Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

MATHEMATICS

GRADE 12

LAST PUSH P2
2023

COMPILATION OF:
ALL PROVINCIAL
SEPTEMBER 2023 PAPERS

TABLE OF CONTENTS

STATISTICS

QUESTION 1: MPUMALANGA

The table below shows the mass (in kg) of 15 randomly chosen weight lifters of a certain gymnasium.

71	83	88	91	92	92	95	97	104	108	109	110	111	115	129

1.1 Calculate the mean mass of the weight lifters. (2)

1.2 Calculate the standard deviation of the masses of the weight lifters. (1)

1.3 What percentage of weight lifters fall in the feather weight division, if the criteria is that your mass must be below one standard deviation below the mean? (3)

1.4 If all weight lifters lose 3 kg, what will be the new:

1.4.1 Mean mass of the weight lifters? (1)

1.4.2 Standard deviation of their masses? (1)

[8]

QUESTION 2: LIMPOPO

The following set of data: $3\,;4\,;4\,;4\,;6\,;10\,;12\,;12\,;y$ has a mean of 7.

2.1 Determine:

2.1.1 The value of y. (2)

2.1.2 The median of this set of data points. (1)

2.2 Two additional number, $7-n$ and $7+n$, are added to the data set.

2.2.1 Calculate the of the eleven numbers. (2)

2.2.2 Determine the standard deviation if the data points, that are within ONE standard deviation of the mean, lie in the interval $3 \leq x \leq 11$. (2)

[7]

QUESTION 3: LIMPOPO

A mathematics teacher wants to make an unbiased prediction of her Grade 12 learners' final marks. She uses SBA mark and notes the final mark. The results are as follows:

SBA MARK (%)	FINAL MARK (%)	SBA MARK (%)	FINAL MARK (%)
42	51	48	59
35	43	72	85
69	76	57	63
62	73	25	35
83	85	65	59
75	72	68	75

3.1 Draw the scatter plot for the data on the grid provided in the ANSWER BOOK. (4)

3.2 Calculate the correlation coefficient for the data. (2)

3.3 Is the SBA mark a reliable predictor of the final mark? Provide a reason for your answer. (2)

3.4 Determine the equation of the least squares regression line. (3)

3.5 Predict Toby's final mark if his SBA mark was 66%. (2)

[13]

QUESTION 4: NORTHERN CAPE

Nine (9) Grade 12 learners were asked about the number of times they visited the library in the past month. Their responses were as follows:

2 3 4 5 5 8 9 10 12

4.1 Use the number line given in the answer book to draw a box-and-whisker diagram for the above data. (3)

4.2 Describe the skewness of the data. (1)

4.3 Calculate the mean of the data. (2)

4.4 How many data values lie outside one standard deviation of the mean? (3)

[9]

QUESTION 5: NORTHERN CAPE

The effect that the number of hours without sleep has on the number of mistakes that a person makes was investigated. The table below compares the number of hours without sleep (x) with the number of mistakes made (y).

NUMBER OF HOURS WITHOUT SLEEP $(\boldsymbol{x})$	10	11	12	13	14	15	16	17	18	19	21	24	25
NUMBER OF MISTAKES MADE $(\boldsymbol{y})$	4	4	5	8	8	10	11	11	13	15	21	23	25

5.1 Calculate the range of the number of hours without sleep. (2)

5.2 Determine the equation of the least squares regression line for the data. (3)

5.3 Write down the correlation coefficient. (1)

5.4 Predict the number of mistakes that a person would make is he/she went without sleep for 23 hours. Write your answer to the nearest integer. (2)

5.5 Is your prediction in QUESTION 5.4 reliable? Motivate your answer. (2)

[10]

QUESTION 6: NORTH WEST

To celebrate Pi Day at school, learners participate in a competition where they have to write down the value of Pi (π) up to the most correct decimal places. Eleven learners make it to the final round of the competition where their number of correct decimal places are counted. The judges stop counting after the first mistake. The results of the eleven learners are shown in the table below.

63	79	50	74	75	66	150	86	72	74	60

6.1 Calculate the:

6.1.1 Mean of the data (2)

6.1.2 Standard deviation for the given data. (1)

6.1.3 Number of results that lie outside ONE standard deviation of the mean. (3)

6.2 Identify the outlier in the given results. (1)

6.3 The result with the number of most correct decimal places is increased by $k\%$, while the result with the number of the lowest correct decimal places is decreased by $t\%$. The other nine results remain unchanged.

Only one of the options below correctly reflects the new range of the data in terms of k and t. Only write down the letter of the correct option as your answer.

A. $100 + k - t$

B. $150k - 50t$

C. $150k + 50t$

D. $100 + \frac{3}{2}k + \frac{1}{2}t$ (2)

6.4 It was established that a judge made a mistake with one of six lowest results. The result was corrected and changed to double its original value. How will this change impact on the median of the data? Motivate your answer. (2)

[11]

QUESTION 7: FREE STATE

The speeds, in kilometre per hour, of cyclists that passed a point on the route of the Ironman Race were recorded and summarised in the table below:

Speed (km/h)	**Frequency** (f)	**Cumulative Frequency**
$0 < x \leq 10$	10	10
$10 < x \leq 20$		30
$20 < x \leq 30$	45	
$30 < x \leq 40$	72	
$40 < x \leq 50$		170

7.1 Complete the above table in the ANSWER BOOK provided. (2)

7.2 Make use of the axes provided in the ANSWER BOOK to draw a cumulative frequency curve for the above data. (3)

7.3 Indicate clearly on your graph where the estimates of the lower quartile (Q_1) and median (M) speeds can be read off. Write down these estimates. (2)

7.4 Draw a box-and-whisker diagram for the data. Use the number line in the ANSWER BOOK. (2)

7.5 Use your graph to estimate the number of cyclists that passed the point with speeds greater than 35 km/h. (1)

[10]

QUESTION 8: FREE STATE

During the month of June, patients visited a number of medical facilities for treatment. The table below shows the number of patients treated on certain dates during the month of June.

Dates in the month of June	3	5	8	12	15	19	22	26
Number of patients treated.	270	275	376	420	602	684	800	820

8.1 On **DIAGRAM SHEET 2**, draw a scatter plot of the given data. (3)

8.2 Determine the equation of the least squares regression line of patients treated (y) against date (x). (3)

8.3 Estimate how many patients have been treated on the 24^{th} of June. (2)

8.4 Draw the least squares regression line on the grid on DIAGRAM SHEET 2. (3)

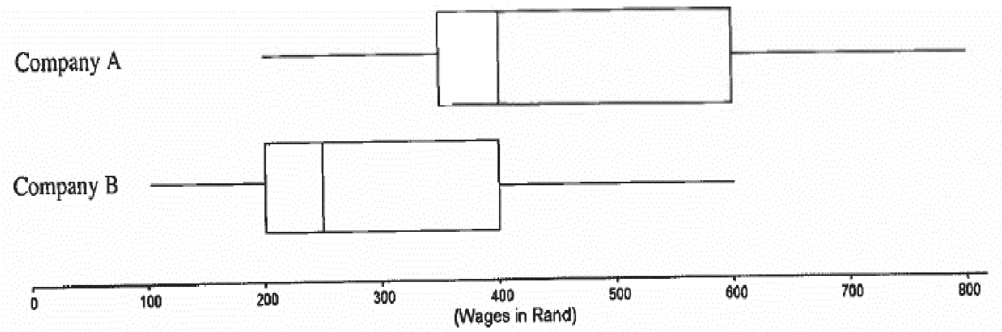
8.5 Calculate the correlation coefficient of the data. Comment on the strength of the relationship between the variables. (3)

8.6 Given that the mean for patients treated on certain dates is 528,63, calculate how many patients were within one deviation of the mean. (3)

[17]

QUESTION 9: MPUMALANGA

9.1 The box-and-whisker diagram plots the wages of workers (in Rands) at two companies for the same type of work. Both companies have 20 workers.



9.1.1 State whether the following statement is TRUE or FALSE: (1)
All the workers at Company A earn more than 25% of the workers at Company B.

9.1.2 Comment on the skewness of the data for company B. (1)

9.1.3 Which company has the biggest range? Motivate your answer with the necessary calculations. (2)

9.1.4 How many workers at Company B earn more than R200? (2)

9.2 A table of data, showing the price of crude oil at the end of each year is US Dollars ($) (to the nearest dollar) per barrel is given.

Year	2011	2012	2013	2014	2015	2016	2017	2019	2020	2021
Price	90	92	98	54	37	54	60	61	49	75

9.2.1 Determine the equation of the least squares regression line for the price of crude oil per year. (3)

9.2.2 Calculate the value of the correlation coefficient. (1)

9.2.3 Use calculations to predict the price of crude oil (in US $) per barrel at the end of 2018. Discuss the validity of the answer that you obtained by using your answer in 9.2.2. (2)

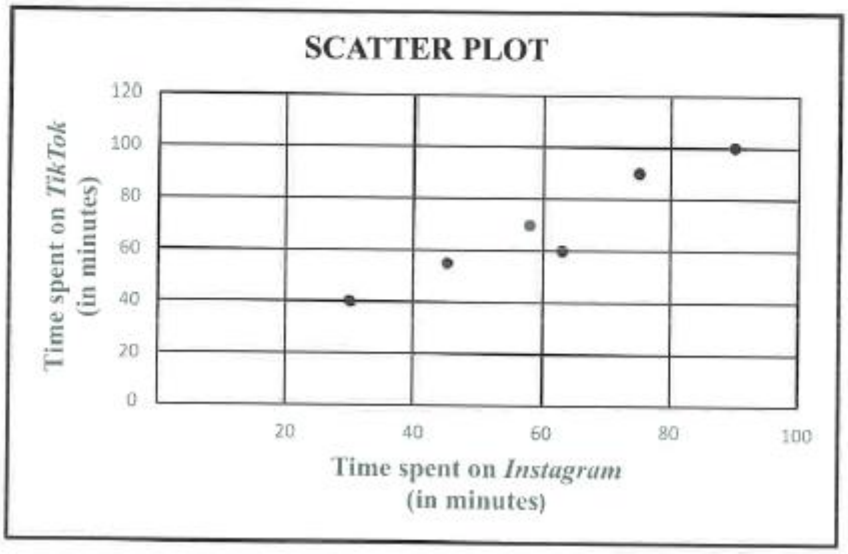
[12]

QUESTION 10: GAUTENG

A survey was conducted among a group of learners to compare the time spent on *Instagram* to the time spent on *TikTok.*

The results are shown in the table below:

TIME SPENT ON *INSTAGRAM* (in minutes)	30	45	58	63	75	90
TIME SPENT ON *TIKTOK* (in minutes)	40	55	70	60	90	100



10.1 Calculate the correlation coefficient of the data. (1)

10.2 Comment on the strength of the correlation between the time spent on *Instagram* and the time spent on *TikTok*. (1)

10.3 Determine the equation of the least squares regression line of the data. (3)

10.4 Predict the time that will be spent on *TikTok* if a leant spent 115 minutes on *Instagram*. (2)

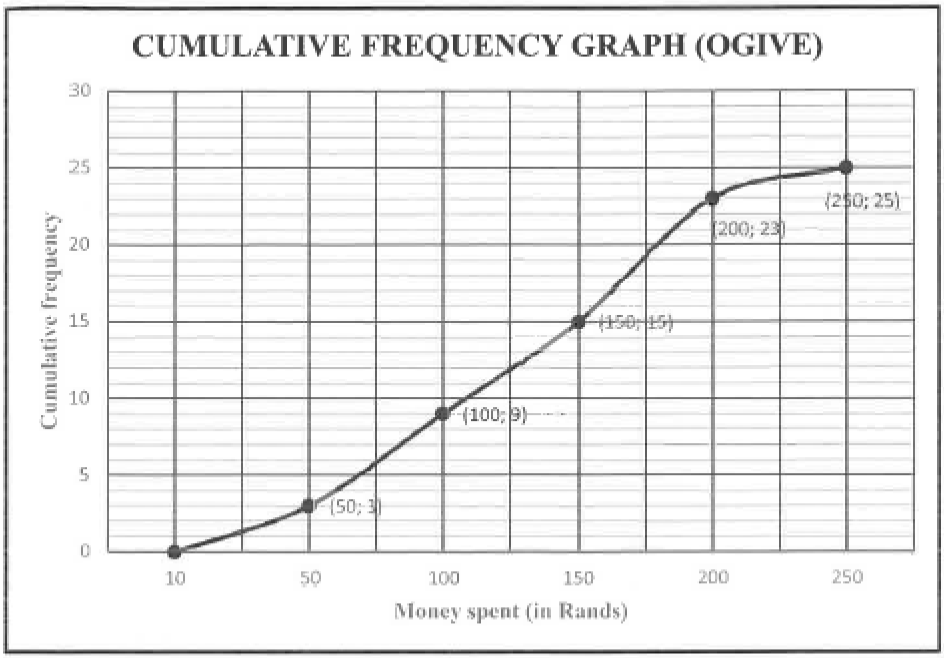
10.5 It was noticed that 4 learners' data was not recorded. The mean time of the *TikTok* users and *Instagram* users 73,4 minutes each. The researcher commented that the total amount of time spent on the two social media platforms was more than a full day. Do you agree with the researcher?

Motivate your answer by using necessary calculations. (3)

[10]

QUESTION 11: GAUTENG

The amount of money (in rands) that a group of learners spent at a theme park on a specific day was recorded. The data is represented in the cumulative frequency graph (ogive) below.



11.1 The data from the cumulative frequency graph (ogive) is represented in the incomplete frequency table below.

AMOUNT OF MONEY (IN RANDS)	NUMBER OF LEARNERS
$10 \leq x < 50$	a
$50 \leq x < 100$	6
$100 \leq x < 150$	b
$150 \leq x < 200$	8
$200 \leq x < 250$	2

11.1.1 How many learner visited the theme park on that specific day. (1)

11.1.2 Determine the values of a and b in the frequency table. (2)

11.1.3 Use the ogive to determine the percentage of learners that spent more than R175. (2)

11.2 It is further given that there are two rides at theme park, *The Intimidator* and *Terror Thrills*.

The mean amount of money spent on these rides was analysed and is given below.

Rides	***The Intimidator***	***Terror Thrills***
Mean amount spent	R13,20	R12,70

The two standard deviations interval about the mean for *The Intimidator* was calculated as $(4{,}8; 9{,}2)$. If the standard deviation of *Terror Thrills* is double the standard deviation of *The Intimidator*, calculate the interval for the one standard deviation about the mean for *Terror Thrills*. (4)

[9]

QUESTION 12: EASTERN CAPE

12.1 A school's hockey team recorded the number of push-ups each player completed in a minute. The numbers for seven players were:

29 27 24 31 22 19 30

12.1.1 Calculate the:

(a) Mean (2)

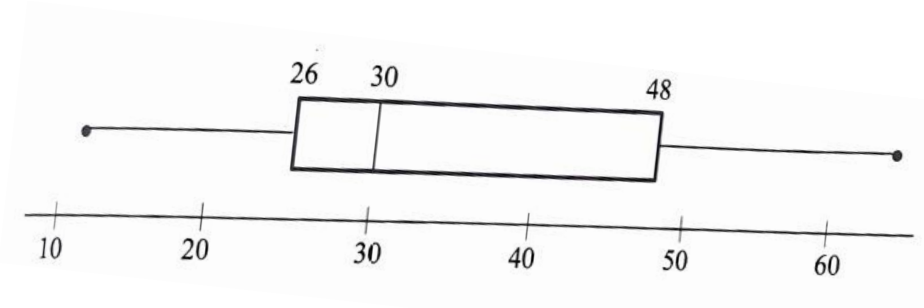
(b) Standard deviation (1)

12.1.2 How many players were within one deviation of the mean? (3)

12.1.3 Seven players in the school's rugby team also recorded the number of push-ups they completed in a minute. Their numbers gave a mean of 26 and a standard deviation of 3,2.

Use the standard deviations and the means to compare the number of push-ups of the players in the rugby and hockey teams. (2)

12.2 The number points scored by a rugby team in each of 10 matches is represented in the box-and-whisker diagram below. The scores of the 10 matches were different.



12.2.1 In what percentage of the matches did the team score over 30 points? (1)

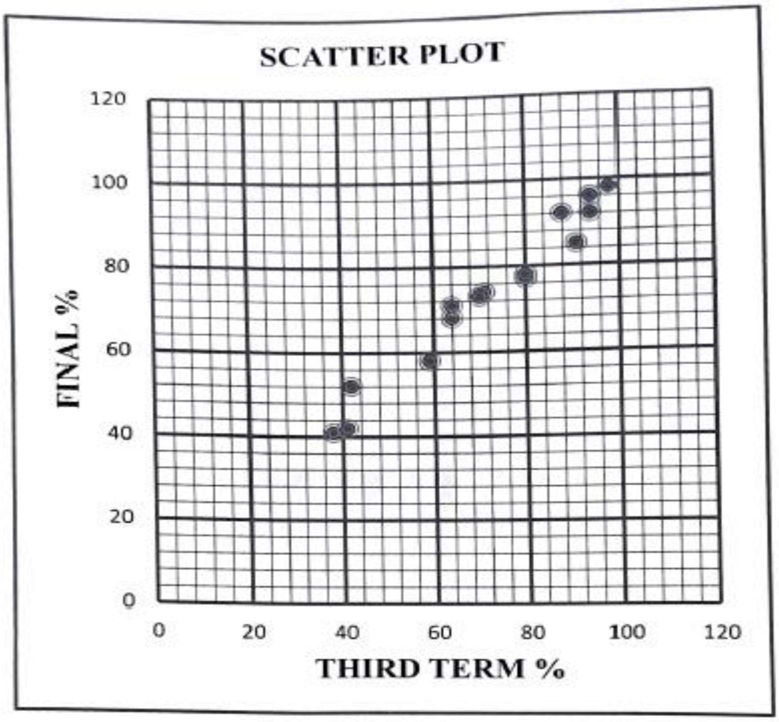
12.2.2 Which of the mean or median is likely to be greater? Give a reason for your answer. (2)

[11]

QUESTION 13: EASTERN CAPE

The table shows the percentages scored by a sample of 15 candidates in the third terms and final examinations of 2022. The table and the scatter plot below represent these marks.

Third term	71	80	59	38	41	98	80	88	91	94	64	94	70	42	64
Final examinations	74	77	58	41	42	98	78	92	85	92	68	96	73	52	71



13.1 Determine the equation of the least squares regression line for the data, rounding off your answer to 3 decimal places. (3)

13.2 Write down the value of the correlation coefficient, r, between the 3rd term and final exam percentages. (1)

13.3 A candidate scored 48% in the third term.

13.3.1 Use the equation of the least squares regression line to predict his final percentage. Round your answer off to the nearest whole number. (2)

13.3.2 Give a reason why the prediction can be regarded as reliable. (1)

13.4 The least squares regression line is used to predict that the final percentage of a candidate who scored 50% in the third term is 80%.

13.4.1 Why would this prediction be unreliable? (1)

13.4.2 Would adding the point $(20; 10)$ to the original data set increase or decrease the gradient of the least squares regression line? (1)

[9]

QUESTION 14: KWA-ZULU NATAL

Mr Siphokazi supplements his pension by mowing lawns for customers. He measures the areas (x) (in m^2) of 12 of his customers' lawns and the time (y) in minutes, it takes him to these lawns. He works 8 hours a day. He recorded the data.

Area $(\boldsymbol{x})$ **(in** $\mathbf{m^2}$**)**	360	120	845	602	1190	530	245	486	350	1005	320	250
Time $(\boldsymbol{y})$ **(minutes)**	50	28	130	75	120	95	55	70	48	110	55	60

14.1 Determine the equation of the least squares regression line. (3)

14.2 Calculate the value of r, the correlation coefficient for the data. (2)

14.3 Given that Mr Siphokazi charges a flat call out fee of R150, as well as R50 per half hour (of part thereof), estimate the charge for mowing a customer's lawn that has an area of 560 m^2.

(For example: 100 minutes would be taken as 2 hours). (3)

14.4 The local high school wants Mr Siphokazi to mow their rugby field which is rectangular, 100 metres long and 70 metres wide.

14.4.1 Use the regression equation found in 14.1 to calculate the time it would take to mow this area. (1)

14.4.2 Is it possible for him to complete this job in a day? Give a reason for your answer. (1)

[10]

QUESTION 15: KWA-ZULU NATAL

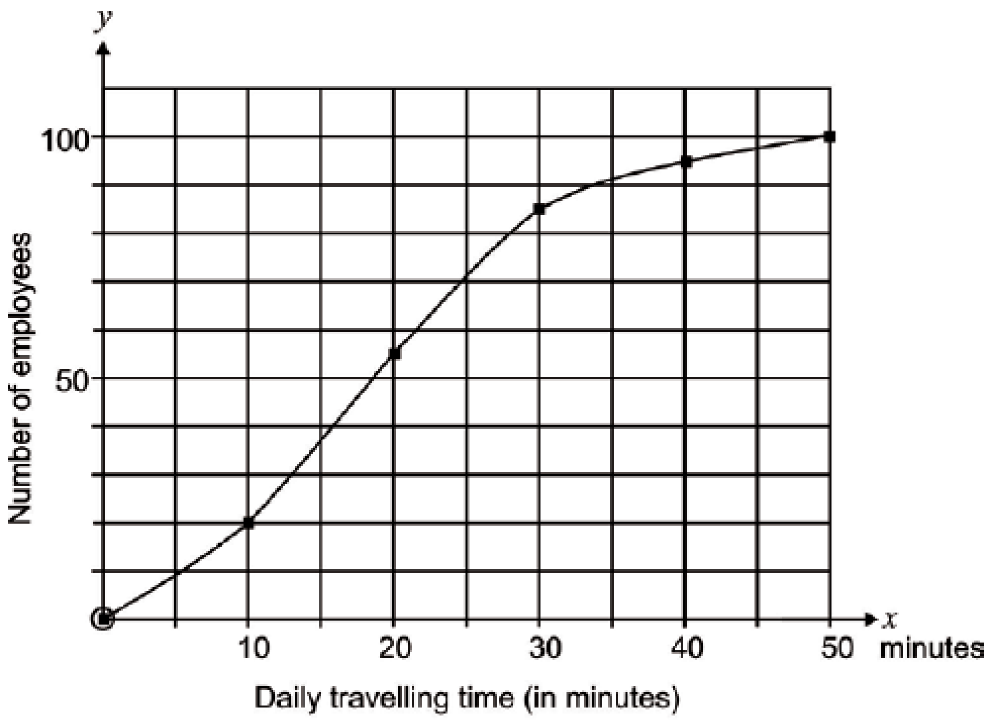
The following table gives the frequency distribution of the daily travelling time (in minutes) from home to work for the employees of a certain company.

Daily travelling time x (in minutes)	**Number of employees (f)**	**Midpoint of Interval**	
$0 \leq x < 10$	20		
$10 \leq x < 20$	35		
$20 \leq x < 30$	30		
$30 \leq x < 40$	10		
$40 \leq x < 50$	5		

15.1 Calculate the estimated mean travelling time. (3)

15.2 Write the modal class of the data. (2)

15.3 An ogive was drawn for the given data. (3)
Construct a box-and-whisker plot for the data in the ANSWER BOOK.



15.4 State whether the following are True or FALSE.

15.4.1 The distribution of these travelling times is positively skewed. (1)

15.4.2 The inter-quartile range for the data is 2,5. (1)

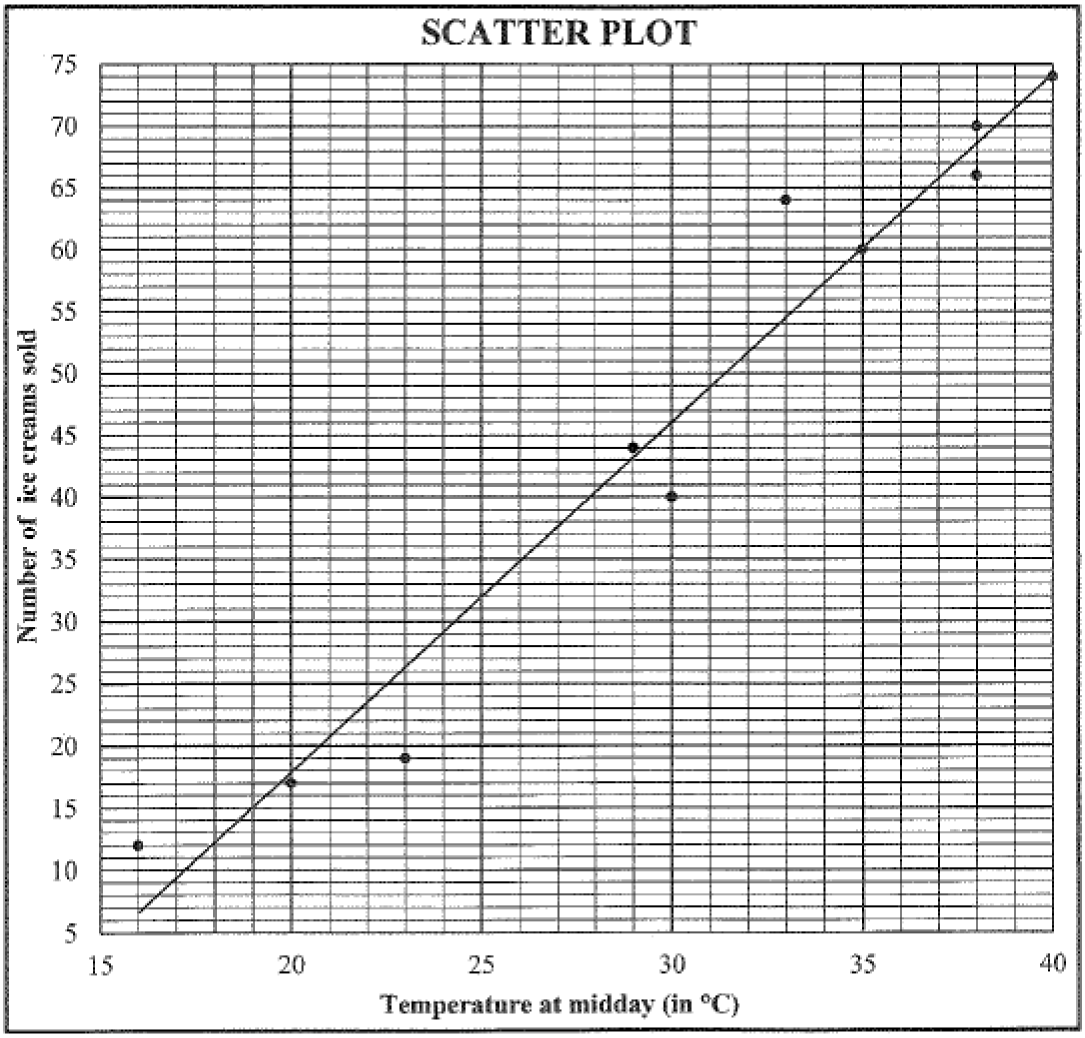
15.4.3 35 employees take less than 20 minutes. (1)

[11]

QUESTION 16: NORTH WEST

On the first Saturday of a month, for a period of ten months, information was recorded about the temperature at midday (in ℃) and the number of ice creams that were sold at and ice cream stand at a certain beach. The data is shown in the table below and represented on the scatter plot. The least squares regression line is drawn on the scatter plot.

Temperature at midday (in ℃)	21	20	23	29	33	38	40	38	35	30
Number of ice creams sold	12	17	19	44	64	70	74	66	60	40



16.1 Refer to the scatter plot. Would you say that the relationship between the temperature at midday and the number of ice creams sold is weak or strong? Motivate your answer. (2)

16.2 Determine the equation of the least squares regression line. (3)

16.3 Predict the number of ice creams that will be sold on a Saturday if the temperature is 26℃ at midday. (2)

16.4 On another first Saturday of the month, the temperature at midday was 24℃ and 40 ice creams were sold. If the data is added to the data set, how will the prediction of the number of ice creams sold within the given domain be affected? Motivate your answer. (2)

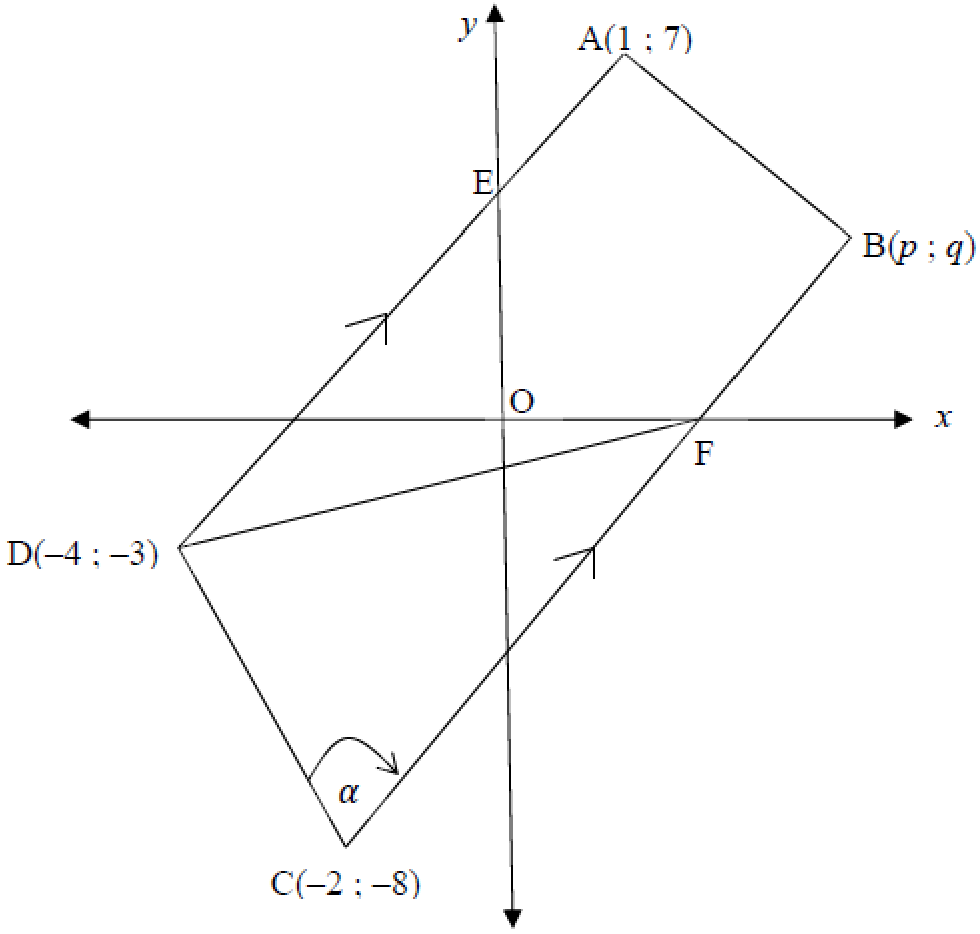
[9]

ANALYTICAL GOEMETRY

Part 1: Lines and polygons

QUESTION 1: KWA-ZULU NATAL

Trapezium ABCD is drawn below with AD ∥ BC is drawn. The coordinates of the vertices are $A(1;7)$, $B(p;q)$, $C(-2;-8)$ and $D(-4;-3)$. BC intersects the x-axis at F. $D\hat{C}B = \alpha$. AD intersects the y-axis at E.

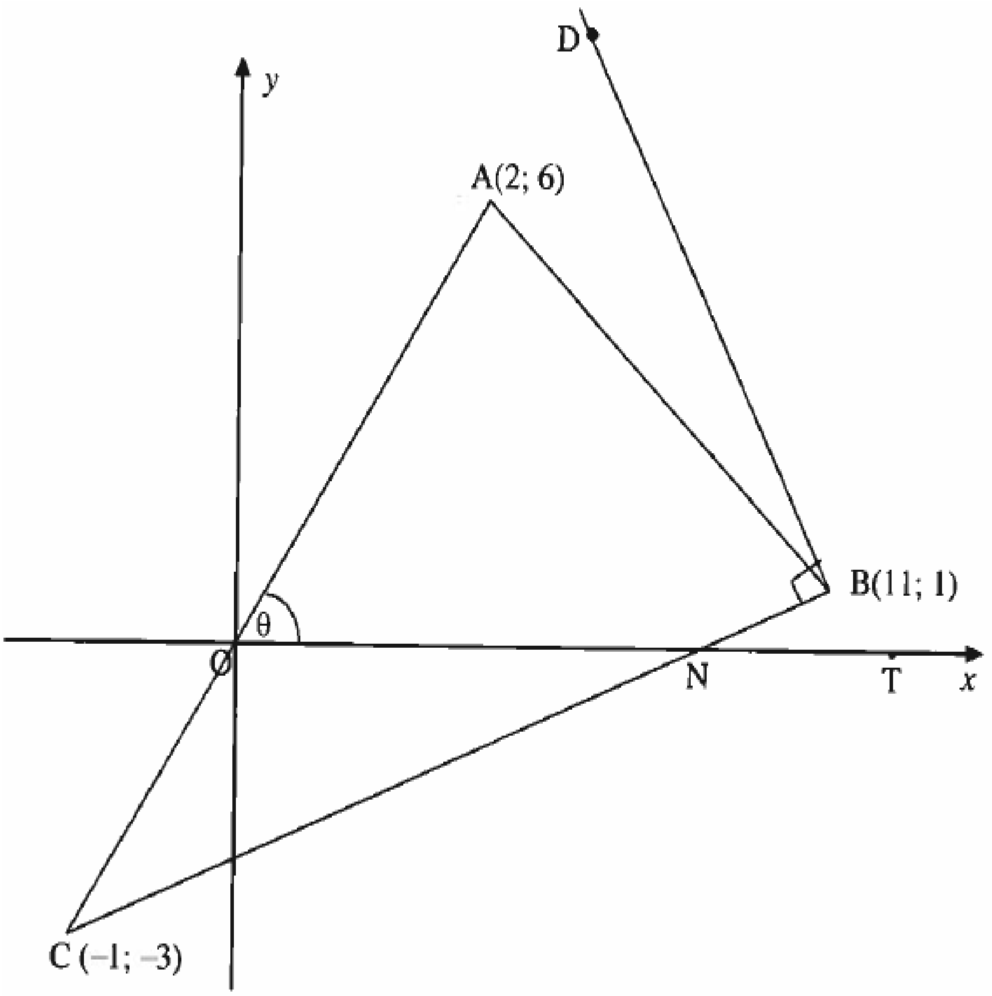


1.1	Calculate the gradient of AD.	(2)
1.2	Determine the equation of BC in the form $y = mx + c$.	(3)
1.3	Determine the coordinates of F.	(2)
1.4	AMCD is a parallelogram, with M on BC. Determine the coordinates of M.	(2)
1.5	Show that: $\alpha = 48{,}37°$.	(4)
1.6	Calculate the area of ΔDCF.	(4)

[17]

QUESTION 2: MPUMALANGA

In the diagram, $A(2;6)$, $B(11;1)$ and $C(-1;-3)$ are the vertices of ΔABC. Point D is shown in the diagram such that $BD \perp BC$. N is the x-intercept of BC. $A\hat{O}N = \theta$.



2.1 If the gradient of BC is $\frac{1}{3}$ and the gradient of AC is 3, calculate:

2.1.1 The x-coordinate of N. (2)

2.1.2 The size of $A\hat{C}B$. (5)

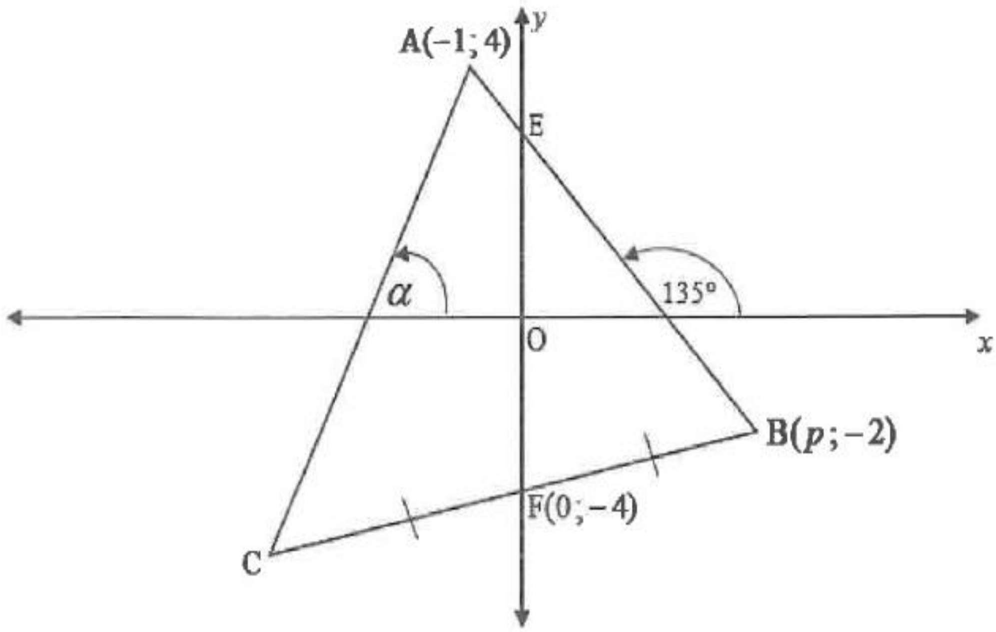
2.2 Determine the equation of AC. (2)

2.3 If it is further given that point D lies on AC produced such that $BC \perp BD$, calculate the coordinates of D. (5)

[14]

QUESTION 3: GAUTENG

In the diagram below, $A(-1; 4)$, $B(p; -2)$ and C, are the vertices of ΔABC. E is the y-intercept of AB. $F(0; -4)$ is the midpoint of BC. The angles of inclination of AB an AC are $135°$ and α respectively.

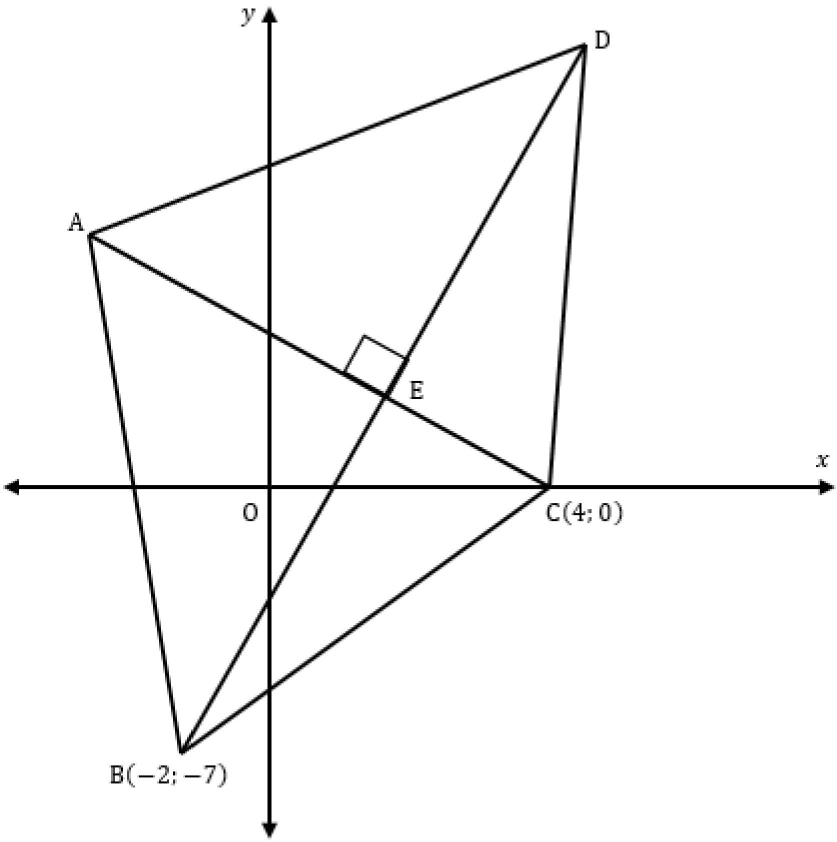


3.1	Calculate the gradient of AB.	(2)
3.2	Show that the value of p is 5.	(2)
3.3	Calculate the coordinates of C.	(2)
3.4	Determine the equation of AC in the form $y = mx + c$.	(4)
3.5	Calculate the size of $C\hat{A}B$	(3)
3.6	Calculate the area of ΔBEF.	(3)
3.7	Another point $K(t; t)$ where $t < 0$, is plotted such that $AK = 5\sqrt{5}$. Calculate the coordinates of K.	(5)

[21]

QUESTION 4: LIMPOPO

In the diagram below, A, $B(-2;-7)$, $C(4;0)$ and D are the vertices of a kite . E is the midpoint of the diagonal BD and $AC \perp BD$ at E. the equation of AC is $y = -\frac{1}{2} + 2$.



Determine:

4.1 The equation of BD. (4)

4.2 The coordinates of E. (3)

4.3 If the ratio $CE : EA = 1 : 3$, determine the coordinates of A. (2)

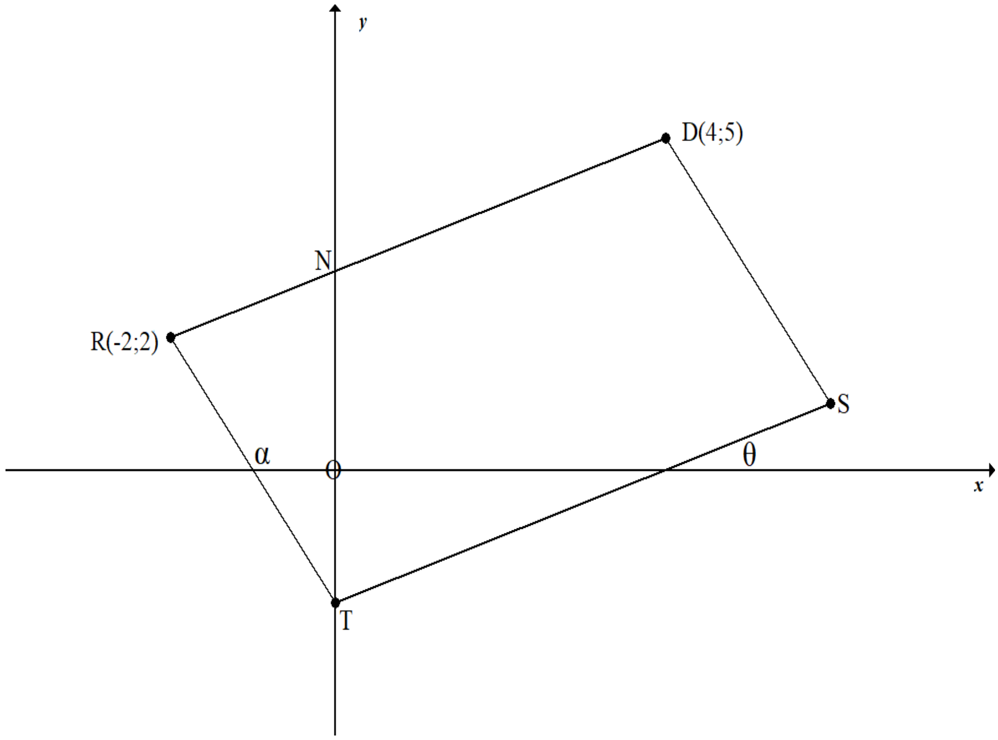
4.4 Kite PQRS is obtained after the measurements of kite ABCD is enlarged by a scale factor 2. Calculate the area of kite PQRS. (5)

[14]

QUESTION 5: EASTERN CAPE

In the diagram below, $D(4; 5)$, $R(-2; 2)$, T and S form a quadrilateral.
RD cuts the y-axis at N and T is a point on the y-axis.
The inclinations of RT and TS are α and θ respectively.

RD ∥ TS and the equation of TS is $y = \frac{1}{2}x - 2$.



5.1 Write down the coordinates of T. (1)

5.2 Calculate:

5.2.1 The gradient of RT (2)

5.2.2 The size of $R\hat{T}S$ (5)

5.3 Determine the equation of RD in the form $y = mx + c$. (3)

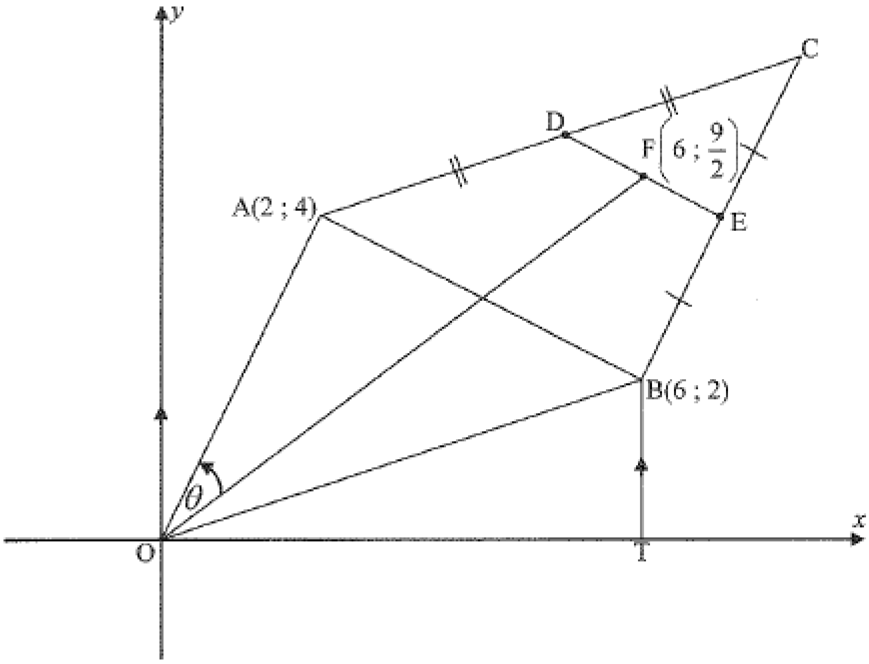
5.4 If RT ∥ DS, calculate the coordinates of M, the midpoint of RS. (3)

5.5 Calculate the area of ΔRTN. (4)

[18]

QUESTION 6: NORTH WEST

In the diagram below, $A(2; 4)$, O, $B(6; 2)$ and C are the vertices of a quadrilateral. D and E are the midpoints of AC and BC respectively. F $\left(6; \frac{9}{2}\right)$ is a point on DE. From B, the straight line drawn parallel to the y-axis cuts the x-axis in T. $A\hat{O}F = \theta$.



6.1 Calculate:

6.1.1 The length of AB. Leave your answer in surd form. (2)

6.1.2 The gradient of AB. (2)

6.2 Prove that $OA \perp AB$ (2)

6.3 Determine the equation of DE (4)

6.4 Determine the coordinates of C such that AOBC, in this order, is a parallelogram. (3)

6.5 Calculate the:

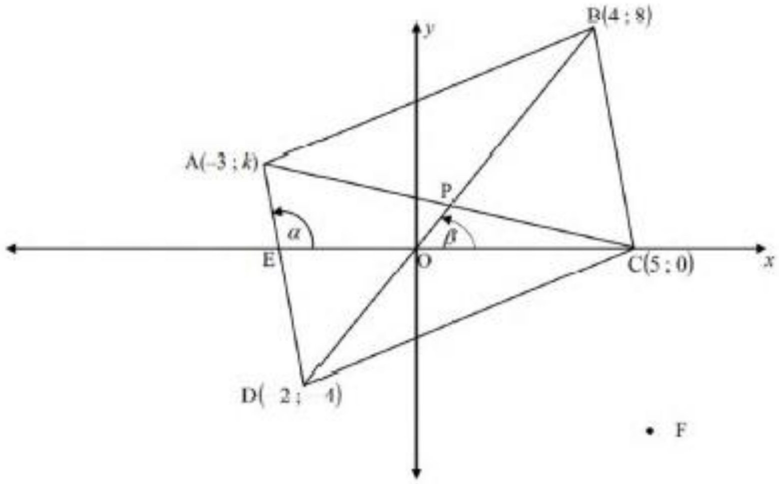
6.5.1 Size of θ (5)

6.5.2 Area of ΔABT, if A and TB are joined to form ΔABT (4)

[22]

QUESTION 7: FREE STATE

In the diagram below, $A(-3; k)$, $B(4; 8)$, $C(5; 0)$ and $D(-2; -4)$ are vertices of the parallelogram ABCD. Diagonals AC and BD bisect each other at P. The angles of inclination of AD and BD are α and β respectively. AD cuts the x-axis at E. F is a point in the fourth quadrant.



7.1 Determine the gradient of BC. (2)

7.2 If the distance between points $A(-3; k)$ and $B(4; 8)$ is 65, calculate the value of k. (4)

7.3 Prove, using analytical methods, that $BP \perp AC$. (3)

7.4 Calculate the coordinates of F if it is given that ACFD is a parallelogram. (2)

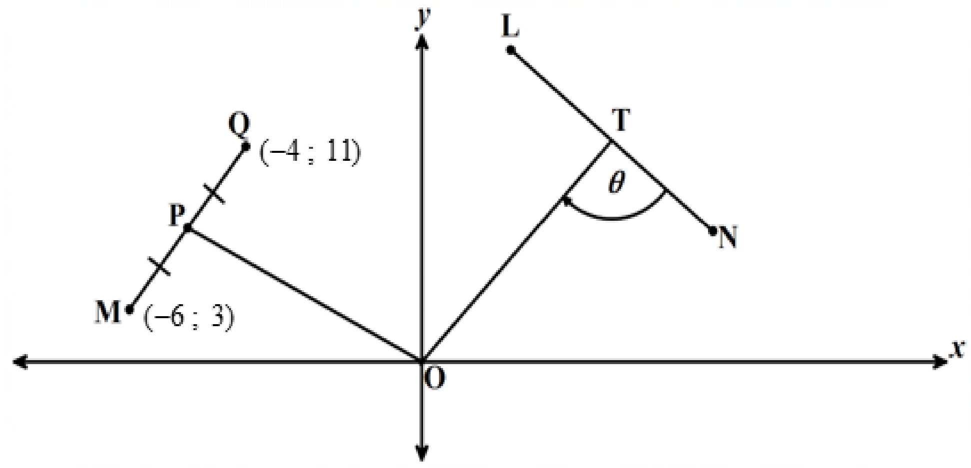
7.5 Calculate the size of $E\hat{D}O$ (Correct to ONE decimal place). (6)

7.6 Calculate the area of ΔADC. (4)

[21]

QUESTION 8: NORTHERN CAPE

8.1 In the diagram below, P is the midpoint of the line segment joining $M(-6; 3)$ and $Q(-4; 11)$. The equation of OT is $y = \frac{3}{2}x$. The equation of LN is $x + y = 15$.



8.1.1 Write down the coordinates of P, the midpoint of MQ. (2)

8.1.2 Determine the coordinates of T. (3)

8.1.3 Calculate the size of θ. (4)

8.2 The distance between the origin and point $A(-2; k-1)$ is $2k$ units. Calculate the value of k. (4)

8.3 Given: $S(2; 3)$, $Y(2+4a; 3-5a)$ and $U(2+4b; 3-5b)$ with $a \neq 0$, $b \neq 0$ and $a \neq b$.

8.3.1 Prove that the points S, Y and U are collinear. (3)

8.3.2 Hence, determine the equation of the straight line SYU in the form $y = mx + c$. (3)

[19]

Part 2: Circle-Analytical geometry

QUESTION 1: KWA-ZULU NATAL

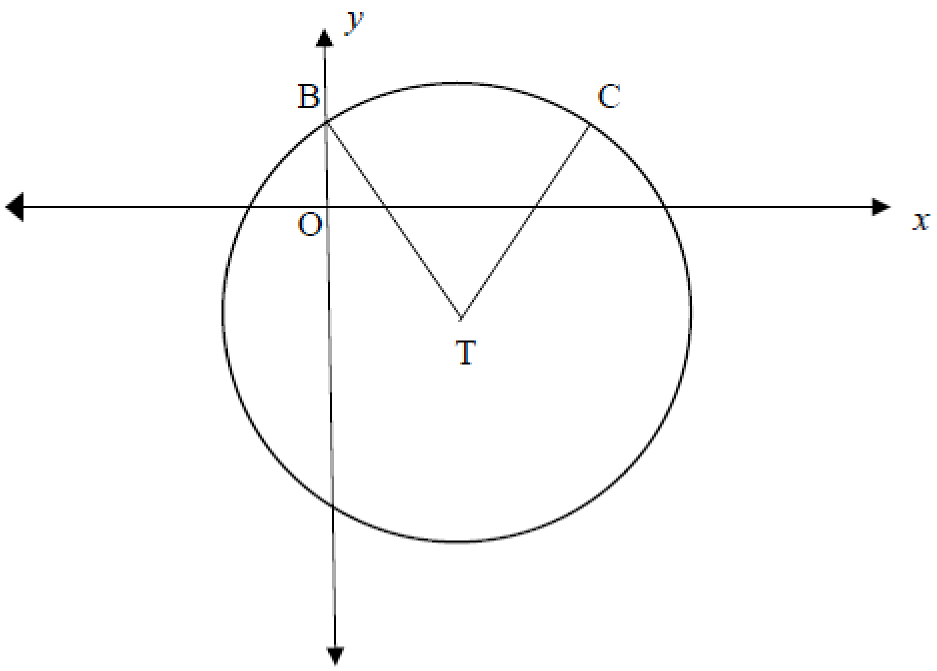
1.1 The equation of a circle is $x^2 + y^2 - 8y + 6y = 15$.

1.1.1 Show that P$(2; -9)$ lies on the circle. (2)

1.1.2 Determine the equation of the tangent to the circle at point P$(2; -9)$. (6)

1.1.3 A tangent is drawn from Q$(-10; 12)$ to the circle. Calculate the length of the tangent. (4)

1.2 The circle, with centre T, and equation $(x - 3)^2 + (y + 2)^2 = 25$ is given below. B is the y-intercept of the circle.



1.2.1 Determine the coordinate of B. (4)

1.2.2 Write down the coordinates of C, if C if the reflection B in the line $y = 3$. (2)

1.2.3 Another circle with centre M and equation, $(x - 12)^2 + (y - 10)^2 = 100$ is given

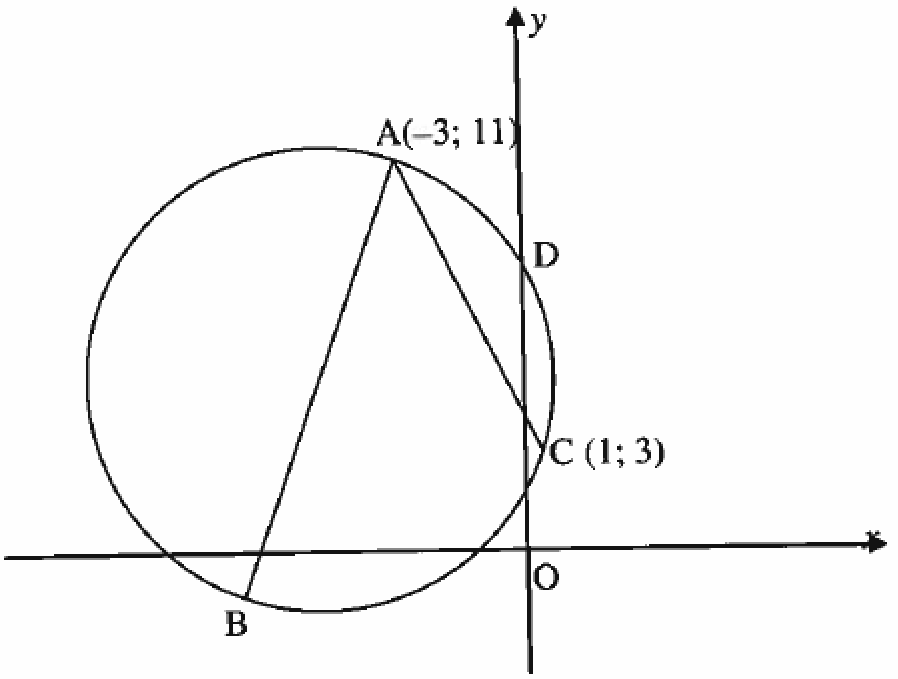
(a) Calculate the distance, TM, between the centres. (2)

(b) Do these circles touch or intersect each other? Justify your answer. (2)

[22]

QUESTION 2: MPUMALANGA

In the diagram, $A(-3; 11)$ and $C(1; 3)$ are points on the circumference of a circle with diameter AB and centre T. The equation of AB is given by $y = 3x + 20$.



2.1 Determine the equation of the perpendicular bisector of AC. (4)

2.2 Show that the coordinates of the centre of the circle are $(-5; 5)$. (3)

2.3 Calculate the length of diameter AB. (3)

2.4 Write down the equation of the circle. (2)

2.5 The tangent to the circle at A cuts the y-axis at $(0; p)$. Calculate the numerical value of p. (4)

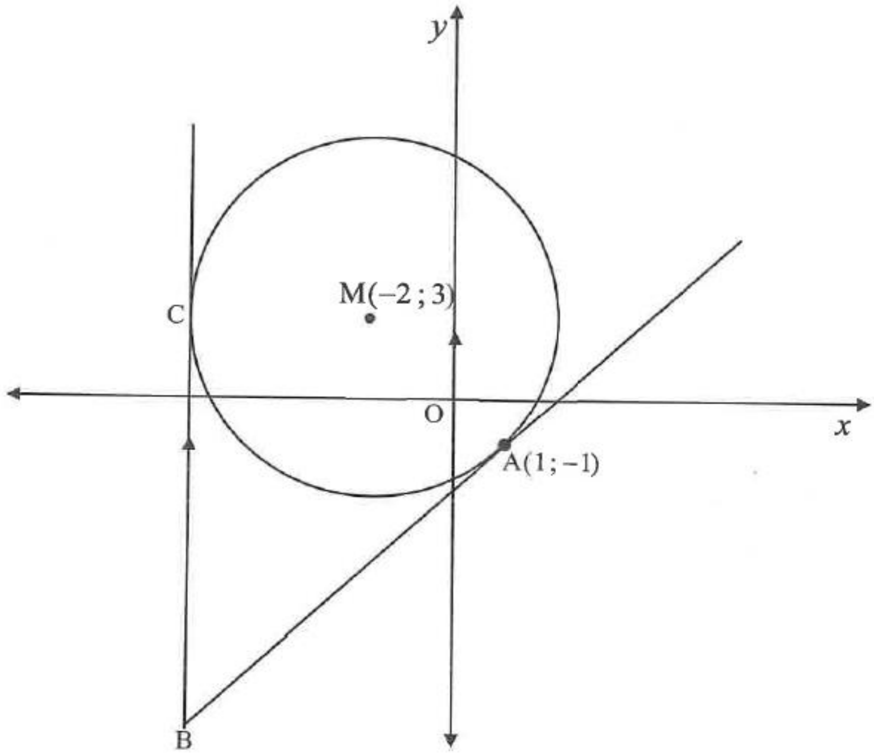
2.6 If the circle through A, B, and C is moved 3 units to the right and 2 units upwards, and the radius is halved, write down the equation of the new circle. (3)

2.7 A new circle with equation $(x-2)^2 + (y-3)^2 = 4$ and centre P is given. Will this circle intersect the original circle or not? Motivate your answer with the necessary calculations. (4)

[23]

QUESTION 3: GAUTENG

In the diagram below, the circle centred at $M(-2; 3)$ passes through $A(1; -1)$ and C. BA an BC are tangents to the circle at A and C respectively, with BC parallel to the y-axis.



3.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

3.2 Write down the coordinates of C. (2)

3.3 Determine the equation of the tangent AB in the form $y = mx + c$. (5)

3.4 Determine the length of BC. (3)

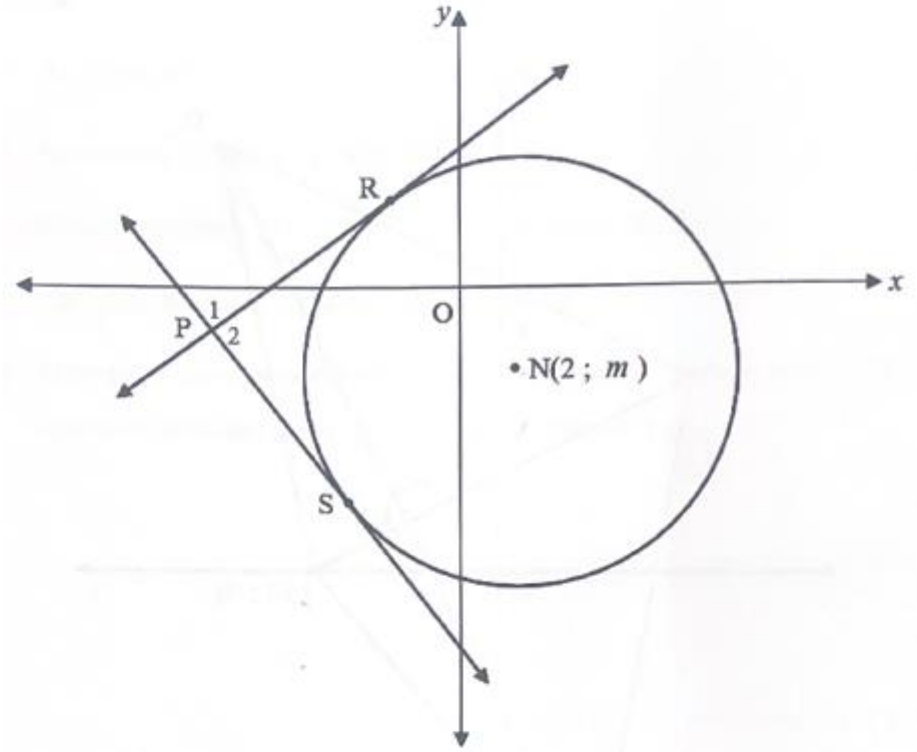
3.5 Determine the equation of the circle centred at A that has the x- and y- as tangents. (2)

3.6 If another circle with centre $N(p; 3)$ and a radius of 4 intersects the circle centred at M at two distinct points, determine all the possible values of p. (5)

[20]

QUESTION 4: LIMPOPO

4.1 In the diagram, the centre of the circle is $\mathrm{N}(2;m)$ where $m<0$.the radius of the circle is 17 units. $\mathrm{R}(-13;5)$ and $\mathrm{S}(-13;-11)$ are two points on the circle.



4.1.1 (a) Determine the numerical value of m. (4)

(b) Determine the equation of the circle in the form: (1)
$(x-a)^2+(y-b)^2=r^2$

4.1.2 Determine the gradients of:

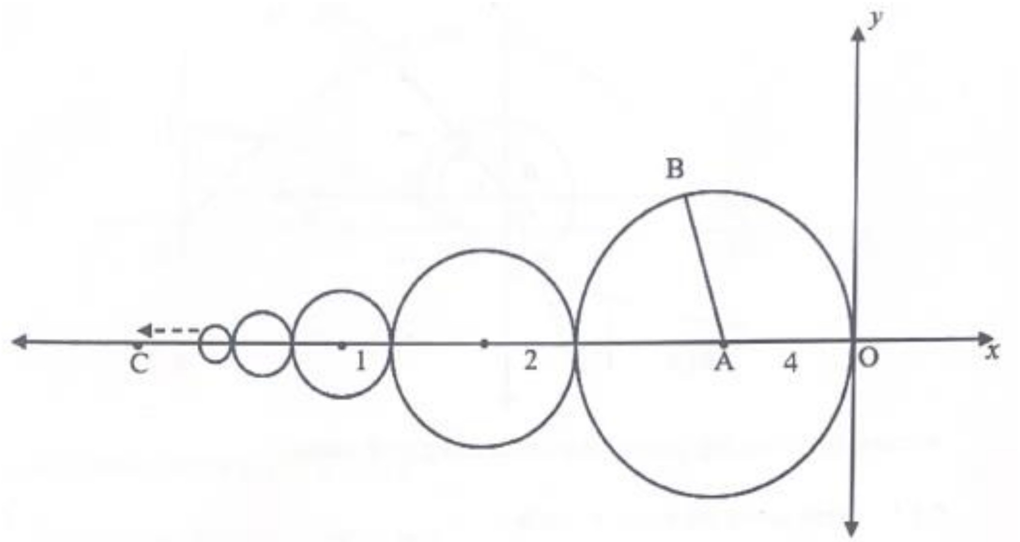
(a) NR (2)

(b) NS (1)

4.1.3 The tangents at S and R intersect at P. calculate the size of $\hat{\mathrm{P}}_2$. (6)

4.1.4 Circle N is reflected about the x-axis and then translated 2 units upwards to obtain circle M. Determine the equation of circle M in the form $(x-c)^2+(y-d)^2=r^2$. (2)

4.2 An infinite number of circles, each touching the next, are drawn between C and O. The centres of all the circles lie on the negative x-axis, the radius of the largest circle, centred at A, is 4 units and the radius of each thereafter is halved. B is a point on the largest circle.



4.2.1 Show that $OC = 16$ units. (2)

4.2.2 If BC is a tangent to circle A at B, write down the size of $A\hat{B}C$, providing a reason for your answer. (2)

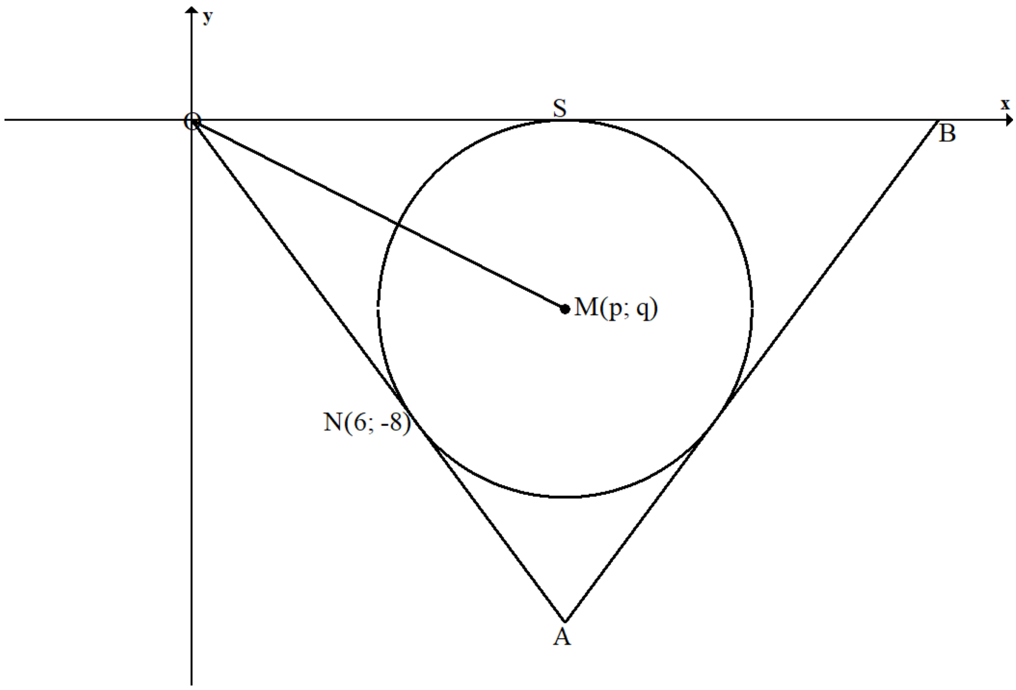
4.2.3 Hence, determine $\tan \hat{C}$. (4)

4.2.4 Determine the equation of BC. (3)

[27]

QUESTION 5: EASTERN CAPE

In the diagram below, a circle, centred at $M(p; q)$, touches the x-axis at S and the line OA is a tangent to the circle at $N(6; -8)$.



5.1 Calculate:

5.1.1 The length of ON (2)

5.1.2 The value of p. (2)

5.1.3 The gradient of NM (3)

5.1.4 The value of q. (2)

5.2 Determine the equation of the circle in the form: $(x - a)^2 + (y - b)^2 = r^2$ (3)

5.3 $x = k$ is a tangent to the circle. Write down the value(s) of k. (2)

5.4 The line $y = -\frac{4}{3}x + t$ cuts the circle at two different points. (6)

Determine the values of t.

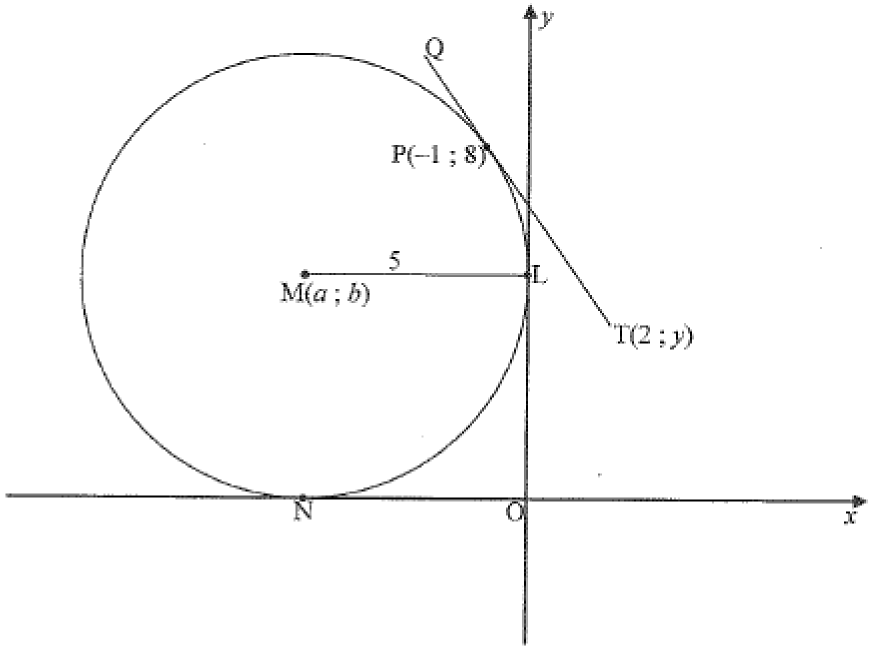
5.5 Another circle with equation $(x - 10)^2 + (y - 6)^2 = 25$ is given. (2)

Will the two circles touch, cut or not? Give a reason for your answer.

[22]

QUESTION 6: NORTH WEST

In the diagram, a circle centred at $M(a; b)$ with a radius of 5 units touches the x-axis and the y-axis at points N and L respectively. QPT is a tangent to this circle at $P(-1; 8)$. The coordinates of T are $(2; y)$.



6.1 Give a reason why ML $\perp$ y-axis. (1)

6.2 Determine the:

6.2.1 Coordinates of M (2)

6.2.2 Equation of the circle having centre M. (2)

6.2.3 Equation of the tangent QOT in the form $y = mx + c$. (5)

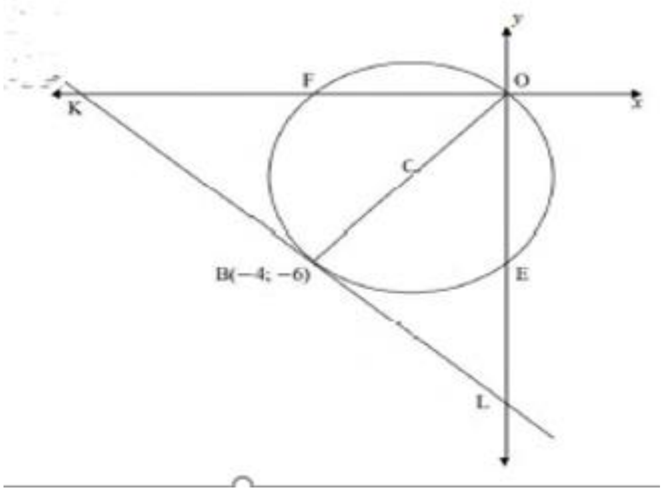
6.3 Another circle having point T as the centre, touches the circle having M as centre, externally. Determine the equation of the circle centred at T in the form $(x-h)^2 + (y-k)^2 = r^2$ (6)

6.4 The circle with centre M is translated across the Cartesian plane in such a way that both horizontal and vertical axes remain tangents to the circle simultaneously. Write down all the possible coordinates of the centres of the newly translated circles, given than $\sqrt{xy}$ must be real for ALL values of x and y. (4)

[20]

QUESTION 7: FREE STATE

A circle with centre at C passes through the origin, O, and also intersects the x-axis at F and the y-axis at E. the tangent to the circle at B$(4; 6)$ intersects the x-axis at K and the y-axis at L.



7.1 Calculate the length of the radius of the circle. (3)

7.2 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$ (4)

7.3 What type of a triangle is ΔOBL? Give a reason for your answer. (2)

7.4 Determine the equation of the tangent KL. (4)

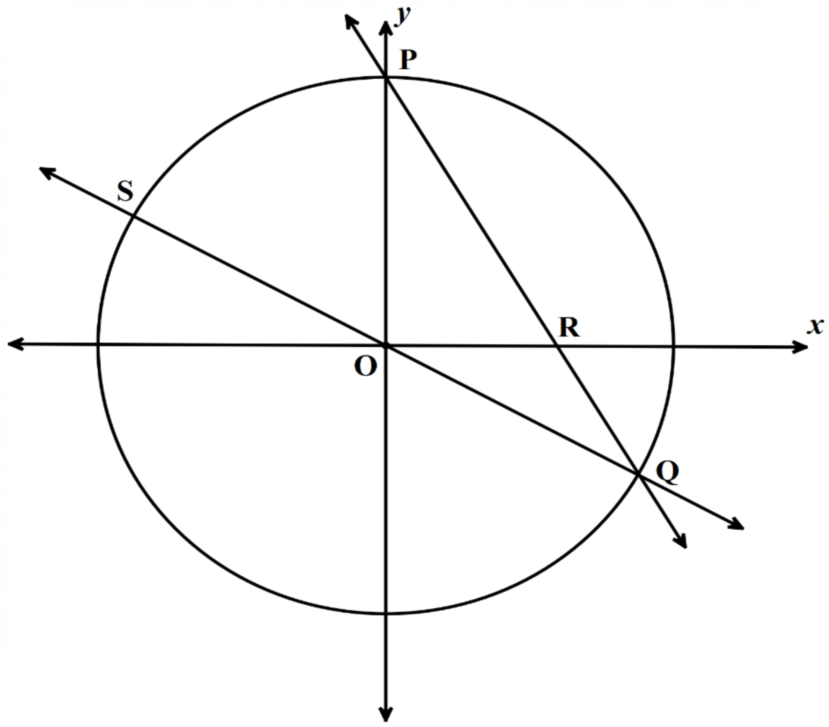
7.5 Determine the coordinates of E. (2)

7.6 Determine whether EF is a diameter of the circle. Show all working. (5)

[20]

QUESTION 8: NORTHERN CAPE

In the diagram, R and P are the x-intercept and y-intercept respectively of the line $y = -2x + 4$. The circle centred at O with equation $x^2 + y^2 = 16$ intersects the line at P and Q. S is a point on the circle such that SOQ is a straight line.



8.1 Write down the coordinates of P. (1)

8.2 Show that the coordinates of Q are: $\left(\frac{16}{5}; -\frac{12}{5}\right)$ (4)

8.3 Determine the equation of the circle with R as the centre that touches the y-axis in the form: $(x-a)^2 + (y-b)^2 = r^2$ (4)

8.4 Determine with how many degrees of inclination of PQ must be adjusted so that the adjusted line is parallel to SQ. (4)

8.5 The equation of another circle is $x^2 + 2x + y^2 - 6y = 6$.

8.5.1 Write down the equation in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

8.5.2 Write down the coordinates of the centre of this circle. (1)

8.6 Do the circles $x^2 + y^2 = 16$ and $x^2 + 2x + y^2 - 6y = 6$ intersect each other? Justify your answer with calculations. (4)

[21]

TRIGONOMETRY

Part 1: Trigonometric identities

QUESTION 1: EASTERN CAPE

1.1 If $\sin 54° = p$, express each of the following in terms of p, **without the use of a calculator.**

1.1.1 $\sin 594°$ (2)

1.1.2 $\cos 36°$ (2)

1.1.3 $\cos 18°$ (4)

1.2 Simplify the following **without the use of a calculator**.

$$\frac{\cos 140° - \sin(90° - \theta)}{\sin 410° + \cos(-\theta)}$$ (6)

1.3 Determine, **without the use of a calculator**, the value of the following trigonometric expression.

$$\cos(x + 65°) . \cos(x + 20°) - \sin(x + 245°) . \sin(x + 20°)$$ (4)

1.4 Determine the general solution of :

$$\cos^2 x - \sin^2 x = \frac{1}{2}$$

1.5 Given the identity:

$$\frac{\sin 2\theta . \tan\theta}{\cos 2\theta + 1} = tan^2\theta$$

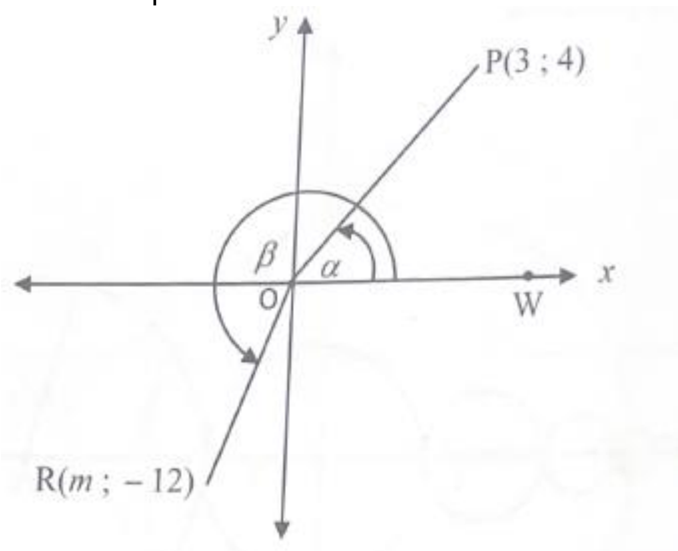
1.5.1 Prove the identity (4)

1.5.2 Determine the values of θ for which the identity is undefined if $0° < \theta < 180°$. (4)

[30]

QUESTION 2: LIMPOPO

2.1 In the diagram below $P(3\,;4)$ and $R(m\,;-12)$ are two points as indicated.
$\hat{POW} = \alpha$ and $\hat{ROW} = \beta$.



Answer the following questions without using a calculator.

2.1.1 Write down the value of $\tan\alpha$. (1)

2.1.2 Determine the value of $\sin(90° + \alpha)$. (3)

2.1.3 Determine the value of m if it is given that $12 + 13\sin\beta = 0$. (4)

2.1.4 Determine the value of $\sin(\alpha + \beta)$. (3)

2.3 Simplify the following:

2.1.2 $\sqrt{4^{\sin 150°}.\,2^{3\tan 225°}}$ without using a calculator. (5)

2.1.2 $\dfrac{\tan(180°+x).\cos x}{\sin(180°+x)\cos x - \cos(540°+x).\cos(90°+x)}$ to a single trigonometric expression. (6)

2.3 Prove that: $\dfrac{1-\cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$ (4)

2.4 It is given that P and Q are both acute angles, solve for P and Q if:

$\sin P\sin Q - \cos P\cos Q = \dfrac{1}{2}$ and $\sin(P - Q) = \dfrac{1}{2}$ (7)

[33]

QUESTION 3: MPUMALANGA

3.1 If θ is a reflex angle, and $\tan\theta = -\frac{3}{4}$, determine without the use of a calculator and with the aid of a sketch, the value of:

3.1.1 $\sin\theta$ (2)

3.1.2 $\cos 2\theta$ (3)

3.1.3 $\cos(\theta + 30°)$ (3)

3.2 If $x = 4\sin\alpha$ and $y = 4\cos\alpha$, calculate the value of $x^2 + y^2$. (2)

3.3 Simplify the expression to a single trigonometric ratio:

$\sin(900° - x) . \cos(-x) - \sin(x - 180°) . \sin(90° + x)$ (6)

3.4 Given the following identity:

$$\frac{\sin 7x + \sin x}{2\cos 3x} = \sin 4x$$

3.4.1 Prove the identity. (4)

3.4.2 For which values of x is the identity above, undefined? Determine the general solution of x for which the identity is undefined. (3)

3.5 Calculate the general solution of x if $2\sin(3x + 20°) = 2\cos x$ (6)

[29]

QUESTION 4: FREE STATE

4.1 If $\tan 58° = m$, determine the following in terms of m without using a calculator.

4.1.1 $\sin 58°$ (2)

4.1.2 $\sin 296°$ (3)

4.1.3 $\cos 2°$ (3)

4.2 If $5\tan\theta + 2\sqrt{6} = 0$ and $0° < \theta < 270°$, determine with the aid of a sketch and without using a calculator, the value of:

4.2.1 $\sin\theta$ (2)

4.2.2 $\cos\theta$ (1)

4.2.3 $\dfrac{14\cos\theta + 7\sqrt{6}\sin\theta}{\cos(-240°)\,.\tan 225°}$ (4)

4.3 Determine the value of: (6)

$$\frac{\cos(180° + x)\,.\tan(360° - x)\,.\sin^2(90° - x)}{\sin(180° - x)} + \sin^2 x$$

4.5 Prove the identity: $\cos(\mathrm{A} - \mathrm{B}) - \cos(\mathrm{A} + \mathrm{B}) = 2\sin A \sin B$ (3)

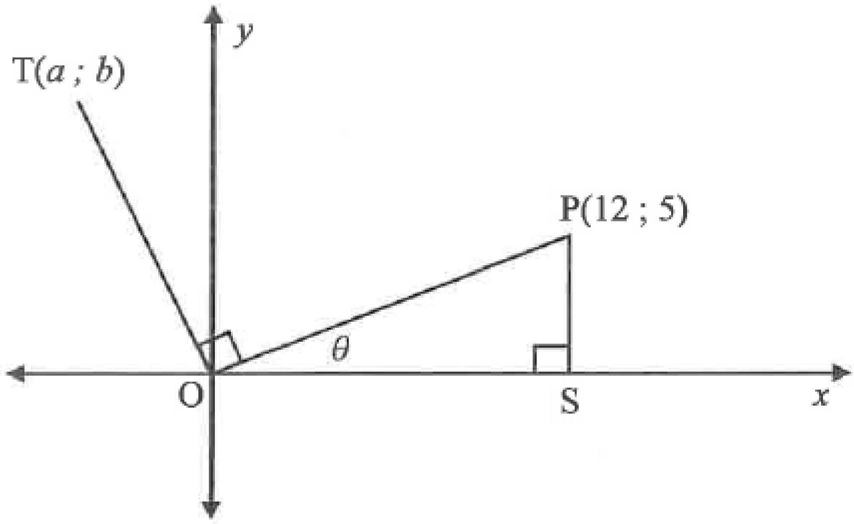
4.5 Hence calculate, without using a calculator, the value of : $\cos 15° - \cos 75°$ (4)

4.6 Find the value of $\tan\theta$, if the distance between $\mathrm{A}(\cos\theta\ ; \sin\theta)$ and $\mathrm{B}(6\,;7)$ is $\sqrt{86}$ units. (4)

[32]

QUESTION 5: GAUTENG

5.1 In the diagram below, P is the point $(12\,;5)$ and $T(a\,;b)$. $OT \perp OP$, $PS \perp x$-axis and $P\hat{O}S = \theta$.



Without using a calculator, determine, the value of:

5.1.1 $\tan\theta$ (1)

5.1.2 $\sin\theta$ (2)

5.1.3 a, if $TO = 19{,}5$ units (4)

5.2 Determine the value of the following, without using a calculator:

$$\frac{\sin(360° - 2x)\,.\sin(-x)}{\sin(90° + x)} + 2\cos^2(180° + x)$$ (6)

5.3 Given: $\cos 42° = \sqrt{k}$
Without using a calculator, determine the value of $\sin^2 69°$ in terms of k. (3)

5.4 Given the identity: $\dfrac{\sin 5x\,.\cos 3x - \cos 5x \sin 3x}{\tan 2x} - 1 = -2\sin^2 x$

5.4.1 Prove the identity. (4)

5.4.2 Determine the values of x for which the identity will be undefined in the interval $x \in [0°\,;60°]$. (2)

5.5 Given: $f(x) = 2\cos x - \sin^2 x$

5.5.1 Show that $f(x)$ can be expressed as $f(x) = (\cos x +)^2 - 2$. (2)

5.5.2 Hence, or otherwise, find the maximum value of f. (2)

[26]

QUESTION 6: KWA-ZULU NATAL

6.1 If $\sin 38° = p$, determine the value of the following, **without using a calculator**:

6.1.1 $\cos 218°$ (3)

6.1.2 $\cos 14°$ (3)

6.1.3 $\sin 26° \cos 26°$ (2)

6.2 Evaluate the following trigonometric expression **without using a calculator**: (5)

$$\frac{2\sin 165° \cos 195°}{\cos 45° \sin 15° - \cos 15° \sin 45°}$$

6.3 Given: $\text{K} = \sqrt{3}\cos x + \sin x$

6.3.1 Write K in the form of $t\sin(x+\theta)$. (3)

6.3.2 Hence, calculate the values of t and θ. (1)

6.3.3 Write down the maximum value of K. (1)

6.4 Prove the identity:

(6)

6.4.1 $\dfrac{2\tan\theta - \sin 2\theta}{2\sin^2\theta} = \tan\theta$

6.4.2 Hence, determine the values of θ, $\theta \in [180°; 360°]$ which will make the above identity undefined. (2)

[26]

QUESTION 7: NORTH WEST

7.1 **WITHOUT using a calculator**, determine the following in terms of $\sin 25°$:

7.1.1 $\sin 335°$ (1)

7.1.2 $\cos 50°$ (2)

7.2 Simplify the following expression to ONE trigonometric ration: (6)

$$\frac{\sin(-2x).(1-\sin^2 x)}{\sin(90°+x).\tan x}$$

7.3 **WITHOUT using a calculator**, simplify $(p\tan 30° + q\sin 60°)$ to a single fraction in terms of p and q. (3)

7.4 Given: $\cos(\mathrm{A}-\mathrm{B}) = \cos\mathrm{A}\cos\mathrm{B} + \sin\mathrm{A}\sin\mathrm{B}$

7.4.1 Use the formula for $\cos(\mathrm{A}-\mathrm{B})$ to derive a formula for $\sin(\mathrm{A}-\mathrm{B})$. (4)

7.4.2 Prove that $\sin 9\mathrm{A} + \sin\mathrm{A} = 2\sin 5\mathrm{A}.\cos 4\mathrm{A}$ (3)

7.4.3 Write down the maximum value of $3^{2\sin 3\mathrm{A}\cos 4\mathrm{A}}$ (2)

7.5 Determine the general solution of $\cos 2x - 5\cos x - 2 = 0$ (6)

7.6 Given: $\tan x = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ ...}}}$, with $x \in [0°; 90°)$ (5)
WITHOUT using a calculator, show that : $2\sin^2 x = \sin x.(\cos x + 1)$

[32]

QUESTION 8: NORTHERN CAPE

8.1 Given: $\cos 48° = t$

Determine EACH of the following in terms of t, **without using a calculator**:

8.1.1 $\cos 228°$ (2)

8.1.2 $\sin 48°$ (2)

8.1.3 $\cos 96°$ (3)

8.1.4 $\sin 93°$ (4)

8.2 Calculate the value of the following expression **without using a calculator**:

$$\frac{\sin 36°\,.\sin\theta.\cos(90° - \theta)}{\sin 756°} + \frac{\sin 2\theta\,.\cos\theta}{2\sin\theta}$$ (5)

8.3 Given: $\tan^2 x\left(\frac{1}{\tan^2 x} - 1\right) = \frac{\cos 2x}{\cos^2 x}$

8.3.1 Prove the identity. (4)

8.3.2 For which value(s) of x in the interval $x \in [0°; 360°]$ will the identity be undefined? (2)

8.3.3 Determine the general solution of: $\tan^2 x\left(\frac{1}{\tan^2 x} - 1\right) = 0$ (4)

8.4 Given: $f(x) = \sin x - 1$ and $g(x) = 1 - 2\sin^2 x$

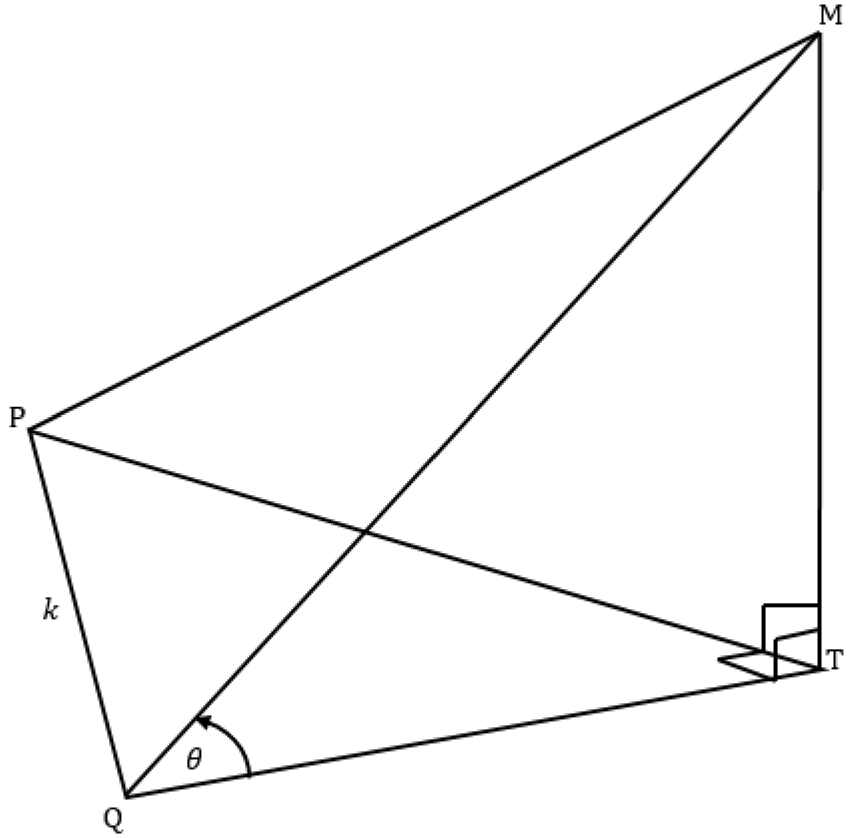
Determine the value(s) of x in the interval $x \in [270°; 360°]$ for which the value of $g(x) - f(x)$ will be maximum. (5)

[31]

Part 2: 2D & 3D

QUESTION 1: GAUTENG

In the diagram below, P, Q and T are three points in the same horizontal plane and MT is a vertical mast. MP and MQ are two straight stay wires. The angle of elevation of M from Q is θ. PQ $= k$ metres. PM $=$ 2PQ.
The area of $\Delta\text{MPQ} = 2k^2 \sin\theta \cos\theta$



1.1 Show that $\text{M}\hat{\text{P}}\text{Q} = 2\theta$ (3)

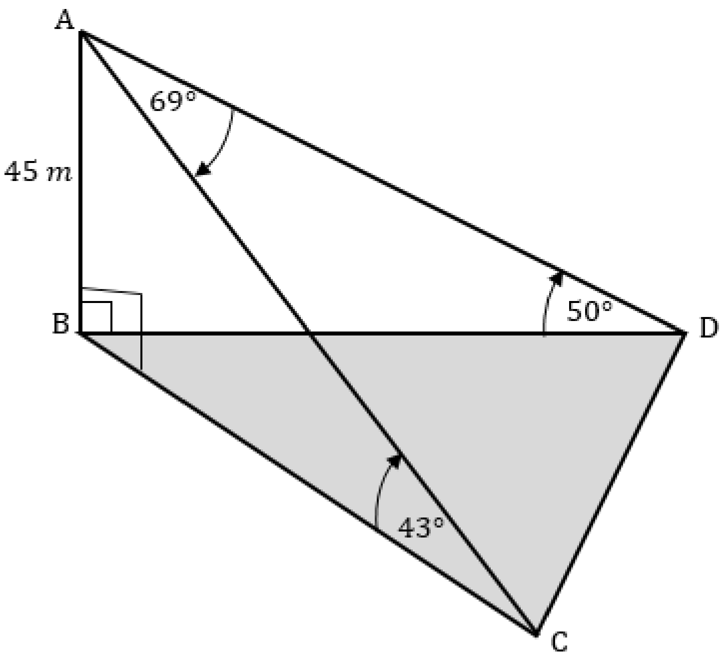
1.2 Hence, show that $\text{MQ} = k\sqrt{1 + 8\sin^2\theta}$ (4)

1.3 If $k = 139{,}5\ m$ and $\theta = 42°$, determine the length of MT, correct to the nearest metre. (3)

[10]

QUESTION 2: FREE STATE

In the figure below, Thabo is standing at point A on top of building AB that 45 m high. He observes two cars at C and D respectively. The cars at C and D are in the same horizontal plane as B. The angle of elevation from C to A is 43° and the angle of elevation from D to A is 50°. $C\hat{A}D = 69°$.



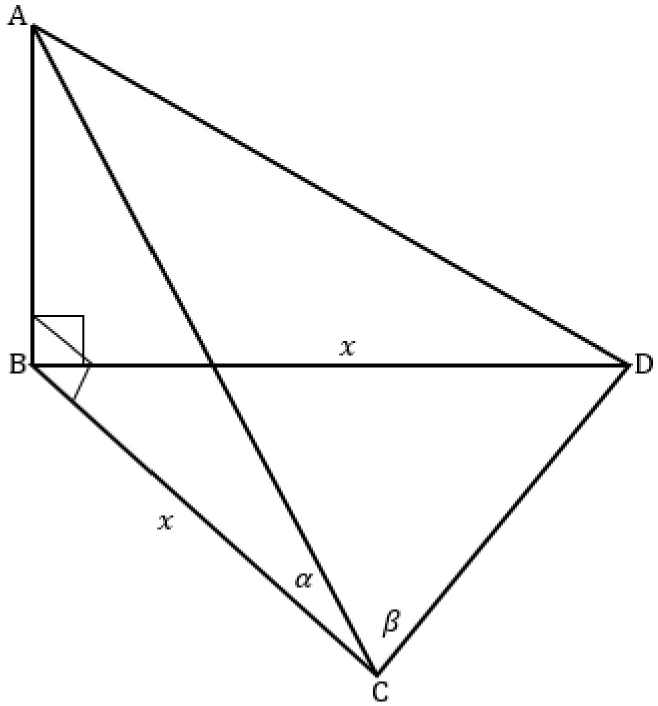
2.1 Calculate the lengths of AC and AD, correct to TWO decimal places. (4)

2.2 Calculate the distance between the two cars, the length of CD. (3)

[7]

QUESTION 3: EASTERN CAPE

In the figure below, B, C and D are points in the same horizontal plane. AB is a vertical tower with the angle of elevation from C to A equal to α $\mathrm{A\hat{C}D} = \beta$.
$\mathrm{BD} = \mathrm{BC} = x$.



3.1 Why is $\mathrm{AC} = \mathrm{AD}$? (1)

3.2 Write AC in terms of x and α. (2)

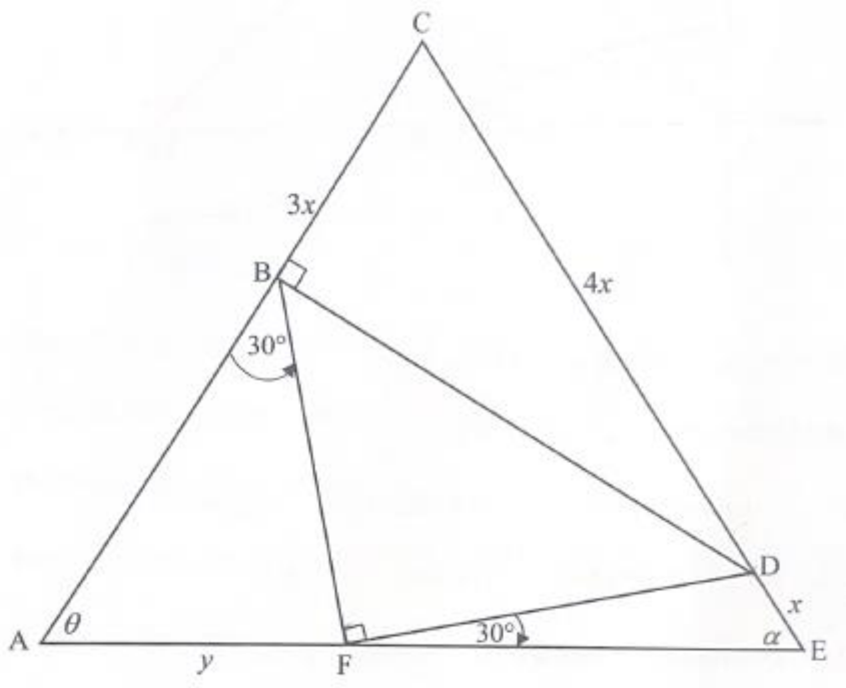
3.3 Show that: $\mathrm{CD} = \dfrac{2x\cos\beta}{\cos\alpha}$ (4)

3.4 Hence, determine the length of CD if $x = 25\ cm$, $\alpha = 30°$ and $\beta = 65{,}62°$. (2)

[9]

QUESTION 4: LIMPOPO

The diagram below shows ΔACE with $\hat{A} = \theta$ and $\hat{E} = \alpha$. Points B, D and F lie on AC, CE and AE respectively so that BC $= 3x$, CD $= 4x$, DE $= x$ and AF $= y$. BD $\perp$ BC and $B\hat{F}D = 90°$. $A\hat{B}F = 30°$ and $D\hat{F}E = 90°$.



4.1 Write BF in terms of θ and y. (3)

4.2 Write DF in terms of α and x. (2)

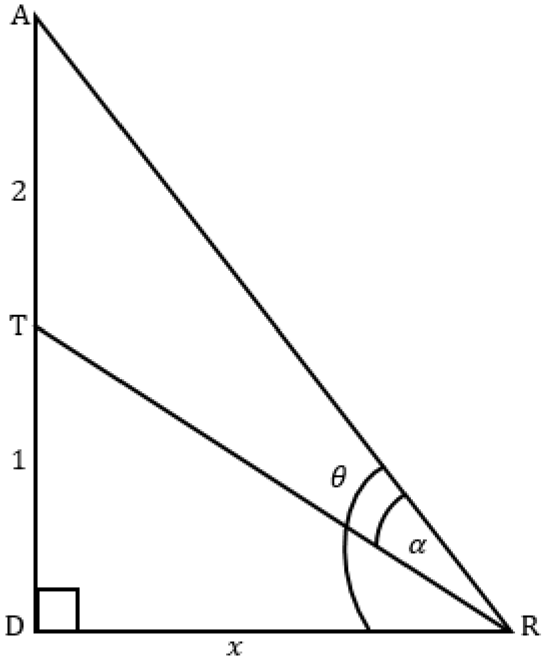
4.3 Hence, prove that $BD^2 = 4x^2.\sin^2\alpha + 4y^2.\sin^2\theta$ (1)

4.4 Hence, prove that: $x = \sqrt{\dfrac{4y^2.\sin^2\theta}{7 - 4sin^2\alpha}}$ (3)

[9]

QUESTION 5: MPUMALANGA

In the diagram, AD is a 3 m vertical pole with support cables attached at T and A to a point R which is x metres from the foot of the pole, D. D and R are in the same horizontal plane. T is 1 m above the foot of the pole. The angle of elevation of A from R is θ. $\text{T}\hat{\text{R}}\text{A} = \alpha$



5.1 Show that: $\text{TR} = \dfrac{x}{\cos(\theta - \alpha)}$ (2)

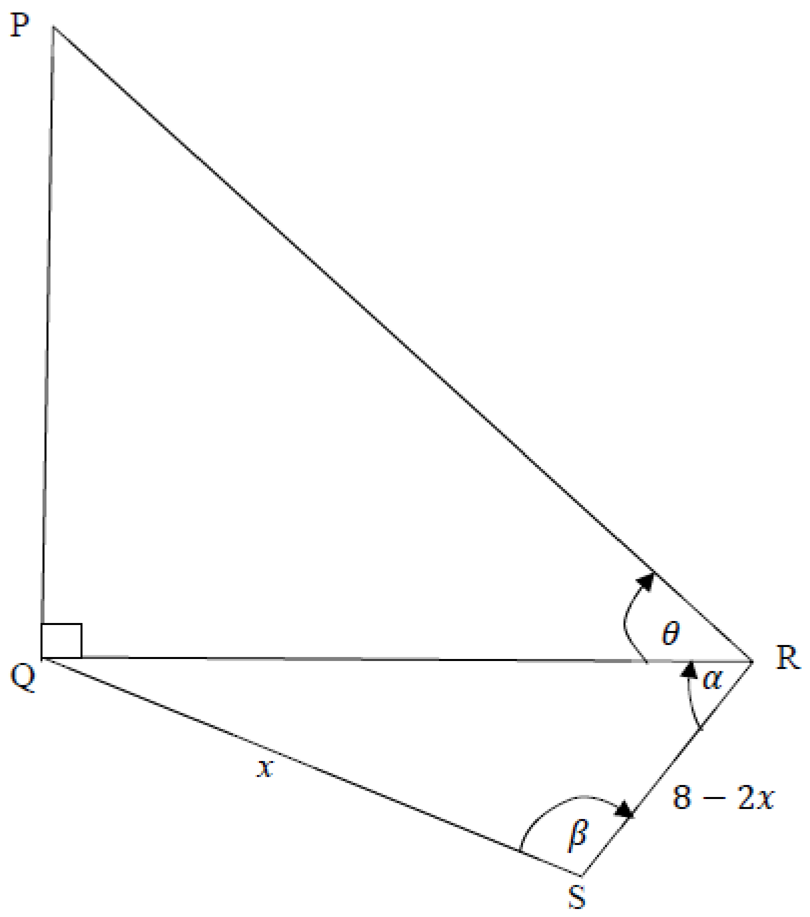
5.2 Prove that: $x = \dfrac{2\cos\theta\cos(\theta - \alpha)}{\sin\alpha}$ (4)

5.3 If it is further given that $\theta = 68{,}33°$ and $\alpha = 28°$, calculate the area of ΔATR **by using the formulas in 5.1 and 5.2.** (4)

[10]

QUESTION 6: KWA-ZULU NATAL

I n the diagram below, PQ is a vertical mast. R and S are two points in the same horizontal plane as the foot of the mast, Q. $Q\hat{S}R = \alpha$, $Q\hat{S}R = \beta$, $SR = 8 - 2x$ and $SQ = x$. The angle of elevation of P, the top of the mast from R, is θ.



6.1 Express PQ in terms of QR and a trigonometric ratio of θ. (1)

6.2 Show that: $PQ = \frac{x \sin\beta \tan\theta}{\sin\alpha}$ (4)

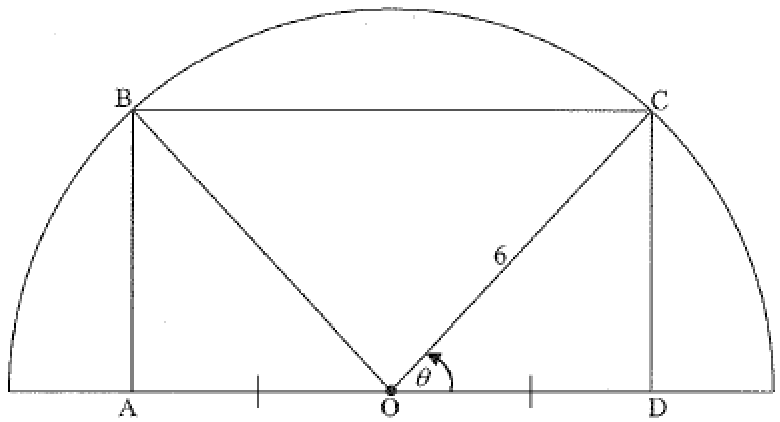
6.3 If $\beta = 60°$, show that the area of $\Delta QSR = 2\sqrt{3x} - \frac{\sqrt{3}}{2}x^2$. (3)

6.4 Determine the value of x for which the area of ΔQSR will be at maximum. (3)

[11]

QUESTION 7: NORTH WEST

In the diagram below, O is the centre of the circle. A, B, C and D are points on the semi-circle such that ABCD is a rectangle. The radius of the semi-circle is 6 units, $\hat{COD} = \theta$ and $AO = OD$.



7.1 Write $B\hat{O}C$ in terms of θ. (2)

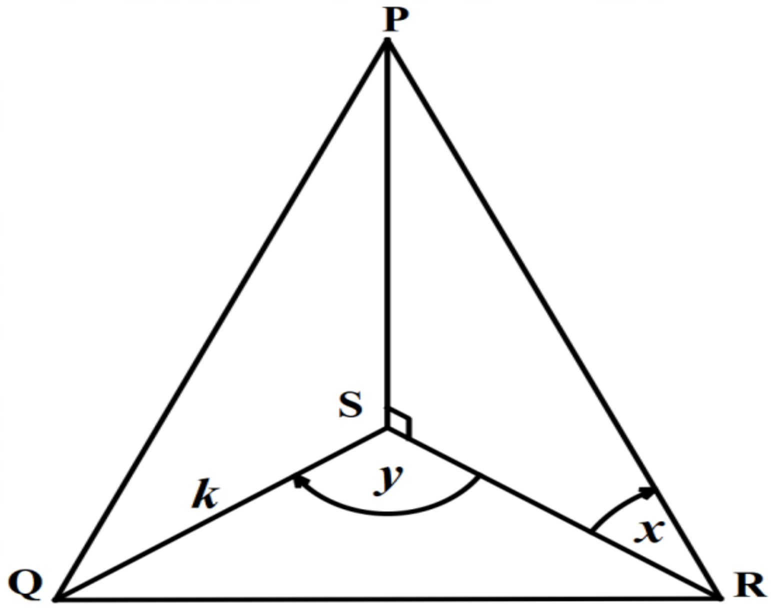
7.2 If $\theta = 43°$, calculate the length of BC. (3)

7.3 Points A, B, C and D are shifted along the semi-circle. Calculate the value of θ, if ABCD now form a square. (4)

[9]

QUESTION 8: NORTHERN CAPE

The diagram shows a vertical pole PS held in position by two anchor cables PQ and PR respectively. S, Q and R lie on the same horizontal plane. The area of $\Delta QRS = A \text{ m}^2$. The angle of elevation from R to P is $x°$. $Q\hat{S}R = y°$ and $QS = k$ metres.



8.1 Express SR in terms of x and PS. (2)

8.2 Prove that: $PS = \dfrac{2A.\tan x}{k.\sin y}$ (4)

8.3 If it is given that $A = 480{,}9 \text{ m}^2$; $x = 46{,}5°$; $k = 87{,}36 \text{ m}$ and $PS = 76{,}8$ m. Determine the value of y. (3)

[9]

Part 3: Trigonometric functions

QUESTION 1: KWA-ZULU NATAL

1.1 Sketch the graphs of $f(x) = 2\sin x$ and $g(x) = \cos(x - 30°)$ for $x \in [-180°; 180°]$ on the grid in the ANSWER BOOK. Indicate the intercepts with the axes and also the turning points. (6)

1.2 Use your graphs to estimate the following questions:

1.2.1 Write down the period of g. (1)

1.2.2 Determine the values of x for which $f(x) > g(x)$. (4)

1.2.3 Write down the values of x for which $f(x) = 1{,}5 + g(x)$. (2)

[13]

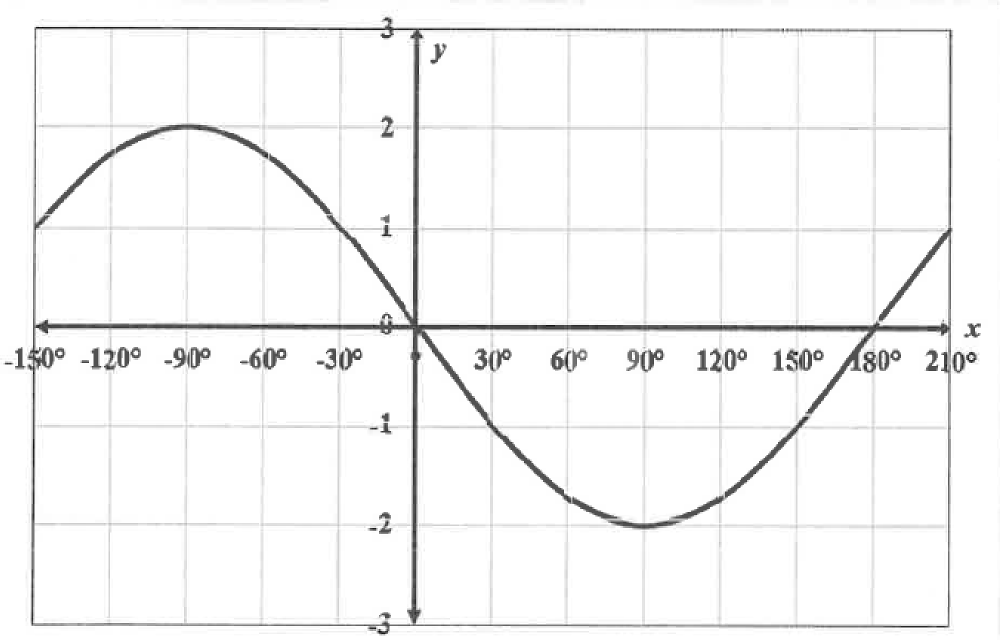
QUESTION 2: GAUTENG

Given the equation: $\cos(x - 30°) + 2\sin x = 0$

2.1 Show that the equation can be written as $\tan x = -\frac{\sqrt{3}}{5}$. (4)

2.2 Determine the solutions of the equation $\cos(x - 30°) + 2\sin x = 0$ in the interval $-180° \leq x \leq 180°$. (3)

2.3 In the diagram below, the graph of $f(x) = -2\sin x$ is drawn for $[-150°; 210°]$.



2.3.1 Write down the amplitude of f. (1)

2.3.2 Draw the graph of $g(x) = \cos(x - 30°)$ for the interval $[-150°; 210°]$ on the grid provided in the ANSWER BOOK. Clearly show ALL intercepts with the axes and endpoint(s) of the graph. (3)

2.3.3 Use the graphs to determine the values of x, in the interval $[-150°; 210°]$ for which:

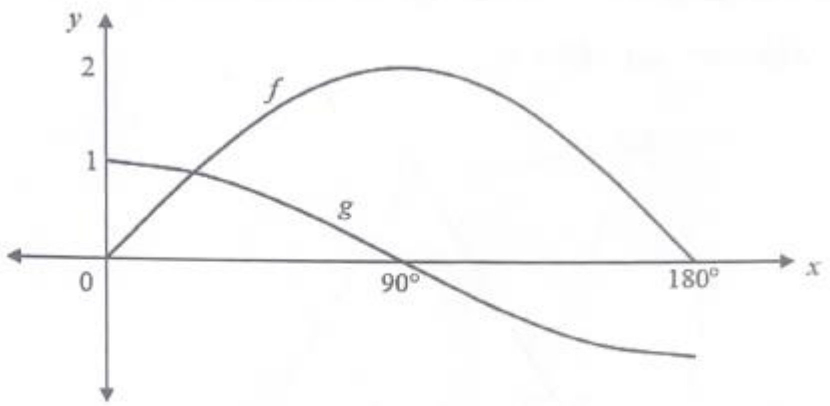
(a) $g(x) > f(x)$ (2)

(b) $f'\left(\frac{1}{2}x\right) = 0$ (1)

[14]

QUESTION 3: LIMPOPO

In the diagram, thr graphs of $f(x) = a\sin x$ and $g(x) = \cos bx$ are drawn for $x \in [0°; 180°]$.



3.1 Determine the values of a and b. (2)

3.2 Consider the interval $x \in [0°; 180°]$:

3.2.1 Calculate the value(s) of x where $a\sin x - \cos bx = 0$. (2)

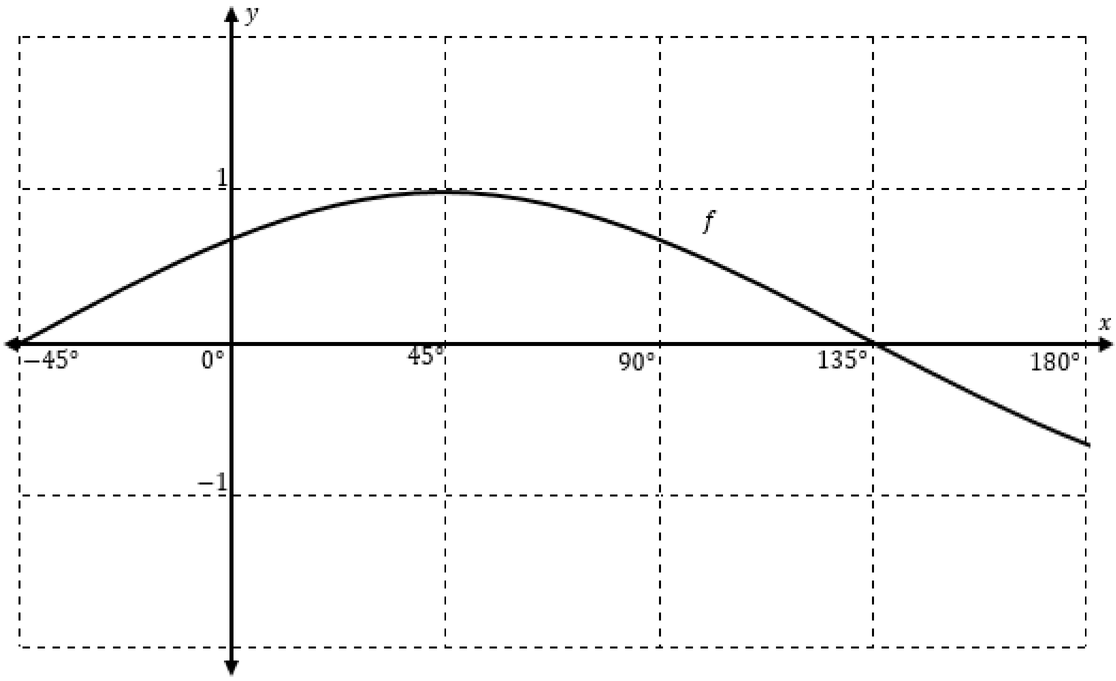
3.2.2 For which value(s) of x will $g(x).f'(x) \geq 0$? (2)

3.2.3 Determine the value(s) of y for which $y = 2^{2f(x)-1}$. (2)

[8]

QUESTION 4: EASTERN CAPE

Sketched below is the graph of $f(x) = \cos(x - 45°)$ for $-45° \leq x \leq 180°$.
Use the graph to answer the questions that follow



4.1 Write down the range f, for the given interval. (2)

4.2 Draw the graph of $h(x) = \sin 2x$, for $x \in [-45°; 180°]$ on the same set of axes as f in the ANSWER BOOK. Indicate the coordinates of all intercepts with the axes as well as turning points. (3)

4.3 State the period of h. (1)

4.4 Use your graph to determine the values of x for which f and h are both increasing. (2)

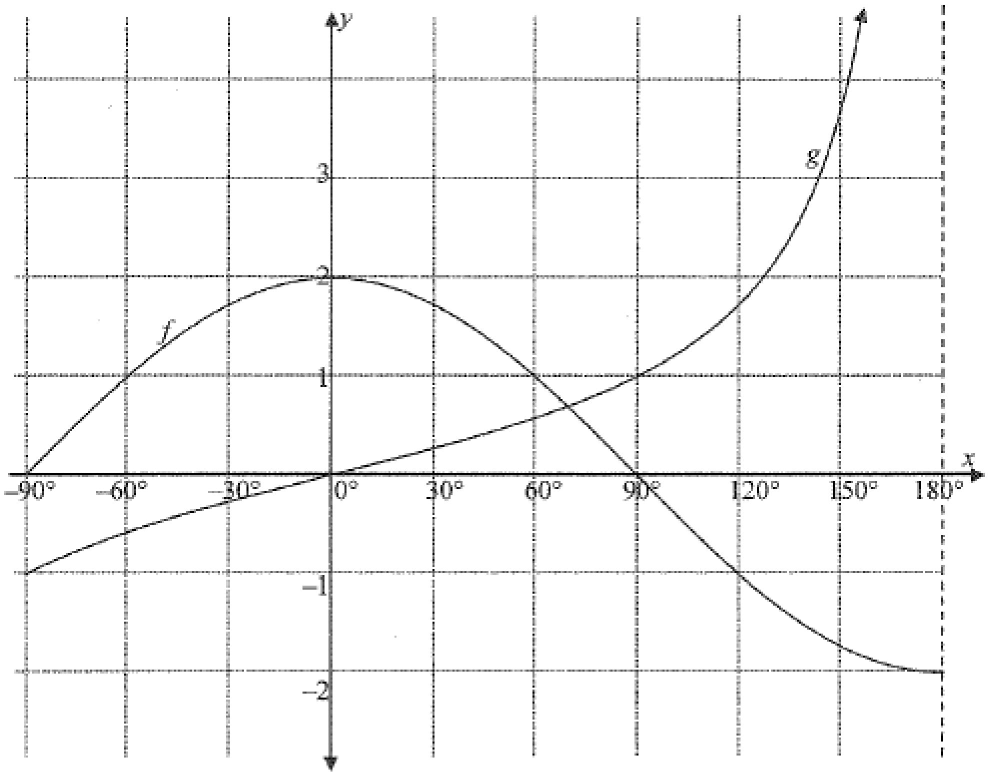
4.5 Determine the values of x for which $f(x) - h(x) = 1$. (2)

4.6 The graph of f is translated 60° to the left to form the graph of g. Write down the equation of g in the form: $g(x) =$ ... (1)

[11]

QUESTION 5: NORTH WEST

In the diagram below, the graphs of $f(x) = 2\cos x$ and $g(x) = \tan bx$ are drawn for the interval $x \in [-90°; 180°]$.



Use the graphs to answer the following questions.

5.1 Write down the value of b. (1)

5.2 Write down the range of g for the interval $x \in [-90°; 180°]$. (2)

5.3 Write down the period of g. (1)

5.4 Write down the value of x, in the interval, where $g(x+5) - f(x+5°) = 1$ (1)

5.5 Write down the value of x, in the interval, where $\frac{g(x)}{f'(x)}$ is undefined. (2)

5.6 Write down the value of p, if $\sum_{x=0°}^{p} 2\cos x = 0$ (2)

[9]

QUESTION 6: FREE STATE

Consider: $f(x) = \cos(x - 45°)$ and $g(x) = \tan\frac{1}{2}x$ for $x \in [-180°; 180°]$

6.1 Use the grid provided to draw sketch graphs of f and g on the same set of axes for $x \in [-180°; 180°]$. Show clearly all the intercepts on the axes, the coordinates of the turning points and the asymptotes. (6)

6.2 Use your graphs to answer the following questions for $x \in [-180°; 180°]$.

6.2.1 Write the solutions of $\cos(x - 45°) = 0$ (2)

6.2.2 Write down the equations of the asymptote(s) of g. (2)

6.2.3 Write down the range of f. (1)

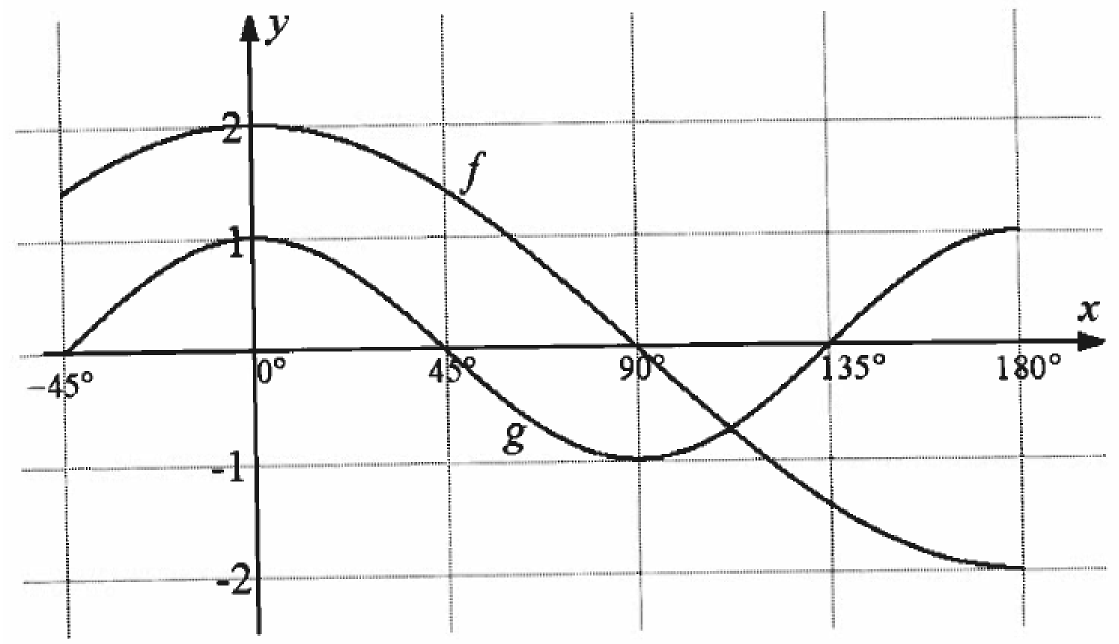
6.2.4 How many solutions exist for the equation $\cos(x - 45°) = \tan\frac{1}{2}x$? (1)

6.2.5 For what value(s) of x is $f(x).g(x) > 0$? (3)

[15]

QUESTION 7: MPUMALANGA

In the diagram below, the graphs of $f(x) = 2\cos x$ and $g(x) = \cos 2x$ are drawn for the interval $x \in [-45°; 180°]$.



7.1 Write down the period of g. (1)

7.2 Write down the values of x for which the graph of f is increasing in the given interval. (2)

7.3 Write down the of $y = 3g(x) - 1$. (2)

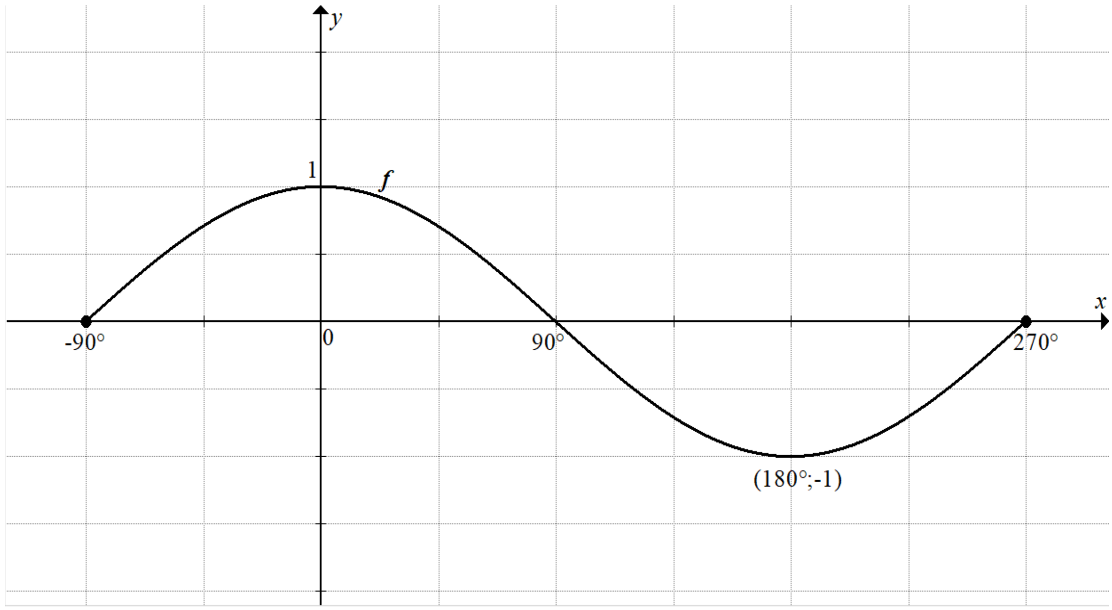
7.4 Determine the values of x for which $f(x) \geq \frac{1}{2}$ in the given interval. (4)

7.5 Determine the minimum value of $\frac{1}{2}\cos^2 x - \frac{1}{4}$ in the interval $x \in [0°; 180°]$. (4)

[13]

QUESTION 8: NORTHERN CAPE

In the diagram below, the graph of $f(x) = \cos x$ is drawn for the interval $x \in [-90°; 270°]$.



8.1 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \sin 2x - 1$ for $x \in [-90°; 270°]$. Clearly label ALL intercepts with the axes and turning points of the graph. (3)

8.2 Write down the amplitude of $g(x)$. (1)

8.3 Determine the coordinates of the turning point of $f(x - 30°)$ in the interval $x \in [180°; 270°]$. (2)

8.4 Use your graph to determine 3 possible solutions for $\sin 2x \cos x = \cos x$ for $x \in [90°; 270°]$. (4)

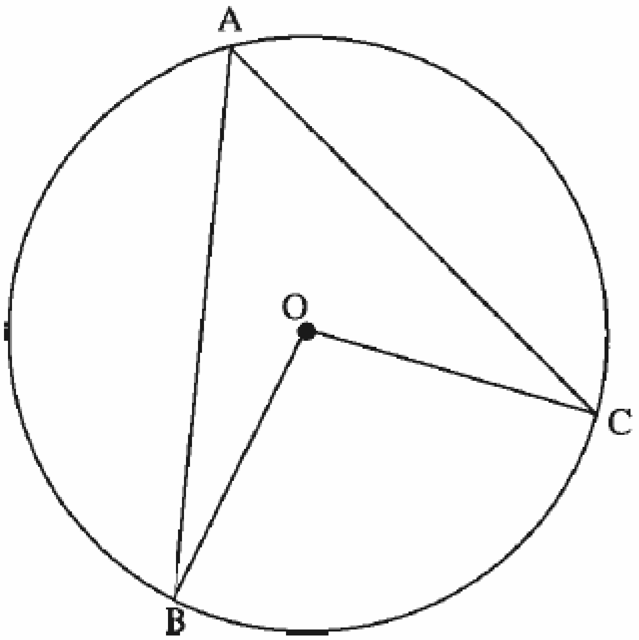
[10]

EUCLIDEAN GEOMETRY

Part 1: Proofs of theorems

QUESTION 1: MPUMALANGA

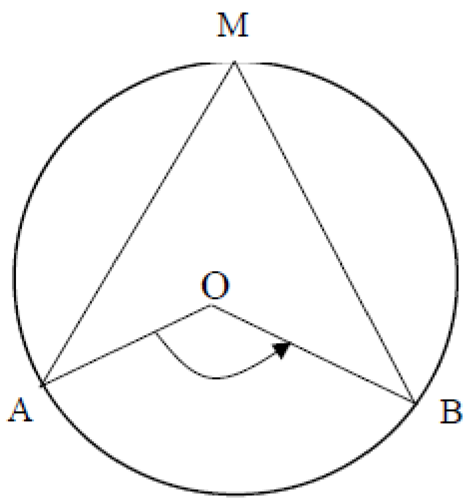
In the diagram below, O is the centre of the circle ABC. AB and AC are chords. OB and OC are joined.



Prove the theorem that states that $B\hat{O}C = 2 \times B\hat{A}C$ **[5]**

QUESTION 2: KWA-ZULU NATAL

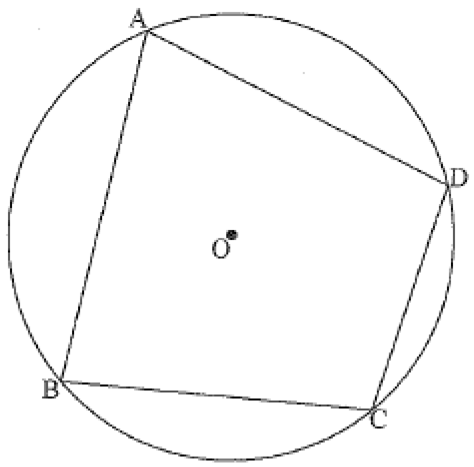
In the diagram, O is the centre of the circle and M is the point on the circumference of the circle. Arc AB subtends $A\hat{O}B$ at the centre of the circle and $\hat{M}$ at the circumference of the circle.



Use the diagram to prove the theorem that state that $A\hat{O}B = 2\hat{M}$ **[5]**

QUESTION 3: NORTH WEST

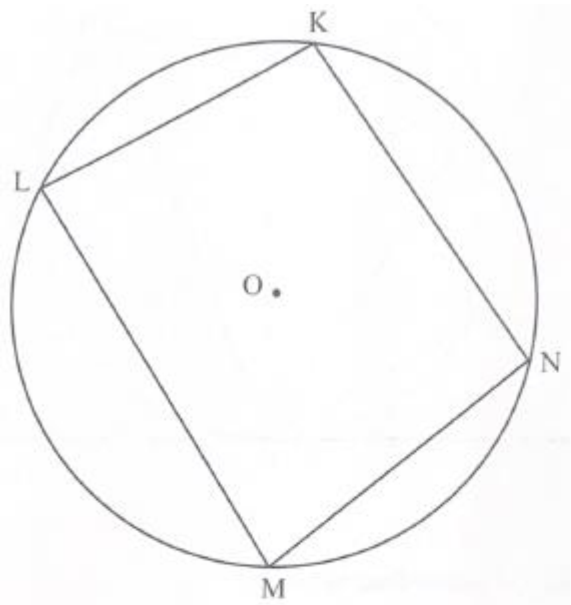
In the diagram, ABCD is a cyclic quadrilateral and the circle has centre O.



Prove the theorem that states that $\widehat{A} + \widehat{C} = 180°$ **[5]**

QUESTION 4: NORTHERN CAPE

O is the centre of the circle. Points K, L, M and N are on the circle.

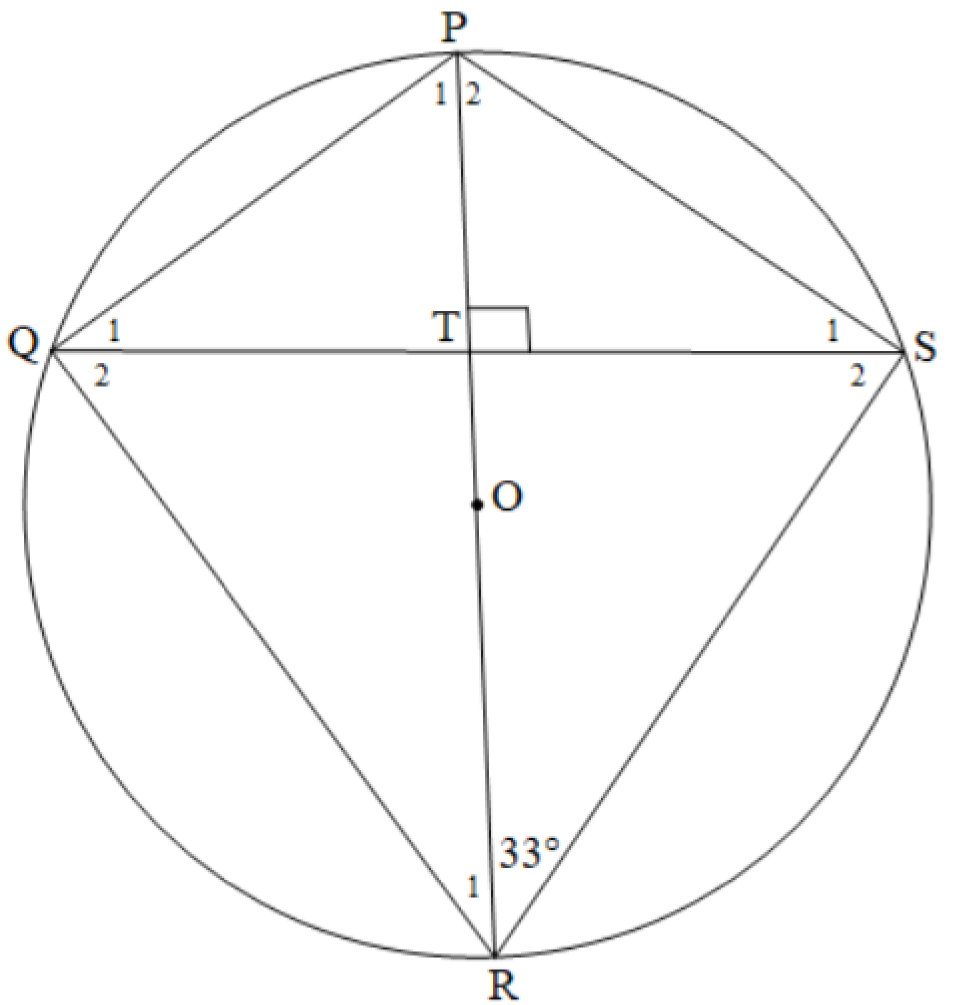


Use the diagram to prove the theorem that states that the opposite angles of a cyclic quadrilateral are supplementary, i.e. $\widehat{K} + \widehat{M} = 180°$. **[5]**

Part 2: Circle geometry

QUESTION 1: EASTERN CAPE

In the diagram below, PR is a diameter of circle PQRS with centre O. PR intersects with chord QS at T such that $P\hat{T}S = 90°$. $P\hat{R}S = 33°$.



1.1 Determine, with reasons, the size of:

1.1.1 $\hat{P}_1$ (3)

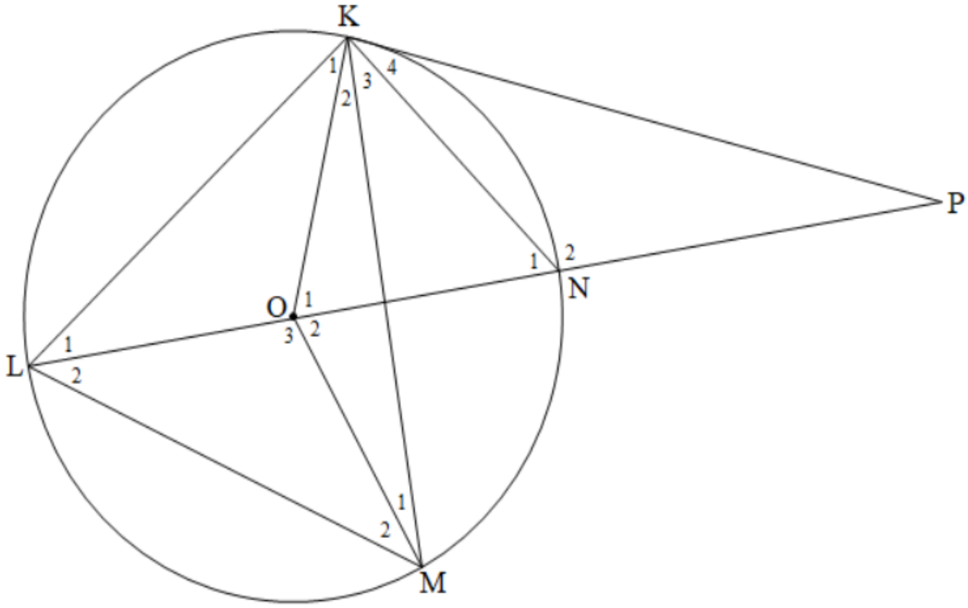
1.1.2 $\hat{Q}_2$ (2)

1.2 If $QS = 16$ cm and $PR = 20$ cm, determine, with reasons, the length of TO. (4)

[9]

QUESTION 2: EASTERN CAPE

In the diagram below, O is the centre of the circle and KP is the tangent to the circle. LN, the diameter of the circle, is extended to meet KP at P. Straight lines OK, OM, KM and KN are drawn.



2.1 Write down two angles equal to $90°$. (2)

2.2 If $\hat{K}_4 = x$, write down the following angles in terms of x, giving reasons.

2.2.1 $\hat{L}_1$ (2)

2.2.2 $\hat{K}_1$ (2)

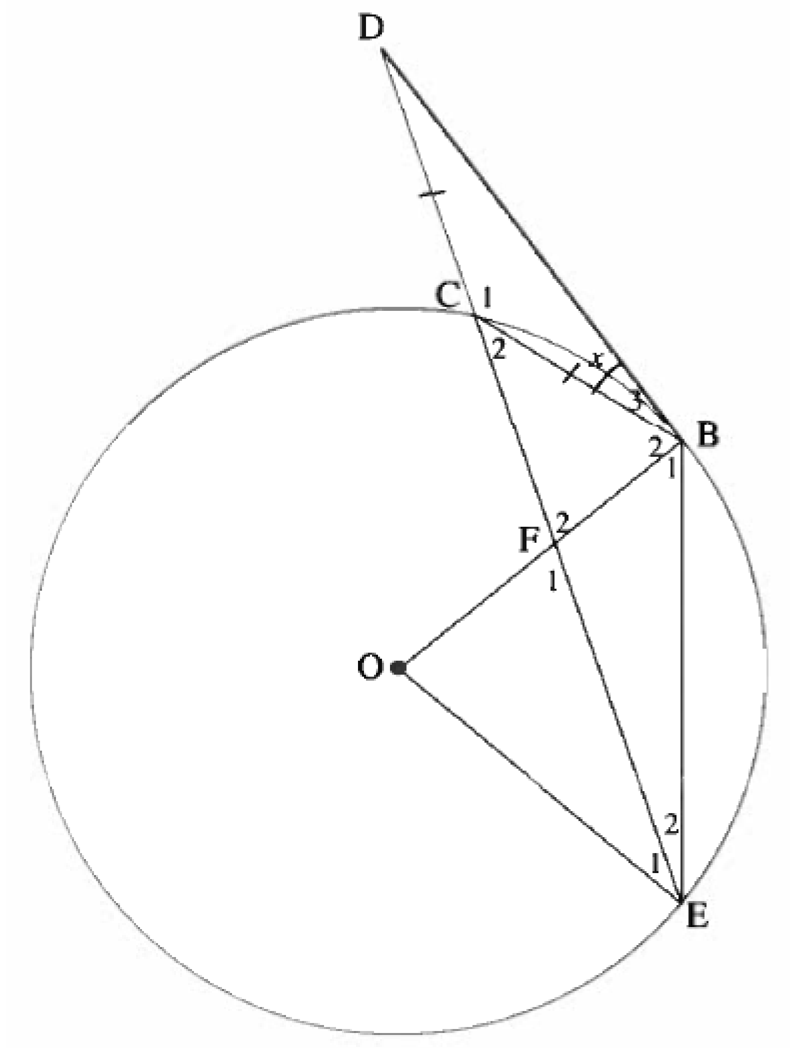
2.2.3 $\hat{P}$ (2)

2.3 Join MP, which is a tangent to the circle, and hence prove that KOMP is a cyclic quadrilateral. (3)

[11]

QUESTION 3: MPUMALANGA

O is the centre of circle CBE. DB is a tangent to the circle at B. EC produced meets BD in D and intersects OB at F. $CD = CB$. OE and BE are joined. $\hat{B}_3 = x$.



3.1 Name, with reasons, TWO other angles each equal to x. (4)

3.2 Express $E\hat{O}B$ in terms of x. (3)

3.3 Determine, with reasons, the size of $\hat{B}_2$ in terms of x. (2)

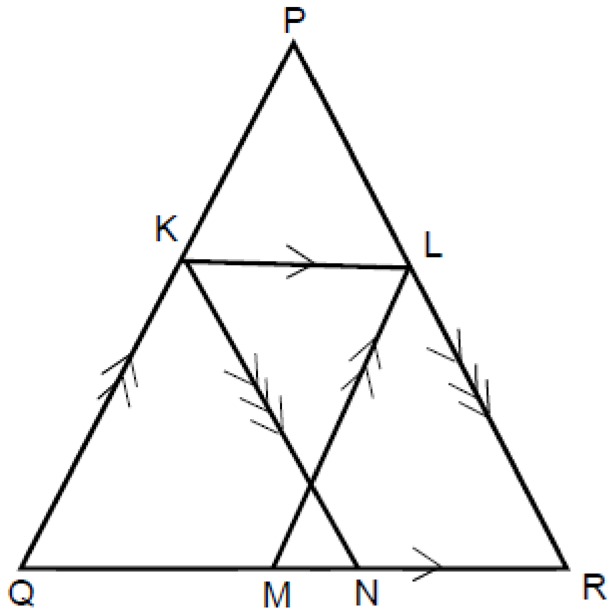
3.4 Show that $DC = CF$. (3)

[12]

QUESTION 4: KWA-ZULU NATAL

4.1 Compete the following statement: A line drawn parallel to one side of a triangle … (2)

4.2 In the figure, KL ∥ QR. M and N are points on QR such that KN ∥ PR and LM ∥ PQ. PK = 3 units, PL = 4 units, LR = 6 units and MN = 1,8 units.



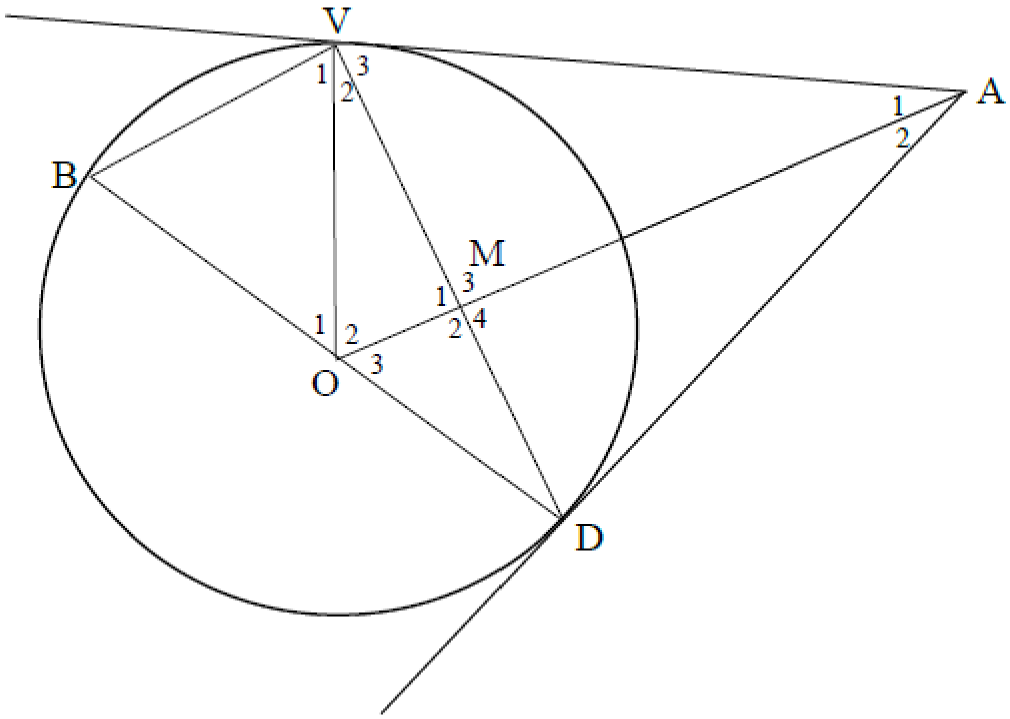
4.2.1 Calculate the length of KQ. (2)

4.2.2 Prove that QM = NR. (2)

[6]

QUESTION 5: KWA-ZULU NATAL

From a point A outside the circle, centre O, two tangents AD and AV are drawn. AO and VD intersect at M. BOD is a diameter of the circle. BV and VO are drawn. $V\hat{A}D = 40°$



5.1 Prove that quadrilateral VODA is cyclic. (2)

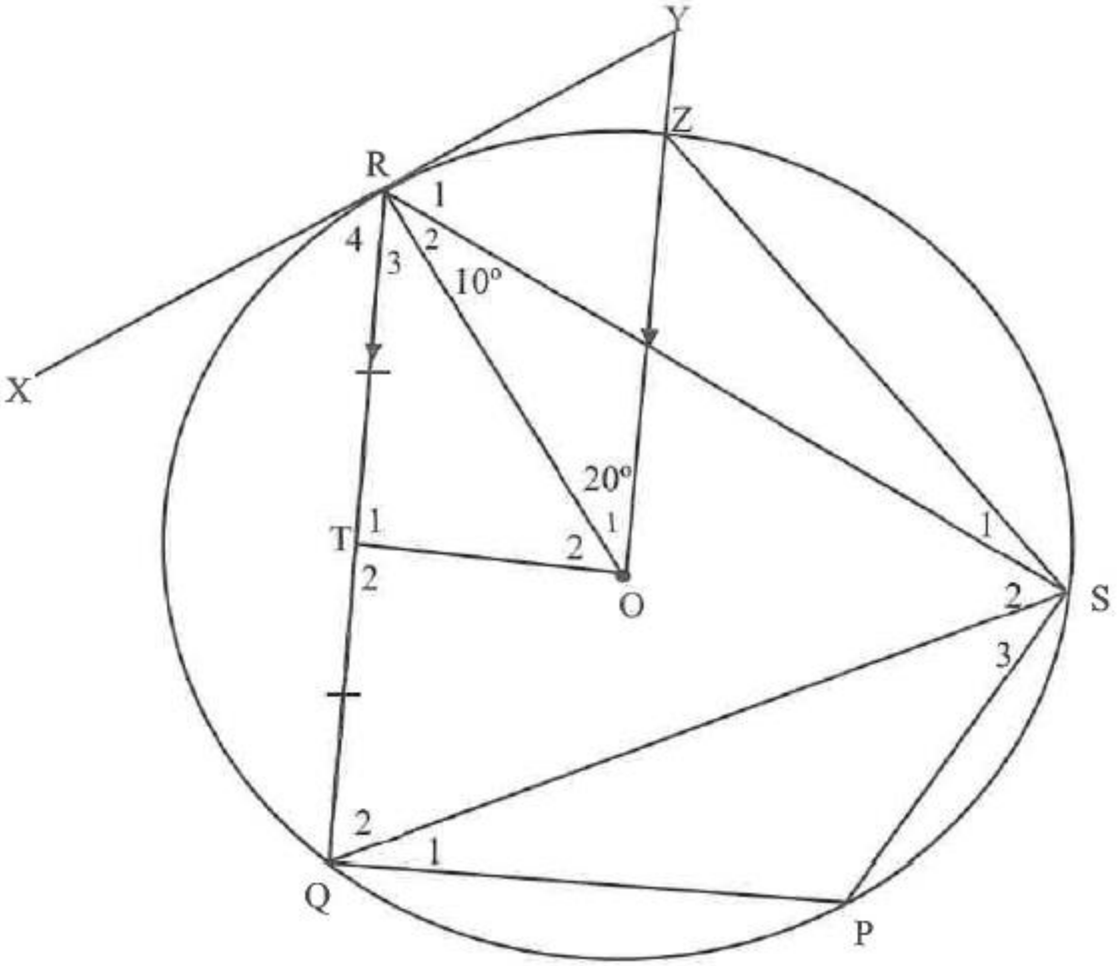
5.2 Calculate the magnitude of $\hat{O}_1$. (2)

5.3 Prove that $BV \parallel OA$. (5)

[14]

QUESTION 6: GAUTENG

In the diagram below, points P, Q, R and S are points on a circle with centre O. OT bisects chord QR at T. XRY is a tangent to the circle at point R. OZ is produced to meet at Y where OY ∥ QR. $R\hat{O}Y = 20°$ and $S\hat{R}O = 10°$. Chord SZ is drawn.



6.1 Calculate, with reasons, the size of the following angles:

6.1.1 $\hat{S}_1$ (2)

6.1.2 $\hat{R}_3$ (1)

6.1.3 $\hat{P}$ (2)

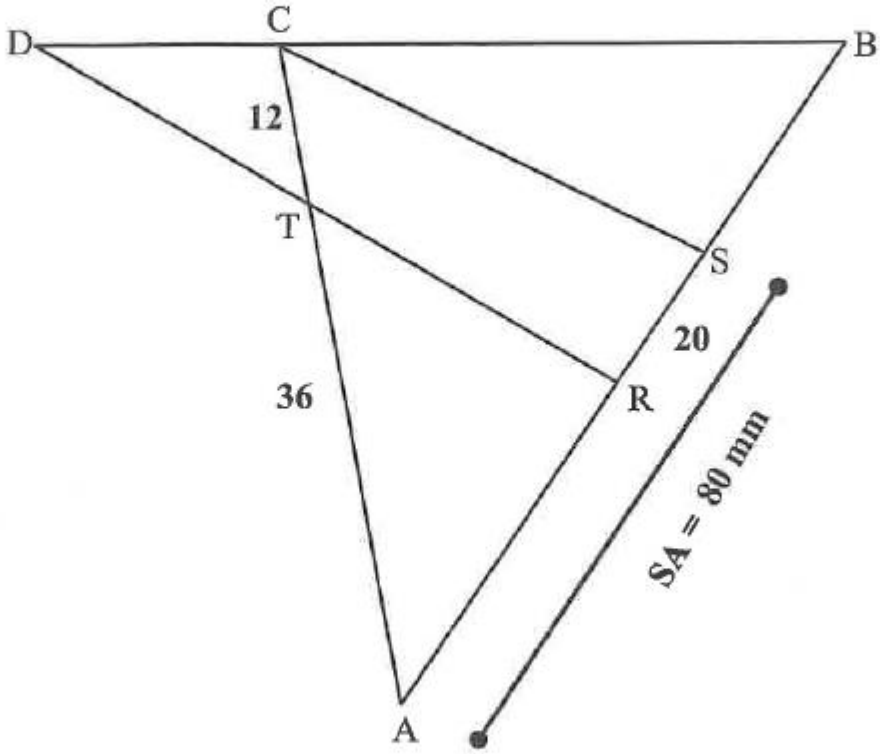
6.1.4 $\hat{S}_2$ (4)

6.2 Prove that XRY is a tangent to the circle passing through R, T and O. (3)

[12]

QUESTION 7: GAUTENG

In the diagram below, ΔABC is constructed such that BC is produced to D.
DR is drawn, with point T on AC and R on BA. CS is drawn. CT $= 12$ mm, TA $= 36$ mm, SR $= 20$ mm and SA $= 80$ mm.



7.1 Prove that CS $\parallel$ TR. (3)

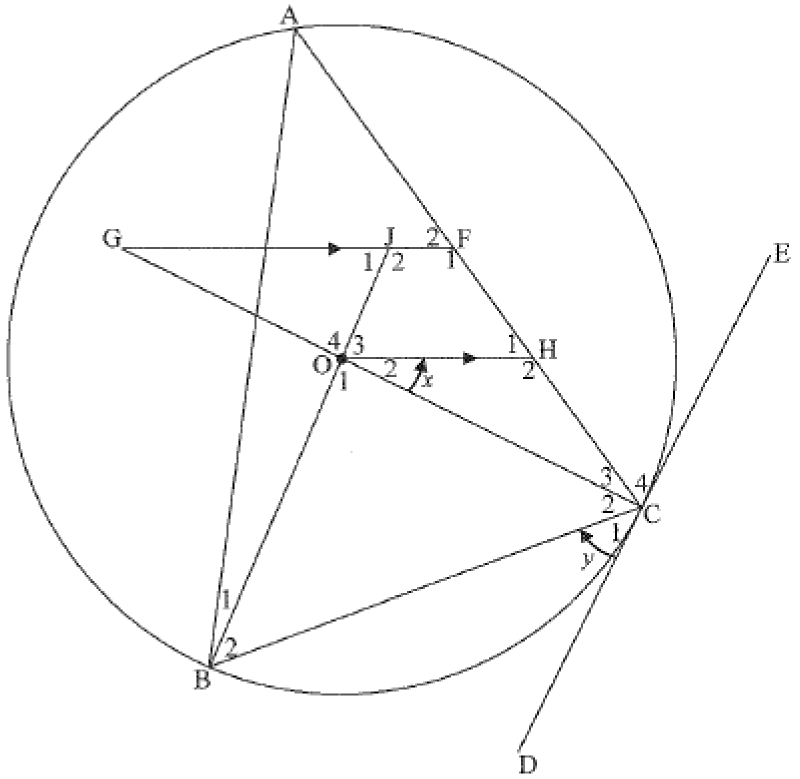
7.2 It is further given that $\text{AR} = \frac{2}{3}\text{RB}$, BC $= 2x$ and $\text{CD} = \frac{1}{2}x + 1$

Calculate the value of x. (6)

[9]

QUESTION 8: NORTH WEST

In the diagram below, O is the centre of the circle with points A, B and C on the circle. DCE is a tangent to the circle at C. GOC, BOJ and GJF are straight lines. F and H are points on AC such that $GF \parallel OH$. $\hat{C}_1 = y$, $\hat{O}_2 = x$ and $FH : HC = 2 : 3$.



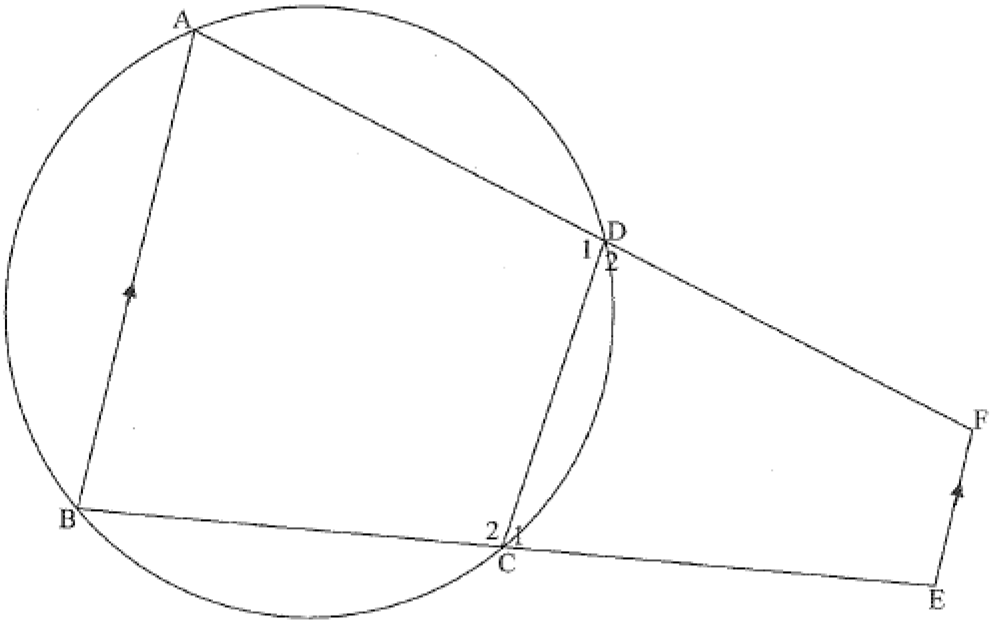
8.1 Calculate, giving reasons, $\hat{J}_1$ in terms of x and y. (6)

8.2 Determine, giving reasons, the value of: $\frac{GO}{GC}$ (3)

[9]

QUESTION 9: NORTH WEST

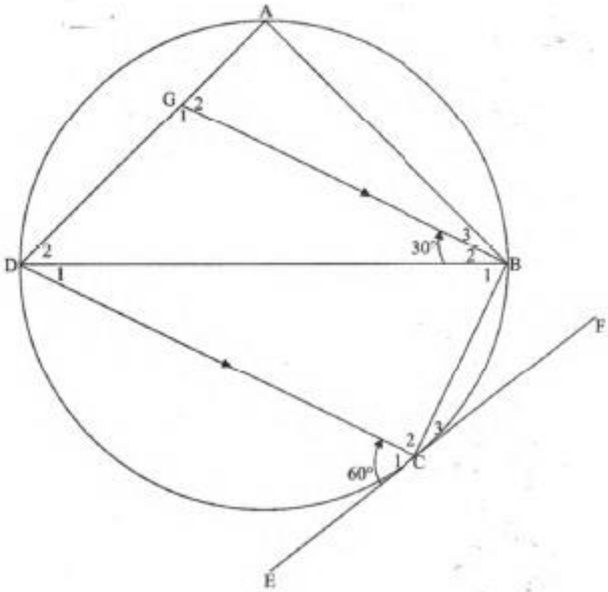
In the diagram below, ABCD is a cyclic quadrilateral. Chords AD and BC are produced to meet at F and E respectively. F and E are joined such that EF ∥ AB.



Prove that CEFD is a cyclic quadrilateral. **[5]**

QUESTION 10: FREE STATE

In the diagram, ABCD is a cyclic quadrilateral. G is a point on AD such that $BG \parallel CD$.
ECF is a tangent to the circle at C. BD is a chord of the circle. $G\hat{B}D = 30°$ and $D\hat{C}E = 60°$



10.1 Calculate, with reasons, the size of:

10.1.1 $\hat{D}_1$ (1)

10.1.2 $\hat{B}_1$ (2)

10.1.3 $\hat{C}_2$ (1)

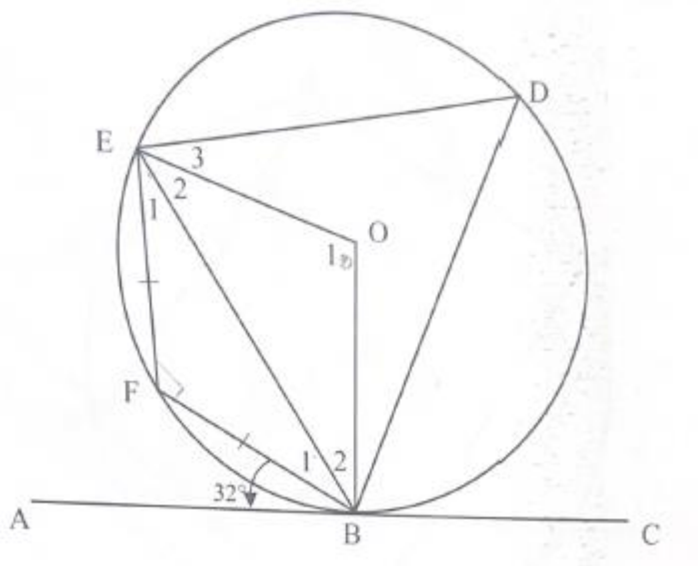
10.1.4 $D\hat{A}B$ (2)

10.2 Is BD a diameter of the circle? Motivate your answer. (2)

[8]

QUESTION 11: LIMPOPO

In the diagram, ABC is the tangent to the circle centre O at B. F, E and D are points on the circle. $EF = BF$. $A\hat{B}F = 32°$



Determine, with reasons, the sizes of the following:

11.1 $\hat{E}_1$ (2)

11.2 $\hat{F}$ (2)

11.3 $\hat{D}$ (2)

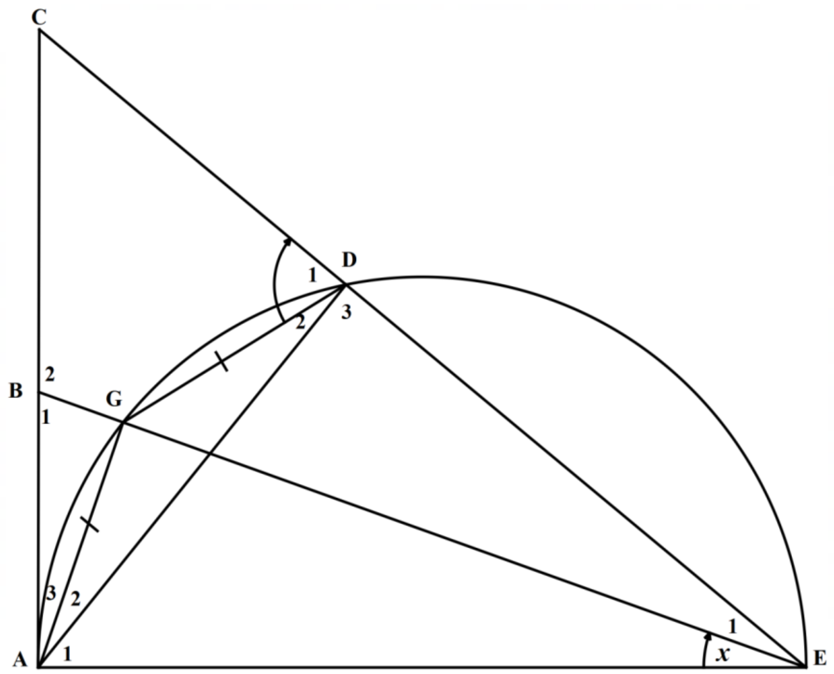
11.4 $\hat{O}_1$ (2)

11.5 $\hat{E}_2$ (2)

[10]

QUESTION 12: NORTHERN CAPE

A, G, D and E are points on a semi-circle having AE as the diameter. CA is a tangent to the semi-circle at A. ED produced meets the tangent at C. AG, GD and EG are drawn. EG produced meets the tangent at B. $AG = GD$. $A\hat{E}B = x$.



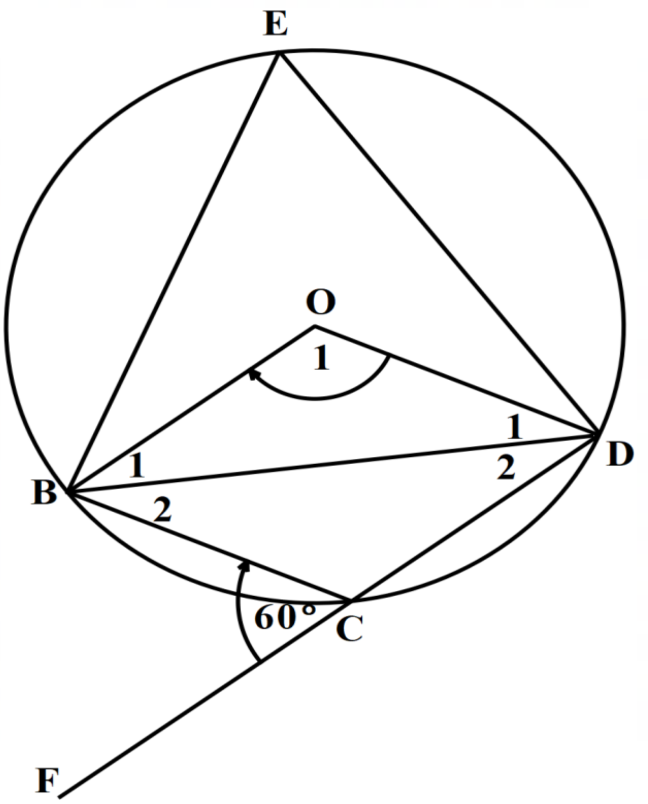
12.1 Name, with reasons, THREE other angles each equal to x. (5)

12.2 Prove that BCDG is a cyclic quadrilateral. (5)

[10]

QUESTION 13: NORTHERN CAPE

The diagram below shows a circle with centre O. BEDC is a cyclic quadrilateral. OB, OD and BD are drawn. DCF is a straight line. $B\hat{C}F = 60°$.



Determine, giving reasons, the size of the following angles:

13.1 $\hat{E}$ (2)

13.2 $\hat{O}_1$ (2)

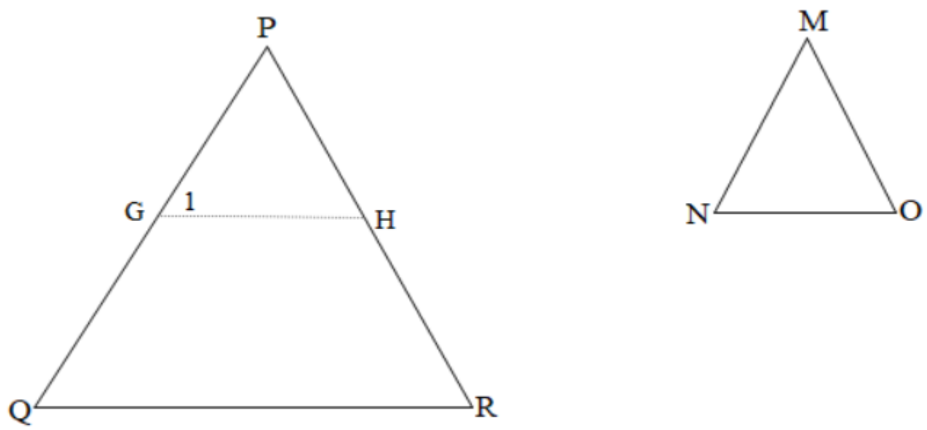
13.2 $\hat{D}_1$ (2)

[6]

Part 3: Similarity and Proportionality

QUESTION 1: EASTERN CAPE

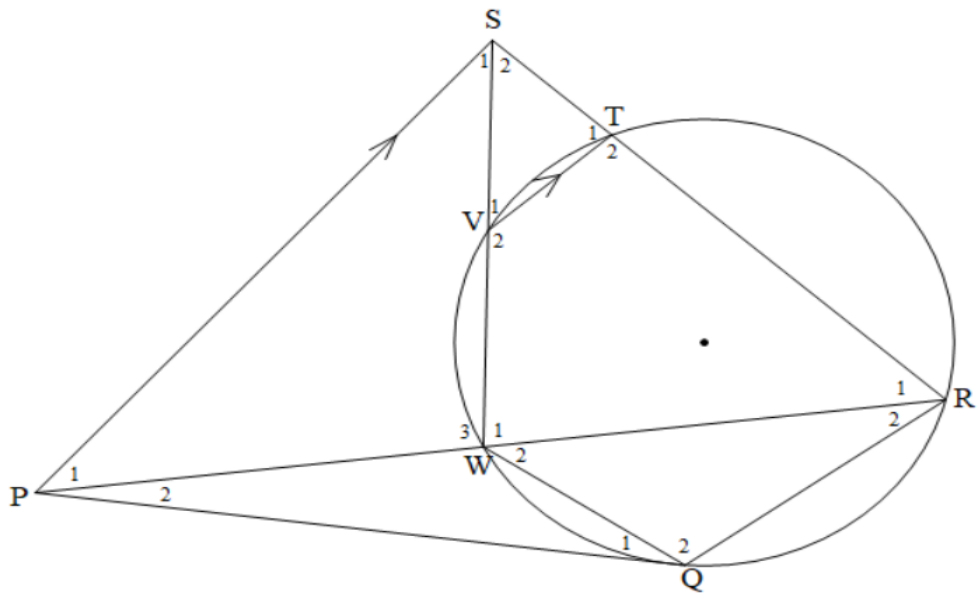
1.1 In the diagram below, ΔPQR and ΔMNO are given with $\hat{P} = \hat{M}$, $\hat{Q} = \hat{N}$ and $\hat{R} = \hat{O}$.



Use the diagram in your answer book to prove the theorem which states that:

$$\frac{MN}{PQ} = \frac{MO}{PR}$$ (6)

1.2 In the diagram below, PQ is a tangent to the circle at Q. R is a point on the circle and S lies outside the circle. PR cuts the circle in W and RS cuts the circle in T. SW cuts the circle in V. VT ∥ PS.



Prove that:

1.2.1 $\hat{S}_1 = \hat{R}_1$ (3)

1.2.2 $\Delta PWS\ |||\ \Delta PSR$ (3)

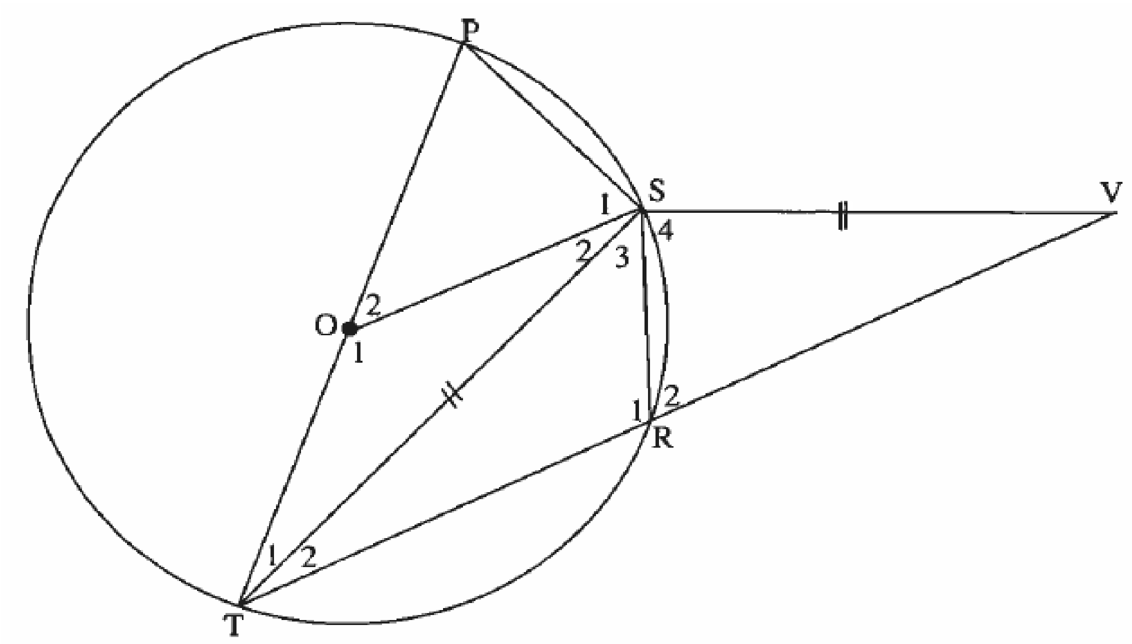
1.2.3 $PQ^2 = PW.PR$ (5)

1.2.4 $PQ = PS$ (3)

[20]

QUESTION 2: MPUMALANGA

In the diagram, O is the centre of circle PSRT. TR produced intersects SV in V. ST bisects $P\hat{T}R$ and $TS = SV$. TOP, OS and SR are joined.



2.1 Determine with reasons, the size of $P\hat{S}T$. (2)

2.2 Determine the size of $\hat{S}_4$. (5)

2.3 Prove that $\Delta TSO\ |||\ \Delta TVS$ (3)

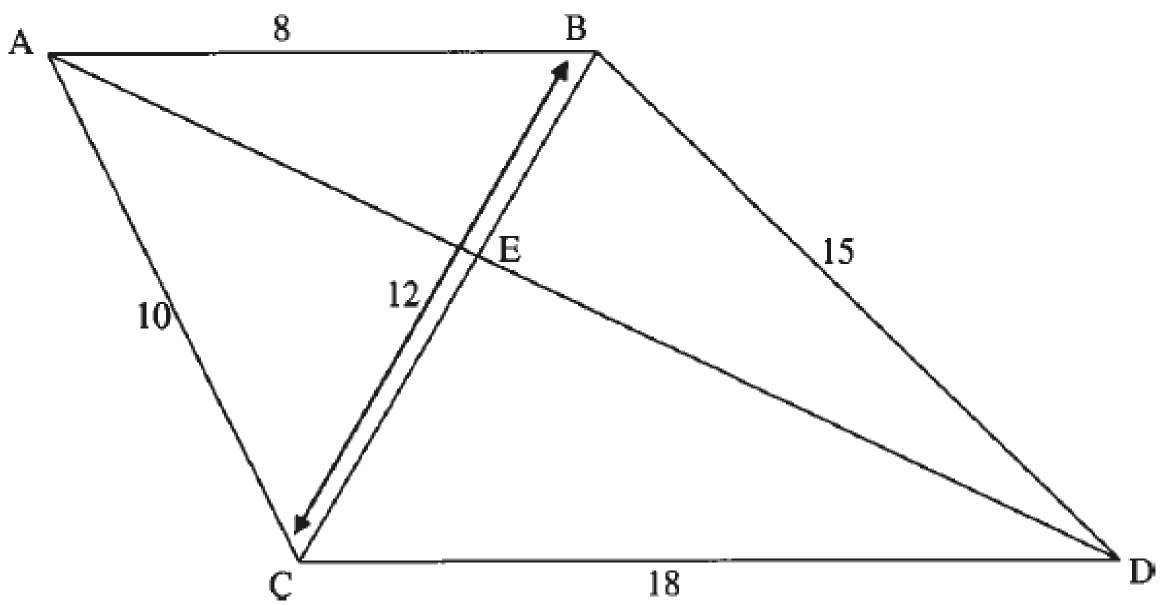
2.4 Show that $2VS^2 = PT.TV$ (4)

[14]

QUESTION 3: MPUMALANGA

In quadrilateral ABCD, diagonals AD and BC intersect at E.

$AB = 8$ cm
$BD = 15$ cm
$DC = 18$ cm
$AC = 10$ cm
$BC = 12$ cm



3.1 Prove that $\Delta BCA \,|||\, \Delta CDB$. (4)

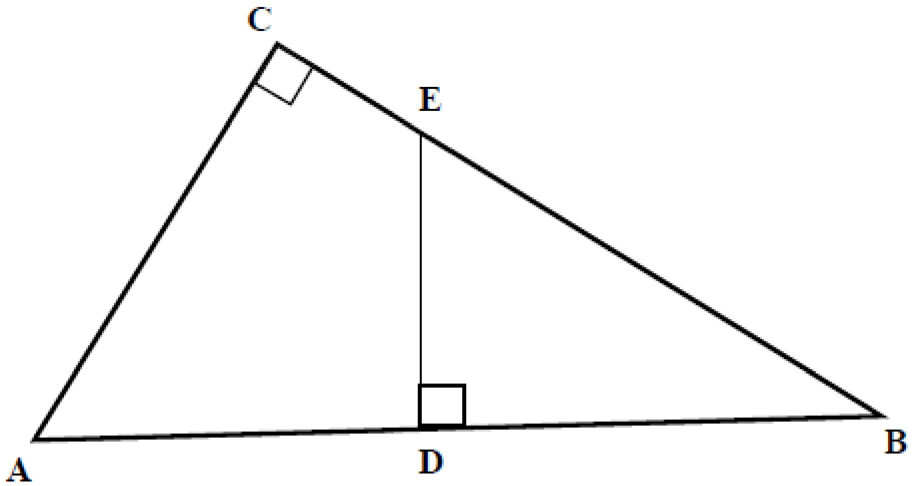
3.2 Hence, prove that $AB \parallel CD$. (2)

3.3 Calculate the length of CE. (4)

[10]

QUESTION 4: KWA-ZULU NATAL

ΔABC is right angled at C. $ED \perp AB$ with E on CB and D on AB. $AC = 4{,}8$ cm and $AB = 8$ cm. $AD = DB$.



4.1 Calculate BC, correct to 1 decimal digit. (2)

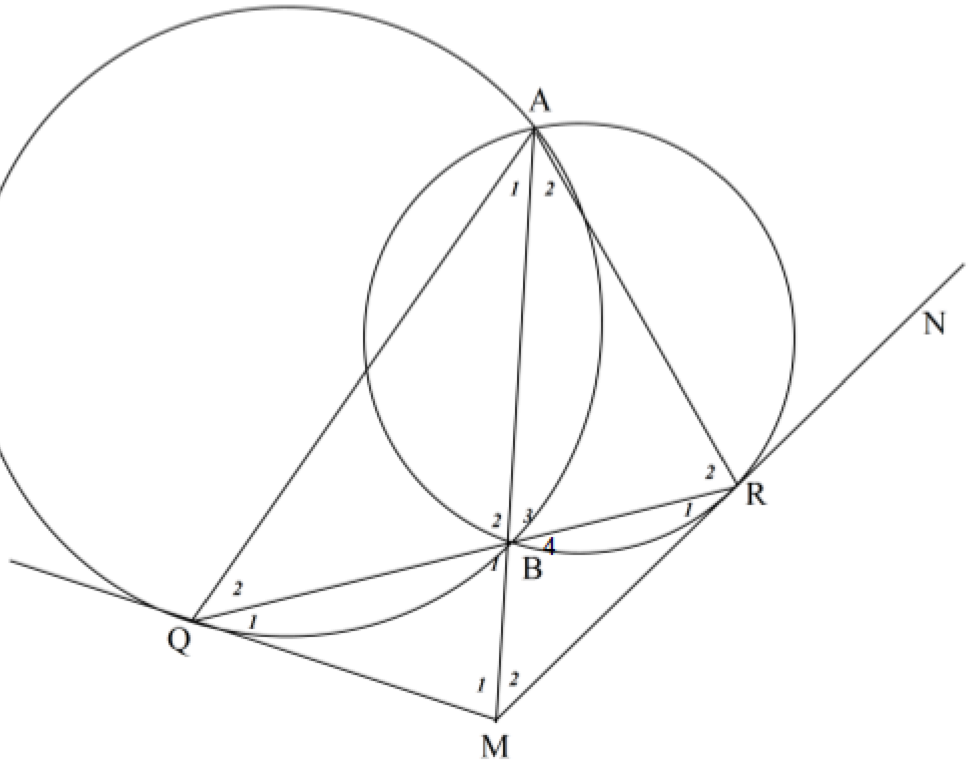
4.2 Complete: $\Delta BAC\ |||$ .... (1)

4.3 Hence, or otherwise calculate the area of ADEC. (5)

[8]

QUESTION 5: KWA-ZULU NATAL

In the figure, two circles intersect at A and B. AB produced to M bisects $Q\hat{A}R$. Tangents MQ and MR meet the circles at Q and R respectively. QBR is a straight line. AQ and AR are drawn.



Prove:

5.1 $\Delta MQA \mid\mid\mid \Delta MBQ$ (3)

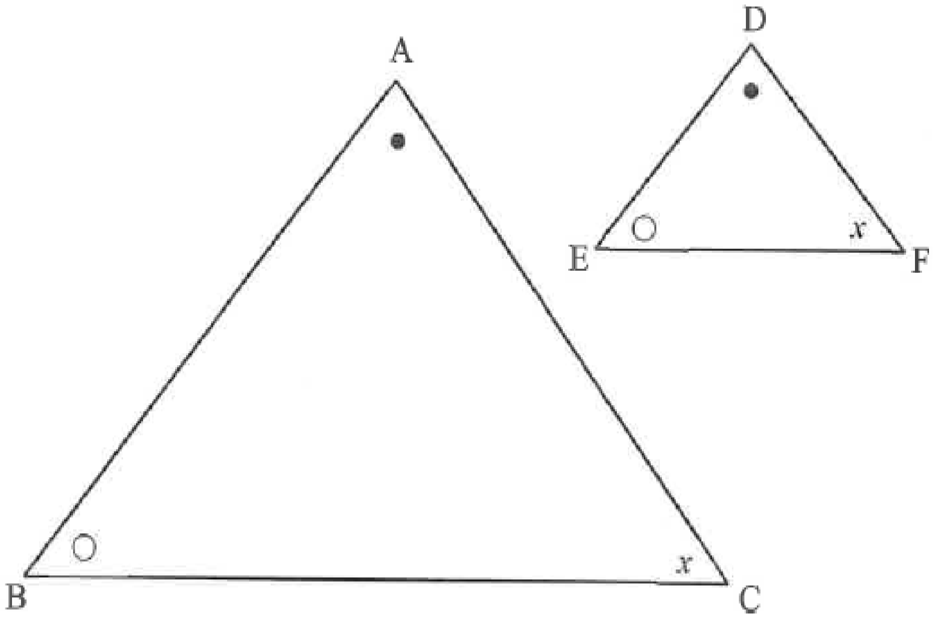
5.2 $MR^2 = AM.MB$ (5)

5.3 $\frac{MQ^2}{MR^2} = 1$ (4)

[12]

QUESTION 6: GAUTENG

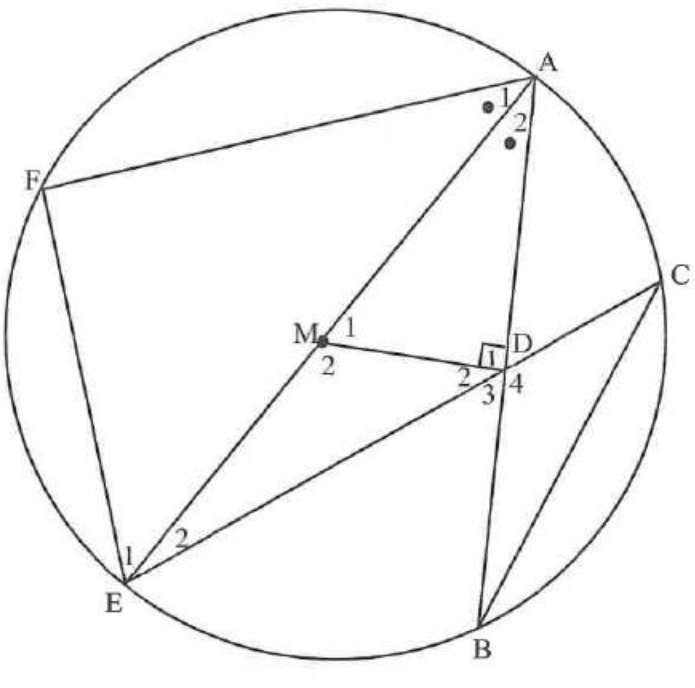
6.1 In the diagram below, ΔABC and ΔDEF are drawn, such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is:

$$\frac{AB}{DE} = \frac{AC}{DF}$$ (6)

6.2 In the diagram below, diameter EMA of a circle with centre M bisects $F\hat{A}B$. MD is perpendicular to the chord AB. ED produced meets the circle at C. CHORDS CB and FE are drawn.



6.2.1 Prove that $\Delta AEF\ |||\ \Delta AMD$ (4)

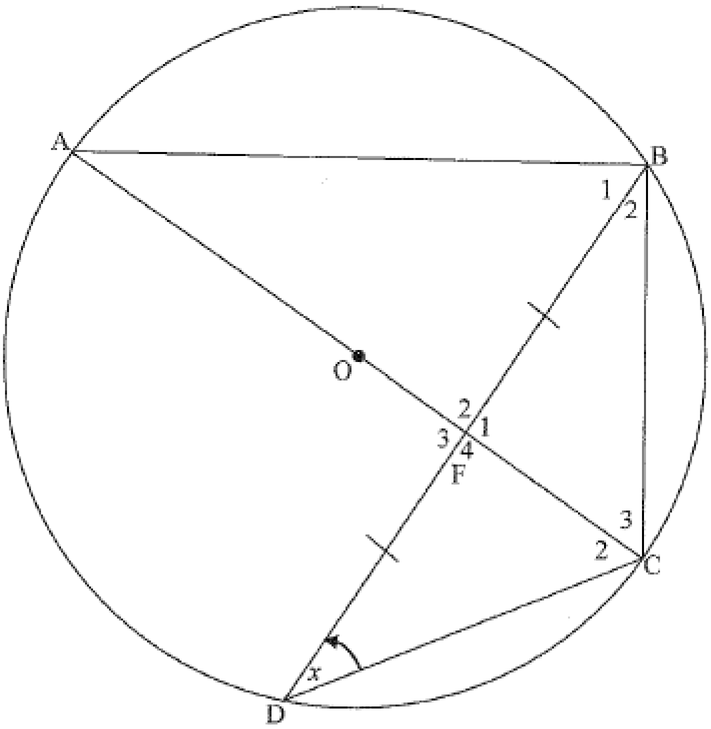
6.2.2 Determine the numerical value of: $\frac{AF}{AD}$ (3)

6.2.3 Prove that $AD^2 = CD \times DE$. (6)

[19]

QUESTION 7: NORTH WEST

In the diagram below, A, B, and D lie on the circle with centre O. AOFC and DFB are straight lines, $DF = FB$, $\hat{D} = x$.



7.1 Determine, with reasons, the size of EACH of the following in terms of x.

7.1.1 $\hat{A}$ (2)

7.1.2 $\hat{C}_3$ (3)

7.2 Prove, giving reasons, that:

7.2.1 $\hat{F}_2 = \hat{F}_3$ (2)

7.2.2 $\Delta CFB\ |||\ \Delta CBA$ (3)

7.2.3 $DC^2 = FC.AC$. (4)

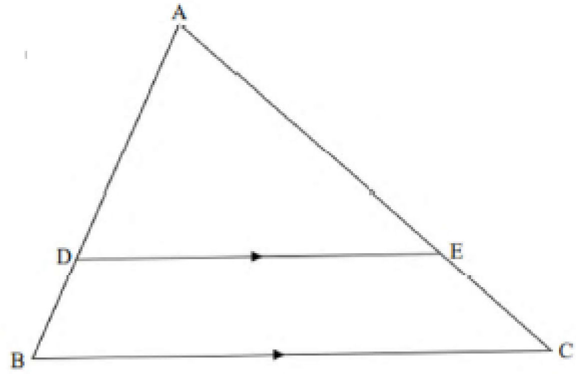
7.2.4 $\frac{FC}{AC} = \left(1 - \frac{AB}{AO + OC}\right)\left(1 + \frac{AB}{AO + OC}\right)$ (5)

[19]

QUESTION 8: FREE STATE

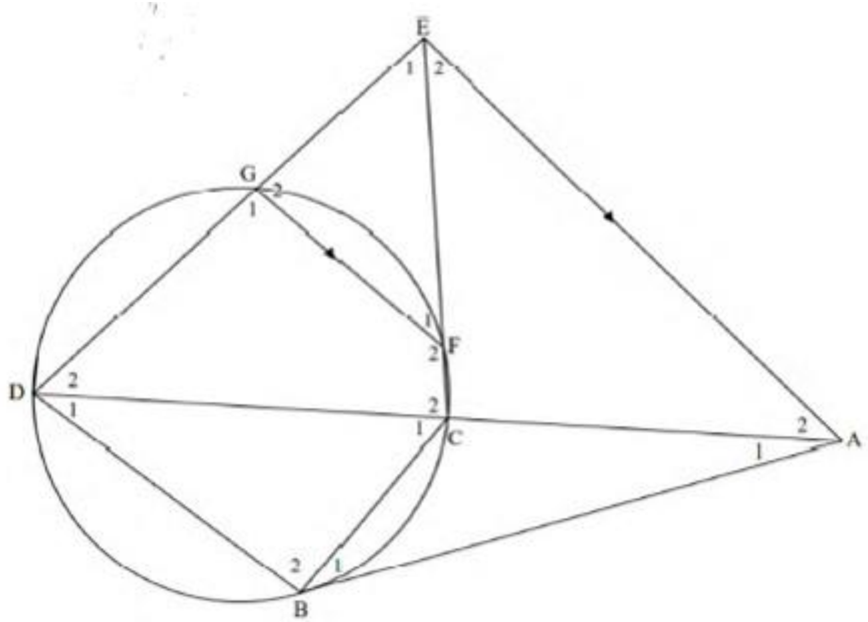
8.1 In ΔABC below, D and E are points on AB and AC respectively such that DE ∥ BC. Prove the theorem that states that:

$$\frac{AD}{DB} = \frac{AE}{EC}$$



(6)

8.2 In the diagram below, DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B. 2 chords BD and BC are drawn. DG and CF produced meet at E and DC is produced to A. EA ∥ GF.



8.2.1 Give a reason why $\hat{B}_1 = \hat{D}_1$ (1)

8.2.2 Prove that $\Delta ABC \,|||\, \Delta ADB$ (3)

8.2.3 Prove $\hat{E}_2 = \hat{D}_2$ (4)

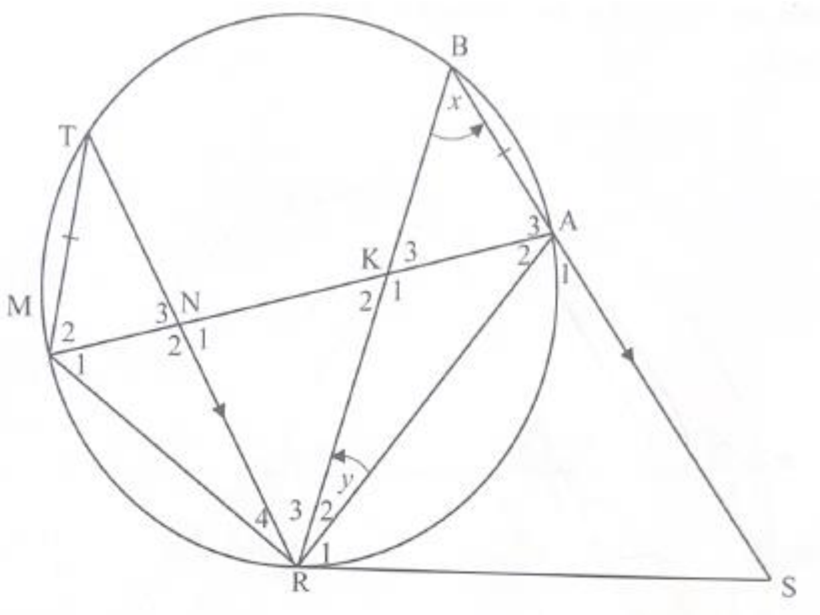
8.2.4 Prove $AE^2 = AD \times AC$ (4)

8.2.5 Hence, deduce that $AE = AB$. (3)

[21]

QUESTION 9: LIMPOPO

In the diagram, RS is a tangent to the circle at R. SAB is a line that passes through the circle and $RT \parallel SAB$. $MT = AB$.



9.1 If $A\hat{B}R = x$, write down THREE other angles in the diagram which are also equal to x. Provide reasons. (6)

9.2 If $A\hat{R}B = y$, provide a reason why $A\hat{R}B = y$? (1)

9.3 9.3.1 Write $\hat{A}_1$ in terms of x and y. (1)

9.3.2 Write $\hat{N}_1$ in terms of x and y. (1)

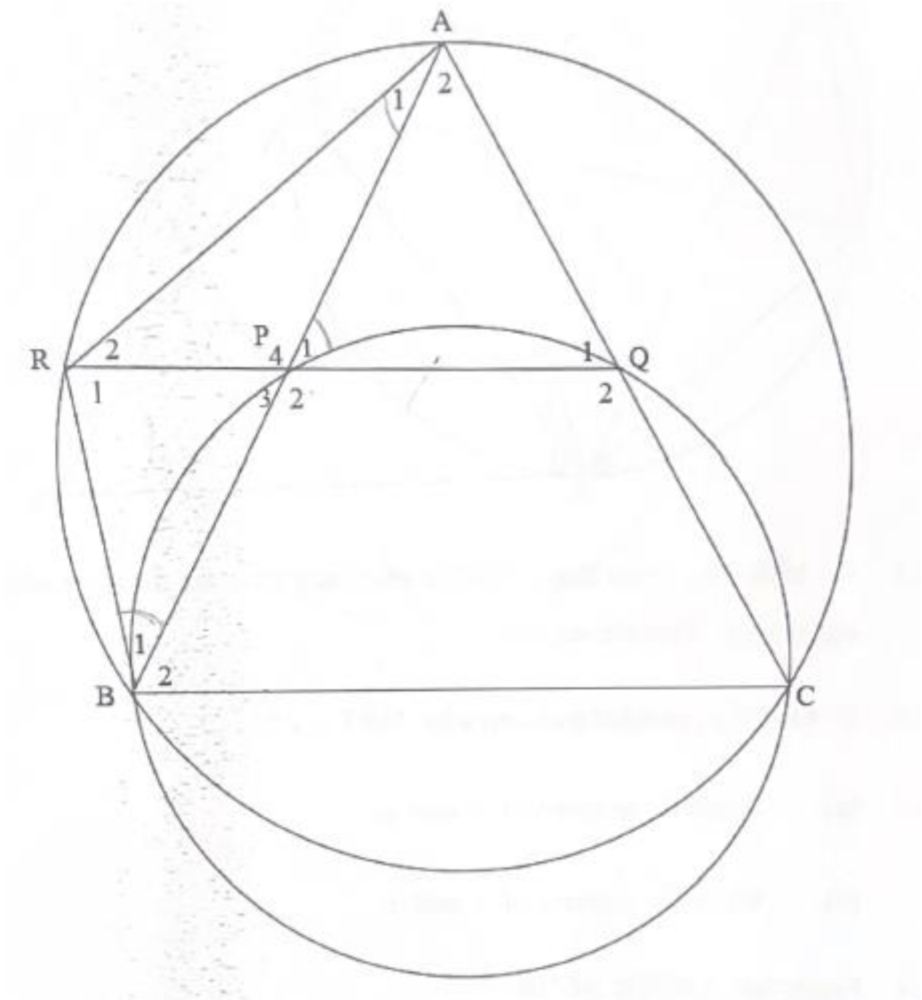
9.4 Prove that $\Delta SAR \,|||\, \Delta KNR$ (3)

9.5 Prove that SAKR is a cyclic quadrilateral. (2)

[19]

QUESTION 10: LIMPOPO

In the diagram, P is a point on side AB of ΔABC. The circle through P, B and C cuts AC at Q. QP produced cuts the circle passing through A, B and C at R.



Prove that:

10.1 $\hat{P}_1 = \hat{A}_1 + \hat{B}_1$ (5)

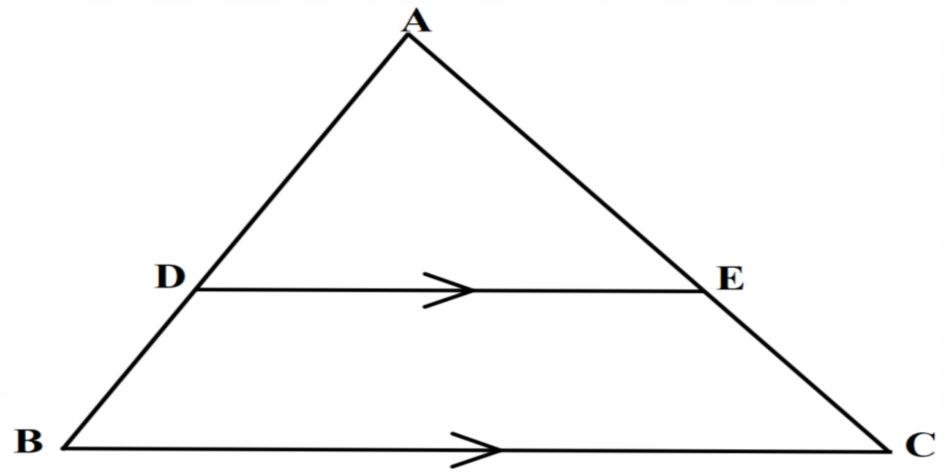
10.2 $AR^2 = AP.AB$ (5)

[10]

QUESTION 11: NORTHERN CAPE

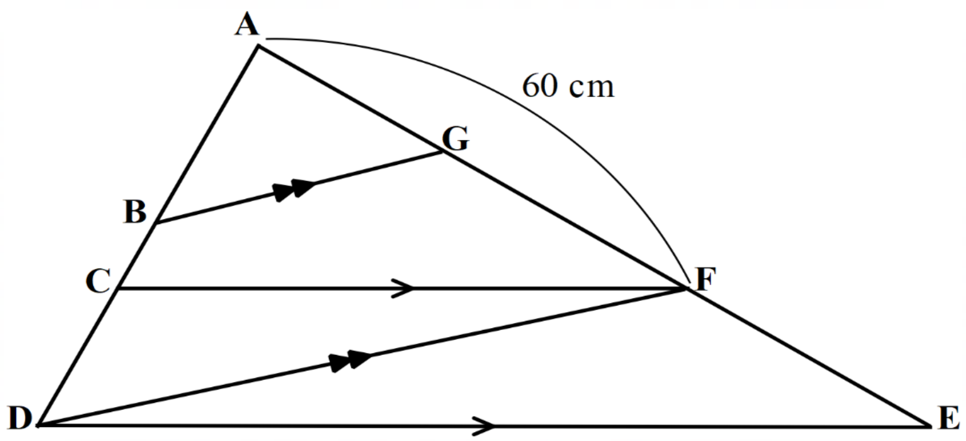
11.1 In ΔABC below, D is a point on AB and E is a point on AC such that DE $\parallel$ BC. Prove the theorem that states that:

$$\frac{AD}{DB} = \frac{AE}{EC}$$



(5)

11.2 In ΔADE, $BG \parallel DF$ and $CF \parallel DE$. $AF = 60\text{ cm}$ and $AF : FE = 3 : 2$



Determine, with reasons:

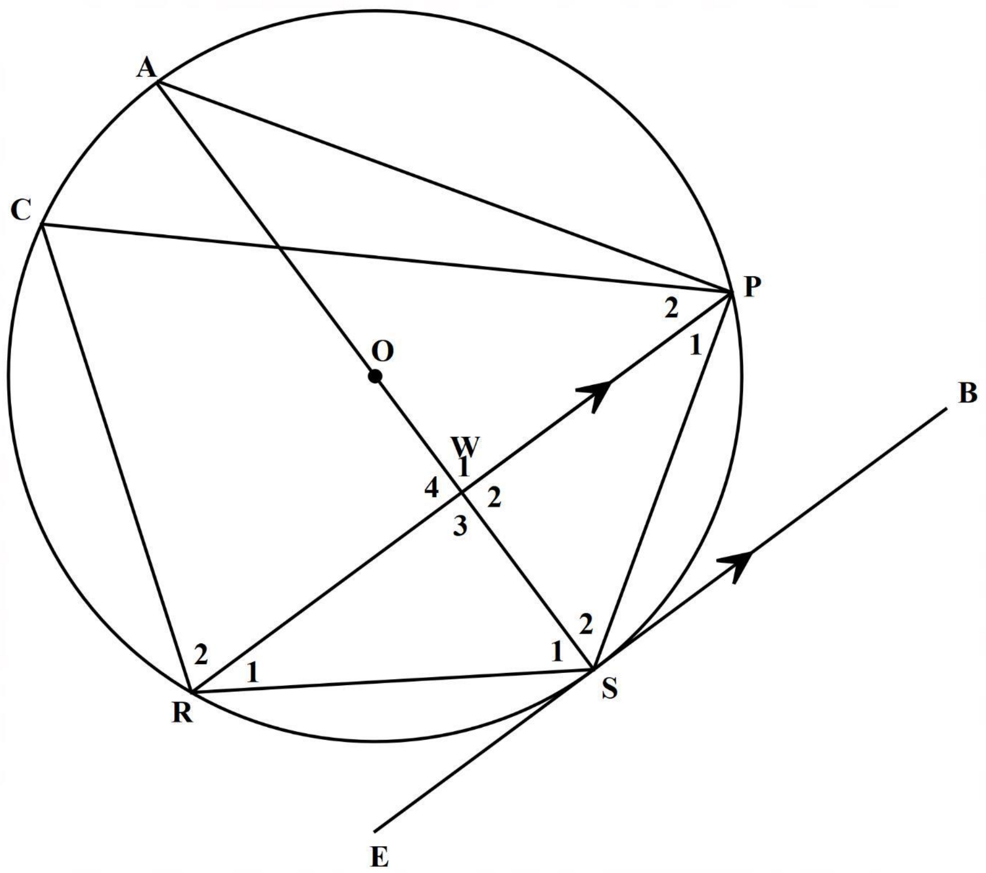
11.2.1 The length of FE. (2)

11.2.2 The value of $\frac{BC}{CD}$, if it is given that $AG : GF = 7 : 8$. (4)

[11]

QUESTION 12: NORTHERN CAPE

EB is a tangent to the circle with centre O at S. SOA is a diameter of the circle. R, C, A and P lie on the circle such that chord RP ∥ EB. Chords SR, SP, RC, PC and PA are drawn.



Prove, giving reasons, that:

12.1 $PW = WR$ (4)

12.2 $\Delta WRS \mathbin{|||} \Delta PAS$ (4)

12.3 Hence, deduce that $PS^2 = AS.WS$ (6)

[14]