KWV EDUCATION

KWV MATHS GRADE 12

CALCULUS QUESTIONS

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WHERE TO START IN MATHS AND SCIENCE

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x)=2x^2-1$. (5)

7.2 Determine:

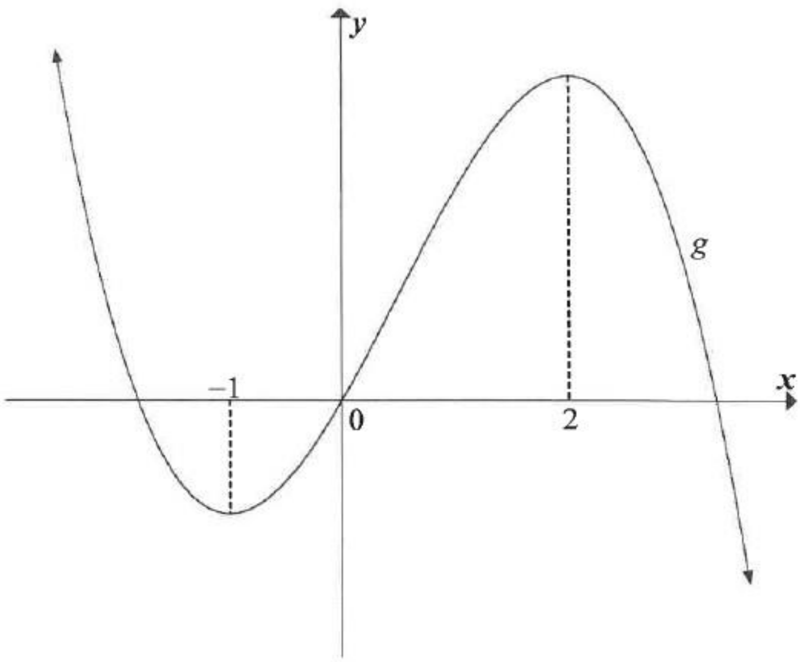
7.2.1 $\frac{d}{dx}\left(\sqrt[5]{x^2}+x^3\right)$ (3)

7.2.2 $f'(x)$ if $f(x)=\frac{4x^2-9}{4x+6}$; $x\neq-\frac{3}{2}$ (4)

[12]

QUESTION 8

The graph of $g(x)=ax^3+bx^2+cx$, a cubic function having a y-intercept of 0, is drawn below. The x-coordinates of the turning points of g are -1 and 2.



8.1 For which values of x will g increase? (2)

8.2 Write down the x-coordinate of the point of inflection of g. (2)

8.3 For which values of x will g be concave down? (2)

8.4 If $g'(x)=-6x^2+6x+12$, determine the equation of g. (4)

8.5 Determine the equation of the tangent to g that has the maximum gradient. Write your answer in the form $y=mx+c$. (5)

[15]

QUESTION 9

A closed rectangular box has to be constructed as follows:

- Dimensions: length (l), width (w) and height (h).
- The length (l) of the base has to be 3 times its width (w).
- The volume has to be $5\ m^3$.

The material for the top and the bottom parts costs R15 per square metre and the material for the sides costs R6 per square metre.

9.1 Show that the cost to construct the box can be calculated by: $\text{Cost} = 90w^2 + 48wh$ (4)

9.2 Determine the width of the box such that the cost to build the box is a minimum. (6)
[10]

KWV 02

QUESTION 7

7.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 4 - 7x$. (4)

7.2 Determine $\frac{dy}{dx}$ if $y = 4x^8 + \sqrt{x^3}$ (3)

7.3 Given: $y = ax^2 + a$

Determine:

7.3.1 $\frac{dy}{dx}$ (1)

7.3.2 $\frac{dy}{da}$ (2)

7.4 The curve with equation $y = x + \frac{12}{x}$ passes through the point A(2 ; b). Determine the equation of the line perpendicular to the tangent to the curve at A. (4)
[14]

QUESTION 8

After flying a short distance, an insect came to rest on a wall. Thereafter the insect started crawling on the wall. The path that the insect crawled can be described by $h(t) = (t-6)(-2t^2+3t-6)$, where h is the height (in cm) above the floor and t is the time (in minutes) since the insect started crawling.

8.1 At what height above the floor did the insect start to crawl? (1)

8.2 How many times did the insect reach the floor? (3)

8.3 Determine the maximum height that the insect reached above the floor. (4)
[8]

QUESTION 9

Given: $f(x) = 3x^3$

9.1 Solve $f(x) = f'(x)$ (3)

9.2 The graphs f, f' and f'' all pass through the point $(0\,;0)$.

9.2.1 For which of the graphs will $(0\,;0)$ be a stationary point? (1)

9.2.2 Explain the difference, if any, in the stationary points referred to in QUESTION 9.2.1. (2)

9.3 Determine the vertical distance between the graphs of f' and f'' at $x = 1$. (3)

9.4 For which value(s) of x is $f(x) - f'(x) < 0$? (4)
[13]

KWV 03

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given $f(x) = x^2 - 5$. (5)

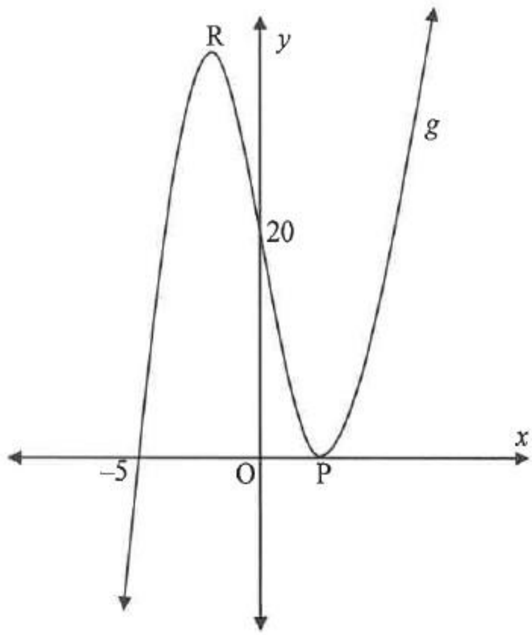
8.2 Determine $\frac{dy}{dx}$ if:

8.2.1 $y = 3x^3 + 6x^2 + x - 4$ (3)

8.2.2 $yx - y = 2x^2 - 2x \quad ; \quad x \neq 1$ (4)
[12]

QUESTION 9

9.1 The graph of $g(x) = x^3 + bx^2 + cx + d$ is sketched below.
The graph of g intersects the x-axis at $(-5 ; 0)$ and at P, and the y-axis at $(0 ; 20)$.
P and R are turning points of g.



9.1.1 Show that $b = 1$, $c = -16$ and $d = 20$. (4)

9.1.2 Calculate the coordinates of P and R. (5)

9.1.3 Is the graph concave up or concave down at $(0 ; 20)$? Show ALL your calculations. (3)

9.2 If g is a cubic function with:

- $g(3) = g'(3) = 0$
- $g(0) = 27$
- $g''(x) > 0$ when $x < 3$ and $g''(x) < 0$ when $x > 3$,

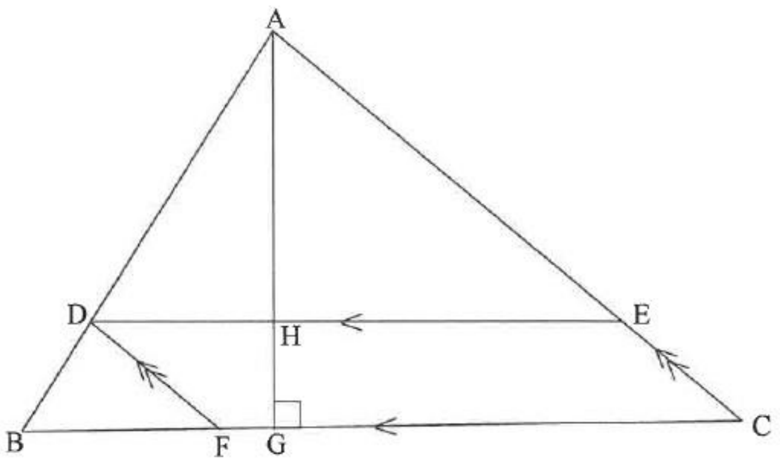
draw a sketch graph of g indicating ALL relevant points. (3)

[15]

QUESTION 10

In ΔABC:

- D is a point on AB, E is a point on AC and F is a point on BC such that DECF is a parallelogram.
- BF : FC = 2 : 3.
- The perpendicular height AG is drawn intersecting DE at H.
- AG = t units.
- BC = $(5 - t)$ units.



10.1 Write down AH : HG. (1)

10.2 Calculate t if the area of the parallelogram is a maximum.
(**NOTE:** Area of a parallelogram = base × $\perp$ height) (5)

[6]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=4x^2$. (5)

8.2 Determine:

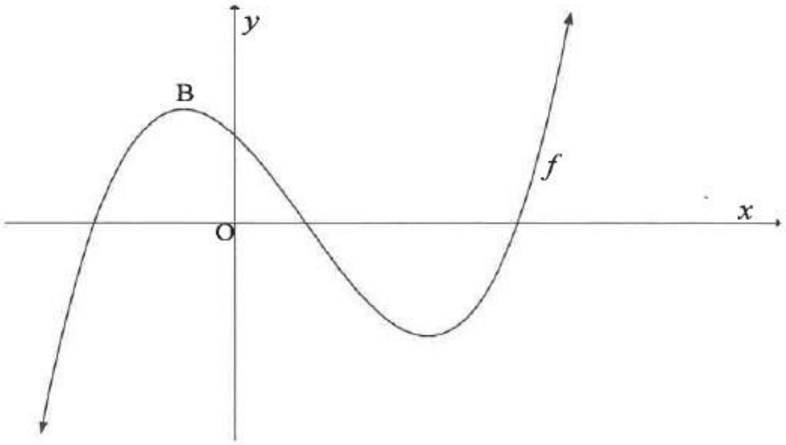
8.2.1 $D_x\left[\dfrac{x^2-2x-3}{x+1}\right]$ (3)

8.2.2 $f''(x)$ if $f(x)=\sqrt{x}$ (3)

[11]

QUESTION 9

The sketch below represents the curve of $f(x)=x^3+bx^2+cx+d$. The solutions of the equation $f(x)=0$ are -2 ; 1 and 4.



9.1 Calculate the values of b, c and d. (4)

9.2 Calculate the x-coordinate of B, the maximum turning point of f. (4)

9.3 Determine an equation for the tangent to the graph of f at $x=-1$. (4)

9.4 In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x- and y-intercepts on your sketch. (3)

9.5 For which value(s) of x is $f(x)$ concave upwards? (2)

[17]

QUESTION 10

Given: $f(x)=-3x^3+x$.

Calculate the value of q for which $f(x)+q$ will have a maximum value of $\frac{8}{9}$. **[6]**

QUESTION 7

7.1 Given: $f(x)=2x^2-x$

Determine $f'(x)$ from first principles. (6)

7.2 Determine:

7.2.1 $D_x[(x+1)(3x-7)]$ (2)

7.2.2 $\frac{dy}{dx}$ if $y=\sqrt{x^3}-\frac{5}{x}+\frac{1}{2}\pi$ (4)

[12]

QUESTION 8

Given: $f(x)=x(x-3)^2$ with $f'(1)=f'(3)=0$ and $f(1)=4$

8.1 Show that f has a point of inflection at $x=2$. (5)

8.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

8.3 For which values of x will $y=-f(x)$ be concave down? (2)

8.4 Use your graph to answer the following questions:

8.4.1 Determine the coordinates of the local maximum of h if $h(x)=f(x-2)+3$. (2)

8.4.2 Claire claims that $f'(2)=1$.

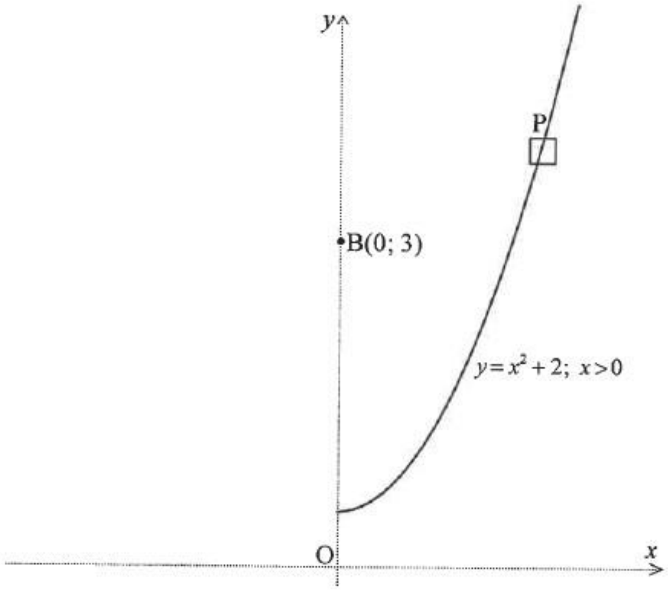
Do you agree with Claire? Justify your answer. (2)

[15]

QUESTION 9

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point B(0 ; 3) and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny. **[7]**

KWV 06

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 - 5$. (5)

7.2 Determine the derivative of: $g(x) = 5x^2 - \frac{2x}{x^3}$ (3)

7.3 Given: $h(x) = ax^2$, $x > 0$.
Determine the value of a if it is given that $h^{-1}(8) = h'(4)$. (6)

[14]

QUESTION 8

Given: $f(x) = 2x^3 - 5x^2 + 4x$

8.1 Calculate the coordinates of the turning points of the graph of f. (5)

8.2 Prove that the equation $2x^3 - 5x^2 + 4x = 0$ has only one real root. (3)

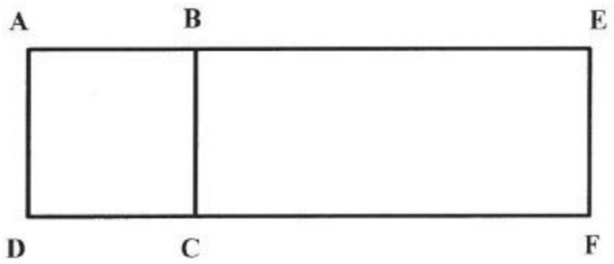
8.3 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (3)

8.4 For which values of x will the graph of f be concave up? (3)

[14]

QUESTION 9

A piece of wire 6 metres long is cut into two pieces. One piece, x metres long, is bent to form a square ABCD. The other piece is bent into a U-shape so that it forms a rectangle BEFC when placed next to the square, as shown in the diagram below.



Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum. **[7]**

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2$ (5)

8.2 John determines $g'(a)$, the derivative of a certain function g at $x = a$, and arrives at the answer: $\lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$
Write down the equation of g and the value of a. (2)

8.3 Determine $\frac{dy}{dx}$ if $y = \sqrt{x^3} - \frac{5}{x^3}$ (4)

8.4 $g(x) = -8x + 20$ is a tangent to $f(x) = x^3 + ax^2 + bx + 18$ at $x = 1$. Calculate the values of a and b. (5)

[16]

QUESTION 9

For a certain function f, the first derivative is given as $f'(x) = 3x^2 + 8x - 3$

9.1 Calculate the x-coordinates of the stationary points of f. (3)

9.2 For which values of x is f concave down? (3)

9.3 Determine the values of x for which f is strictly increasing. (2)

9.4 If it is further given that $f(x) = ax^3 + bx^2 + cx + d$ and $f(0) = -18$, determine the equation of f. (5)

[13]

QUESTION 10

The number of molecules of a certain drug in the bloodstream t hours after it has been taken is represented by the equation $M(t) = -t^3 + 3t^2 + 72t$, $0 < t < 10$.

10.1 Determine the number of molecules of the drug in the bloodstream 3 hours after the drug was taken. (2)

10.2 Determine the rate at which the number of molecules of the drug in the bloodstream is changing at exactly 2 hours after the drug was taken. (3)

10.3 How many hours after taking the drug will the rate at which the number of molecules of the drug in the bloodstream is changing, be a maximum? (3)

[8]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -x^2 + 4$. (5)

8.2 Determine the derivative of:

8.2.1 $y = 3x^2 + 10x$ (2)

8.2.2 $f(x) = \left(x - \frac{3}{x}\right)^2$ (3)

8.3 Given: $f(x) = 2x^3 - 23x^2 + 80x - 84$

8.3.1 Prove that $(x-2)$ is a factor of f. (2)

8.3.2 Hence, or otherwise, factorise $f(x)$ fully. (2)

8.3.3 Determine the x-coordinates of the turning points of f. (4)

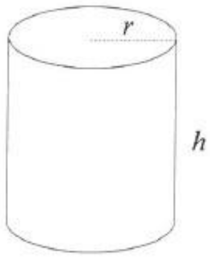
8.3.4 Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axes. (3)

8.3.5 Determine the coordinates of the y-intercept of the tangent to f that has a slope of 40 and touches f at a point where the x-coordinate is an integer. (6)

[27]

QUESTION 9

A soft drink can has a volume of 340 cm^3, a height of h cm and a radius of r cm.



9.1 Express h in terms of r. (2)

9.2 Show that the surface area of the can is given by $A(r) = 2\pi r^2 + 680r^{-1}$. (2)

9.3 Determine the radius of the can that will ensure that the surface area is a minimum. (4)

[8]

QUESTION 8

8.1 If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = \left(x^2 - \frac{1}{x^2}\right)^2$ (3)

8.2.2 $D_x\left(\frac{x^3 - 1}{x - 1}\right)$ (3)

[11]

QUESTION 9

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q(2 ; 10) are the turning points of h. The graph of h passes through the origin.

9.1 Show that $a = \frac{3}{2}$ and $b = 6$. (5)

9.2 Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P. (4)

9.3 Show that the concavity of h changes at $x = \frac{1}{2}$. (3)

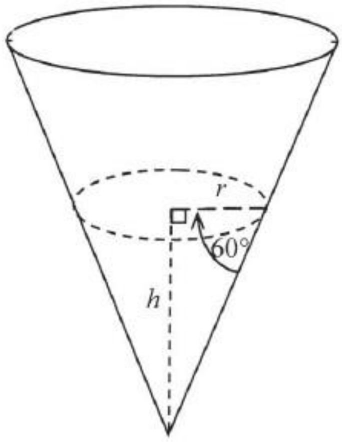
9.4 Explain the significance of the change in QUESTION 9.3 with respect to h. (1)

9.5 Determine the value of x, given $x < 0$, at which the tangent to h is parallel to g. (4)

[17]

QUESTION 10

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is 60°, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $\qquad$ $V = \frac{1}{3}\pi r^2 h$

$V = lbh$ $\qquad$ $V = \frac{4}{3}\pi r^3$

10.1 Determine r in terms of h. Leave your answer in surd form. (2)

10.2 Determine the derivative of the volume of water with respect to h when h is equal to 9 cm. (5)

[7]

QUESTION 8

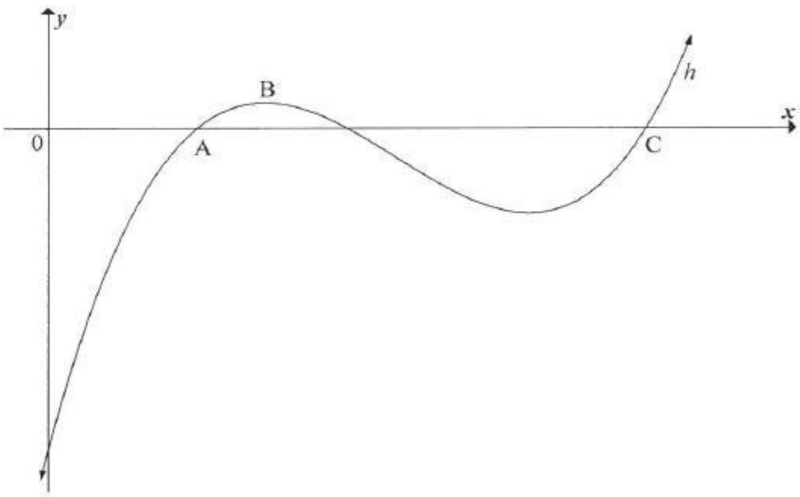
8.1 Determine the derivative of $f(x) = 2x^2 + 4$ from first principles. (4)

8.2 Differentiate:

8.2.1 $f(x) = -3x^2 + 5\sqrt{x}$ (3)

8.2.2 $p(x) = \left(\dfrac{1}{x^3} + 4x\right)^2$ (4)

8.3 The sketch below shows the graph of $h(x) = x^3 - 7x^2 + 14x - 8$. The x-coordinate of point A is 1. C is another x-intercept of h.



8.3.1 Determine $h'(x)$. (1)

8.3.2 Determine the x-coordinate of the turning point B. (3)

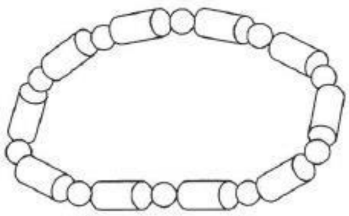
8.3.3 Calculate the coordinates of C. (4)

8.3.4 The graph of h is concave down for $x < k$. Calculate the value of k. (3)

[22]

QUESTION 9

A necklace is made by using 10 wooden spheres and 10 wooden cylinders. The radii, r, of the spheres and the cylinders are exactly the same. The height of each cylinder is h. The wooden spheres and cylinders are to be painted. (Ignore the holes in the spheres and cylinders.)



$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$
$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$

9.1 If the volume of a cylinder is 6 cm^3, write h in terms of r. (1)

9.2 Show that the total surface area (S) of all the painted surfaces of the necklace is equal to $S = 60\pi r^2 + \frac{120}{r}$ (4)

9.3 Determine the value of r so that the least amount of paint will be used. (4)
[9]

KWV 11

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = x^3$. (5)

8.2 Determine the derivative of: $f(x) = 2x^2 + \frac{1}{2}x^4 - 3$ (2)

8.3 If $y = (x^6 - 1)^2$, prove that $\frac{dy}{dx} = 12x^5\sqrt{y}$, if $x > 1$. (3)

8.4 Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up. (4)
[14]

QUESTION 9

Given: $f(x) = (x+2)(x^2 - 6x + 9)$
$= x^3 - 4x^2 - 3x + 18$

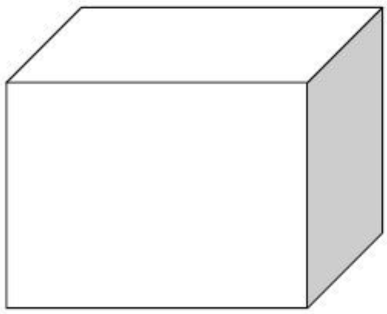
9.1 Calculate the coordinates of the turning points of the graph of f. (6)

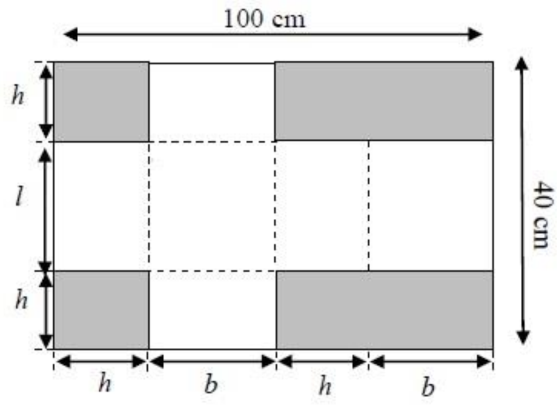
9.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

9.3 For which value(s) of x will $x.f^{\prime}(x) < 0$? (3)

[13]

QUESTION 10





A box is made from a rectangular piece of cardboard, 100 cm by 40 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram above.

10.1 Express the length l in terms of the height h. (1)

10.2 Hence prove that the volume of the box is given by $V = h(50-h)(40-2h)$ (3)

10.3 For which value of h will the volume of the box be a maximum? (5)

[9]

QUESTION 10

10.1 Given: $f(x) = -\frac{2}{x}$

10.1.1 Determine $f'(x)$ from first principles. (5)

10.1.2 For which value(s) of x will $f'(x) > 0$? Justify your answer. (2)

10.2 Evaluate $\frac{dy}{dx}$ if $y = \frac{1}{4}x^2 - 2x$. (2)

10.3 Given: $y = 4\left(\sqrt[3]{x^2}\right)$ and $x = w^{-3}$

Determine $\frac{dy}{dw}$. (4)

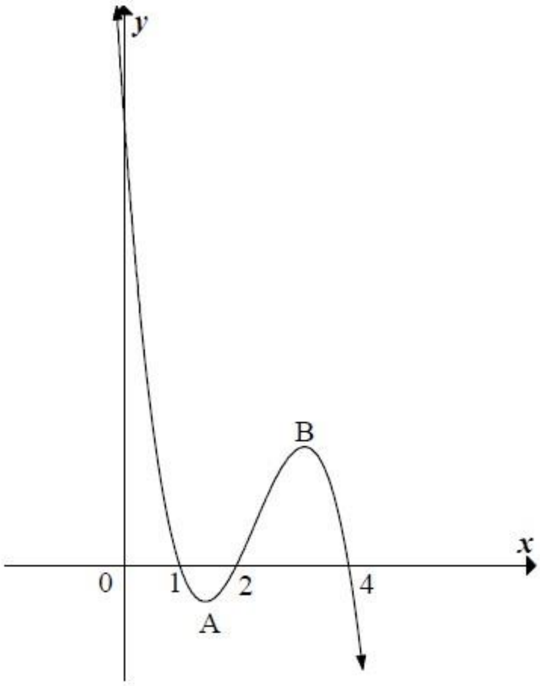
10.4 Given: $f(x) = ax^3 + bx^2 + cx + d$

Draw a possible sketch of $y = f'(x)$ if a, b and c are all NEGATIVE real numbers. (4)

[17]

QUESTION 11

The graph of $f(x) = -x^3 + ax^2 + bx + c$ is sketched below. The x-intercepts are indicated.



11.1 Calculate the values of a, b and c. (4)

11.2 Calculate the x-coordinates of A and B, the turning points of f. (5)

11.3 For which values of x will $f'(x) < 0$? (3)

[12]

QUESTION 12

A small business currently sells 40 watches per year. Each of the watches is sold at R144. For each yearly price increase of R4 per watch, there is a drop in sales of one watch per year.

12.1 How many watches are sold x years from now? (1)

12.2 Determine the annual income from the sale of watches in terms of x. (3)

12.3 In what year and at what price should the watches be sold in order for the business to obtain a maximum income from the sale of watches? (4)

[8]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 2$. (5)

8.2 Determine $\frac{dy}{dx}$ if $y = 2x^{-4} - \frac{x}{5}$. (2)

[7]

QUESTION 9

Given: $f(x) = x^3 - 4x^2 - 11x + 30$.

9.1 Use the fact that $f(2) = 0$ to write down a factor of $f(x)$. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (4)

9.3 Calculate the coordinates of the stationary points of f. (5)

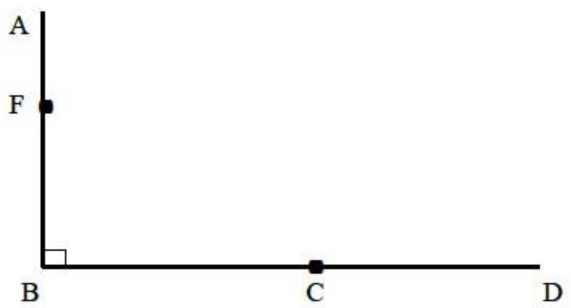
9.4 Sketch the curve of f in your ANSWER BOOK. Show all intercepts with the axes and turning points clearly. (3)

9.5 For which value(s) of x will $f'(x) < 0$? (2)

[15]

QUESTION 10

Two cyclists start to cycle at the same time. One starts at point B and is heading due north to point A, whilst the other starts at point D and is heading due west to point B. The cyclist starting from B cycles at 30 km/h while the cyclist starting from D cycles at 40 km/h. The distance between B and D is 100 km. After time t (measured in hours), they reach points F and C respectively.



10.1 Determine the distance between F and C in terms of t. (4)

10.2 After how long will the two cyclists be closest to each other? (4)

10.3 What will the distance between the cyclists be at the time determined in QUESTION 10.2? (2)

[10]

QUESTION 9

9.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 2x^2 - 3x$. (5)

9.2 Determine:

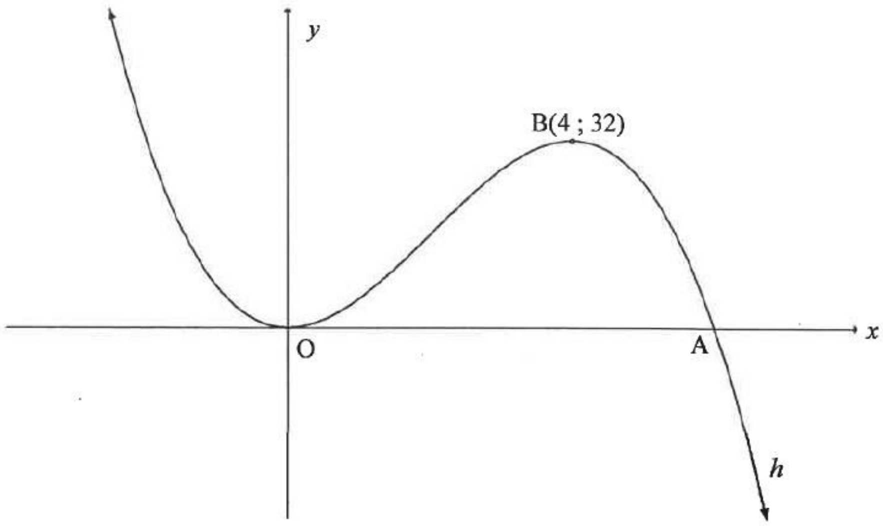
9.2.1 $\frac{dy}{dx}$ if $y = 4x^5 - 6x^4 + 3x$ (3)

9.2.2 $D_x\left[-\frac{\sqrt[3]{x}}{2} + \left(\frac{1}{3x}\right)^2\right]$ (4)

[12]

QUESTION 10

The graph of $h(x) = ax^3 + bx^2$ is drawn.
The graph has turning points at the origin, O(0 ; 0) and B(4 ; 32).
A is an x-intercept of h.



10.1 Show that $a = -1$ and $b = 6$. (5)

10.2 Calculate the coordinates of A. (3)

10.3 Write down the values of x for which h is:

10.3.1 Increasing (2)

10.3.2 Concave down (2)

10.4 For which values of k will $-(x-1)^3 + 6(x-1)^2 - k = 0$ have one negative and two distinct positive roots? (3)

[15]

QUESTION 11

After travelling a distance of 20 km from home, a person suddenly remembers that he did not close a tap in his garden. He decides to turn around immediately and return home to close the tap.

The cost of the water, at the rate at which water is flowing out of the tap, is R1,60 per hour.

The cost of petrol is $\left(1{,}2 + \frac{x}{4000}\right)$ rands per km, where x is the average speed in km/h.

Calculate the average speed at which the person must travel home to keep his cost as low as possible. **[7]**

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 5$ (5)

7.2 Determine $\frac{dy}{dx}$ if:

7.2.1 $y = 2x^5 + \frac{4}{x^3}$ (3)

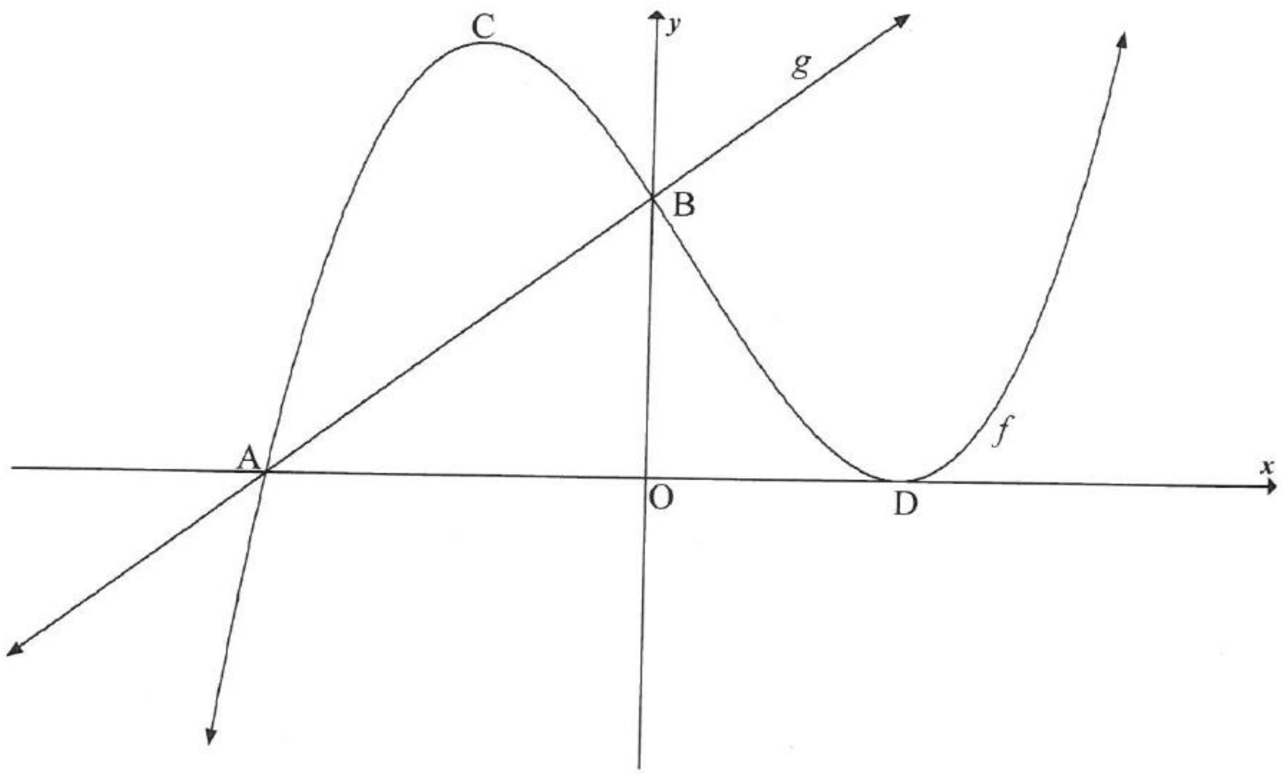
7.2.2 $y = \left(\sqrt{x} - x^2\right)^2$ (4)

[12]

QUESTION 8

Sketched below are the graphs of $f(x) = (x-2)^2(x-k)$ and $g(x) = mx + 12$

- A and D are the x-intercepts of f.
- B is the common y-intercept of f and g.
- C and D are turning points of f.
- The straight line g passes through A.



8.1 Write down the y-coordinate of B. (1)

8.2 Calculate the x-coordinate of A. (3)

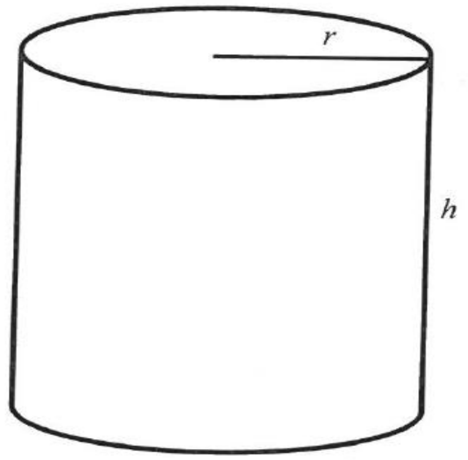
8.3 If $k = -3$, calculate the coordinates of C. (6)

8.4 For which values of x will f be concave down? (3)

[13]

QUESTION 9

A 340 mℓ can with height h cm and radius r cm is shown below.



$1 \text{ m}\ell = 1 \text{ cm}^3$

9.1 Determine the height of the can in terms of the radius r. (3)

9.2 Calculate the length of the radius of the can, in cm, if the surface area is to be a minimum. (6)

[9]

KWV 16

QUESTION 8

8.1 Given $f(x) = 3 - 2x^2$. Determine $f'(x)$, using first principles. (5)

8.2 Determine $\frac{dy}{dx}$ if $y = \frac{12x^2 + 2x + 1}{6x}$. (4)

8.3 The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at (2 ; 4). Calculate the values of b and c. (7)

[16]

QUESTION 9

Given: $f(x) = x^3 - x^2 - x + 1$

9.1 Write down the coordinates of the y-intercept of f. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (5)

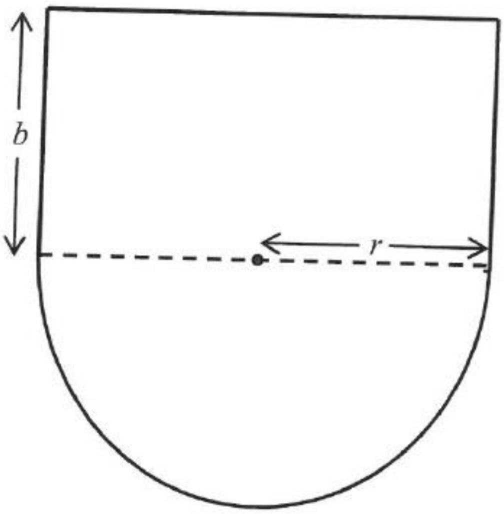
9.3 Calculate the coordinates of the turning points of f. (6)

9.4 Sketch the graph of f in your ANSWER BOOK. Clearly indicate all intercepts with the axes and the turning points. (3)

9.5 Write down the values of x for which $f'(x) < 0$. (2)

[17]

QUESTION 10



The figure above shows the design of a theatre stage which is in the shape of a semicircle attached to a rectangle. The semicircle has a radius r and the rectangle has a breadth b. The perimeter of the stage is 60 m.

10.1 Determine an expression for b in terms of r. (2)

10.2 For which value of r will the area of the stage be a maximum? (6)

[8]

KWV 17

QUESTION 7

7.1 Given: $f(x)=2-3x^2$

Determine $f'(x)$ from first principles. (5)

7.2 Determine:

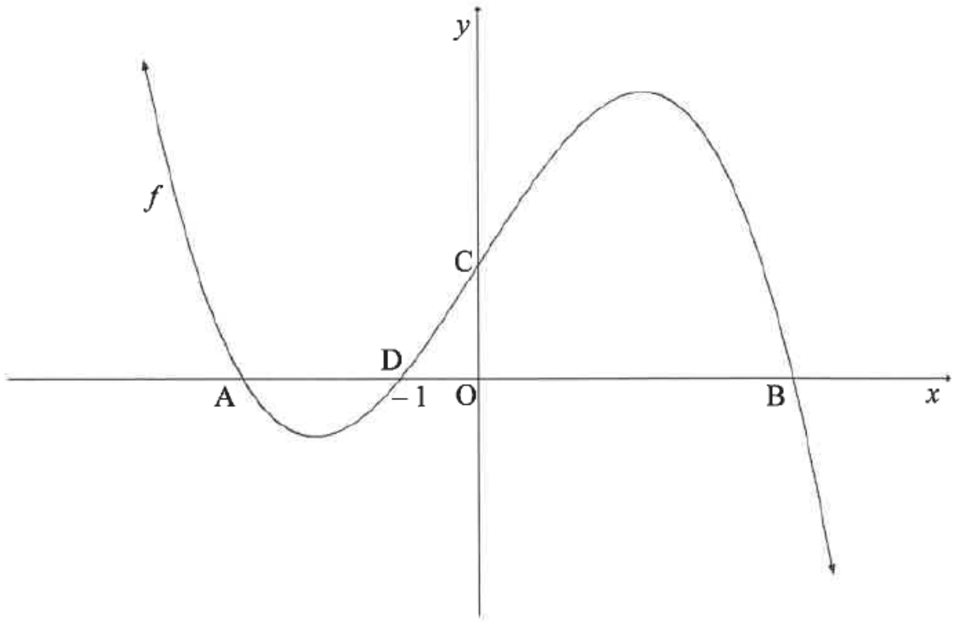
7.2.1 $D_x\left[(4x+5)^2\right]$ (3)

7.2.2 $\frac{dy}{dx}$ if $y=\sqrt[4]{x}+\frac{x^2-8}{x^2}$ (4)

[12]

QUESTION 8

The graph of $f(x) = -x^3 + 13x + 12$ is sketched below.
A, B and D(–1 ; 0) are the x-intercepts of f.
C is the y-intercept of f.



8.1 Write down the coordinates of C. (1)

8.2 Calculate the coordinates of A and B. (5)

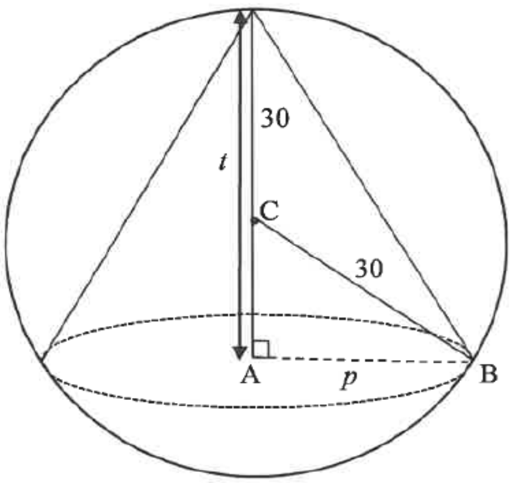
8.3 Determine the point of inflection of g if it is given that $g(x) = -f(x)$. (4)

8.4 Calculate the value(s) of x for which the tangent to f is parallel to the line $y = -14x + c$. (4)

[14]

QUESTION 9

A right circular cone with radius p and height t is machined (cut out) from a solid sphere (with centre C) with a radius of 30 cm, as shown in the sketch.



Sphere: $V = \frac{4}{3}\pi r^3$
Cone: $V = \frac{1}{3}\pi r^2 h$

9.1 From the given information, express the following:

9.1.1 AC in terms of t. (1)

9.1.2 p^2, in its simplest form, in terms of t. (3)

9.2 Show that the volume of the cone can be written as $V(t) = 20\pi t^2 - \frac{1}{3}\pi t^3$. (1)

9.3 Calculate the value of t for which the volume of the cone will be a maximum. (3)

9.4 What percentage of the sphere was used to obtain this cone having maximum volume? (4)

[12]

KWV 18

QUESTION 7

7.1 Given $f(x) = x^2 + 2$.

Determine $f'(x)$ from first principles. (4)

7.2 Determine $\frac{dy}{dx}$ if:

7.2.1 $y = 4x^3 + \frac{2}{x}$ (3)

7.2.2 $y = 4.\sqrt[3]{x} + (3x^3)^2$ (4)

7.3 If g is a linear function with $g(1) = 5$ and $g'(3) = 2$, determine the equation of g in the form $y = \ldots..$ (3)

[14]

QUESTION 8

A cubic function $h(x) = -2x^3 + bx^2 + cx + d$ cuts the x-axis at $(-3\ ;\ 0)$; $\left(-\frac{3}{2}\ ;\ 0\right)$ and $(1\ ;\ 0)$.

8.1 Show that $h(x) = -2x^3 - 7x^2 + 9$. (3)

8.2 Calculate the x-coordinates of the turning points of h. (3)

8.3 Determine the value(s) of x for which h will be decreasing. (2)

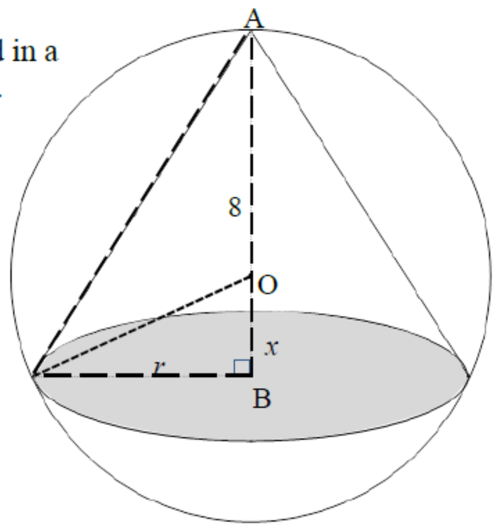
8.4 For which value(s) of x will there be a tangent to the curve of h that is parallel to the line $y - 4x = 7$. (4)

[12]

QUESTION 9

A cone with radius r cm and height AB is inscribed in a sphere with centre O and a radius of 8 cm. OB $= x$.

Volume of sphere $= \frac{4}{3}\pi r^3$
Volume of cone $= \frac{1}{3}\pi r^2 h$



9.1 Calculate the volume of the sphere. (1)

9.2 Show that $r^2 = 64 - x^2$. (1)

9.3 Determine the ratio between the largest volume of this cone and the volume of the sphere. (7)

[9]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 3x^2$. (5)

8.2 Determine:

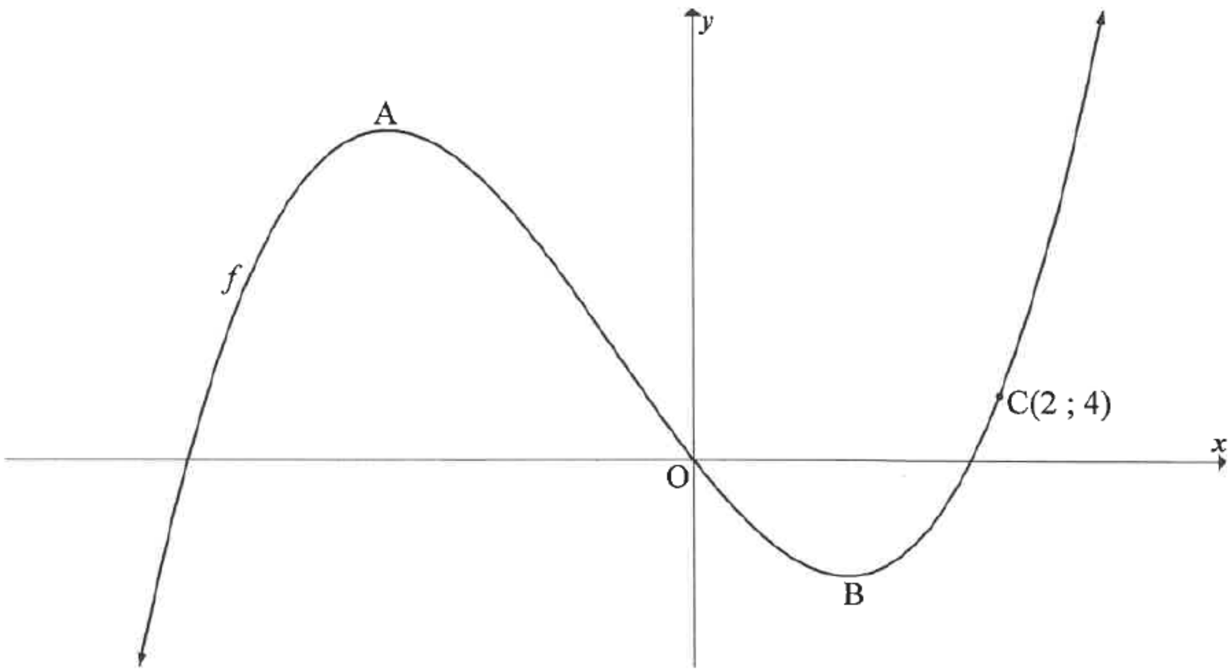
8.2.1 $f'(x)$ if $f(x) = x^2 - 3 + \frac{9}{x^2}$ (3)

8.2.2 $g'(x)$ if $g(x) = (\sqrt{x} + 3)(\sqrt{x} - 1)$ (4)

[12]

QUESTION 9

The graph of $f(x) = 2x^3 + 3x^2 - 12x$ is sketched below.
A and B are the turning points of f. C(2 ; 4) is a point on f.



9.1 Determine the coordinates of A and B. (5)

9.2 For which values of x will f be concave up? (3)

9.3 Determine the equation of the tangent to f at C(2 ; 4). (3)

[11]

QUESTION 10

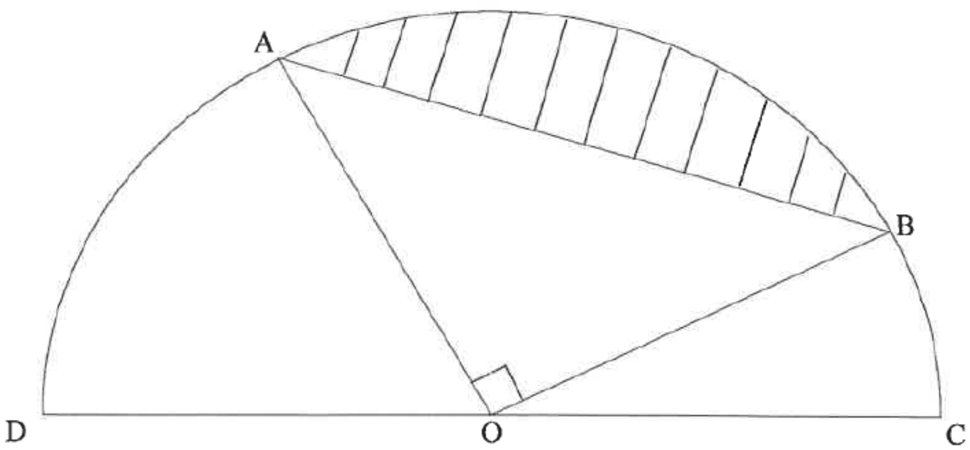
10.1 The graph of $f(x) = ax^3 + bx^2 + cx + d$ has two turning points.

The following information about f is also given:

- $f(2) = 0$
- The x-axis is a tangent to the graph of f at $x = -1$
- $f'(1) = 0$
- $f'\left(\frac{1}{2}\right) > 0$

Without calculating the equation of f, use this information to draw a sketch graph of f, only indicating the x-coordinates of the x-intercepts and turning points. (4)

10.2 O is the centre of a semicircle passing through A, B, C and D. The radius of the semicircle is $(x - x^2)$ units for $0 < x < 1$. ΔAOB is right-angled at O.



10.2.1 Show that the area of the shaded part is given by:

$$\text{Area} = \left(\frac{\pi - 2}{4}\right)\left(x^4 - 2x^3 + x^2\right)$$

(5)

10.2.2 Determine the value of x for which the shaded area will be a maximum. (4)

[13]

QUESTION 8

8.1 Determine $f^{/}(x)$ from first principles if it is given that $f(x)=-x^2$. (5)

8.2 Determine:

8.2.1 $f^{/}(x)$, if it is given that $f(x)=4x^3-5x^2$ (2)

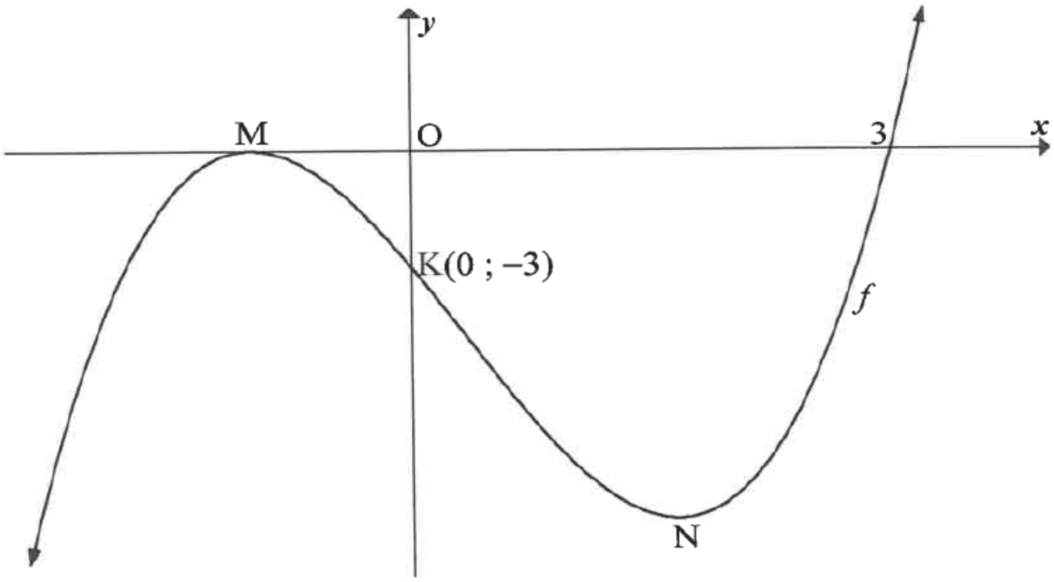
8.2.2 $D_x\left[\dfrac{-6\sqrt[3]{x}+2}{x^4}\right]$ (4)

[11]

QUESTION 9

Sketched below is the graph of $f(x)=x^3+ax^2+bx+c$.
The x-intercepts of f are at $(3\,;0)$ and M, where M lies on the negative x-axis.

$K(0\,;-3)$ is the y-intercept of f.
M and N are the turning points of f.



9.1 Show that the equation of f is given by $f(x)=x^3-x^2-5x-3$. (5)

9.2 Calculate the coordinates of N. (5)

9.3 For which values of x will:

9.3.1 $f(x)<0$ (2)

9.3.2 f be increasing (2)

9.3.3 f be concave up (3)

9.4 Determine the maximum vertical distance between the graphs of f and $f^{/}$ in the interval $-1<x<0$. (6)

[23]

QUESTION 7

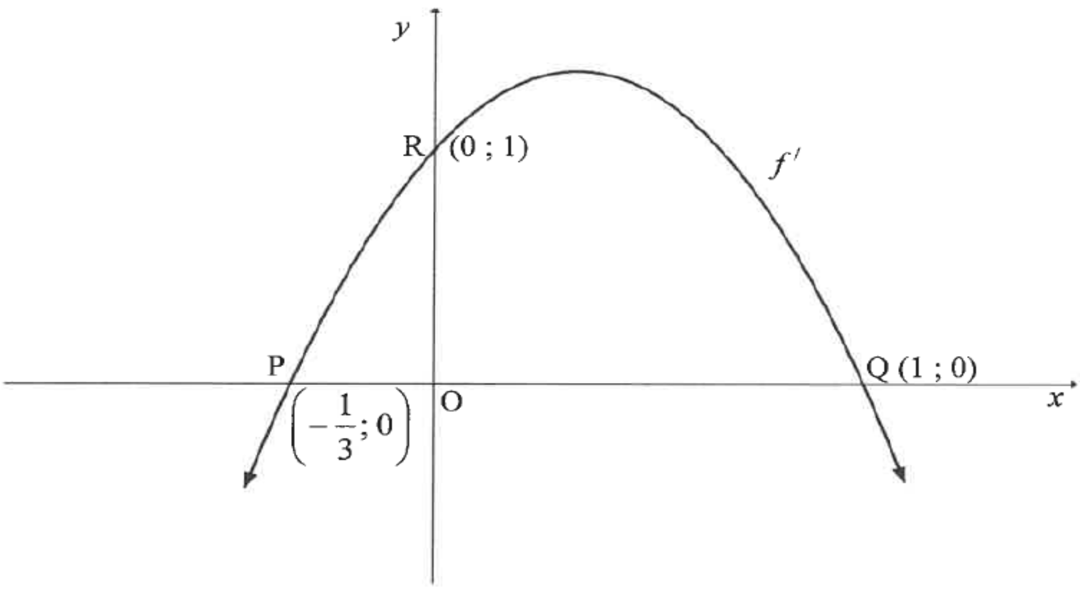
7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 + x$. (5)

7.2 Determine $f'(x)$ if $f(x) = 2x^5 - 3x^4 + 8x$. (3)

7.3 The tangent to $g(x) = ax^3 + 3x^2 + bx + c$ has a minimum gradient at the point $(-1\,;-7)$. For which values of x will g be concave up? (4)
[12]

QUESTION 8

The graph of $y = f'(x) = mx^2 + nx + k$ is drawn below.
The graph passes the points $P\left(-\frac{1}{3}\,;0\right)$, $Q(1\,;0)$ and $R(0\,;1)$.



8.1 Determine the values of m, n and k. (6)

8.2 If it is further given that $f(x) = -x^3 + x^2 + x + 2$:

8.2.1 Determine the coordinates of the turning points of f. (3)

8.2.2 Draw the graph of f. Indicate on your graph the coordinates of the turning points and the intercepts with the axes. (5)

8.3 Points E and W are two variable points on f' and are on the same horizontal line.

- h is a tangent to f' at E.
- g is a tangent to f' at W.
- h and g intersect at $D(a\,;b)$.

8.3.1 Write down the value of a. (1)

8.3.2 Determine the value(s) of b for which h and g will no longer be tangents to f'. (2)
[17]

QUESTION 9

Given $f(x) = x^2$.

Determine the minimum distance between the point $(10 ; 2)$ and a point on f. **[8]**

KWV 22

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = -2x^2 - 1$. (5)

7.2 Determine:

7.2.1 $f'(x)$, if it is given that $f(x) = -2x^3 + 3x^2$ (2)

7.2.2 $\frac{dy}{dx}$ if $y = 2x + \frac{1}{\sqrt{4x}}$ (4)

7.3 The graph $y = f'(x)$ has a minimum turning point at $(1 ; -3)$.
Determine the values of x for which f is concave down. (2)
[13]

QUESTION 8

Given: $f(x) = x^3 + 4x^2 - 7x - 10$

8.1 Write down the y-intercept of f. (1)

8.2 Show that 2 is a root of the equation $f(x) = 0$. (2)

8.3 Hence, factorise $f(x)$ completely. (3)

8.4 If it is further given that the coordinates of the turning points are approximately at $(0,7\ ;\ -12,6)$ and $(-3,4\ ;\ 20,8)$, draw a sketch graph of f and label all intercepts and turning points. (3)

8.5 Use your graph to determine the values of x for which:

8.5.1 $f'(x) < 0$ (2)

8.5.2 The gradient of a tangent to f will be a minimum (2)

8.5.3 $f'(x) . f''(x) \leq 0$ (3)

[16]

QUESTION 9

A wire, 12 metres long, is cut into two pieces. One part is bent to form an equilateral triangle and the other a square. A side of the triangle has a length of $2x$ metres.

9.1 Write down the length of a side of the square in terms of x. (2)

9.2 If this square is now used as the base of a rectangular prism with a height of $4x$ metres, determine the maximum volume of the rectangular prism. (7)

[9]