1.3	196602
3.2	6
3.3.1	-105
3.3.2	-21
3.3.3	89
4.1	51
4.2	$T_n=n^2-26n+120$
4.3	32
4.4	$n=49$ or $n\neq -20$
5.1	$T_n=n^2+2n$
5.2	17 drawers
5.3	15 complete rows will have handles

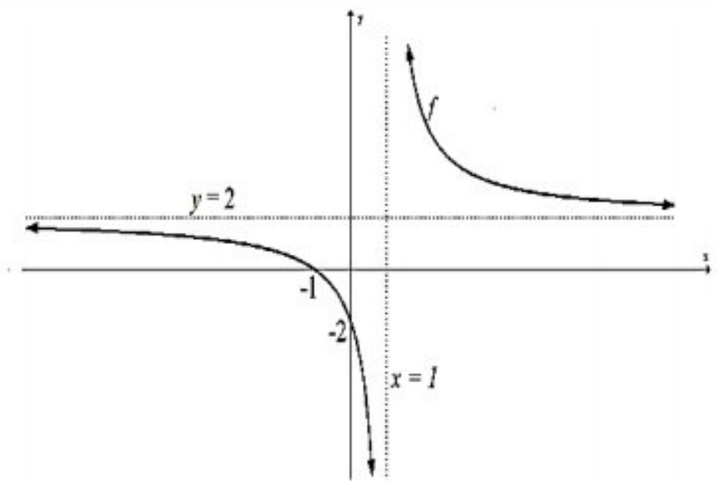
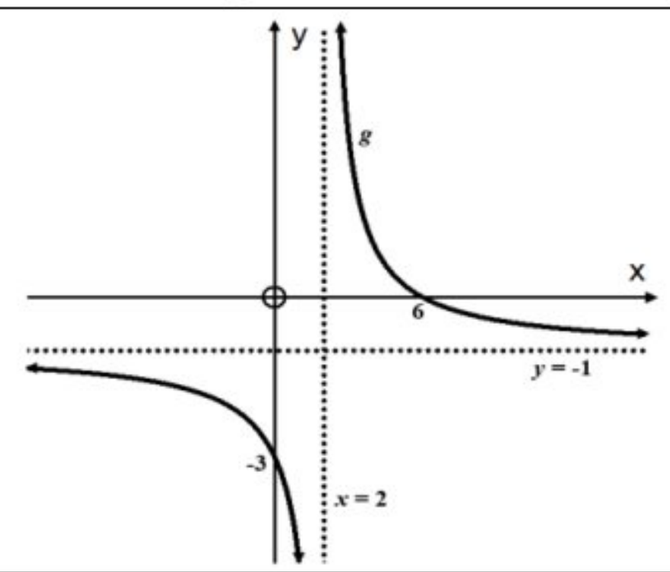
6.1.	$x=1$ $y=-5$
6.2.1	$T_n=6n^2-9n$
6.2.2	14550
6.2.3	
6.2.4	59 1st differences must be added
7.1.1	$\frac{1}{4}$
7.1.2	128
7.2.1	
7.2.2	$T_n=3n^2+5n-6$
7.3	$\sum_{r=8}^{20}(3r-7)p^r k^{23-r}$ *or* $\sum_{r=1}^{13}(3r+14)p^{r+7}k^{16-r}$
8.1.2	$n>11$
8.2	
9.1	$\frac{1}{3}$
9.2	$n=13$ *terms*
10.1.1	11;14
10.1.2	No term is a perfect square
10.2.1	$T_5=15;T_6=24$
10.2.2	$T_n=n^2-2n$
10.4	$n=91$
11.1.1	1426
11.1.2	117

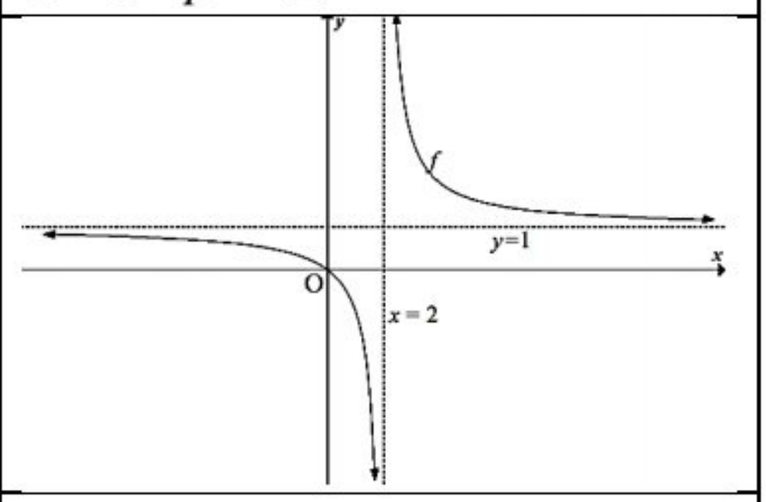
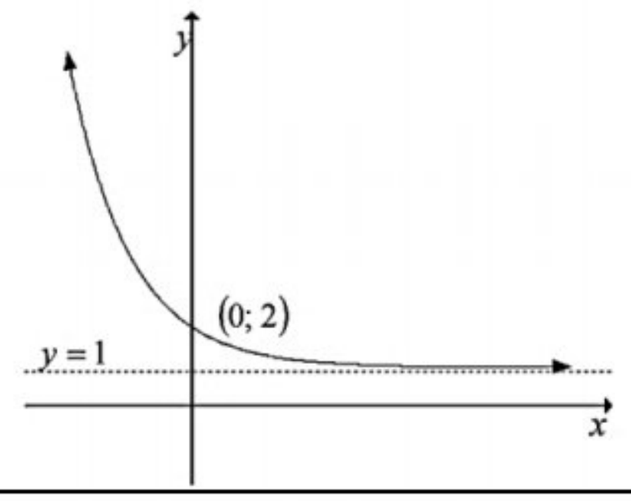
11.2	$r=\frac{3}{2}$ *or* $r=\frac{1}{2}$
12.1	-558
12.2.1	216 l
12.2.2	180 l
12.3.1	$0<x<\frac{2}{3}$
12.3.2	2
12.4	$x=4;y=26$
13.2	$0^\circ<\theta<30^\circ$
14.1.1.	$T_1=19;T_2=7$
14.1.2	$a=2;b=18;c=3$
14.1.3	$n=13$
15.1.1	49
15.1.2	2459

16	Value = -1
17	$a=9\ ;\ T_{16}=99$
18	$\frac{1}{1458}$
19.1	$x=14$
19.2	$T_n=4n+15$
20.1	$n=20$
20.2	$n=10$
20.3	$a=2$
21.1	k=5
21.2	k=18
22.1	3553
22.2	$\sum_{n=1}^{34}(7n-18)$

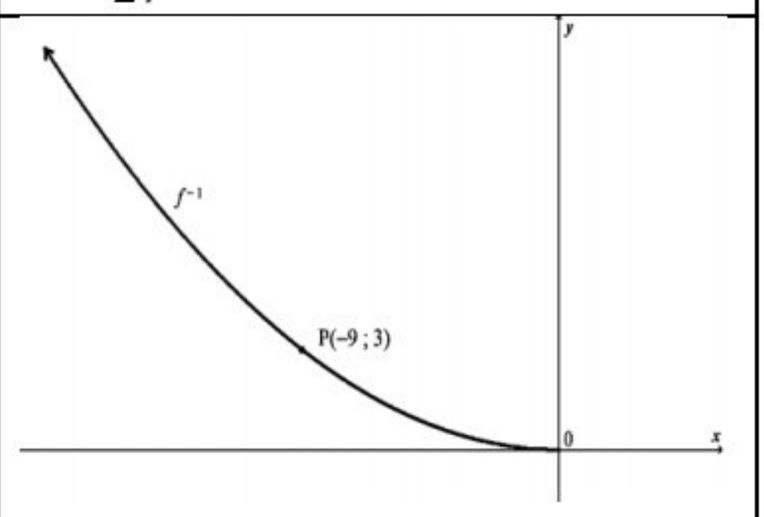
TOPIC: FUNCTIONS

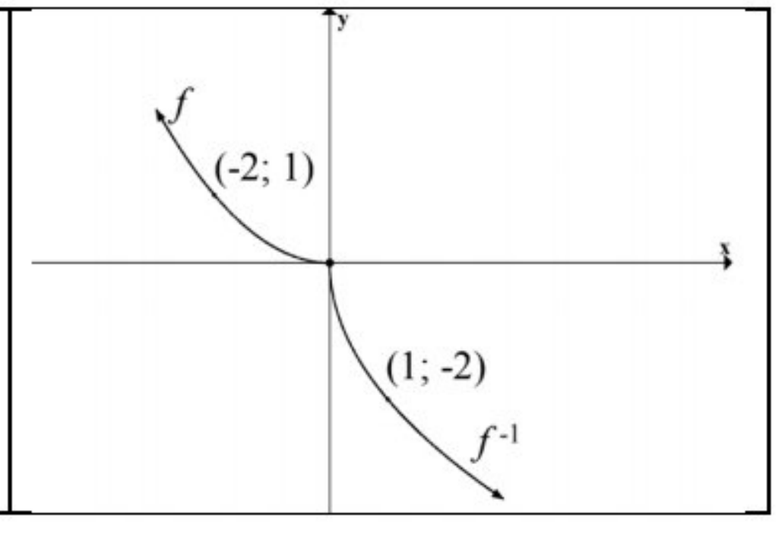
1		
	1.1	$a=\frac{1}{2}\quad b=2$
	1.2	$(-2;0)\qquad(6;0)$
	1.3	$(2;8)$
	1.4 1.6	
	1.5	$0<x<4\quad$ or $\quad x\in(0;4)$
	1.7	$x\le -2\quad$ or $\quad -1\le x\le 6$ OR $x\in(-\infty;-2]\quad$ or $\quad x\in[-1;6]$
2		
	2.1	$S(2;0)$
	2.2	$y=-2(x+1)^2+18$
	2.3	$T(-3;10)$
	2.4	$x<-3$ or $x>2$ $(-\infty;-3)\cup(2;\infty)$
	2.5.1	$x<-1$ or $(-\infty;-1)$
	2.5.2	Graph 
3		
	3.1	$y=-5$
	3.2	$D\ (2;-9)$
	3.3	$q=-5\quad a=-1$
	3.4	$y\in(-\infty;-5)$ OR $y<-5;y\in R$
	3.5	$k<-9$
4		
	4.1	$(-3;4)$
	4.2	$y\in(-\infty;4]$ OR $y\le 4;y\in R$
	4.4	$-5<c<-2$
	4.5	$h(x)=x+\frac{29}{4}$
5		
	5.1	$A(-2;0)$ and $B(4;0)$
	5.2	$C(1;18)$
	5.3	$y\le 18\quad$ or $\quad y\in(-\infty;18]$
	5.4	$p=-1\quad q=-3$
	5.5	$y=\frac{1}{2}x-2$
	5.6	$x=4\quad$ or $\quad x=-2$
	5.7	$k<-12,5$
6		
	6.1	E(−3;16)
	6.2	$k=12$
	6.3	$y=-x+7$
	6.4	$\mathrm{P}\left(-\frac{5}{2};\frac{63}{4}\right)$
	6.5	$-5<x<-1$ or $(-5;-1)$
7		
	7.1	$(0;3)$
	7.2	C(−1;4)
	7.3	A(−3;0)
	7.4	$\mathrm{CE}=\sqrt{5}/2,24\,\text{units}$
	7.5	$k=7$
	7.6	$y=\frac{x-6}{2}\quad$ or $\quad y=\frac{x}{2}-3$

	7.7	$x \geq -6$
8		
	8.1	$x = 1$ and $y = 2$
	8.2	$(0;-2)$
	8.3	$(-1;0)$
	8.4	
	8.5	$x \leq -1$ or $x > 1$
	8.6	$y = -x + 5$
9	9.1	$x = 1$ and $y = 1$
	9.2.1	$p = -1$ and $q = 2$
	9.2.2	$\left(\frac{1}{2};0\right)$
	9.2.3	$x = \frac{-5}{2}$
	9.2.4	$t = 1$
	9.2.5	$x \leq \frac{1}{2}$ or $x > 1$ OR $x \in \left(-\infty;\frac{1}{2}\right]$ or $x \in (1;\infty)$
10	10.1	$x = 2$ and $y = 1$
	10.2	
11		
	11.1	A(4;3)
	11.2	$B\left(0;\frac{3}{2}\right)$
	11.3	C(2;0)
	11.4	$m = -\frac{3}{4}$
	11.5	$y = -x + 7$
12		

	12.1	$q = 1$
	12.2	$a = 2 \quad p = -2$
	12.3	
13		
	13.2	$y \in R; \quad y \neq 1$
	13.3	$y = x + 3$
	13.4	K′ (−5;2)
14		
	14.1	$x \in R; \quad x \neq -1$
	14.2	$x = -\frac{3}{2}$
	14.3	$k = 2$
	14.4	C(2;4)
	14.5	$y = -(x-2)^2 + 4$
	14.6	$x \leq -\frac{3}{2}$ or $-1 < x < 0$ or $x > 4$
	14.7	One real root
15		
	15.1	$a = 4 \quad p = -1 \quad q = -8$
	15.2	$k = 3$
	15.3	$0 \leq x \leq 1$
	15.4	$-8 < k < -4$
	15.5	$y = -x - \frac{1}{2}$
	15.6	$Q\left(\frac{15}{2};-\frac{3}{2}\right)$
16		
	16.1	(0;2)
	16.2	
	16.3	$m = -\frac{7}{6}$
	16.4	$y = 3$
17		
	17.1	Decreasing function

	17.2	$y = \log_{\frac{1}{3}} x$	
	17.3	$x > 0;\ x \in R$	
	17.4.	$y = -5$	
18			
	18.1	$y = \log_3 x$	
	18.2	$P'(2;9)$	
	18.3	$q = -16 \quad p = 2$	
19			
	19.1	$T(0;1)$	
	19.2	$a = 3$	
	19.3	$h(x) = \left(\frac{1}{3}\right)^x$	
	19.4	$1 < x < 3$	
20			
	20.1	$y = \log_b x$	
	20.2	$y = x$	
	20.3	$P(1;0)$	
	20.4	$y = -x + 1$	
	20.5	$b = \frac{1}{4}$	
21			
	21.1	$0 < x \leq 1$ or $(0;1]$	
	21.2	$p = 2$	
	21.3	$y = \left(\frac{4}{3}\right)^x$	
	21.4	$y > 0$ or $y \in (0;\infty)$	
	21.5	$B''\left(-2;\frac{16}{9}\right)$	
22			
	22.1	$a = \frac{1}{3}$	
	22.2	$\left(0;\frac{22}{3}\right)$	
	22.3		
		22.3.1	7 unit down
		22.3.2	$y = \log_{\frac{1}{3}} x$
23			
	23.1	$U(1;0)$	
	23.2	$x = 1$ $y = 1$	
	23.3	$T(-1;0)$	
	23.4	$y = 5^x$	
	23.5	$y = 0$	
	23.6	$V(2,41;2,41)$	

	23.7	$T'(3;2)$
24		
	24.1	$r = 2$
	24.2	$p = -3$
	24.3	$A\left(-3;\frac{17}{8}\right)$
	24.4	$-3 < x \leq 0$ or $(-3;0]$
	24.5	$h(x) = \frac{3}{x+1} + 2$
25		
	25.1	$y = 6$
	25.2	$y = \frac{1}{2}x - 3$
	25.3	$A(-6;-6)$
	25.4	$AB = 13,42$
	25.5	Area = 54 square units
26		
	26.1	Yes, for every x-value there is only one corresponding y-value
	26.2	$R(-12;-6)$
	26.3	$a = -\frac{1}{3}$
	26.4	$y = -\sqrt{-3x}$ and $x \leq 0$
27		
	27.1	$0 \leq x \leq 3$ or $x \in [0;3]$
	27.2	$y = \frac{x^2}{27} \quad x \leq 0$
	27.3	
	27.4	Reflection about the x-axis
28		
	28.2	$y = -x^2,\ x \leq 0$
29		
	29.1	$f^{-1}(x) = -2\sqrt{x}$ or $f^{-1}(x) = -\sqrt{4x}$

	29.2	
	29.3	Yes, one to one function.

ANSWERS
FINANCIAL MATHEMATICS

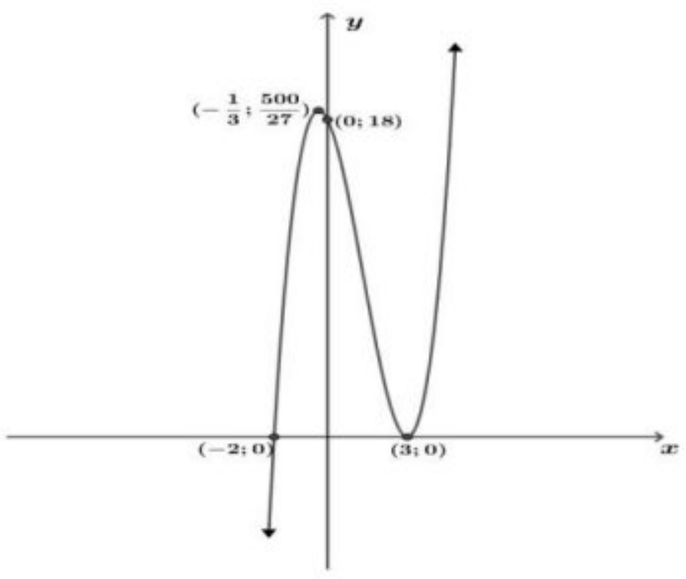
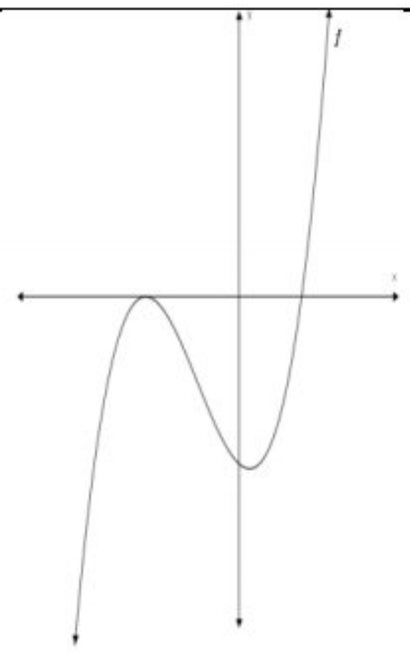
1	R3 037,50
2.1	155 payments
2.2	R3 230,50
2.3	R3 278,96
2.4	R773 278,96
3.1	R19 694,79
3.2	R1 588 473,03
3.3	R1 181 687,40
3.4	R770 160,43
4	12%
5.1	234 payments
5.2	R10 632,39
6.1	R1 034 939,44
6.2	R2 944 096.27
6.3	R1 909 156,83
6.4	R26 666,85
6.5	R27 070,32
7.1	R74 883,86
7.2	R168 306,21
7.3	R1 184,62
8	R57 934,44
9	8,24 years
10.1	18,48%
10.2	R678 635,11
10.3	R6 510,36
11	4 years
12	66,04 months
13	1,8:1
14	R791 000
15.1	R718 305,71
15.2	R273421,38

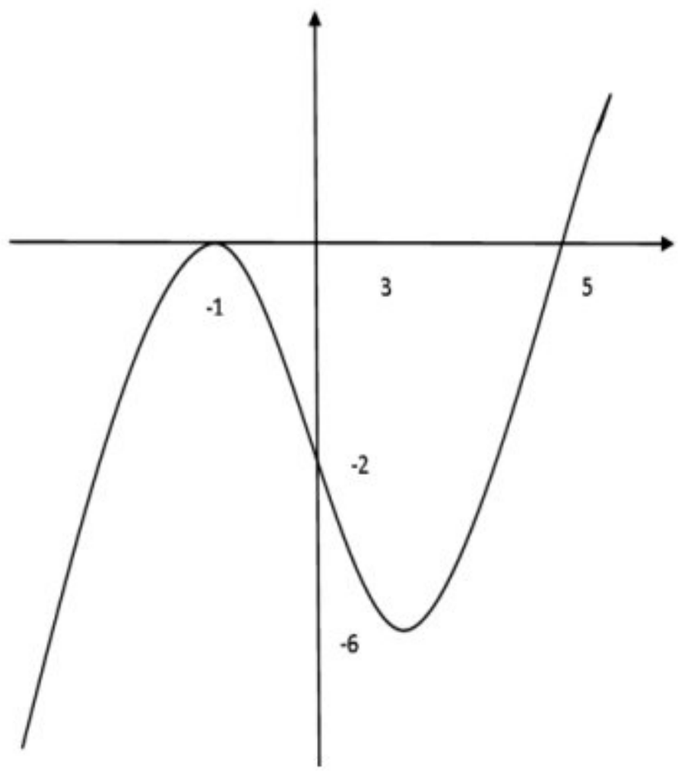
16.1	R7 982,73
16.2	R216 021,16
16.3	27 months
17.1	R14 808,16
17.2	R1555 284,20 or R1555 284,45
18.1	R602 201,51
18.2	R147 456
18.3	R6 029,18
19.1	R225 577,53
19.2	160 Months
20.1	R7 647,67
20.2	R173 507,13
21.1.1	R91 086,77
21.1.2	38 years
21.1.3	R454 299,12
21.2	R35 320,26
22 (a)	H 52%
(b)	OPTION 1 would yield the largest amount.
23.1	R12 378,21
23.2	3 Years
23.3	R7 409,40
23.4	Balance outstanding = R4 631,57 and Depreciated amount = R7 409,40. Therefore it would be sufficient
24.1	R50 636,01
24.2	R44 764,10
25.3	R1 674 501,40
25.2	75 Full payments
25.3	BO=R1 326 862,62; No.

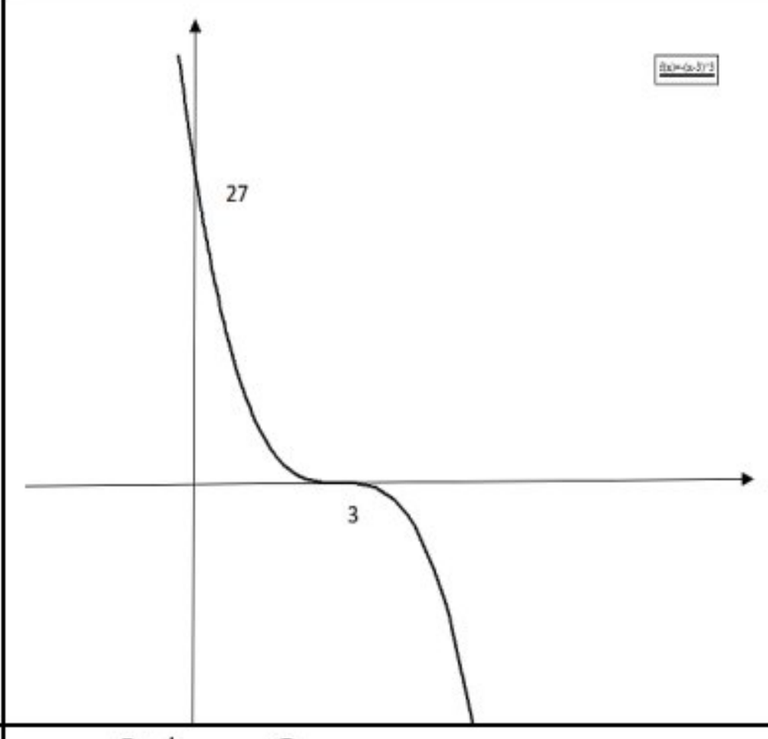
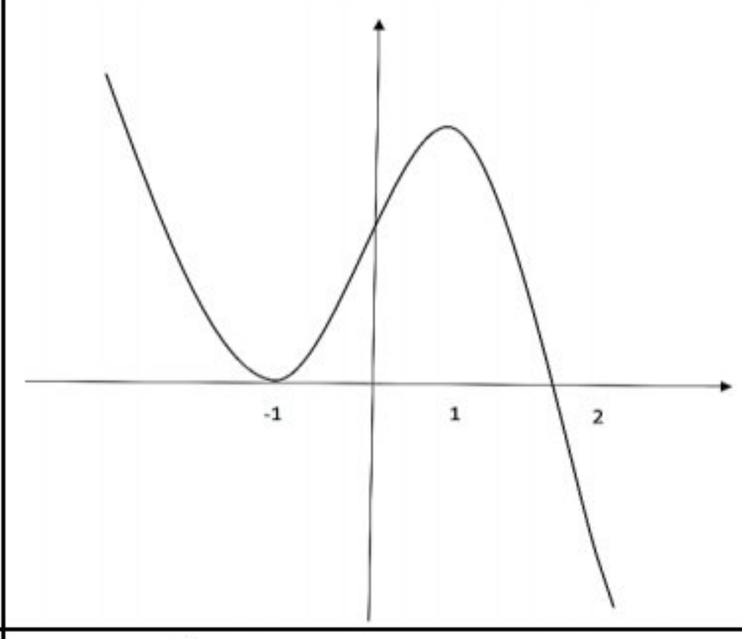
ANSWERS: CALCULUS

1.1	1
1.2	$2x+2$
1.3	$\frac{2}{x^2}$
1.4	$-6x^2$
1.5	$2ax+b$

1.6	0
1.7	$y=-30x+76$
1.8	$4x^2,\ f'(-1)=4$
2	

2.1	$\frac{1}{4}t^{-\frac{1}{2}}+\frac{3t^{-\frac{9}{2}}}{2}$
2.2	1
2.3	$2x+2$
2.4	$-\frac{1}{4s^{\frac{1}{2}}}+\frac{2}{s^3}$
2.5	3
2.6	$9x^2\beta^4-\beta^5$
2.7	$\frac{22}{x^{\frac{1}{2}}}-\frac{8}{x^5}$
2.8	2
2.9	
2.9.1	-5
2.9.2	$-\frac{1}{5}$
3.	
3.1	$y=3x-4$
3.2	$\left(-\frac{1}{2};-\frac{1}{2}\right)$
3.3	It is a tangent at $(5;-5)$
3.4	$a=-2$ $b=7$
3.5	$y=2x+3$
3.6	$b=-6$ $c=12$
3.7	$y=\frac{1}{2}x+7$
3.8	
3.8.1	$4\frac{1}{2}$
3.8.2	For $x<\frac{1}{2}$,h is concave up and for $x>\frac{1}{2}$,h is concave down $\therefore$ concavity changes at $x=\frac{1}{2}$.
3.8.3	The graph of h has a point of inflection at $x=\frac{1}{2}$
3.8.4	$x=-2$
4	
4.1	$(-\frac{1}{3};\frac{500}{27})$ and $(3;0)$
4.2	
4.3	$x<-\frac{1}{3}$ or $0<x<3$ OR $(-\infty;--\frac{1}{3})\cup(0;3)$
5	
5.1	$x=\frac{1}{3}$ or $x=-3$
5.2	
	
5.3	$x\le-3$ or $x\ge\frac{1}{3}$
5.4	$x^3+4x^2-3x-18$
6.1	$D(0;12)$
6.2	$B\left(-\frac{4}{3};\frac{500}{27}\right)$ $C(2;0)$
6.4	$C'(-2;1)$
6.5.1	$k=0$ or $k=\frac{500}{27}$
6.5.2	$k=12$
7.2	$L\left(-\frac{2}{3};\frac{400}{27}\right)$ $M(4;-36)$
7.3	$g(x)=4x-24$

7.4	$x=-3$ $AM=7units$
7.5.1	$x<-\frac{2}{3}$ or $x>4$
8.1	
8.2	$x=1$
8.3	$-1<x<3$
9.2	$x=2.12$
9.3	f is decreasing between A and B
9.4	$x>\frac{2}{3}$
9.5	Max = 10
10.1	Proof
10.2	$B\left(\frac{5}{3};-\frac{256}{27}\right)$
10.3.1	$-1\le x\le 3$
10.3.2	$x<-1$ or $x>\frac{5}{3}$
10.3.3	$x>\frac{1}{3}$
10.4	Length=2.52 *units*
11.2	$x=0$ $-\frac{1}{2}$ $y=24x-44$ $x=3$ $f(x)$ or $x=-\frac{7}{3}$ $f'(x)$ $\rightarrow$ $x>3$ $x=\frac{1}{2}$
17.2	$x=10$
17.3	360*square units*

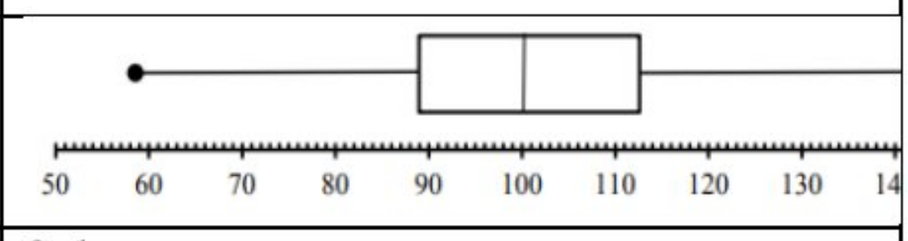
7.5.2	$x<\frac{5}{3}$
11.3	$x<-\frac{7}{3}$ or $x>0$
11.4	$x=0$ or $x=-3$
12	
13.1	$x=0$ / $x=3$
13.2.1	$f(x)$ & $f'(x)$
13.2.2	$f(x)\rightarrow$ point of inflection $f'(x)\rightarrow$ min TP
13.3	Length = 9 *units*
13.4	$x>3$
14	
15.1	$A=-x^2+3x+10$
15.2	$x=\frac{1}{3}$
16.1	Proof
16.2	$x=7$
16.3	73,5 *square units*

18.1	36 *cm*
18.2	Once

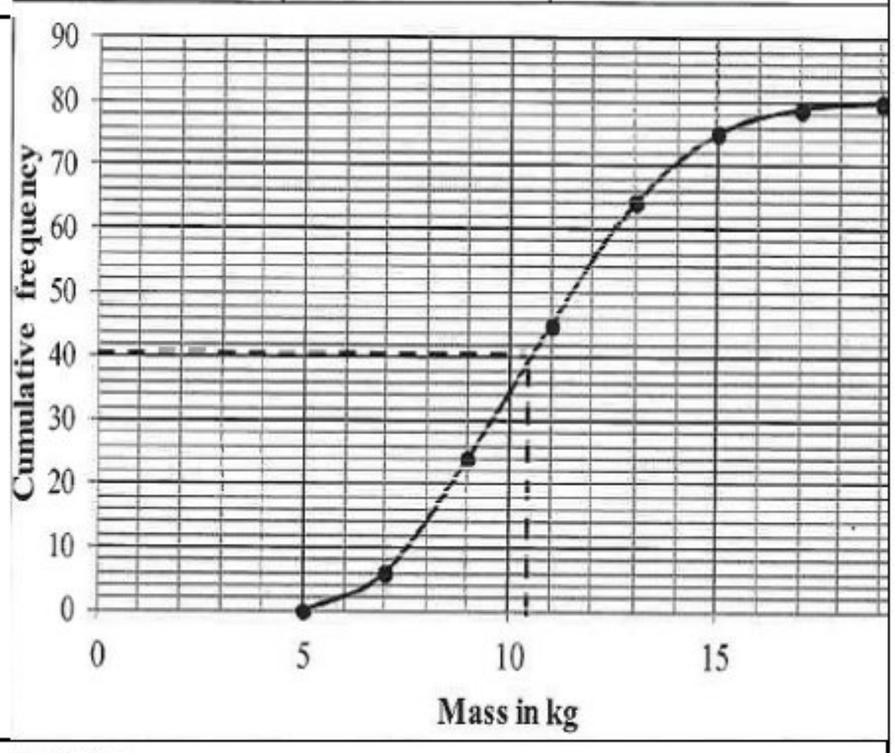
18.3	$52\ m$
19.1	3:2
19.2	$t = 5$
21.1	$25 - 10t$
21.2	$10 m/s$

21.3	$7{,}5 m/s$
21.4	$31{,}25 m$
22.1	$2\,309 m$
22.2	$0\ km$ or $5\ km$
22.3	$69°$

ANSWERS
TOPIC: STATISTICS

1.1	101,47
1.2	19,07
1.3	Q1 = 89 ; Q3 = 113
1.4	
1.5	2 days
2.1	$9 < m \leq 11$
2.2	

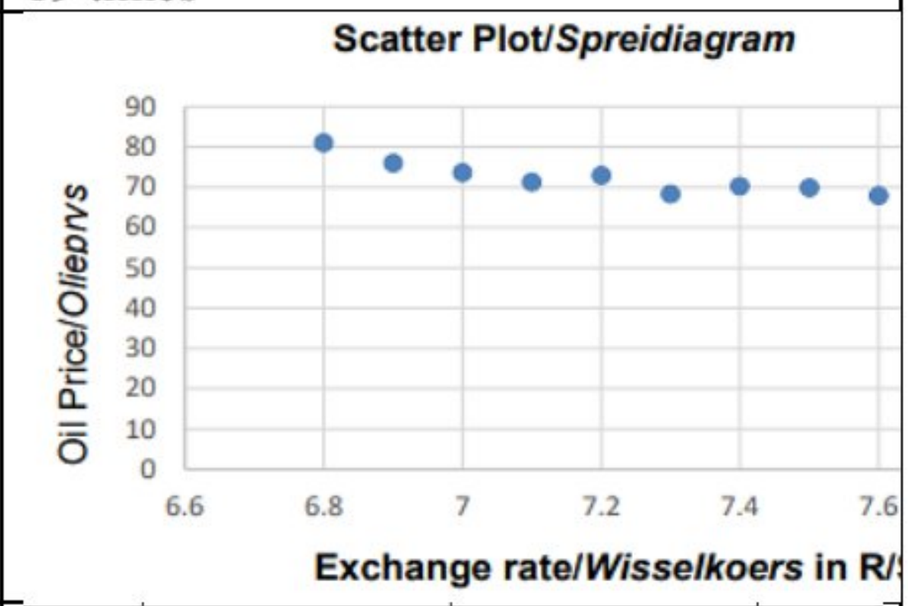
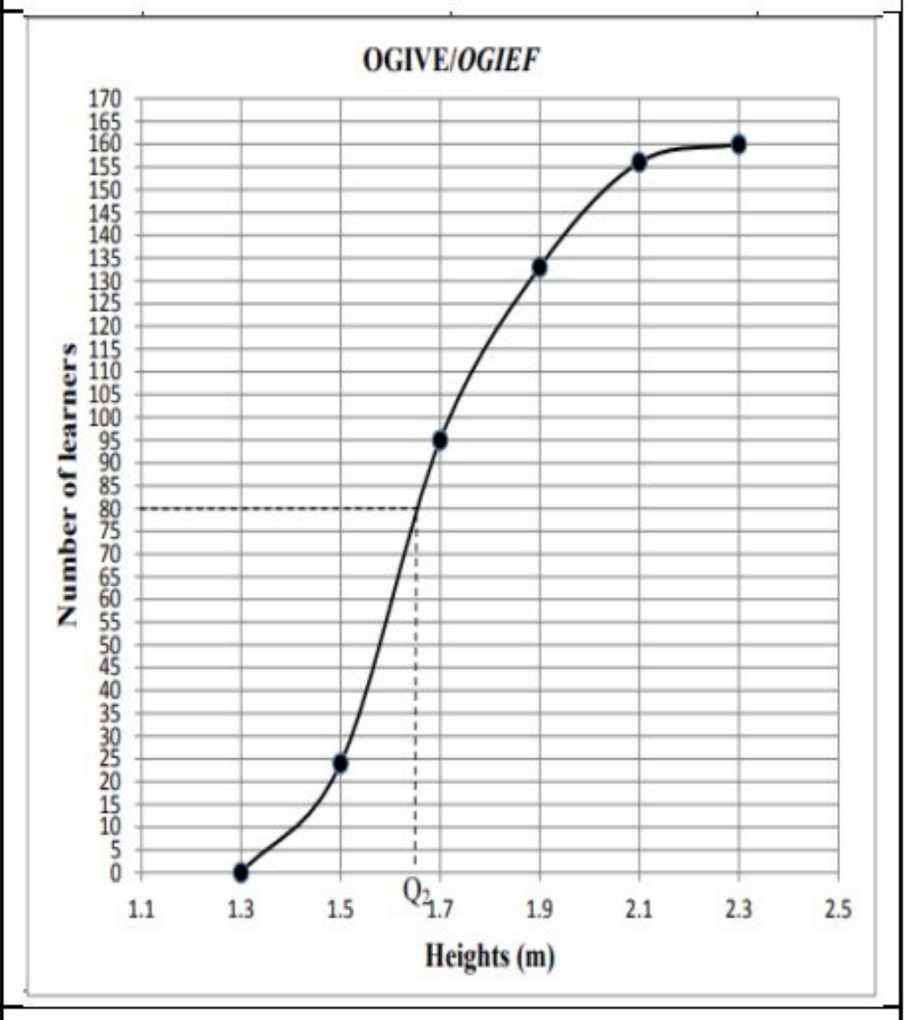
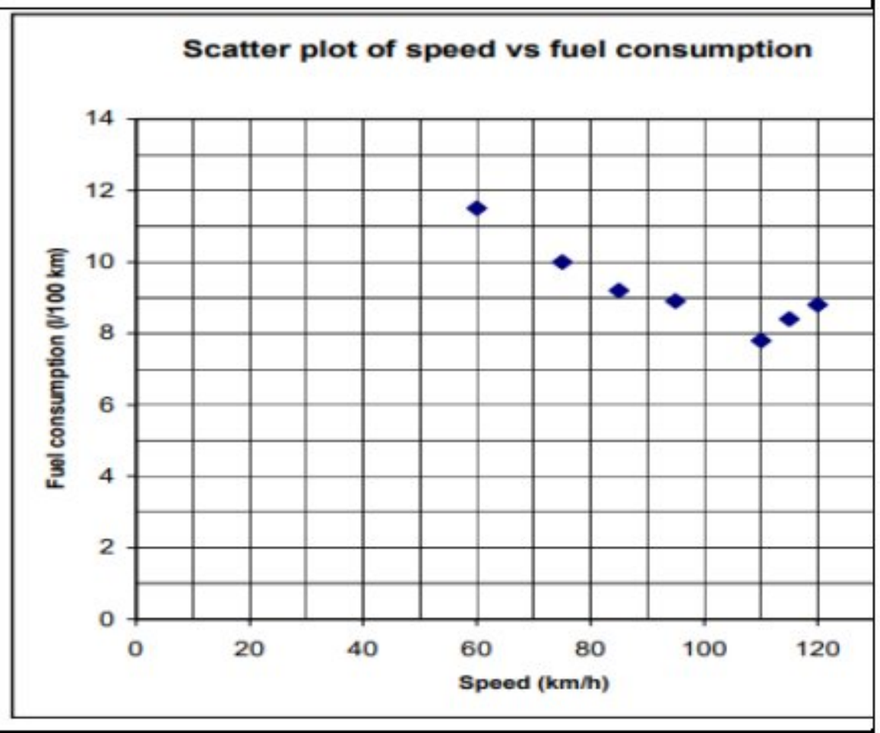
Mass (in kg)	Frequency	Cumulative frequen
$5 < m \leq 7$	6	6
$7 < m \leq 9$	18	24
$9 < m \leq 11$	21	45
$11 < m \leq 13$	19	64
$13 < m \leq 15$	11	75
$15 < m \leq 17$	4	79
$17 < m \leq 19$	1	80

2.3	
2.4	10,5 kg
2.5.1	10,68
2.5.2	Learners' bags are heavier than the stipulated international guidelines. 10, 68 kg is greater than 8 kg.

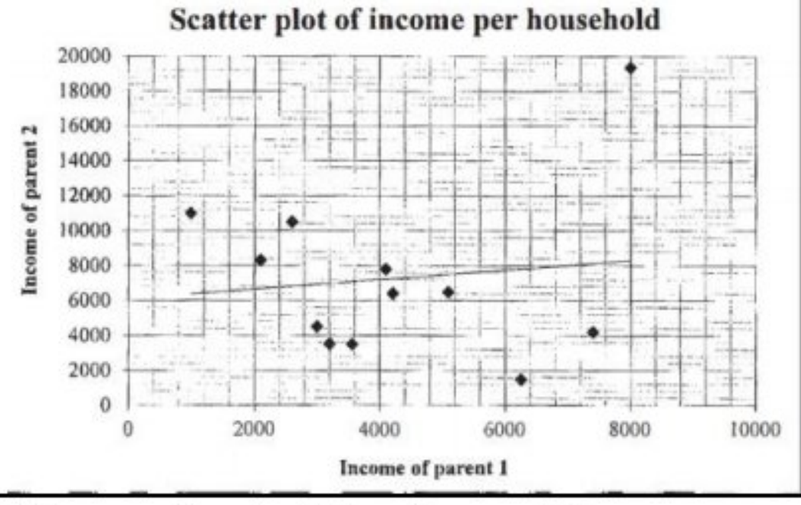
3.1	Modal class is $50 \leq x < 60$ **OR** 50 to 60
3.2	Median position is 15 learners (grouped data). Approximate weight is about 53 kg. (Accept from 52 kg to 54 kg)
3.3	7 learners collected more than 60 kg
4.1	July
4.2	2 245, 08 aircraft landings
4.3	86, 30
4.4	There were 9 months
4.5	The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of arrivals at the King Shaka International Airport **or C**
5.1	Range = 26 – 4 =22
5.2	Mean = 15,25
5.3	Standard deviation = 7,6
5.4.1	Increase in mean is 2°C per month.
5.4.2	Range of the data will decrease, This will result in the SD getting smaller.
6.1	Positively skewed/Skewed to the Right.
6.2	0, 82 m
6.3	

Intervals *Klasse*	Cumulative frequency *Kumulatiewe frekwensie*
$1{,}3 \leq x < 1{,}5$	24
$1{,}5 \leq x < 1{,}7$	95
$1{,}7 \leq x < 1{,}9$	133
$1{,}9 \leq x < 2{,}1$	156
$2{,}1 \leq x < 2{,}3$	160

7.1	6,73
7.2	Median is 7

7.3	Standard deviations is 2,26
7.4	19 times
8.1	
7.4	
7.5	1,65 (accept any value between 1,6 and 1,69)
7.6.1	The mean would change by 0,1 m
7.6.2	No influence/change
8.2	$y = 158{,}67 - 11{,}96x$
8.3	$r = -0{,}91$
8.4	Exchange rate increase, oil price decrease OR Strong Negative correlation
8.5	$y = 71{,}05$
8.6	Standard deviation: $\sigma = 4{,}09$
8.7	December
9.1	Skewed to the left or negatively skewed
9.2	A = 65; B = 99
9.3	Units not counted:150
10.1	150
10.2	R1 590 000
10.3	R10 600
10.4	No, 110 of the 150 employees earn R7 500 or less **OR** Only 10 employees in this company earn more than R10 600. The majority (140) of the employees earn below this amount. It is therefore not a good indicator of the average monthly amount earned by an employee.
11.1	51
11.2	$m = 12$ and $n = 10$
11.3	CUMUATIVE FREQUENCY 3 9 **21** 25 32 41 51
12	$a = 7$ $b = 15$ $c = 17$ $d = 23$ $e = 34$ $f = 37$
13.1	
13.2	Quadratic
13.3	The quadratic pattern shows that the best fuel consumption occurs when the car is driven at 110 km/h. In order for the company to keep its fuel bill to a minimum, drivers should be urged to travel at this speed where possible.
14.1.1	$\bar{y} = \frac{155}{10}$ $= 15{,}5$
14.1.2	SD = 4.59

14.2	$\bar{y} - SD$ $= 15,5 - 4,59$ $= 10,91$ $\therefore 10 - 2 = 8$ learners
14.3	$a = 1,7709\ldots$ $b = 0,2243\ldots$ $\hat{y} = 1,77 + 0,22x$
14.4	$\hat{y} = 1,77 + 0,22(72)$ $= 17,61$ ≈ 18 votes Or $\hat{y} = 17,92 \approx 18$ votes
14.5.1	Points are all scattered therefore low correlation and unrealistic prediction
14.5.2	r = 0.98/ correlation very strong hence a reliable prediction
15.1	60 employees
15.2	20<x<=25
15.3	60-34 = 26 employees
15.4	Salary = 100/7 *R2400 = R34 285.71
15.5	Ogive/ Cumulative frequency graph will shift to the right/ will become steeper

15.5	46 (Accept 45 – 47)
16.1	Median salary = R3885
16.2	Mean = R87 110/12 = R7259.17
16.3	sd = R4579.26 Salaries above one sd = mean + one SD =R7259.17 + R4579.26 = R11 838.43 Therefore only one household
16.4	y = a + bx a = 0.6102.11, b = 0.27 therefore y = 0.6102 + 0.27x
16.5	
16.6	The gradient of the line will become smaller causing the data to be more symmetrical about the regression line, and there will be no outlier.

TOPIC: ANALYITICAL GEOMETRY

		Feb/ March - 2018
1.1		$x = 1$
1.2		$m_{QP} = 3$
1.3		$y = 3x + 19$
1.4		R (3; 0) QR = $\sqrt{104}$ or $2\sqrt{26}$
1.5		$\tan(90^0 - \theta) = m_{QR}$ $= \frac{1}{5}$
1.6		RN = $\sqrt{26}$ SR = 6 Area ΔRSN = 15 units^2
2.1		**May/June – 2019**
	2.1.1	Midpoint of EC = $(0\ ;\ \frac{-3}{2})$
	2.1.2	$M_{DC} = \frac{1}{2}$
	2.1.3	$y = \frac{1}{2}x + 1$
	2.1.4	

		$\theta = 90^0 + 26.57^0$ $\theta = 116.57^0$
2.2		Proof
	2.2.1	Centre of circle = $(0\ ;\ \frac{-3}{2})$
	2.2.2	$x^2 + (y + \frac{3}{2})^2 = \frac{25}{4}$
		Limpopo/ Sep - 2019
	3.1	$m_{AB} = \frac{-1}{6}$
	3.2	$\hat{ABC} = 65.77^0$
	3.3	M(3 ; 0)
	3.4	y = -x +3
	3.5	K(-1 ; 4)
		May/June -2021
	4.1	M (6;-4)
	4.2	$m_{NS} = 2$

	4.3	$y=-\frac{1}{2}x-6$
	4.4	$x^{2}+y^{2}=256$
	4.5	T (0;-1)
4.6		$\frac{LS}{RS}=\frac{PS}{TS}=\frac{2}{3}$ OR $\frac{LS}{RS}=\frac{QS}{MS}=\frac{2}{3}$
4.7		Area of PTMQ = 25 units^2
		May/June 2022
5.1		$\frac{1}{2}$
5.2		$y=\frac{1}{2}x-7$
5.3	5.3.1	C (-6; -10)
	5.3.2	Area $\triangle$BCD = 41,25 units^2
5.4	5.4.1	K (-6; -4)
	5.4.2(a)	Perimeter $\triangle$KEC = 31,42 units
	5.4.2(b)	$\hat{KCE}=63,43^0$
		May/ June - 2023
6.1	6.1.1	$t=-10$
	6.1.2	$\alpha=135°$
	6.1.3	$m_{AC}=2$
6.2		$y=2x-8$
6.3	6.3.1	$C(9;10)$
	6.3.2	108.43°
6.4		$(x-19)^2+y^2=225$
		North West Sept- 2022
7.1		$y=-1$
7.2		$x=7$
7.3		$y=-x+6$
7.4		$\hat{ACB}=45°$
7.5	7.5.1	$K(6;-2)$
	7.5.2	4 sq units
7.6		F(5; 2,46)
		Feb/ March - 2018
8.1	8.1.1	OK = $\sqrt{108}$ or $6\sqrt{5}$
	8.1.2	$a^2+b^2=180$ K (-12;-6)
8.2		
	8.2.1	y = -2x - 3
	8.2.2	M(-16 ; -8)
	8.3.3	$(x+16)^2+(y+8)^2=20$
8.3		$6\sqrt{5}<r<10\sqrt{5}$

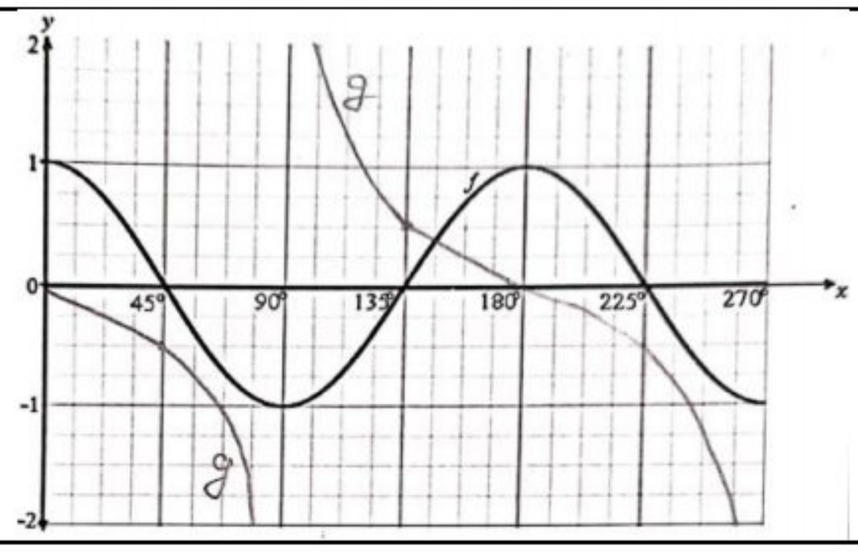
8.4		Proof
		FS Prep Exams- 2019
9.1	9.1.1	$x^2-6x+y^2-2y=30$
	9.1.2	$y=\frac{1}{3}x-\frac{20}{3}$
9.2	9.2.1	$(4;-2)$
	9.2.2	$r=\sqrt{10}$
9.3		EC = $2\sqrt{10}$
		North West Sept 2022
10.1		$M(-3;2)$
10.2		PR=10
10.3		$y=\frac{-3}{4}x+6$
10.4		D$\left(\frac{32}{3};-2\right)$
10.5		$\left(\frac{23}{3};2\right)$ and $\left(\frac{41}{3};2\right)$
		May/June - 2019
11.1		Proof
11.2		
	11.2.1	K(2 ; -4)
	11.2.2	y = $\frac{4}{3}x+\frac{20}{3}$
	11.2.3	Area of $\Delta LPK=\frac{260}{3}$ units^2
11.3		The centre of the two circles lie on the same vertical line x = 2 and the sum of the radii = 10. n = 11 or n = -9
		MAY/ JUNE 2021
12.1		$k=-2$
12.2		BC= 2units
12.3	12.3.1	$\alpha=45°$
	12.3.2	$\hat{VWB}=45°$
12.4	12.4.1	Q(-3;-2)
	12.4.2	$(x+3)^2+(y+2)^2=10$
	12.4.3	$x=-2$ or $x=-4$
		May/ June 2021
13.1	13.1.1	$b=a+1$
	13.1.2	$M(2;3)$
13.2	13.2.1	$r=5$
	13.2.2	TR=8units

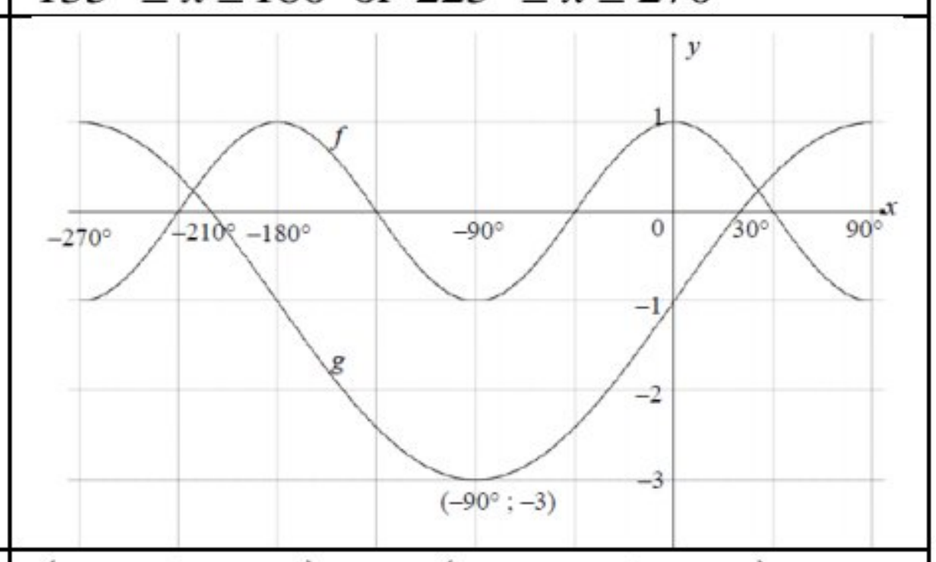
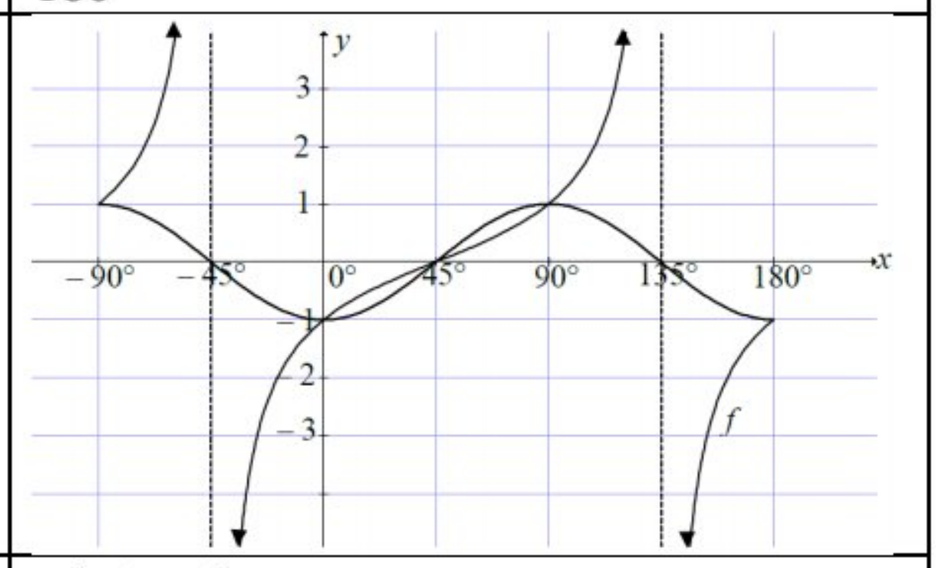
ANSWERS TRIGONOMETRY	
1.1	$k=-7$
1.2	$\frac{-24}{25}$
1.3	$\frac{-24}{7}$
1.4	$\frac{-7}{25}$
2.2	$\frac{1}{\sqrt{1+p^2}}$
3.	$\frac{-72}{25}$
4.1	$\frac{4}{5}$
4.2	$\frac{-28}{15}$
4.3	$\frac{-77}{85}$
5.	$\frac{-4p-3\sqrt{1-p^2}}{5}$
6.1	$\frac{-7}{5}$
6.2	$\frac{24}{7}$
7.1	$\frac{21}{221}$
7.2	$-\frac{7182}{48841}$
8.1	$\sqrt{1-k^2}$
8.2	$1-2k^2$
8.3	$2k\sqrt{1-k^2}$
8.4	$\frac{k^2}{\sqrt{1-k^2}}$
9.1	$\frac{p}{\sqrt{1-p^2}}$
9.2	$2p\sqrt{1-p^2}$
9.3	$\frac{\sqrt{2}}{2}(\sqrt{1-p^2}+p)$
9.4	$\sqrt{\frac{1-\sqrt{1-p^2}}{2}}$
10.1	$-p$
13.3	$y=\frac{-3}{4}x+\frac{43}{4}$

13.4	13.4.1	$N(2;-2)$
	13.4.2	$(x-2)^2+(y+2)^2=20$
		May/ June 2023
14.1		$M(-6;-3)$
14.2	14.2.1	$r=4$units
	14.2.2	TS = 1unit
14.3	14.3.1	$y=-8$
	14.3.2	$y=\frac{3}{4}x+\frac{31}{4}$
14.4	14.4.1	Perimetre PSMR=40 unts
	14.4.2	$\frac{1}{2}$

10.2	$-p$
10.3	$\sqrt{\frac{1-p}{2}}$
11.1	$\sqrt{1-p^2}$
11.2	$p.\sqrt{1-q^2}+q.\sqrt{1-p^2}$
12.1	$p+q$
12.2	$p-q$
13.	$\frac{1}{\sqrt{2}}$
14.1	$\frac{2\sqrt{3}}{3}$
14.2	$\frac{\sqrt{2}}{4}$
14.3	-1
15.1	$\tan x$
15.2	$\frac{1}{\cos x}$
15.3	$2\tan x$
15.4	$\tan x$
15.5	$\tan x$
16.1	$2\sin 4x$
16.2	$67,5^\circ$
17.2	$\frac{1}{3}$
18.2	$x\in[-180°,-90°,0°,90°,180°]$
19.2	$-1+\sqrt{2}$
20.1	$\theta=66,4^\circ+k.360^\circ$; or $\theta=293,6^\circ+k.360^\circ, k\in Z$
20.2	$\theta=103^\circ+k.180^\circ$; or $\theta=160^\circ+k.180^\circ, \quad k\in Z$

20.3	$\theta = 30^\circ + k.180^\circ$; or $\theta = 210^\circ + k.180^\circ, \quad k \in Z$
20.4	$\theta = 90^\circ + k.360^\circ$ or $\theta = 270^\circ + k.360^\circ$ or $\theta = 60° + k.360°$ or $\theta = 300° + k.360°, k \in Z$
20.5	$\theta = 210° + k.360°$; *or* $\theta = 330° + k.360°$; *or* $\theta = 90° + k.360°, \quad k \in Z$
20.6	$\theta = -(\frac{25}{3})° + k.120°$ $\theta = 295° + k.360°, \quad k \in Z$
20.7	$\theta = 30° + k.180°$ *or* $\theta = -240° - k.360°, \quad k \in Z$
20.8	$\theta = 109,47° + k.360°$ *or* $\theta = 250,53° + k.360°, \quad k \in Z$
20.9	$\theta = -60^\circ + k.360^\circ$ *or* $\theta = 80^\circ + k.120°, \quad k \in Z$
20.10	$\theta = 120^\circ + k.360^\circ$ *or* $\theta = 180^\circ + k.360^\circ$ $\theta = 240^\circ + k.120°, \quad k \in Z$
21.2	$\theta = 52,5^\circ + k.90^\circ$ *or* $\theta = 82,5^\circ + k.90^\circ$, $k \in Z$

	TRIGONOMETRIC GRAPHS
1.1	$-1 \le y \le 1$
1.2	720^0
1.3.1	$x = 30^0$ or $x = -150^0$
1.3.2	$-90^0 < x < -60^0$ or $90^0 < x < 120^0$
2.1	720^0
2.2	$p = \frac{1}{2}$; $q = 60^0$
2.3	$-360^0 \le \theta < -240^0$ or $-180^0 \le \theta < -60^0$
3.1	Maximum value = 4
3.2	$-3 \le y \le 5$ or $y \in [-3;5]$
3.3	$h(x) = -4\sin x$
4.1	
.2	$2 \le y \le 4$

4.3	$135^0 \le x \le 180^0$ or $225^0 \le x \le 270^0$
5.1	
5.3	$(38,17^0;0,24)$ and $(-218,17^0;0,24)$
6.1	180^0
6.2.1	$\sqrt{3} = 1,73$
6.2.2	$(-120^0;\sqrt{3})$
6.3	$-4 \le y \le 4$
6.4	$-65^\circ \le x \le -5^\circ$
6.5	$p = -\frac{1}{2}$ or $\frac{1}{2}$
7.1	1 unit
7.2	$\frac{1}{2} \le y \le 3\frac{1}{2}$
7.3	$x = 90^\circ$
7.4.1	$30^\circ < x < 90^\circ$ or $210^\circ < x < 240^\circ$
7.4.2	$-55^\circ < x < 125^\circ$
8.1	180°
8.2	
8.3	$-1 \le y \le 1$
8.4	$g(x + 45^\circ) = \sin 2x$
8.5.1	$-90^\circ < x < -45^\circ$
8.5.2	$-30^\circ < x < -30^\circ$

TOPIC	**2D AND 3D TRIGONOMETRY**
1.1	$\hat{S} = 65,6°$
1.2.1	$12,75\ unit^2$
1.2.2	6,47 units
2.3	$x = 2$
2.4	$2\sqrt{3}$ units
3.1.1	$10\sqrt{3} = 17,32$ units

3.1.2	20,30 units
3.2	$76,58^{\circ}$
4.1	5,85 units
4.2	$100,7^{\circ}$
6.	1,3 m shorter

TOPIC: EUCLIDEAN GEOMETRY ANSWERS

1.1	$\widehat{T}_1 = 90°$ line from centre to midpoint
1.2	$\widehat{O}_1 = 100°$ $\angle$ at the centre $\perp$ chord
1.3	$\widehat{S} = 50°$ $\angle$ in same segment
1.4	$\widehat{P}_1 = 40°$ sum of $\angle$'s in Δ
1.5	NO, $\widehat{P}_1 + \widehat{T}_1 \neq 180°$
2.	OB = 5cm
3.	
3.1	$\widehat{P} = 50°$ $\angle$ sum of $\angle$'s in Δ
3.2	$Q\hat{R}S = 120°$ opp $\angle$'s of a c.q
4.	
4.1	$M\hat{O}S = 62^0$ $\angle$at the cnt 2 X $\angle$ at the circum
4.2	$\hat{L} = 31^0$
4.3	$\hat{P} = 90^0$
4.4	$\hat{N}_2 = 90^0$
5.	
5.1	$\hat{R} = 80°$ co-int $\angle$'s of a c.q
5.2	$\hat{P} = 100°$ opp $\angle$'s of a c.q
5.3	$P\hat{Q}W = 36°$ ext $\angle$ of a c.q
5.4	$\hat{U}_2 = 136°$ alt $\angle$'s; QW $\|\|$ RK
6.	
6.1	$HM = 2x + 6$
6.2	$r = 15$ units
7.1	$\hat{B}_2$ alt $\angle$'s; VC $\|\|$ PQ
	$\hat{A}_1$ tan-chord theorem
	$\hat{B}_1$ $\angle$'s opp = sides
	$\hat{Q}$ corr $\angle$'s; VC $\|\|$ PQ
	$V\hat{B}Q$ alt $\angle$'s; VC $\|\|$ PQ

9.1.2	$\hat{D} = 40°$(ext $\angle$of ΔCDP)
9.1.3	$\hat{A}_1 = 40°$(ext $\angle$of ΔCDP)
10.1	$\hat{B}_1 = \hat{C}_1 = x$ tan-chord theorem
	$\hat{C}_1 = \hat{C}_4 = x$ vert opp. $\angle$'s
	$\hat{C}_4 = \hat{P}_2 = x$ $\angle$'s opp = sides
11.2.1	BD = 4 units
11.2.2	BE = 5 units
11.2.3	$x = 26.57°$
12.1	$\hat{A}_2 = x$ diag of rhombus bisects int $\angle$
12.2.1	$\hat{O}_1 = 90° - x$
12.2.2	$A\hat{D}C = 90° - x$
12.2.3	$\hat{C}_3 = \frac{90°-x}{2}$
13.	$\hat{S}_2 = \hat{R}$ $\hat{R} = \hat{W}_3$ $\hat{V}_3 = \hat{R}$
14.	ext $\angle$of a cyclic quad
14.1.1	$\angle$at the cnt 2 X $\angle$ at the circum
16.2	BF = 5,6 units
17.1	$\frac{AF}{AC} = \frac{4}{9}$ Prop theorem EF $\|\|$ BC
17.2	$\frac{HF}{AF} = \frac{25}{36}$ Prop theorem EF $\|\|$ BC
19.1	$x = 4$
20.1	PS = 9cm
20.2	PU =5cm
20.3	$\frac{VW}{UW} = \frac{6}{4} = \frac{3}{2}$
20.4	UW= $\frac{8}{3}$ cm

8.1	$\hat{B}_2 = \hat{P}_1 = x$ alt $\angle$'s; BC $\|\|$ AP
	$\hat{P}_1 = \hat{Q}_1 = x$ tan-chord theorem
	$\hat{P}_1 = \hat{P}_2 = x$ Given bisect
8.2	$\hat{B}_2 = \hat{P}_1 = x$ alt $\angle$'s; BC $\|\|$ AP
9.1.1	$\hat{P}_2 = 65°$($\angle$'s opp = sides)