DEPARTMENT OF EDUCATION

DEPARTEMENT VAN ONDERWYS

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ISEBE LEZEMFUNDO

PROVINCIAL PREPARATORY EXAMINATION/
PROVINSIALE VOORBEREIDENDE EKSAMEN

GRADE/*GRAAD* 12

MATHEMATICS/*WISKUNDE*

PAPER/*VRAESTEL* 2

SEPTEMBER 2024

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 19 pages./
Hierdie nasienriglyne bestaan uit 19 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark he crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values to solve a problem is NOT acceptable.

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
S/R	**Award a mark if statement AND reason are both correct**

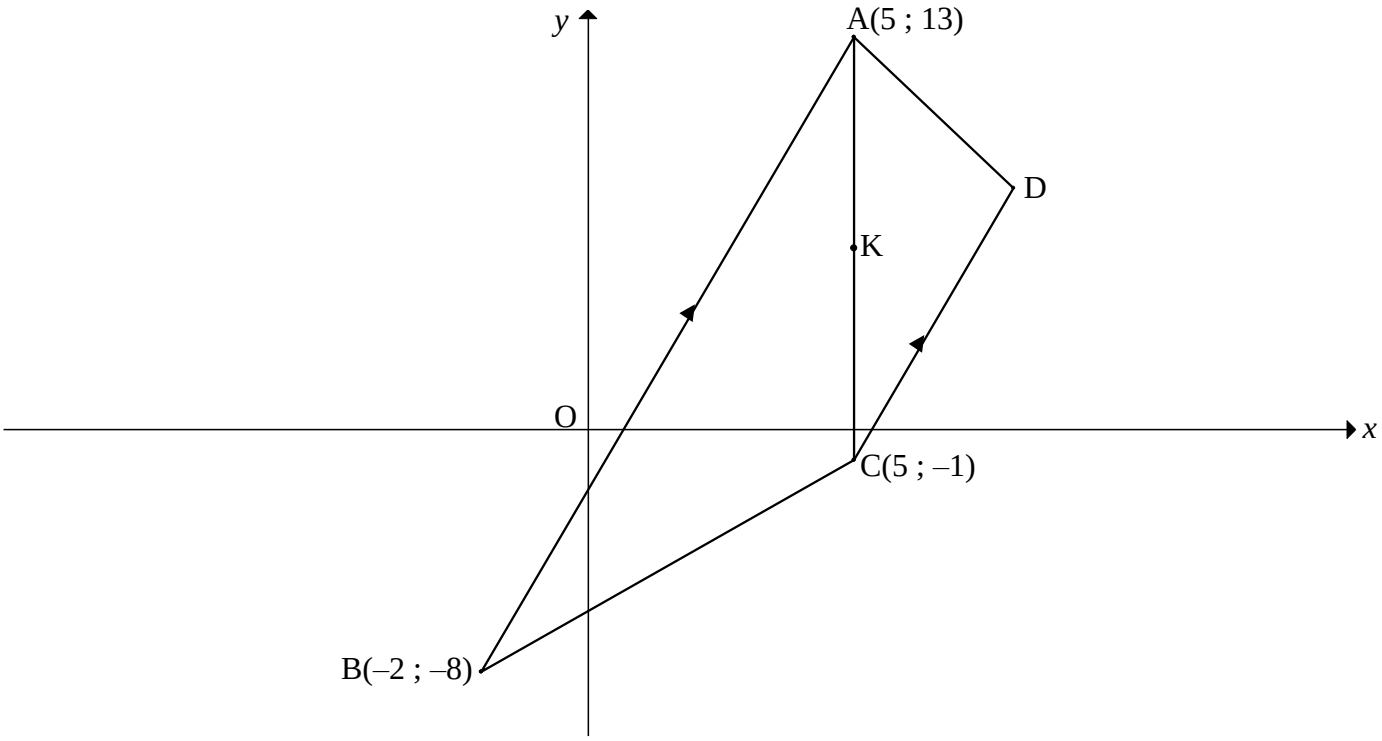
QUESTION/*VRAAG* 1

1.1	$13,96 = \frac{0 + x + 3(x+5) + 10 + 13(6) + 14(3) + 15(2) + 18(4) + 20 + 22(3)}{25}$ $13,96 = \frac{4x + 333}{25}$ $349 - 333 = 4x$ $16 = 4x$ $4 = x$ **If substitutes $x = 4$: only method mark**	✓method ✓simplify (2)
1.2	$\sigma = 5,2419\ldots$ $= 5,24$ Ignore rounding: accept 5,2 or 5,2419…	✓✓ 5,24 (2)
1.3	$13,96 + 5,24 = 19,2$ 4 players	✓ $\bar{x} + \sigma_x$ ✓answer (2)
1.4	$Q_1 = T_6 + T_7$ $\quad Q_3 = T_{19} + T_{20}$ $= \frac{10+13}{2}$ $\quad = \frac{18+18}{2}$ $= 11,5$ visits $\quad = 18$ visits IQR = $18 - 11,5 = 6,5$ visits	✓ Q1 ✓ Q3 ✓Answer (3)
1.5.1	The standard deviation remains the same.	✓Answer (1)
1.5.2	The IQR remains the same.	✓Answer (1)
		[11]

QUESTION/*VRAAG* 2

2.1	$a = 746,06$ (746,064 …) $b = -20,76$ (–20,7575 …) $\hat{y} = 746,06 - 20,76x$ Ignore rounding AO FULL MARKS	✓ $a = 746,06$ ✓ $b = -20,76$ ✓equation (3)
2.2	$r = -0,91$ (–0,9132 …)	✓ $r = -0,91$ (1)
2.3.1	As the temperature increases, the number of cups sold decreases. **OR/*OF*** As the temperature decreases, the number of cups sold increases. Must be answered in context	✓ answer (1)
2.3.2	There is a (very) **<u>strong negative</u>** linear correlation.	✓ strong negative (1)
2.4	$\hat{y} = 746,06 - 20,76(30)$ $\hat{y} = 123,26$ cups = 123 cups Ignore rounding **OR/*OF*** $\hat{y} = 123,34$ cups = 123 cups	✓substitution ✓answer ✓✓ $\hat{y} = 123,34$ cups (2)
		[8]

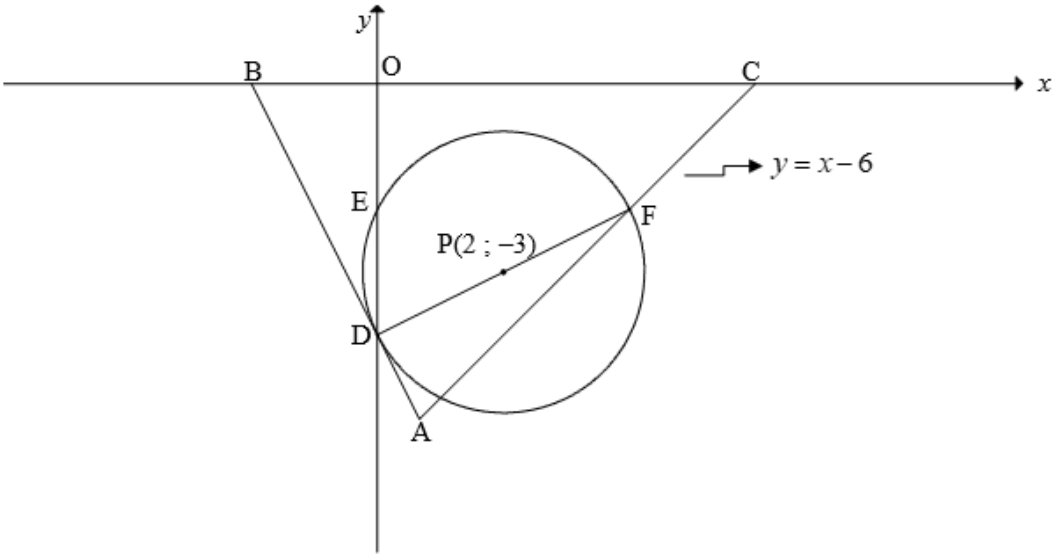
QUESTION/*VRAAG* 3



3.1	K(5 ; 6)	✓ x ✓ y (2)
3.2.1	$m_{AB} = \frac{13-(-8)}{5-(-2)}$ $= \frac{21}{7}$ $= 3$	✓substitution A & B ✓answer (2)
3.2.2	$m_{AB} = m_{CD}(AB \parallel CD)$ $3 = \frac{p+1}{p-5}$ $3p-15 = p+1$ $p = 8$	✓substitute (p ; p) and C and equating to 3.2.1 ✓answer (2)
3.3	D (8 ; 8) $A(x;y) \rightarrow D(x+3\ ;\ y-5)$ $B(-2\ ;\ -8) \rightarrow E(1\ ;\ -13)$	✓x ✓y (2)

3.4	$\perp h: 5-(-2)=7$ units base: $13-(-1)=14$ units Area $=\frac{1}{2}(14)(7)$ $=49$ units^2 OR $b^2=a^2+c^2-ac.\cos B$ $\cos B=\frac{a^2+c^2-b^2}{2ac}$ $\cos B=\frac{98+490-14^2}{2.\sqrt{98}.\sqrt{490}}$ $\hat{B}=26,57^\circ$ Area $=\frac{1}{2}\times\sqrt{98}\times\sqrt{490}\times\sin 26,57^\circ$ $=49,01$ units^2	✓height ✓base ✓area ✓angle ✓area formule ✓area (3)
3.5.1	F(12 ; –8) $d_{AB}=\sqrt{(5-(-2))^2+(13-(-8))^2}$ $=\sqrt{490}$ *or* $7\sqrt{10}$ *or* 22,135... Perimeter $=7\sqrt{10}+7\sqrt{10}+14$ $=58,27$	✓coordinates of F ✓Substitution ✓AB ✓ adding 3 sides ✓Answer (5)
3.5.2	$r^2=(-12)^2+(-8)^2$ $r^2=208$ $x^2+y^2=208$	✓ r^2 ✓equation (2)
		[18]

QUESTION/*VRAAG* 4



4.1.1	Put $x=0$: $y^2+6y+8=0$ $(y+2)(y+4)=0$ $y=-2$ or $y=-4$ E$(0;-2)$ D$(0;-4)$	✓ $y^2+6y+8=0$ ✓ –2 & –4 ✓ E and D in coordinate form (3)
4.1.2	D$(0;-4)$ P$(2;-3)$ $2=\frac{x_F+0}{2}\ \therefore 4=x_F$ $-3=\frac{y_F-4}{2}\ \therefore y_F=-2$ F (4 ; –2) **OR/*OF*** $D\to P:(x+2;\ y+1)$ $P\to F:(2+2;\ -3+1)$ $F:(4;\ -2)$	✓x ✓y ✓x ✓y (2)

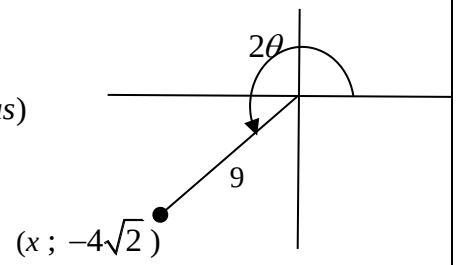
4.2	$r^2=(2+0)^2+(-3+4)^2$ $r^2=5$ $(x-2)^2+(y+3)^2=5$ **OR/*OF*** $(x^2-4x+4)+(y^2+6y+9)=4+9-8$ $(x-2)^2+(y+3)^2=5$	✓substitution ✓ r^2 ✓LHS ✓complete the square. ✓ LHS ✓r^2 (3)

4.3	$m_{DP}=\frac{-4-(-3)}{0-2}$ $=\frac{1}{2}$ $m_{AB}=-2$ (rad $\perp$ tan) $y=-2x-4$	✓substitution of P & **D** ✓ m_{DP} ✓ m_{AB} ✓equation (4)
4.4.1	$\tan\theta=m_{AB}$ $=-2$ $D\hat{B}O=63,43°$ $B\hat{D}E=90°-63,43°$ $=26,57°$ **OR** $\tan\theta=-2$ Inclination AB: $180°-63,43°=116,57°$ $B\hat{D}E=116,57°-90°$ $=26,57°$ **OR** $\tan B\hat{D}E=\frac{1}{2}$ $B\hat{D}E=26,57°$	✓ $\tan\theta=-2$ ✓ $D\hat{B}O$ ✓ answer ✓ $\tan\theta=-2$ ✓ inclination angle ✓ answer ✓ $\tan B\hat{D}E$ ✓ $\frac{1}{2}$ ✓ answer (3)

4.4.2	$-2x-4=x-6$ $2=3x$ $\frac{2}{3}=x$ $y=\frac{2}{3}-6$ $y=\frac{-16}{3}$ or $-5\frac{1}{3}$ $\text{A}\left(\frac{2}{3}\ ;\ -\frac{16}{3}\right)$	✓equating ✓ x ✓substitution ✓ y (4)
4.5	centre: $(2\ ;\ -3)$ and $r=\sqrt{5}$ moving centre $(x+k;\ -3)$ $2+\sqrt{5}=4,24\ \left(x\text{-value on the right}\right)$ $2-\sqrt{5}=-0,24$ (x-value on the left) $k>\sqrt{5}-2$ *OR* $k<-(\sqrt{5}+2)$ $k>0,24$ $k<-2-\sqrt{5}$ $k<-4,24$	✓ x value ✓ x value ✓ $k>0,24$ ✓ $k<-4,24$ (4)
		[23]

QUESTION/*VRAAG* 5

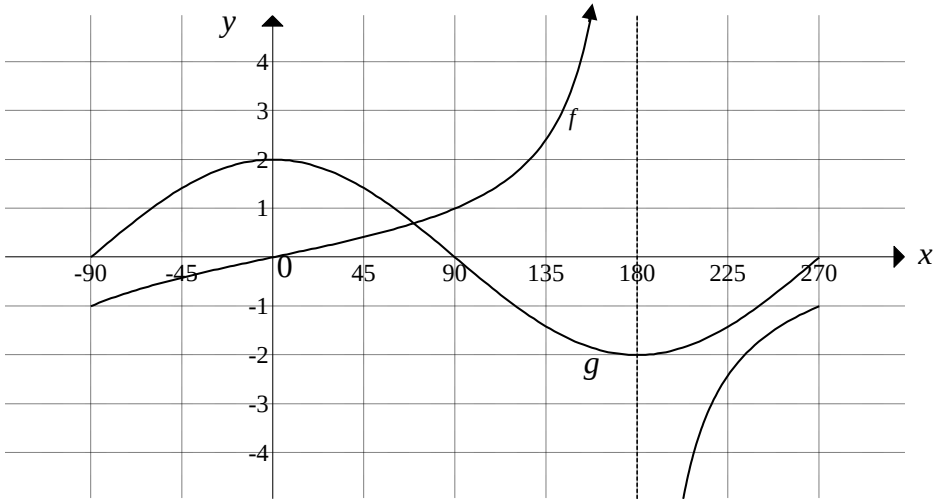
5.1	$\sin(90°-x).\tan(360°-x)-2\sin(180°+x)$ $=\cos x(-\tan x)-2(-\sin x)$ $=\frac{-\cos x.\sin x}{\cos x}+2\sin x$ $=\sin x$	✓ $\cos x$ ✓ $-\tan x$ ✓ $-\sin x$ ✓tan identity ✓answer (5)
5.2	$\sin(23°+x)\cos(7°-x)+\cos(23°+x)\sin(7°-x)$ $=\sin(23°+x+7°-x)$ $=\sin 30°$ $=\frac{1}{2}$	✓compound angle ✓sin 30° ✓answer (3)

5.3.1	$\sin 2\theta = \frac{-4\sqrt{2}}{9}$ and $2\theta \in [90°; 270°]$  $x^2 = 9^2 - (-4\sqrt{2})^2$ (*Pythagoras*) $x^2 = 81 - 32$ $x^2 = 49$ $x = -7$ $\therefore \cos 2\theta = \frac{-7}{9}$	✓using Pythagoras ✓ $x = -7$ ✓answer (3)
5.3.2	$\cos 2\theta = 1 - 2\sin^2\theta$ $-\frac{7}{9} = 1 - 2\sin^2\theta$ $2\sin^2\theta = 1 + \frac{7}{9}$ $\sin^2\theta = \frac{8}{9}$ $\sin\theta = \frac{\sqrt{8}}{3}$ $[\theta \in (45° ; 135°)]$	✓ identity ✓ substitute value of $\cos 2\theta$ ✓ simplification ✓ answer in + surd form (4)

5.4	$\sin 2\alpha = 2(\cos^2\alpha - \sin^2\alpha)$ $= 2\cos 2\alpha$ $\tan 2\alpha = 2$ $2\alpha = 63,43° + 180°k$ $\alpha = 31,72° + 90°k;\ k \in Z$ **OR** $\sin 2\alpha = 2(\cos^2\alpha - \sin^2\alpha)$ $= 2\cos 2\alpha$ $\tan 2\alpha = 2$ $2\alpha = 63,43° + 360°k$ *or* $243,43° + 360°k$ $\alpha = 31,72° + 180°k;\ k \in Z$ *or* $\alpha = 121,72° + 180°k;\ k \in Z$	✓ factorise ✓ double identity ✓ double identity ✓ $\tan 2\alpha = 2$ ✓ solution 2α ✓31,72° ✓general solution period ✓ factorise ✓ double identity ✓ double identity ✓ $\tan 2\alpha = 2$ ✓ solution 2α ✓31,72° & 121,72° ✓general solution period (7)

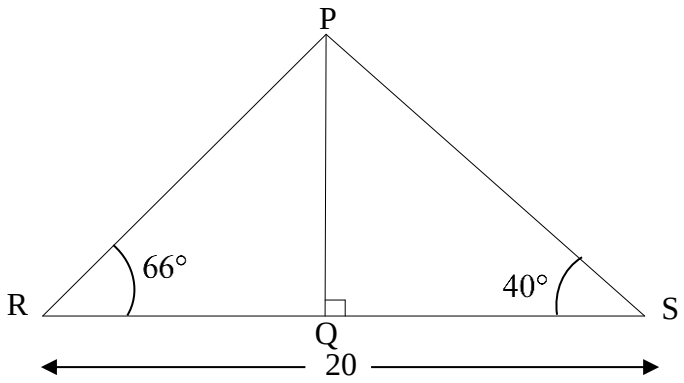
5.5.1	$LHS = \dfrac{\cos 2x + \cos^2 x + 3\sin^2 x}{2(1-\sin^2 x)}$ $= \dfrac{2\cos^2 x - 1 + \cos^2 x + 3(1-\cos^2 x)}{2\cos^2 x}$ $= \dfrac{3\cos^2 x - 1 + 3 - 3\cos^2 x}{2\cos^2 x}$ $= \dfrac{2}{2\cos^2 x}$ $= \dfrac{1}{\cos^2 x} = RHS$	✓ compound identity ✓ identity numerator ✓ identity denominator ✓ simplification (4)
5.5.2	$\cos^2 x = 0$ $x = 90° + 180°k, k \in Z$ $\therefore x = 90°$ or $x = 270°$	✓ 90° ✓ 270° (2)
		[28]

QUESTION/*VRAAG* 6

6.1	360°	✓360° (1)
6.2	$x = 180°$	✓ $x = 180°$ (1)
6.3		✓ *x* intercepts ✓ *y* intercept ✓ (180° ; –2) (3)
6.4	$x = –90°$; 270;° 45°; 225	✓Answer (any 2) (1)
6.5	$–5 \le y \le –1$ **OR/*OF*** $y \in [–5 ; –1]$	✓ critical values ✓answer (2)
6.6	$[5 – 2\sin(90° – x)]^2$ $= [5 – 2\cos x]^2$ $= [5 – (–2)]^2$ $= 49$ AO FULL MARKS	✓co-function ✓minimum value ✓answer (3)
		[11]

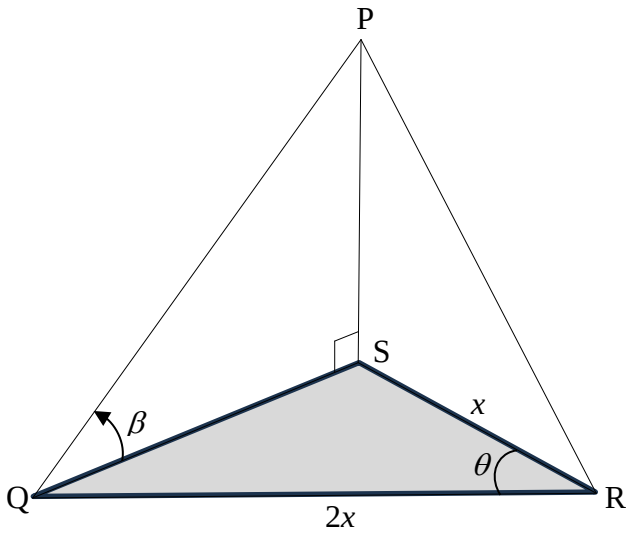
QUESTION/*VRAAG* 7

7.1



7.1.1	$R\hat{P}S = 74°$ (Sum on the int ∠'s of Δ) / (Som v binne∠e v Δ) In ΔPRS: $\frac{PS}{\sin 66°} = \frac{RS}{\sin 74°}$ $PS = \frac{20.\sin 66°}{\sin 74°}$ $PS = 19,01$ units	✓S ✓substitution into sine rule ✓answer (3)
7.1.2	In ΔPQS: $\frac{PQ}{PS} = \sin 40°$ $PQ = 19,01(\sin 40°)$ $PQ = 12,22$ units	✓ ratio ✓answer (2)

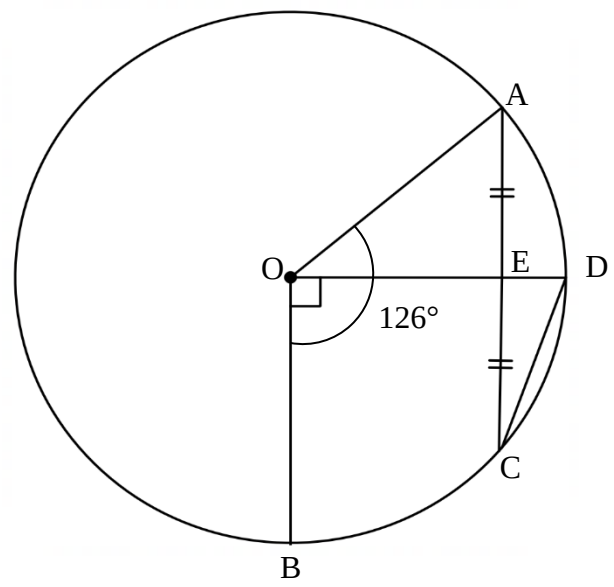
7.2



7.2.1	In ΔQRS: $QS^2 = x^2 + (2x)^2 - 2(x)(2x)\cos\theta$ $QS^2 = x^2 + 4x^2 - 4x^2\cos\theta$ $QS^2 = 5x^2 - 4x^2\cos\theta$ $QS = \sqrt{x^2(5-4\cos\theta)}$ $QS = x\sqrt{5-4\cos\theta}$ In ΔPQS: $\cos\beta = \frac{QS}{PQ}$ $PQ = \frac{QS}{\cos\beta}$ $PQ = \frac{x\sqrt{5-4\cos\theta}}{\cos\beta}$	✓substitution into cosine rule ✓simplification ✓ correct ratio ✓substitution of QS and PQ as subject (4)
7.2.2	$57,36 = \frac{1}{2}(x)(2x)\sin 35°$ $100,004... = x^2$ $10 = x$	✓substitution into area rule ✓answer (2)
		[11]

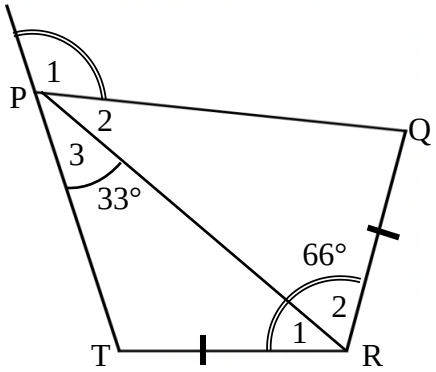
QUESTION/*VRAAG* 8

8.1



8.1.1	$A\hat{O}E = 126° - 90°$ $= 36°$ $O\hat{E}C = 90°$ (line from centre to midpoint of chord / *lyn van midpt v sirkel na middelpunt v koord*) $O\hat{A}C = 180° - 90° - 36°$ ($\angle$s of Δ) $= 54°$	✓S ✓S ✓R ✓S (4)
8.1.2	$A\hat{C}D = 18°$ (Angle at the centre = 2 × angle at circumference / *Middelpts hoek = 2 × omtrekshoek*)	✓S ✓R (2)

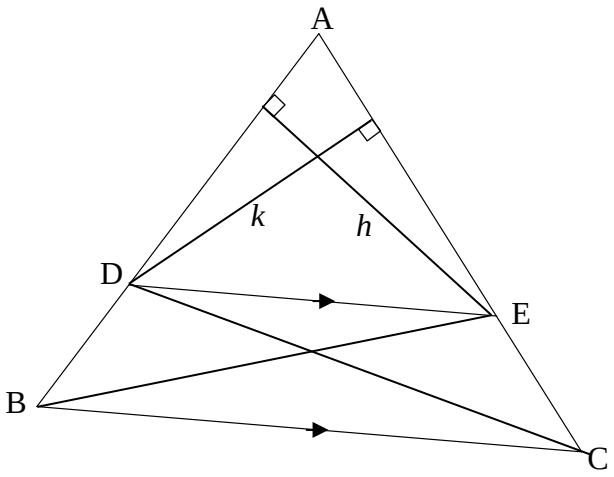
8.2



	$\hat{P}_1 = \hat{R}$ (given/*gegee*) $\therefore$PQRT = cyclic quadrilateral/*koordevh* (ext. $\angle$= opp interior $\angle$ / *buite* $\angle$ = *teenoorst binne*$\angle$) OR (Converse ext $\angle$ cyclic quad) $\hat{P}_2 = \hat{P}_3 = 33°$ (= chords; =$\angle$s / =*koorde*; = $\angle$*e*) $\hat{Q} = 180° - (66° + 33°)$ ($\angle$s of Δ) $= 81°$ $\hat{T} = 99°$ (ext $\angle$ of ΔPTR / *buite* $\angle$ *van* Δ*PTR* **OR**/***OF*** opp$\angle$s cyc quad / *teenoorst* $\angle$*e van koordevh*)	✓S PQRT = cyclic quad ✓R ✓S ✓R ✓S ✓S/R (6)
		[12]

QUESTION/*VRAAG* 9

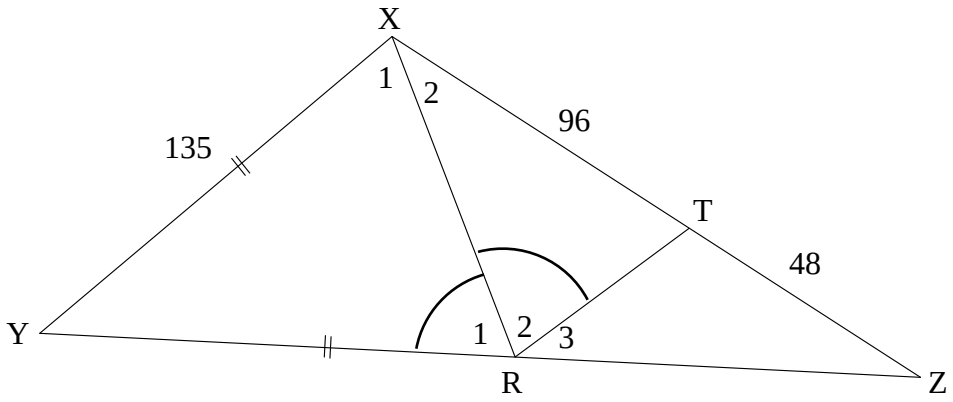
9.1



NOTE: No construction (0 /5)

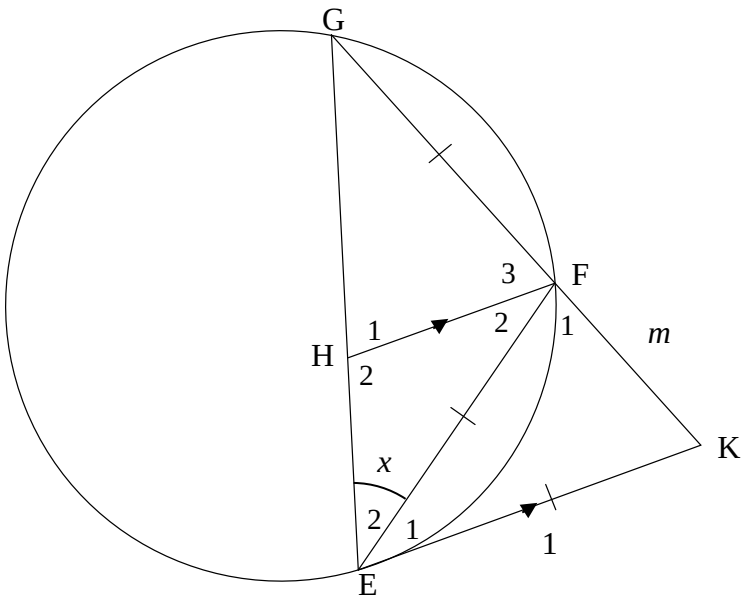
9.1	Constr: Draw DC and BE. Draw Perpendicular height (*h*) from E on AD and perpendicular height (*k)* from D on AE. *Verbind DC en BE. Trek loodregte hoogte (h) van E op AD en loodregte hoogte (k) van D op AE* PROOF/*BEWYS*: $\frac{\text{Area } \Delta ADE}{\text{Area } \Delta BDE} = \frac{\frac{1}{2}AD.h}{\frac{1}{2}DB.h} = \frac{AD}{DB}$ (same base and height) (*Selfde basis en loodregte hoogte)* $\frac{\text{Area } \Delta ADE}{\text{Area } \Delta CDE} = \frac{\frac{1}{2}AE.k}{\frac{1}{2}EC.k} = \frac{AE}{EC}$ (same base and height) Area ΔBDE = Area ΔCDE (Same base, same height/ Dieselfde basis en hoogte) $\therefore \frac{\text{Area } \Delta ADE}{\text{Area } \Delta BDE} = \frac{\text{Area } \Delta ADE}{\text{Area } \Delta CDE}$ $\therefore \frac{AD}{BD} = \frac{AE}{EC}$	✓construction (On diagram or in words) ✓ both S ✓R ✓S /R ✓S (5)

9.2



9.2.1	$\hat{R}_1 = \hat{X}_1$ (∠s opp = sides/*∠e teenoor = sye*) $\therefore\ \hat{R}_2 = \hat{X}_1$ $\therefore$ RT || YX (Alt ∠s = /*Verwisselende ∠'e* =)	✓S/R ✓S ✓R (3)
9.2.2	$\frac{ZR}{RY} = \frac{ZT}{TX}$ (line || one side Δ/*lyn* || *een sy* Δ) OR (Proportionality th, RT || YX) $\frac{RZ}{135} = \frac{48}{96}$ $RZ = 67,5cm$	✓S/R ✓S ✓answer (3)
		[11]

QUESTION/*VRAAG* 10



10.1	$\hat{G} = x$ (∠s opp = sides/∠e teenoor = sye) $\hat{E}_1 = x$ (Tan-chord Th / Raaklyn-koord st) $\hat{F}_2 = x$ (Alt ∠'s, HF $\|\|$ EK / Verw ∠e, HF $\|\|$ EK)	✓ S/R ✓ S ✓R ✓ S/R (4)
10.2	$\hat{K} = 180 - 3x$ OR $\hat{K} = 2x$ OR $\hat{K} = 90 - \frac{1}{2}x$	✓ S (1)
10.3	$\hat{F}_1 = 180° - 3x$ (∠s opp = sides/*∠e teenoor = sye*) $x + 180° - 3x + 180° - 3x = 180°$ (∠s of Δ) $-5x = -180°$ $x = 36°$ **OR/*OF*** $\hat{F}_1 = 180° - 3x$ (∠s opp = sides/*∠e teenoor = sye*) $180° - 3x = 2x$ (ext∠ of Δ) $-5x = -180°$ $x = 36°$	✓ S ✓ S/R ✓Answer (3) ✓ S ✓ S/R ✓Answer (3)

10.4.1	In ΔKEF and ΔKGE: 1. $\hat{K} = \hat{K}$ (common) 2 $K\hat{E}F = \hat{G} = x$ (proven/*bewys*) $\therefore \Delta KGE \,|||\, \Delta KEF \quad (\angle;\angle;\angle)$ **OR/*OF*** In ΔKEF and ΔKGE : 1. $\hat{K} = \hat{K}$ (common) 2. $K\hat{E}F = \hat{G} = x$ (proven/*bewys*) 3. $\hat{F}_1 = G\hat{E}K = 2x$ (proven/*bewys*) $\therefore \Delta KGE \,|||\, \Delta KEF$ (AAA)	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.4.2	$\frac{KG}{KE} = \frac{KE}{KF}$ ($\Delta KGE \,|||\, \Delta KEF$) $\frac{1}{m} = \frac{m+1}{1}$ (KE = GF) $m^2 + m = 1$ $m^2 + m - 1 = 0$	✓S ✓S (2)
10.5	$\frac{GH}{HE} = \frac{GF}{FK}$ (line ‖ side of Δ/*lyn* ‖ *sy van* Δ) $\frac{2}{HE} = \frac{1}{m}$ $HE = 2m$ $m^2 + m - 1 = 0$ $m = \frac{-1 \pm \sqrt{5}}{2}$ $= 0,62$ $\therefore$ HE = 1,24 units **OR/*OF*** $\frac{GH}{HE} = \frac{GF}{FK}$ (line ‖ side of Δ/*lyn* ‖ *sy van* Δ) $\frac{2}{HE} = \frac{1}{m}$ $HE = 2m$ In ΔFEK: $m^2 = (1)^2 + (1)^2 - 2(1)(1)\cos 36°$ $= 0,381...$ $m = 0,62$ $\therefore$ HE = 1,24 units	✓S/R ✓HE = 2*m* ✓*m* ✓Answer (4) ✓S/R ✓HE = 2*m* ✓*m* ✓Answer (4)
		[17]

TOTAL/*TOTAAL*: 150