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PROVINSIALE VOORBEREIDENDE EKSAMEN/
PROVINCIAL PREPARATORY EXAMINATION

GRAAD/GRADE 12

WISKUNDE/MATHEMATICS

VRAESTEL/PAPER 2

SEPTEMBER 2024

PUNTE/MARKS: 150

TYD/TIME: 3 uur/hours

Hierdie vraestel bestaan uit 12 bladsye, 1 inligtingsblad en 'n antwoordeboek van 23 bladsye./
This question paper consists of 12 pages, 1 information sheet and an answer book of 23 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

Twenty five rugby players were asked about the number of times they visited the gymnasium during the December festive season. The responses were as follows:

Number of gym visits	**Number of players**
0	1
x	1
$x + 5$	3
10	1
13	6
14	3
15	2
18	4
20	1
22	3

1.1 The mean of gym visits is 13,96. Use calculations to show that $x = 4$. (2)

1.2 Calculate the standard deviation of the data. (2)

1.3 How many players visited the gym more than one standard deviation above the mean? (2)

1.4 Determine the interquartile range of the data. (3)

1.5 During January, each player increased his number of gym visits with k visits. What impact will this have on the:

1.5.1 standard deviation? (1)

1.5.2 interquartile range? (1)

[11]

QUESTION 2

A restaurant situated on the beach front wants to predict how many cups of coffee will be sold on a given day, depending on the temperature. They collected the following data over 11 days:

Temperature in °C (*x*)	19	21	16	23	26	25	28	19	27	22	35
Cups of coffee sold (*y*)	381	370	400	161	173	184	165	398	182	310	65

2.1 Determine the equation of the least squares regression line for this data. (3)

2.2 Write down the correlation coefficient. (1)

2.3 Hence, comment on the:

2.3.1 trend of the data (1)

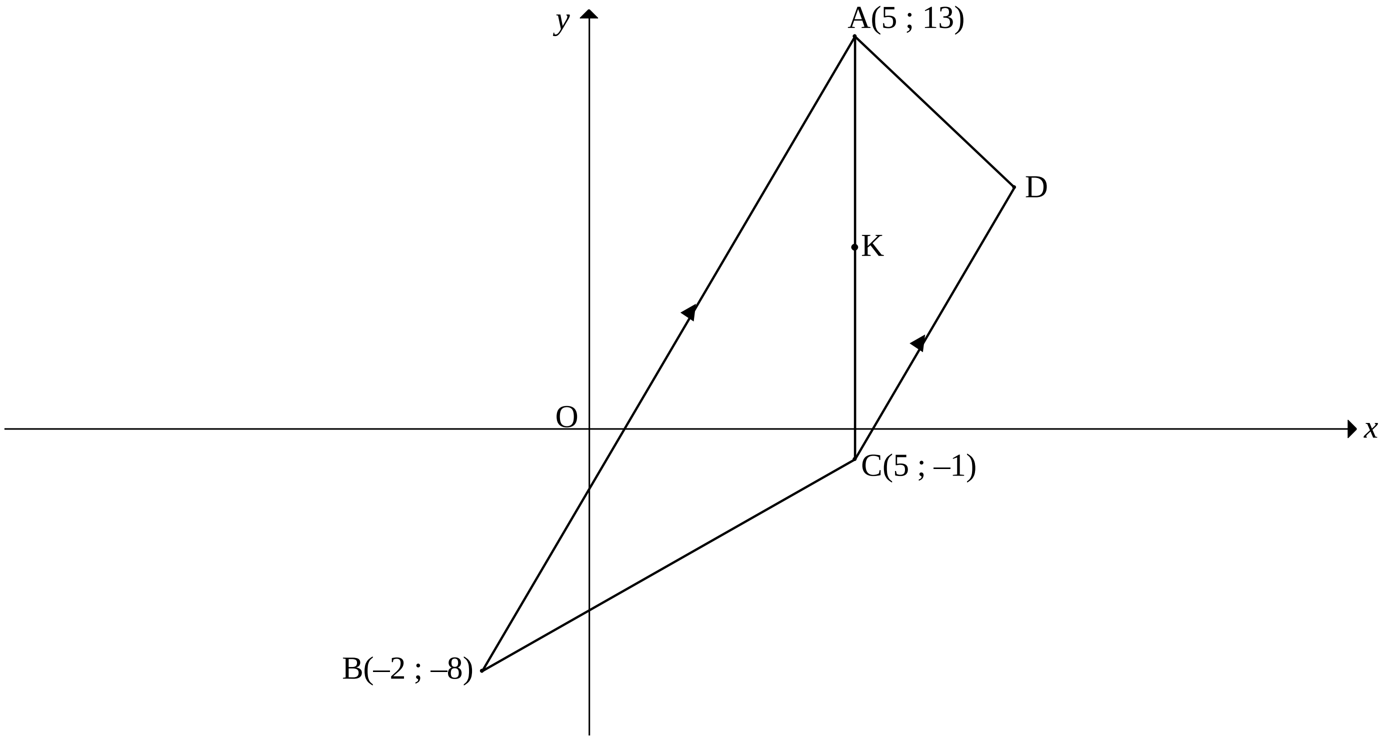
2.3.2 strength of the relationship between the sales and the temperature of the day (1)

2.4 Predict the number of cups of coffee the restaurant will most probably sell if the day temperature is 30°C. (2)

[8]

QUESTION 3

ABCD is a trapezium with vertices A(5 ; 13), B(–2 ; –8) ; C(5 ; –1) and D.
K is the midpoint of AC and BA || CD.



3.1 Write down the coordinates of K. (2)

3.2 Calculate:

3.2.1 The gradient of AB (2)

3.2.2 p, if D(p ; p) (2)

3.3 ABED is a parallelogram with E a point in the fourth quadrant.
Calculate the coordinates of E. (2)

3.4 Calculate the area of obtuse ΔABC. (3)

3.5 ΔABC is reflected in the line $x = 5$ to form ΔAFC.

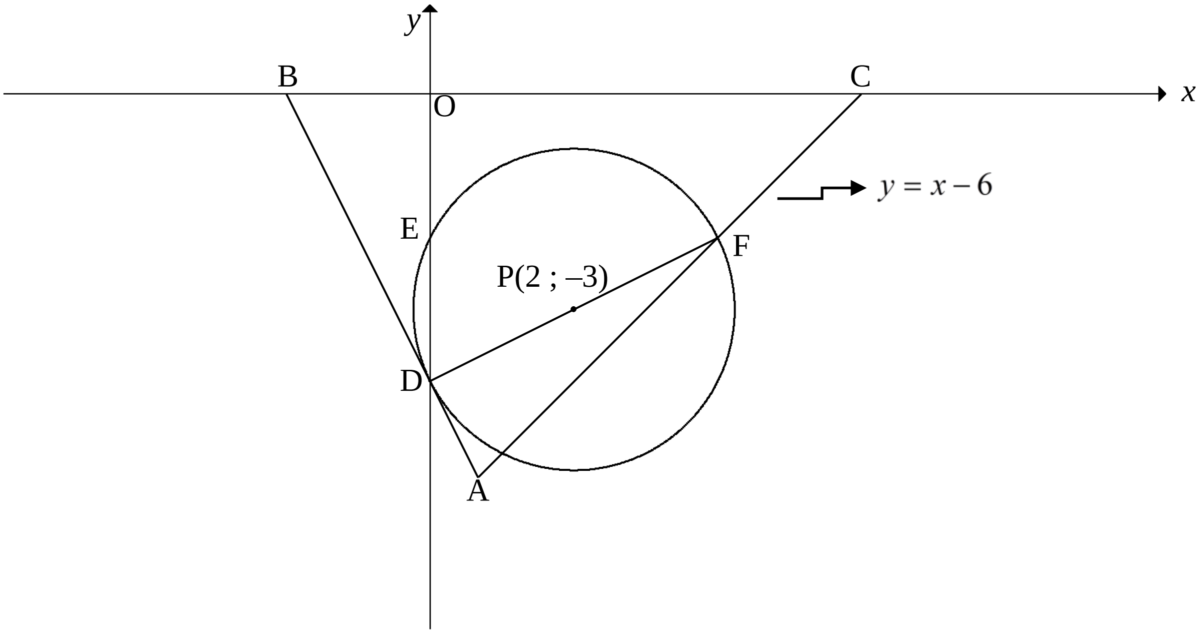
3.5.1 Calculate the perimeter of ΔABF. (5)

3.5.2 Determine the equation of a circle, centred at O, the origin, passing through point F. (2)

[18]

QUESTION 4

In the diagram, P(2 ; –3) is the centre of the circle with equation $x^2 - 4x + y^2 + 6y + 8 = 0$, passing through D, E and F. D and E are *y*-intercepts of the circle. The equation of AC is $y = x - 6$. AB is a tangent to the circle at D. B and C are points on the *x*-axis.



4.1 Calculate the coordinates of:

4.1.1 D and E (3)

4.1.2 F (2)

4.2 Write the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.3 Determine the equation of AB in the form $y = mx + c$. (4)

4.4 Calculate the:

4.4.1 size of $B\hat{D}E$ (3)

4.4.2 coordinates of A (4)

4.5 Circle P is translated such that the new centre has coordinates ($x + k$; –3). Determine all possible values of k for which the translated circle will never intersect the *y*-axis. (4)

[23]

QUESTION 5

5.1 Simplify the following expression **without using a calculator.**

$\sin(90° - x).\tan(360° - x) - 2\sin(180° + x)$ (5)

5.2 Calculate the value of the following expression **without using a calculator**.

$\sin(23° + x)\cos(7° - x) + \cos(23° + x)\sin(7° - x)$ (3)

5.3 If $\sin 2\theta = \frac{-4\sqrt{2}}{9}$, and $2\theta \in [90°; 270°]$, determine the value of the following, **without using a calculator**:

5.3.1 $\cos 2\theta$ (3)

5.3.2 $\sin\theta$ (4)

5.4 Determine the general solution of $2\sin\alpha\cos\alpha = 2\cos^2\alpha - 2\sin^2\alpha$ (7)

5.5 Consider: $\frac{\cos 2x + \cos^2 x + 3\sin^2 x}{2 - 2\sin^2 x} = \frac{1}{\cos^2 x}$

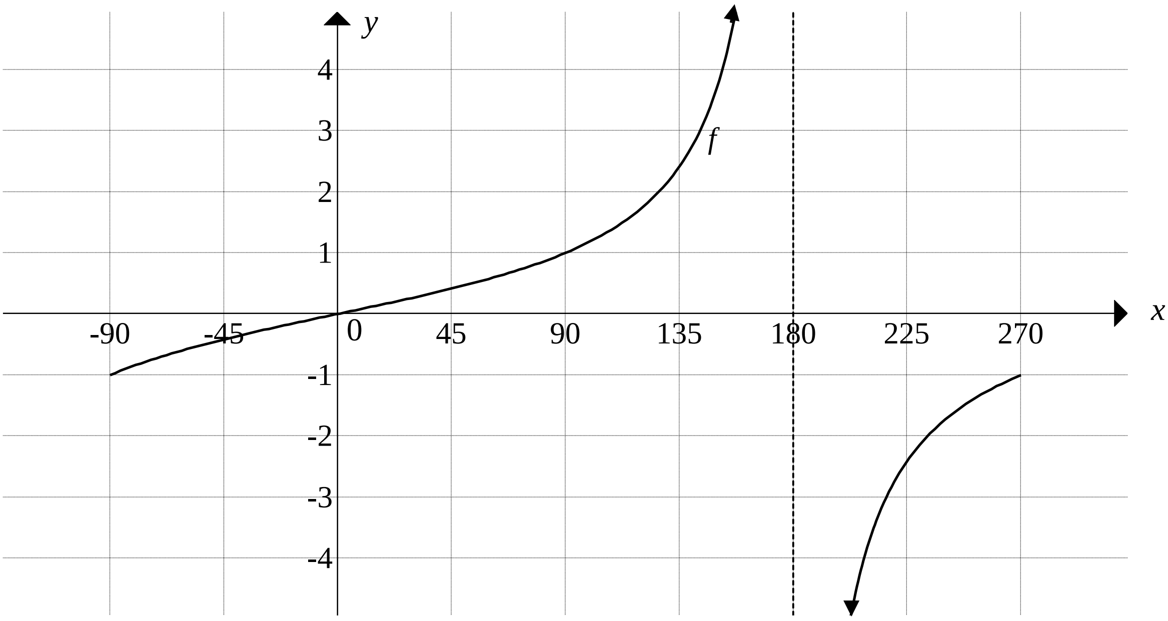
5.5.1 Prove the identity. (4)

5.5.2 For which value(s) of x , $x \in [0°; 360°]$, will the identity be undefined? (2)

[28]

QUESTION 6

The graph of $f(x) = \tan\frac{1}{2}x$ for the interval $x \in [-90°; 270°]$, is drawn below.



6.1 Write down the period of *f*. (1)

6.2 Write down the equation of the asymptote of *f* in the interval $x \in [-90°; 270°]$. (1)

6.3 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = 2\cos x$ for the interval $x \in [-90°; 270°]$. Clearly show all intercepts with the axes, turning points and end points. (3)

6.4 Give 2 values of *x*, in the interval $x \in [-90°; 270°]$, for which $g(x) - f(x) = 1$. (1)

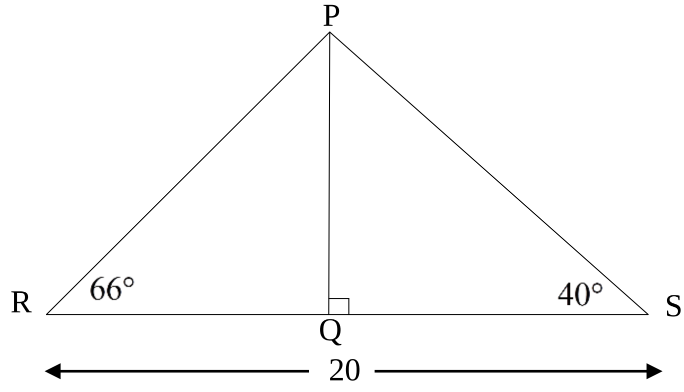
6.5 Write down the range of g(x) – 3 in the interval $x \in [-90°; 270°]$. (2)

6.6 Determine the maximum value of $[5 - 2\sin(90° - x)]^2$ for $x \in R$. (3)

[11]

QUESTION 7

7.1 In the diagram below, ΔPRS is drawn with RS = 20 units, $\hat{R} = 66°$ and $\hat{S} = 40°$. Q is a point on RS such that PQ $\perp$ RS.

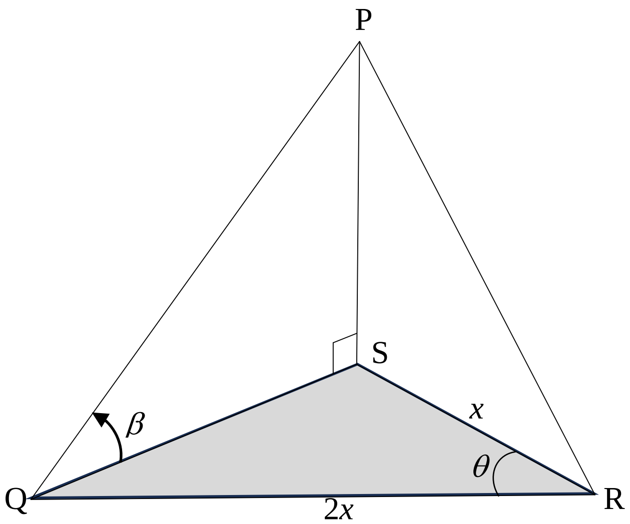


Calculate the length of:

7.1.1 PS (3)

7.1.2 PQ (2)

7.2 In the diagram, PS is a vertical flagpole. Q, R and S lie in the same horizontal plane. PQ and PR are two cables, anchored at Q and R. $P\hat{Q}S = \beta$ and $Q\hat{R}S = \theta$.
$SR = x$ and $QR = 2x$.



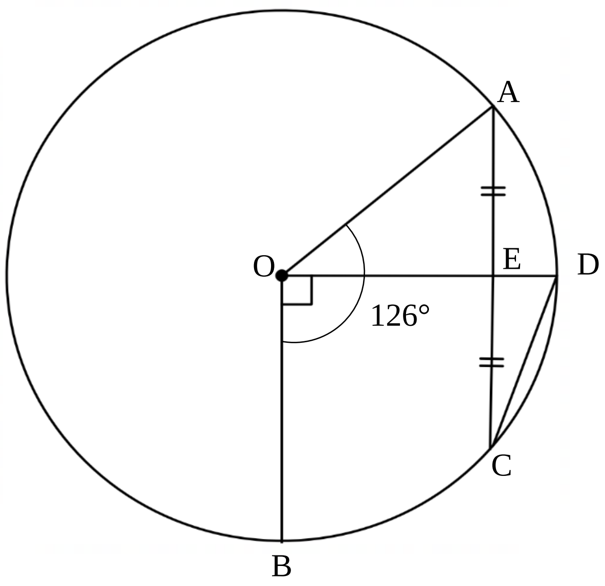
7.2.1 Show that $PQ = \dfrac{x\sqrt{5 - 4\cos\theta}}{\cos\beta}$ (4)

7.2.2 The area of ΔQRS is $57{,}36\,m^2$ and $\theta = 35°$. Calculate the value of x. (2)

[11]

QUESTION 8

8.1 In the diagram, a circle with centre O is drawn. Radii OB, OD and OA are drawn such that $BO \perp OD$ and obtuse angle $A\hat{O}B = 126°$. OD bisects chord AC at E and CD is drawn.

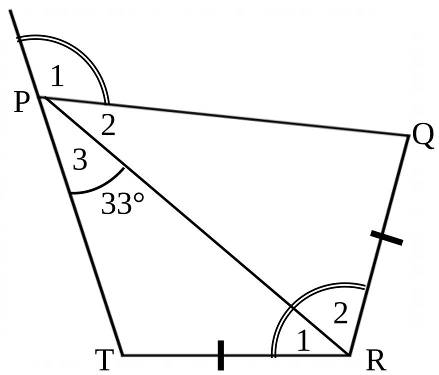


Calculate, giving reasons, the size of:

8.1.1 $O\hat{A}C$ (4)

8.1.2 $A\hat{C}D$ (2)

8.2 In the diagram below, quadrilateral PQRT is drawn with TR = RQ. TP is produced such that $\hat{P}_1 = T\hat{R}Q$. PR is drawn. $\hat{P}_3 = 33°$ and $\hat{R}_2 = 66°$.

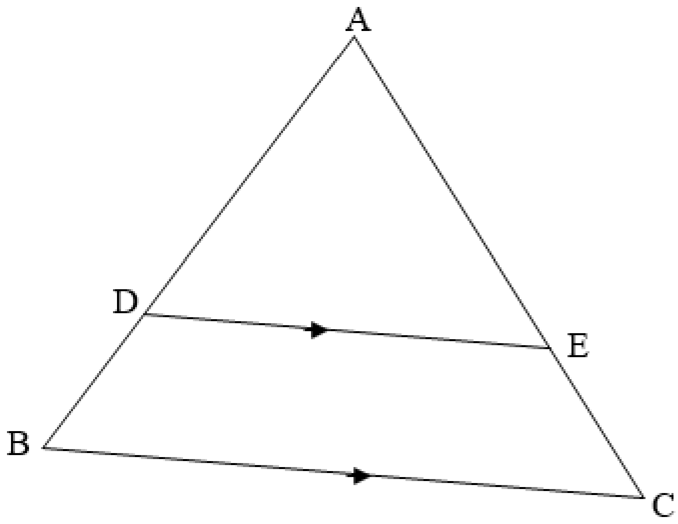


Calculate, giving reasons, the size of $\hat{T}$. (6)

[12]

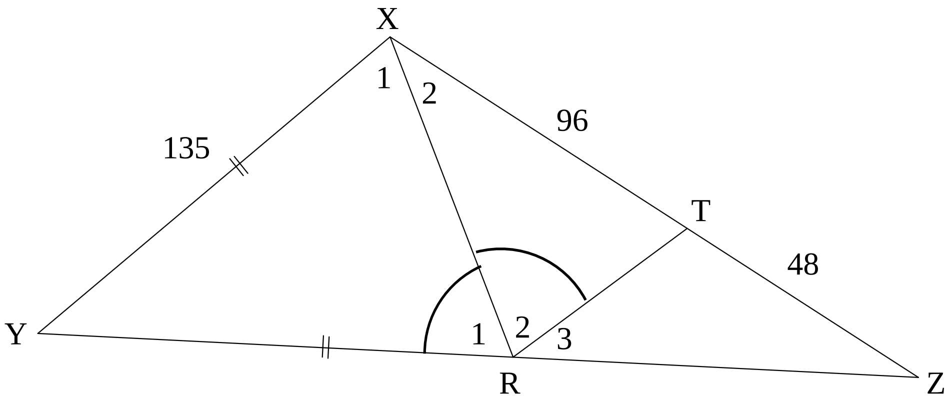
QUESTION 9

9.1 In the diagram, ΔABC is drawn. D and E are points on AB and AC respectively such that DE || BC.



Use the diagram in the ANSWER BOOK to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e. prove that $\frac{AD}{DB} = \frac{AE}{EC}$. (5)

9.2 In the diagram, ΔXYZ is drawn. R and T are points on YZ and XZ respectively and XR and RT are drawn. XY = 135 units, XT = 96 units and TZ = 48 units. $\hat{R}_1 = \hat{R}_2$ and XY = YR.



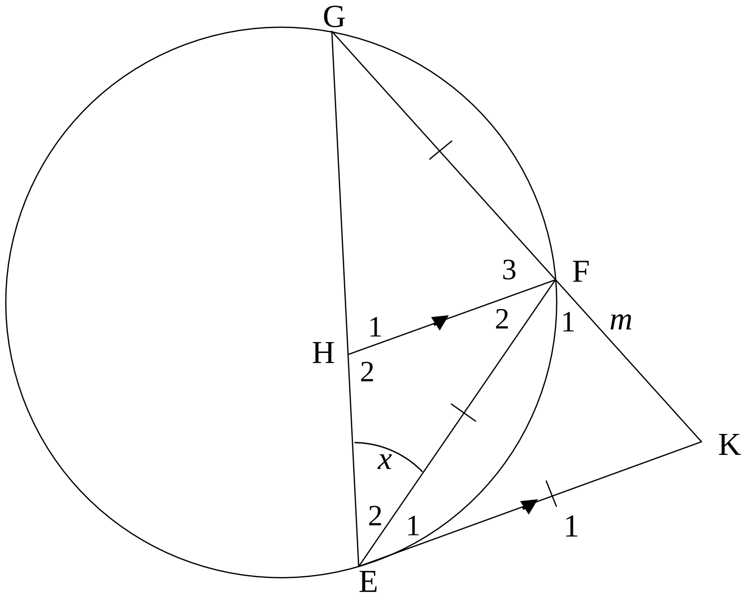
9.2.1 Prove, giving reasons, that RT || YX. (3)

9.2.2 Calculate, giving reasons, the length of RZ. (3)

[11]

QUESTION 10

In the diagram, E, F and G are points on the circle. KE is a tangent to the circle at E. KFG is a straight line. H is a point on chord GE such that HF || EK.
KE = EF = FG. KF = m units and KE = 1 unit. $\hat{E}_2 = x$.



10.1 Name, giving reasons, THREE other angles each equal to x. (4)

10.2 Express $\hat{K}$ in terms of x. (1)

10.3 Calculate, with reasons, the size of x. (3)

10.4 Prove that:

10.4.1 $\Delta KGE \;|||\; \Delta KEF$ (3)

10.4.2 $m^2 + m - 1 = 0$ (2)

10.5 If it is given that GH = 2 units, calculate the length of HE, rounded off to 2 decimal places. (4)

[17]

TOTAL: 150

INFORMATION SHEET

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $A=P(1-ni)$ $A=(1-i)^n$ $A=P(1+i)^n$

$T_n=a+(n-1)d$ $S_n=\frac{n}{2}[2a+(n-1)d]$

$T_n=ar^{n-1}$ $S_n=\frac{a(r^n-1)}{r-1};r\neq 1$ $S_\infty=\frac{a}{1-r};\quad -1<r<1$

$F=\frac{x\left[(1+i)^n-1\right]}{i}$ $P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\mathrm{M}\left(\frac{x_2+x_1}{2};\frac{y_2+y_1}{2}\right)$

$y=mx+c$ $y-y_1=m(x-x_1)$ $m=\frac{y_2-y_1}{x_2-x_1}$ $m=\tan\theta$

$(x-a)^2+(y-b)^2=r^2$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$a^2=b^2+c^2-2bc.\cos A$

area ΔABC $=\frac{1}{2}ab\sin C$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$ $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$$

$\sin 2\alpha=2\sin\alpha\cos\alpha$

$\bar{x}=\frac{\sum x}{n}$ $\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ $P(A\ or\ B)=P(A)+P(B)-P(A\ and\ B)$

$\hat{y}=a+bx$ $b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$