Northern Cape
Department of Education

MATHEMATICS- P2
LAST PUSH

TRIAL PAPERS
PROVINCES-2024

maths

STATISTICS-DATA

A. MEAN; MEDIAN; STANDARD DEVIATION; FIVE NUMBER SUMMARY; OGIVE; ETC.

EC

QUESTION 1

1.1 The number of litres of diesel purchased by 15 truck drivers at a petrol station were recorded as follow.

82	64	55	50	41
71	78	88	98	96
63	66	80	84	88

1.1.1 Write down the mode. (1)

1.1.2 Write down the range. (1)

1.1.3 Calculate the mean. (2)

1.1.4 Calculate the standard deviation of the mean. (1)

1.1.5 Determine how many truck drivers purchased litres of diesel below one standard deviation of the mean. (3)

1.2 The mean weight of 8 people entering into a lift is 75 kg. The lift has weight limit of 1 000 kg.

How many people can still get into the lift assuming that the mean weight remains 75 kg? (4)

[12]

FS

QUESTION 1

A sample of residents were asked how many litres of water they use per week in their household.

The results are summarised in the table below.

Number of litres used	Frequency
$50 \leq x < 100$	20
$100 \leq x < 150$	30
$150 \leq x < 200$	50
$200 \leq x < 250$	100
$250 \leq x < 300$	80
$300 \leq x < 350$	70
$350 \leq x < 400$	50

1.1 Complete the table in the answer book provided. (3)

1.2 Calculate the estimate of the mean litres of water used by each household per week. (2)

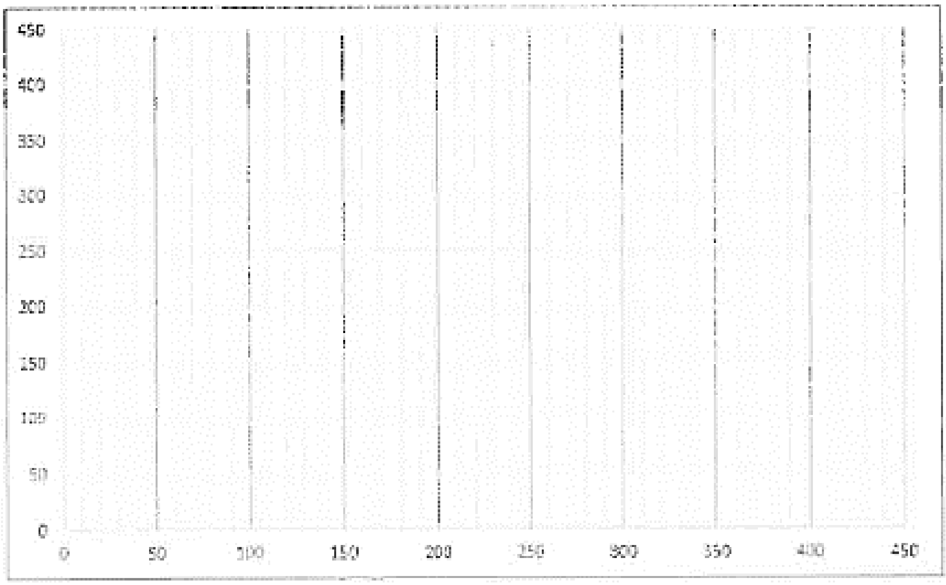
1.3 Draw an ogive of the above data on the grid provided in the answer book. (2)

1.4 Use the ogive to determine the median of the data set. (1)

1.5 Comment on the skewness of the data. Give a reason for your answer. (2)

1.6 The municipality restricted the usage of water due to water shortages. The residents may not use more than 300 litres of water per household per week.
How will this affect the standard deviation? Explain your answer. (2)
[12]

Number of litres used / ***Aantal liter gebruik***	**Frequency (*f*) /** ***Frekwensie (f)***	**Midpoint of interval (x) /** ***Middelpunt van interval (x)***	**Cumulative frequency /** ***Kumulatiewe frekwensie***	***f.(x)***
$50 \leq x < 100$	20			
$100 \leq x < 150$	30			
$150 \leq x < 200$	50			
$200 \leq x < 250$	100			
$250 \leq x < 300$	80			
$300 \leq x < 350$	70			
$350 \leq x < 400$	50			
			TOTAL/*TOTAAL*	



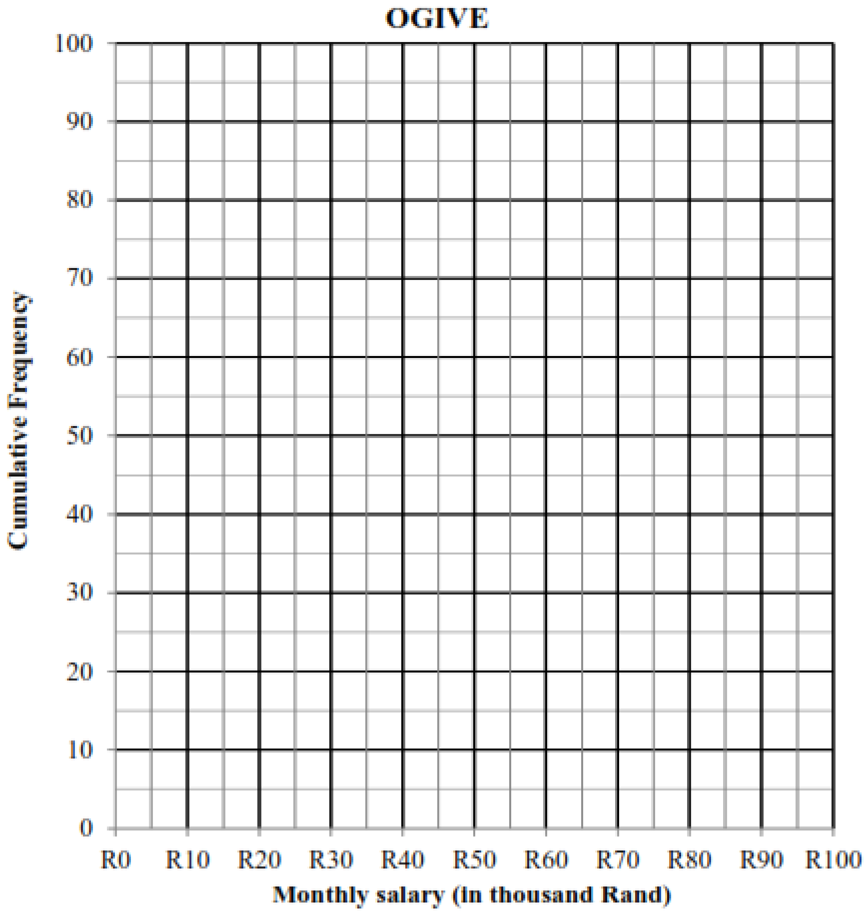
GP

QUESTION 1

The table below shows the monthly salaries of 100 employees at Adams Law.

Monthly salary (in thousand Rand)	**Number of employees**	**Cumulative frequency**
R0 $< x \leq$ R10	3	
R10 $< x \leq$ R20	4	
R20 $< x \leq$ R30	13	
R30 $< x \leq$ R40	20	
R40 $< x \leq$ R50	21	
R50 $< x \leq$ R60	12	
R60 $< x \leq$ R70	12	
R70 $< x \leq$ R80	8	
R80 $< x \leq$ R90	5	
R90 $< x \leq$ R100	2	

1.1 Draw an ogive (cumulative frequency graph) to represent the data given in the table.



(4)

1.2 Use the ogive to determine the number of employees that receive less than R35 000 per month.

(1)

1.3 Determine the median of the data.

(1)

1.4 The monthly salaries of the 5 employees in the interval $R80\,000 < x \leq R90\,000$ are given below:

R84 000	R85 000	R87 000	R89 000	R89 000

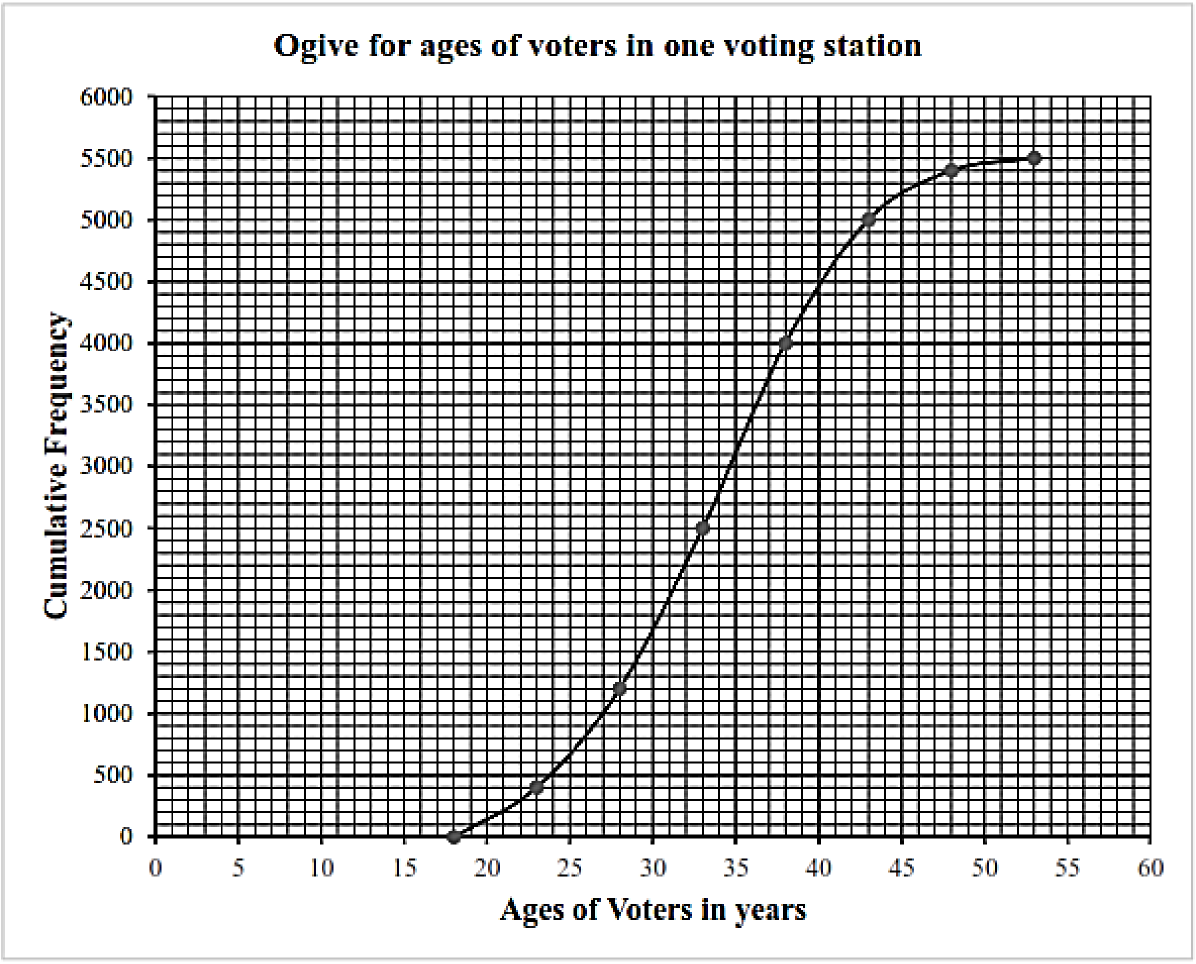
Two employees from the interval R70 000 $< x \leq$ R80 000 are promoted and receive monthly salaries of R83 000 and R84 000 respectively.

1.4.1	Determine the mean monthly salary in the interval R80 000 $< x \leq$ R90 000.	
		(1)
1.4.2	Calculate the standard deviation in this interval.	
		(1)
1.4.3	Determine the percentage of employees whose monthly salary lie within one standard deviation of the mean.	
		(3)
		[11]

KZN

QUESTION 2

The cumulative frequency graph (ogive) drawn below shows the ages of the people who voted in the Local Government elections at one voting station. Use the graph to answer the questions that follow.



2.1	How many people voted at this voting station?	(1)
2.2	Determine the interquartile range of the ages of the voters.	(3)
2.3	What percentage of the voters was 25 years or younger?	(2)

[6]

MP

QUESTION 2

The table below shows the results from a survey of cell phone expenditure for 100 learners from a secondary school in Rustenburg.

Expenditure(in rand)	Frequency	Cumulative frequency
$50 \leq x < 100$	24	
$100 \leq x < 150$	52	
$150 \leq x < 200$	14	
$200 \leq x < 250$	6	
$250 \leq x < 300$	4	

2.1 Complete the cumulative frequency table in the ANSWER BOOK. (2)

2.2 Draw an Ogive (cumulative frequency graph) for the data. (3)

2.3 Calculate the estimated mean of cell phone expenditure. (3)
[8]

NC

QUESTION 1

Twenty five rugby players were asked about the number of times they visited the gymnasium during the December festive season. The responses were as follows:

Number of gym visits	Number of players
0	1
x	1
$x + 5$	3
10	1
13	6
14	3
15	2
18	4
20	1
22	3

1.1 The mean of gym visits is 13,96. Use calculations to show that $x = 4$. (2)

1.2 Calculate the standard deviation of the data. (2)

1.3 How many players visited the gym more than one standard deviation above the mean? (2)

1.4 Determine the interquartile range of the data. (3)

1.5 During January, each player increased his number of gym visits with k visits. What impact will this have on the:

1.5.1 standard deviation? (1)

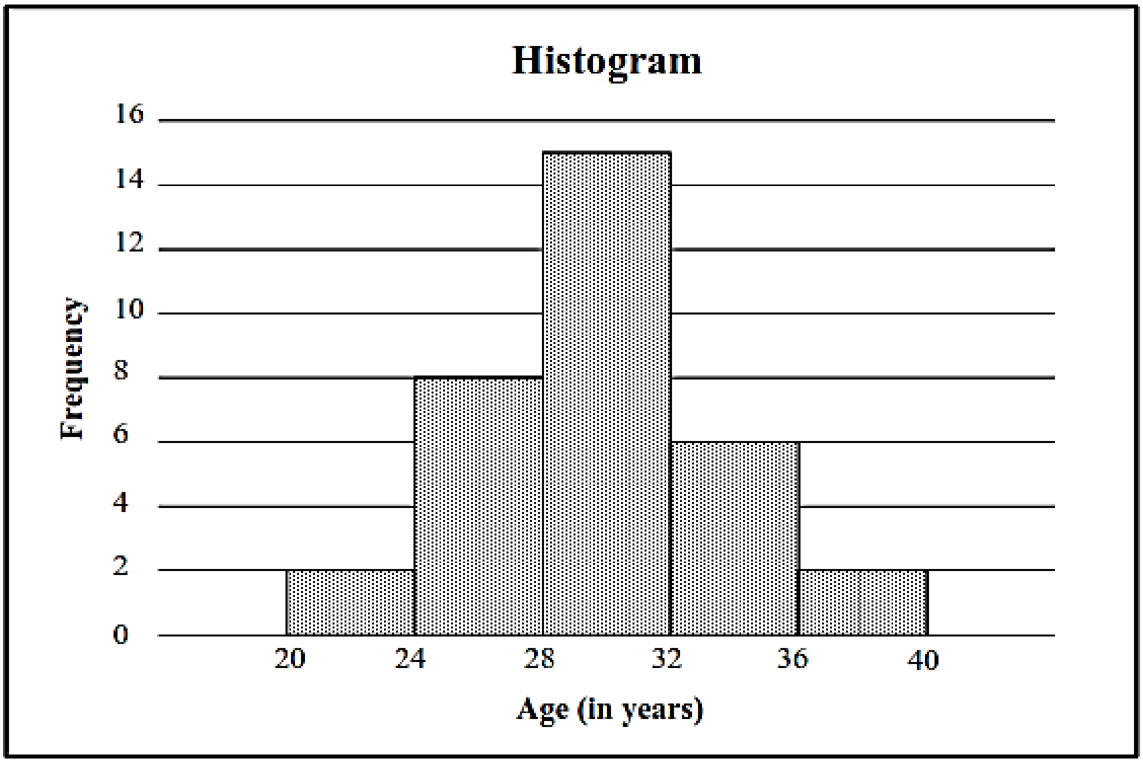
1.5.2 interquartile range? (1)

[11]

NW

QUESTION 1

During the Rugby World Cup of 2023, the ages (in years) of the players of the Springbok rugby squad were recorded. The data is represented in the histogram below.



1.1 How many players were in this rugby squad? (1)

1.2 Calculate the estimated mean age of these rugby players. (2)

1.3 Use the histogram to:

1.3.1 Complete the cumulative frequency column in the ANSWER BOOK (2)

1.3.2 Draw an ogive (cumulative frequency graph) of the above data on the grid that is provided in the ANSWER BOOK (3)

1.4 Write down the estimated median of the above data. (2)

1.5 It was discovered that the frequency of the age data for k player(s) in the modal age interval was recorded incorrectly. The mistake is corrected and the frequency of TWO other intervals are increased. The number of players in the squad remains unchanged. Determine the minimum value of k, if the data of the new histogram is symmetrical. (3)

[13]

WC

QUESTION 1

During a congress that took place in July, the ages of the 100 members who attended, were recorded in the following table:

AGE (x) (IN YEARS)	NUMBER OF PEOPLE
$15 < x \leq 25$	8
$25 < x \leq 35$	14
$35 < x \leq 45$	22
$45 < x \leq 55$	37
$55 < x \leq 65$	a
$65 < x \leq 75$	3

1.1 Calculate the value of a. (1)

1.2 Write down the modal class of the given data. (1)

1.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

1.4 Draw a cumulative frequency graph (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)

1.5 The congress organisers discovered that a mistake was made, and that 3 people counted in the range $45 < x \leq 55$ should have been counted in the range $35 < x \leq 45$. Calculate the estimated average age of the people who attended the congress after the mistake was corrected. (2)

[9]

B. SCATTER PLOT; BOX & WHISKER; REGRESSION LINE; CORRELATION COEFFICIENT; OUTLIERS,ETC.

EC

QUESTION 2

Grade 8 results of two tests each written out of 50 marks are listed below.

TEST A (x)	39	33	35	44	37	40	24	31	30	5
TEST B (y)	41	45	48	40	47	42	37	44	43	24

2.1 Identify an outlier from the given table. (1)

2.2 Determine the equation of the least squares regression line. (3)

2.3 Use the equation of the least squares regression line to predict a mark for TEST B if a learner obtained 14 marks in TEST A. Round off your answer to the nearest whole number. (2)

2.4 Comment on the strength of correlation between TEST A and TEST B. (2)
[8]

FS

QUESTION 2

The leg strength and the number of leg presses (weight training on legs) done per day by a random sample of ten, eighteen-year-old boys were recorded in the table below.

Number of leg presses done per day (x)	36	136	51	126	90	43	77	68	103	124
Strength of upper leg (y)	0,2	0,85	0,35	0,91	0,73	0,34	0,61	0,59	0,78	0,90

2.1 Determine the equation of the regression line and write your answer in the form y = ... (3)

2.2 Use the regression line to predict the leg strength of the eighteen-year-old boy if he does 110 leg presses per day. (2)

2.3 An eighteen-year-old boy does 250 leg presses per day. Can your regression line formula predict the strength of his legs? (Explain your answer). (2)

2.4 Calculate the correlation coefficient of the above data set. (2)

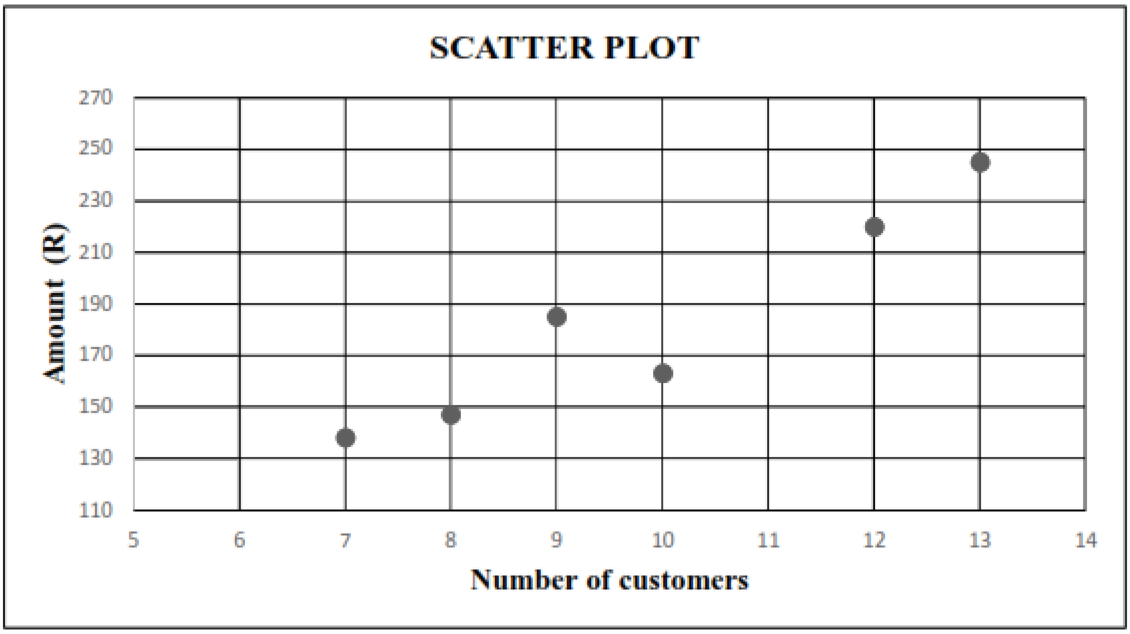
2.5 Use your answer in QUESTION 2.4 to describe the relationship between the number of leg presses and the leg strength of an eighteen-year-old boy. (2)
[11]

GP

QUESTION 2

During a netball tournament, there are several stalls selling food and refreshments. In a period of 10 minutes at 6 stalls, the following was observed:

Stall	Number of customers	Amount of money spent (R)
1	10	163
2	7	138
3	9	185
4	12	220
5	8	147
6	13	245



2.1	Determine the equation of the least squares regression line of the data.	(3)
2.2	Predict the amount of money spent by 11 customers.	(2)

2.3	Determine the correlation coefficient of the data.	
		(1)
2.4	The organisers of the event think that there is a very weak positive correlation between the number of customers and the amount of money received at stalls. Motivate whether you agree or not.	
		(1)
2.5	At another stall, 6 customers spent a total amount of R195. If this point is included in the data, will the gradient of the least squares regression line increase or decrease? Motivate your answer without any further calculations.	
		(2)
		[9]

KZN

QUESTION 1

The Human Resource Department of a company in KwaZulu–Natal wants to create a model to be used in determining the monthly salaries of its employees. Twelve of their current employees were surveyed and the information is displayed in the table below:

Employees' experience in number of years (x)	26	1	3	5	6	6	10	14	12	33	20	8
Salary in R1000s per month (y)	20	9	10,5	11	10	12	16	15	12	23	18	9

1.1 Calculate the

1.1.1 mean of the monthly salaries of these twelve employees. Round your answer off to the nearest rand. (2)

1.1.2 standard deviation of the monthly salaries of these twelve employees. Round your answer off to the nearest rand. (1)

1.2 How many of the twelve employees earn a monthly salary that is more than one standard deviation above the mean? (2)

1.3 Determine the equation of the least squares regression line for the data given in the table. (3)

1.4 Calculate the correlation coefficient between the experience in years and the monthly salary of an employee. (1)

1.5 Predict what the monthly salary will be of an employee who has been working for this company for 30 years. Round your answer off to the nearest rand. (2)

1.6 Is the prediction that is made in question 1.5 likely to be reliable? Give a reason for your answer. (2)

[13]

MP

QUESTION 1

A Mathematics teacher wants to create a model by which she can predict a learner's final marks. She decided to use her 2015 results to create the model.

Preparatory exam(x)	55	35	67	85	91	48	78	72	15	75	69	37
Final exam(y)	57	50	74	80	92	50	80	81	23	80	75	42

1.1 Determine the equation of the least squares regression line in the form $y = a + bx$. (3)

1.2 Draw a scatter plot and show the regression line (3)

1.3 Predict the final mark for a learner who attained 46% in the preparatory examination. (2)

1.4 Determine the correlation coefficient of the data. (1)

1.5 Describe the relationship between preparatory and final exam results. (1)

1.6 Could you use this equation to estimate the preparatory exam mark for a learner who attained 73% in the final exam? Give a reason for your answer. (2)

[12]

NC

QUESTION 2

A restaurant situated on the beach front wants to predict how many cups of coffee will be sold on a given day, depending on the temperature. They collected the following data over 11 days:

Temperature in °C (x)	19	21	16	23	26	25	28	19	27	22	35
Cups of coffee sold (y)	381	370	400	161	173	184	165	398	182	310	65

2.1 Determine the equation of the least squares regression line for this data. (3)

2.2 Write down the correlation coefficient. (1)

2.3 Hence, comment on the:

2.3.1 trend of the data (1)

2.3.2 strength of the relationship between the sales and the temperature of the day (1)

2.4 Predict the number of cups of coffee the restaurant will most probably sell if the day temperature is 30°C. (2)

[8]

NW

QUESTION 2

Mrs Mochini wants to use mathematical modelling to predict the final results of her grade 12 Mathematics learners. She decides to use the Preparatory and Final Mathematics examination results of the previous year to help her in developing such a possible model.

She records in the table below, 10 learners' previous year's results (in %) as follows:

Preparatory examination (x)	38	65	78	23	67	93	39	83	51	66
Final examination (y)	57	72	81	27	59	94	41	85	54	79

2.1 Determine the equation of the least squares regression line. (3)

2.2 A learner obtained 46% for the Preparatory examination.

2.2.1 Calculate the possible final examination results that Mrs Mochine can expect from this learner. (2)

2.2.2 Is the answer in QUESTION 2.2.1 a good indication of the expected final examination result? Motivate your answer. (2)

2.3 The point $(\bar{x}\ ;\ q)$ lies on the regression line of QUESTION 2.1.

Only ONE of the options below correctly reflects the value of q.
Write only the letter of the correct option as your answer.

A $\sqrt{\bar{x}}$

B $\frac{\sum y}{10}$

C σ_x

D σ_y (1)

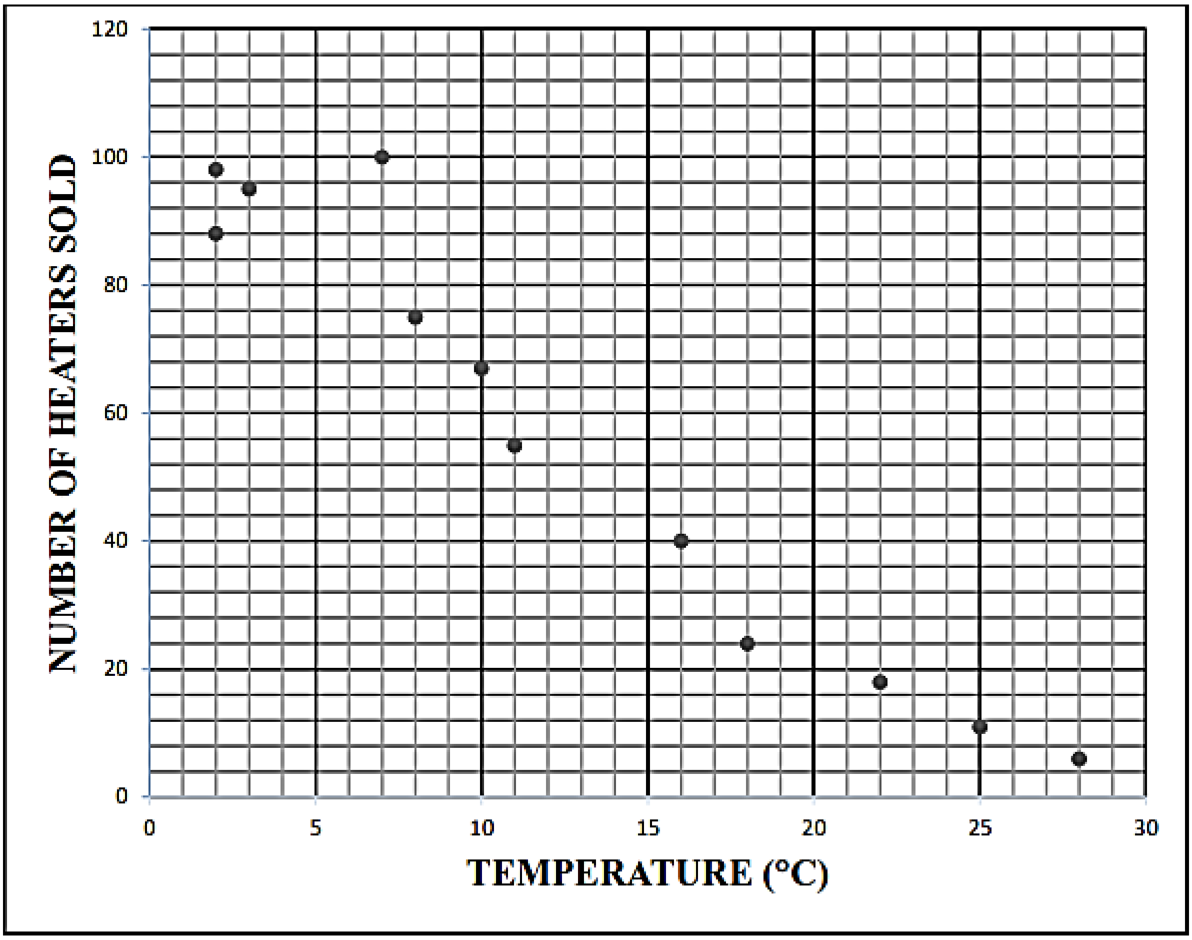
[8]

WC

QUESTION 2

The scatter plot below shows the number of heaters a company sold per month and the average temperature of that month in a certain year.
Both the temperature in °C and the number of heaters sold, are given in the table below.

MONTH	J	F	M	A	M	J	J	A	S	O	N	D
TEMPERATURE (°C)	2	7	8	10	18	22	28	25	16	11	2	3
NUMBER OF HEATERS SOLD	98	100	75	67	24	18	6	11	40	55	88	95



2.1 Describe the correlation between the number of heaters sold and the average temperature per month. Verify your answer by referring to the correlation coefficient. (2)

2.2 Determine the equation of the least squares regression line for the data. (3)

2.3 Predict the number of heaters sold for a month where the average temperature is 20°C. (2)

2.4 Draw the least squares regression line on the grid given in the ANSWER BOOK. (2)

2.5 Calculate the standard deviation of the number of heaters sold. (2)
[11]

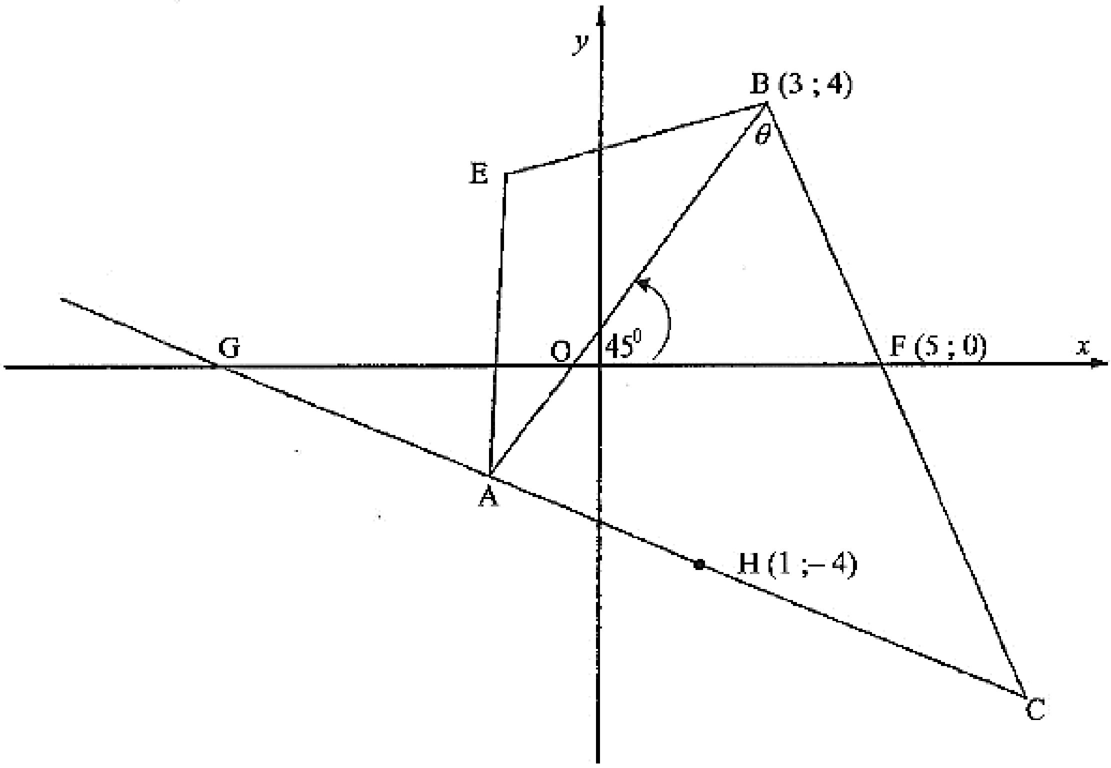
ANALYTICAL GEOMETRY

GRADIENT;DISTANCE,MIDPOINT;EQUATION OF STRAIGHT LINE;INCLINATION ANGLE;COLLINEAR;PARALLEL LINES;PERPENDICULAR LINES;PROPERTIES OF TRIANGLES & QUADRILATERALS;ALTITUDE; MEDIAN;ETC.

EC

QUESTION 3

Quadrilateral AEBC is drawn. Coordinates of B are (3 ; 4). G, O and F(5 ; 0) are x-intercepts of lines AC, AB and BC respectively. H(1 ; – 4) is a point on line AC. $A\hat{B}C = \theta$.
Area of ΔOBF = 12 square units and inclination of line AB is $45°$.
HC = 2AH



3.1 Calculate the length of BF. Leave your answer in simplest surd form. (2)

3.2 Calculate the gradient of BF. (2)

3.3 Calculate the size of θ. (3)

3.4 Prove that $HF \parallel AB$. (4)

3.5 It is further given that, EC bisects AB perpendicularly. What type of quadrilateral is AEBC? (1)

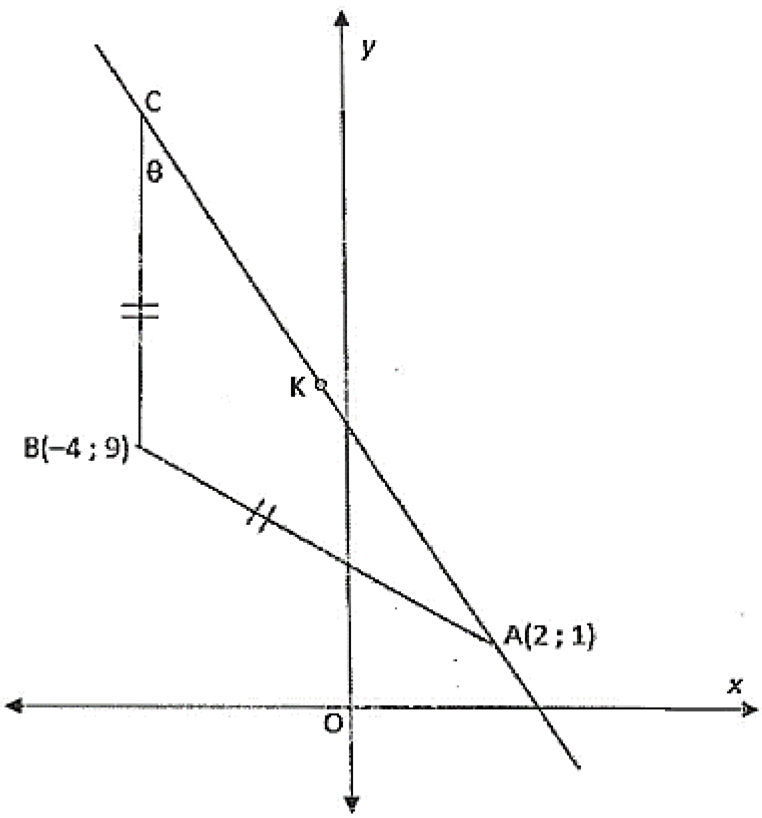
3.6 Hence or otherwise calculate the length of AC. (4)

3.7 Calculate the area of quadrilateral AOFC. (3)
[19]

FS

QUESTION 3

In the diagram, ABC is an isosceles triangle with A(2 ; 1) and B(–4 ; 9).
AB = BC, and BC is parallel to the y-axis.



Calculate:

3.1 The length of AB (2)

3.2 The coordinates of C (2)

3.3 The coordinates of K, the midpoint of AC (2)

3.4 The equation of AC in the form $y = mx + c$ (3)

3.5 The size of θ (3)

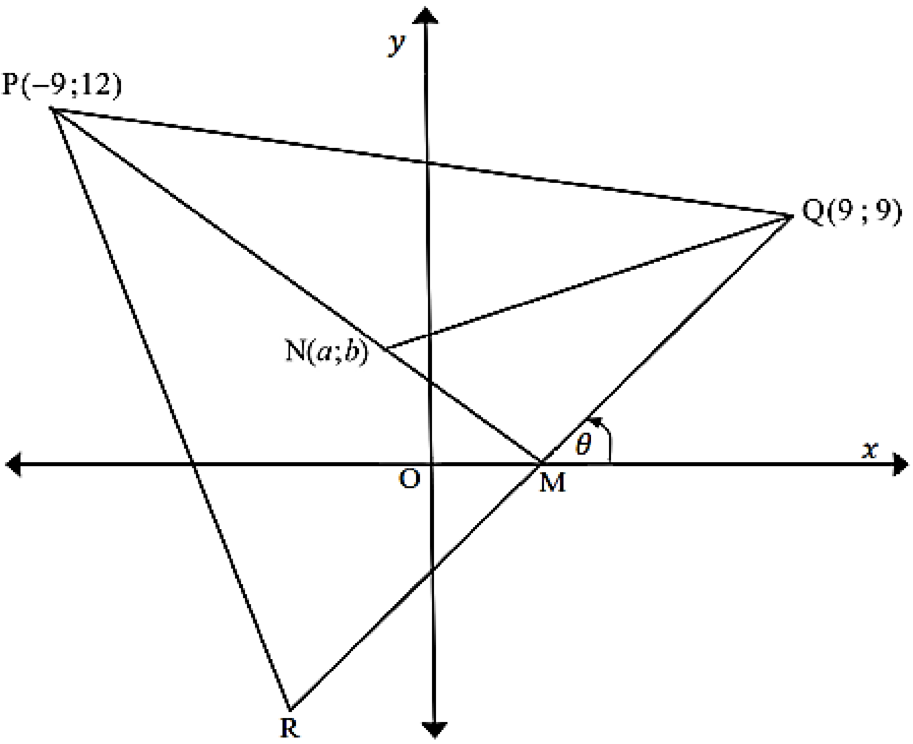
3.6 The area of triangle ABC (4)

3.7 The coordinates of D if ABCD is a rhombus (2)
[18]

GP

QUESTION 3

In the diagram below, P(−9 ; 12), Q(9 ; 9) and R are vertices of ΔPQR. M is the midpoint of QR and N(a ; b) is a point on PM in the second quadrant. The equation of QR is given by $2y - 3x + 9 = 0$. The angle of inclination of QR is θ.



3.1	Calculate the coordinates of M, the x-intercept of line PM.	
		(2)

3.2	Determine the equation of PM in the form $y = mx + c$.	
		(4)
3.3	Calculate the size of θ.	
		(2)
3.4	Show that $b = 3 - a$, if P, N and M are collinear.	
		(1)

3.5	Hence, determine the value of a and b if $NQ = 5\sqrt{5}$ units.	
		(5)

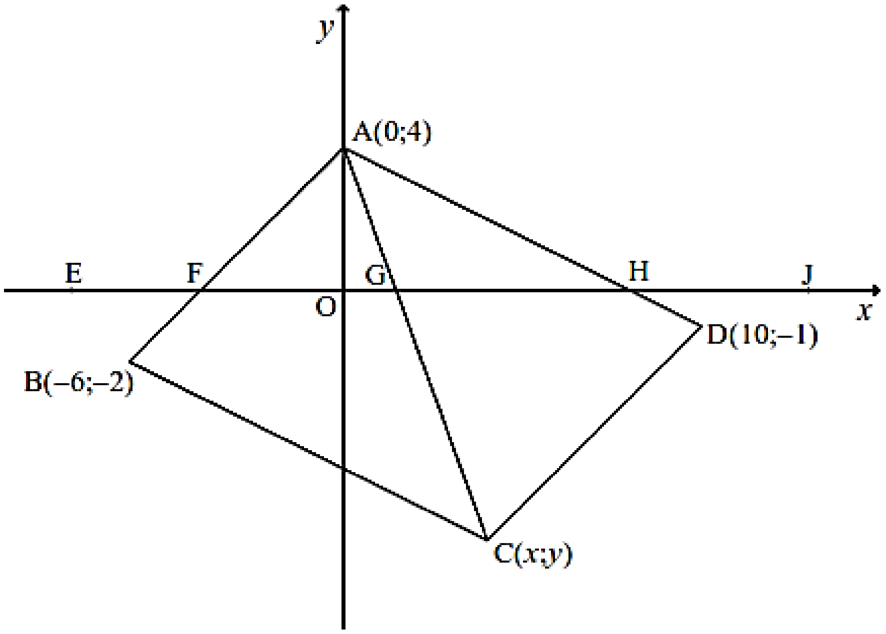
3.6	Determine the equation of a circle having centre at O, the origin, and passing through point R.	
		(4)

3.7	The acute angle between the lines QR and the line with equation $y = mx + 4$ is $45°$. Determine the possible value(s) of m.	
		(4)
		[22]

KZN

QUESTION 3

ABCD is a parallelogram with $A(0;4)$, $B(-6;-2)$, $C(x;y)$ and $D(10;-1)$ as shown below.
AC is drawn. F, G and H are the x-intercepts of AB, AC and AD respectively.
E is a point on the x-axis to the left of F and J a point on the x-axis to the right of H.

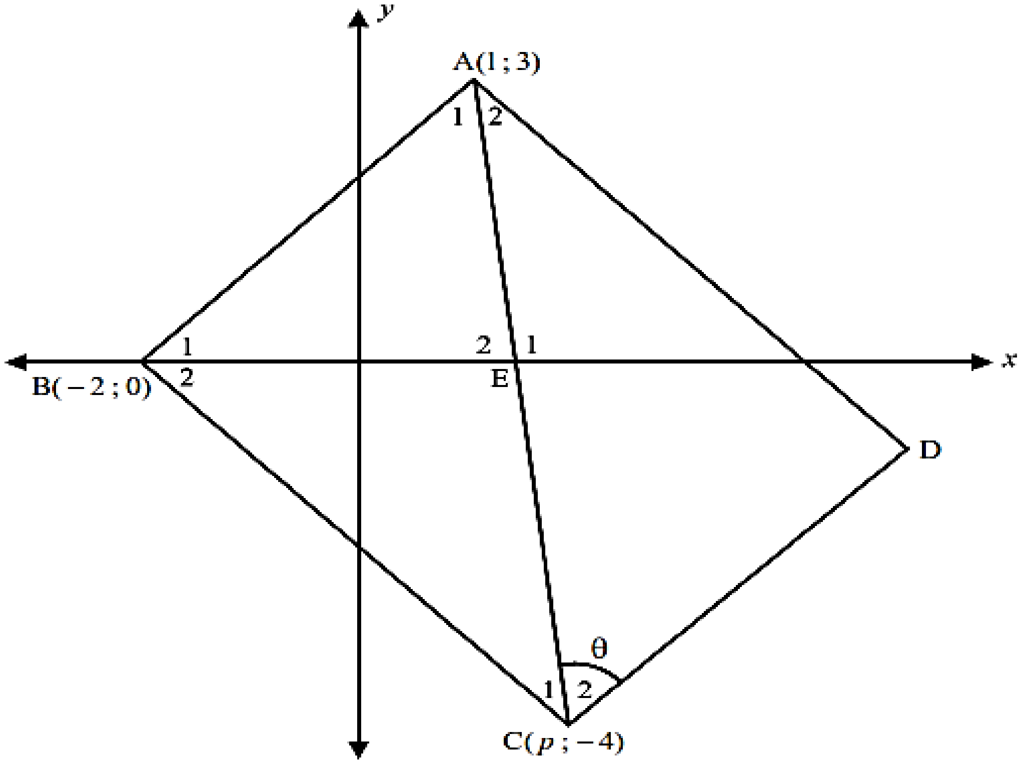


3.1	Determine the gradient of AB.	(2)
3.2	Determine the equation of CD.	(3)
3.3	Determine the coordinates of M, the midpoint of AC.	(3)
3.4	Determine the coordinates of C.	(2)
3.5	Determine the size of $B\hat{C}D$.	(6)
		[16]

MP

QUESTION 3

3.1 ABC is a triangle with vertices $A(1\,;3)$, $B(-2\,;0)$ and $C(p\,;-4)$ where $p>0$.
the length of AC is $\sqrt{50}$ units.



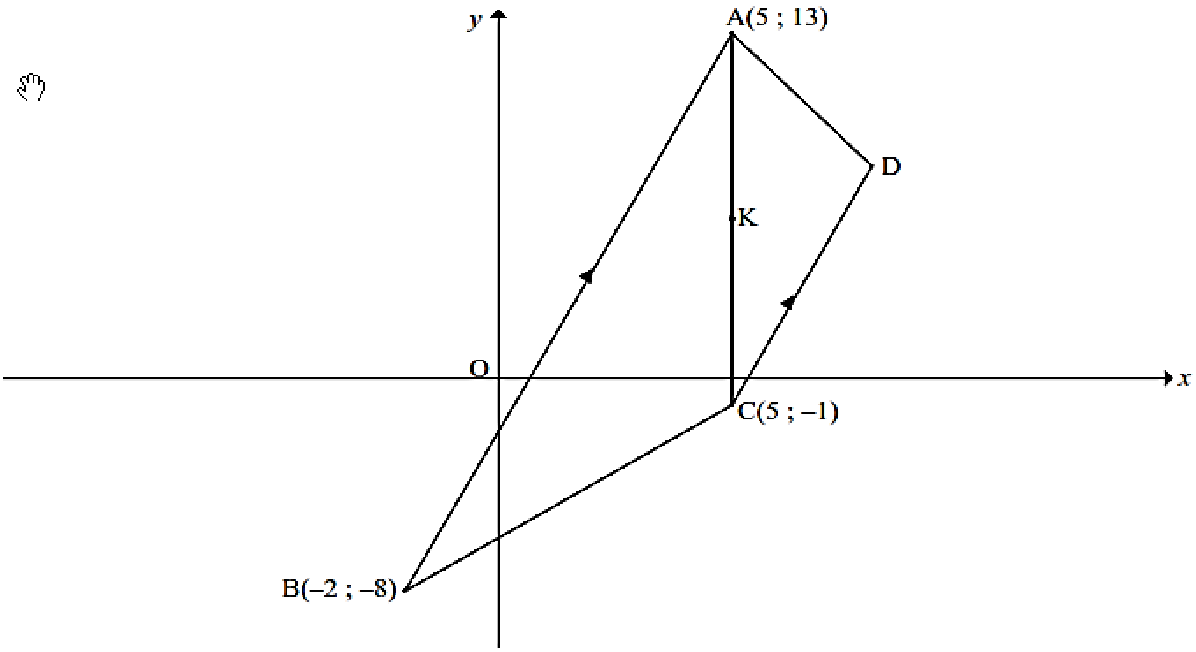
3.1.1	Determine the gradient of AB.	(2)
3.1.2	Show, by calculation, that $p=2$.	(4)
3.1.3.	Determine the equation of the perpendicular bisector of AB.	(4)
3.1.4	Write down the coordinates of D such that ABCD is a rectangle	(2)
3.1.5	Determine the equation of the circle passing through A, B and C.	(4)
3.1.6	Calculate the size of θ rounded off to the nearest whole number	(5)
3.2	Three straight lines AB, RS and $x=-3$ intersect each other. The equation of AB is $3x+by=-2$ with $b\neq 0$, and the equation of RS is $y=-\frac{2}{3}x+2$. Calculate the value of b.	(4)

[25]

NC

QUESTION 3

ABCD is a trapezium with vertices A(5 ; 13), B(–2 ; –8) ; C(5 ; –1) and D.
K is the midpoint of AC and BA || CD.

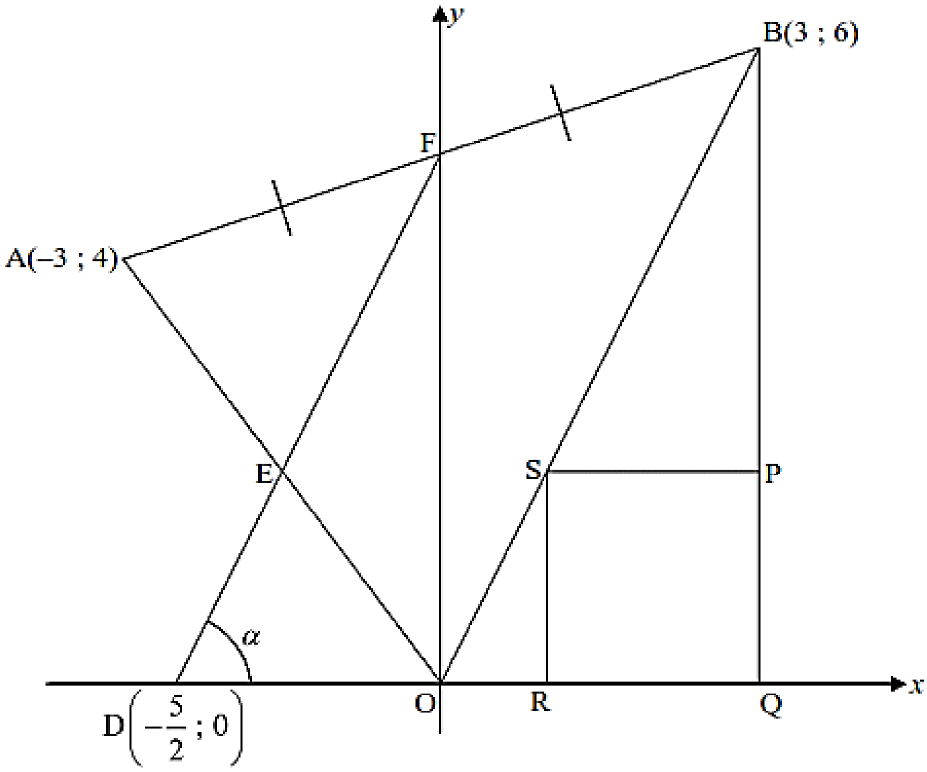


3.1	Write down the coordinates of K.	(2)
3.2	Calculate:	
	3.2.1 The gradient of AB	(2)
	3.2.2 p, if D(p ; p)	(2)
3.3	ABED is a parallelogram with E a point in the fourth quadrant. Calculate the coordinates of E.	(2)
3.4	Calculate the area of obtuse ΔABC.	(3)
3.5	ΔABC is reflected in the line $x = 5$ to form ΔAFC.	
	3.5.1 Calculate the perimeter of ΔABF.	(5)
	3.5.2 Determine the equation of a circle, centred at O, the origin, passing through point F.	(2)
		[18]

NW

QUESTION 3

In the diagram, A(–3 ; 4), B(3 ; 6) and O (origin) are vertices of ΔABO. F is the midpoint of AB and is joined with $D\left(-\frac{5}{2}\,;0\right)$. The angle of inclination of FD is α. The lines AO and DF intersect at E. A quadrilateral PQRS is drawn with QR on the x-axis and S is a point on OB. The side QP is produced to B.



3.1 Calculate the:

3.1.1 Coordinates of F (2)

3.1.2 Gradient of DF (2)

3.1.3 Size of α (2)

3.2 Write down the equation of OB. (1)

3.3 Give a reason why DF || OB. (1)

3.4 It is given that PQRS is a square with an area of $9x^2$ squared units. Calculate the coordinates of S. (6)

3.5 Prove that EDOS forms a parallelogram. (4)

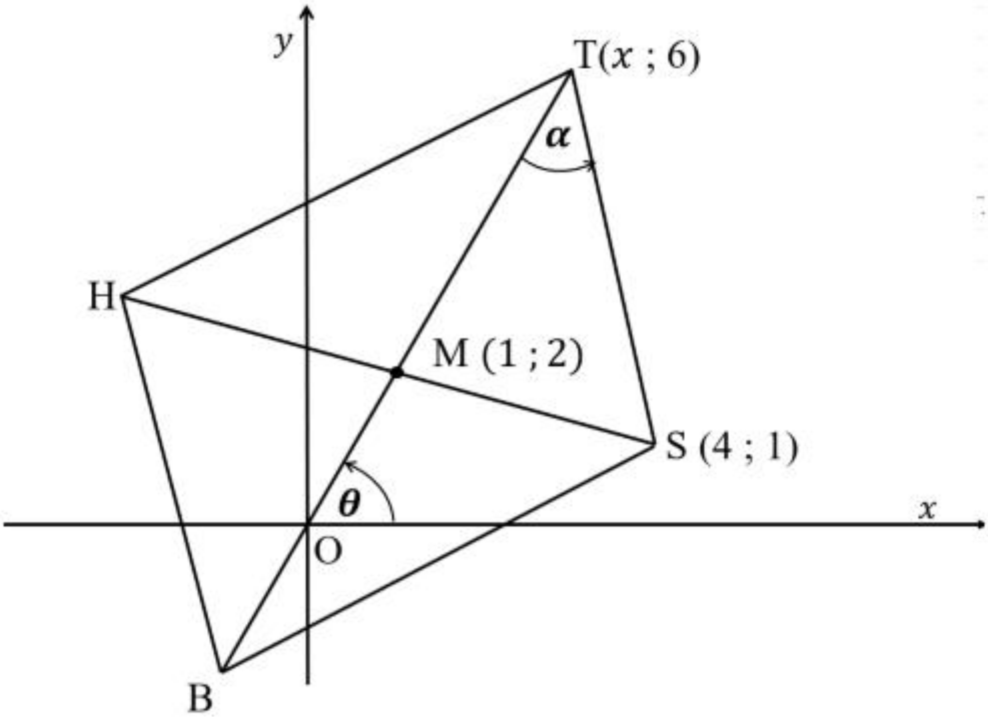
[18]

WC

QUESTION 3

In the diagram below HS and TB are the diagonals of parallelogram HTSB.

- HS and TB intersect at $M(1;2)$.
- TB intersect the x- and y-axes at the origin.
- $T(x;6)$ and $S(4;1)$ are vertices of HTSB.
- $M\hat{T}S = \alpha$ and the inclination angle of TB is θ.



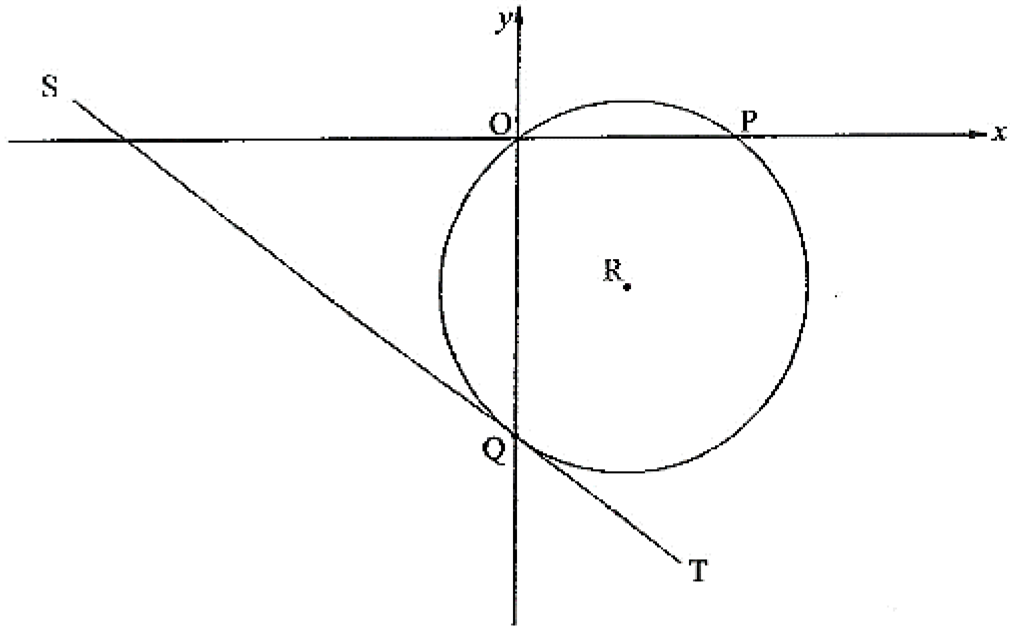
3.1	Determine:		
	3.1.1	the equation of the line TB in the form $y = mx + c$.	(2)
	3.1.2	the size of θ.	(2)
	3.1.3	the coordinates of H.	(3)
3.2	Show that the x-coordinate of T is 3 if it is further given that $TS = \sqrt{26}$.		(3)
3.3	Calculate α, the size of $M\hat{T}S$.		(4)
3.4	Calculate the area of ΔBTS.		(4)
3.5	Determine the perpendicular height of the parallelogram if TS is the base.		(3)
			[21]

CIRCLE ANALYTICAL GEOMETRY

EC

QUESTION 4

4.1 In the diagram below, R is the centre of the circle OPQ. Point Q is the y-intercept of the circle. SQT is the tangent of the circle at Q. The equation of SQT is $y = -\frac{3}{4}x - 8$.



4.1.1 Calculate the coordinates of Q. (2)

4.1.2 Determine the equation of QR in the form $y = mx + c$. (3)

4.1.3 Calculate the coordinates of P, the x-intercept of line QR. (2)

4.1.4 Calculate the coordinates of R, the centre of the circle. (3)

4.1.5 Write down the equation of the circle centred at R in the form: $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.1.6 If $y = k$ is a tangent to the circle, determine the value(s) of k. (3)

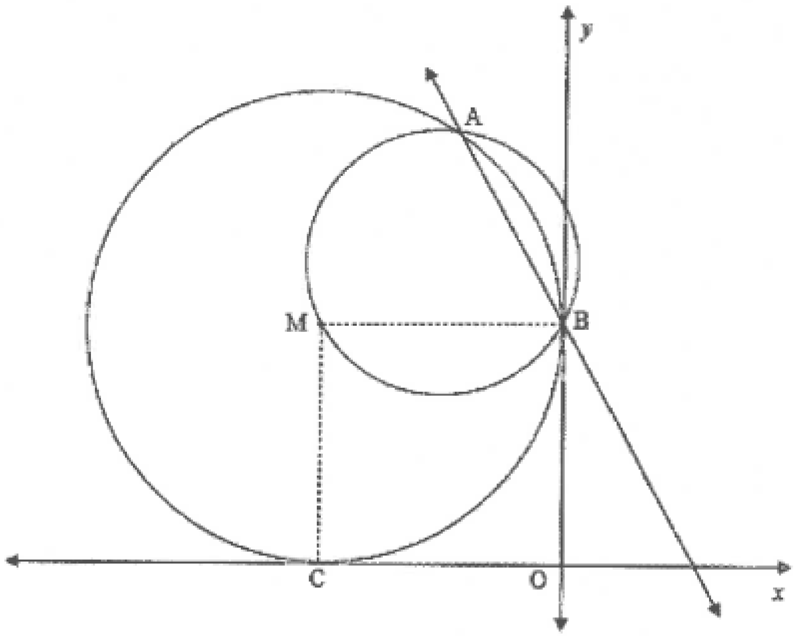
4.2 Calculate the maximum length of the radius of the circle having equation $x^2 + y^2 - 2x\sin\theta - 4y\sin\theta = -2$. (5)

[21]

FS

QUESTION 4

In the diagram, a circle centred at M touches the x-axis at C and the y-axis at point B. A second circle with equation $x^2 + y^2 + x - 3y + 2 = 0$ passes through A and M and intersects circle M at A and B. The equation of the common chord AB is given by $y = -x + 1$.



4.1 Determine the coordinates of the centre and the radius of the circle which passes through B, M and A. (4)

4.2 Calculate the coordinates of A. (5)

4.3 Show that the equation of the circle centred at M, is $x^2 + y^2 + 2x - 2y + 1 = 0$ (5)

4.4 The straight line with equation $y = -x + k$ is a tangent to the circle with centre M.

4.4.1 Show that this equation can be written as: $2x^2 + (4 - 2k)x + (k^2 - 2k + 1) = 0$ (3)

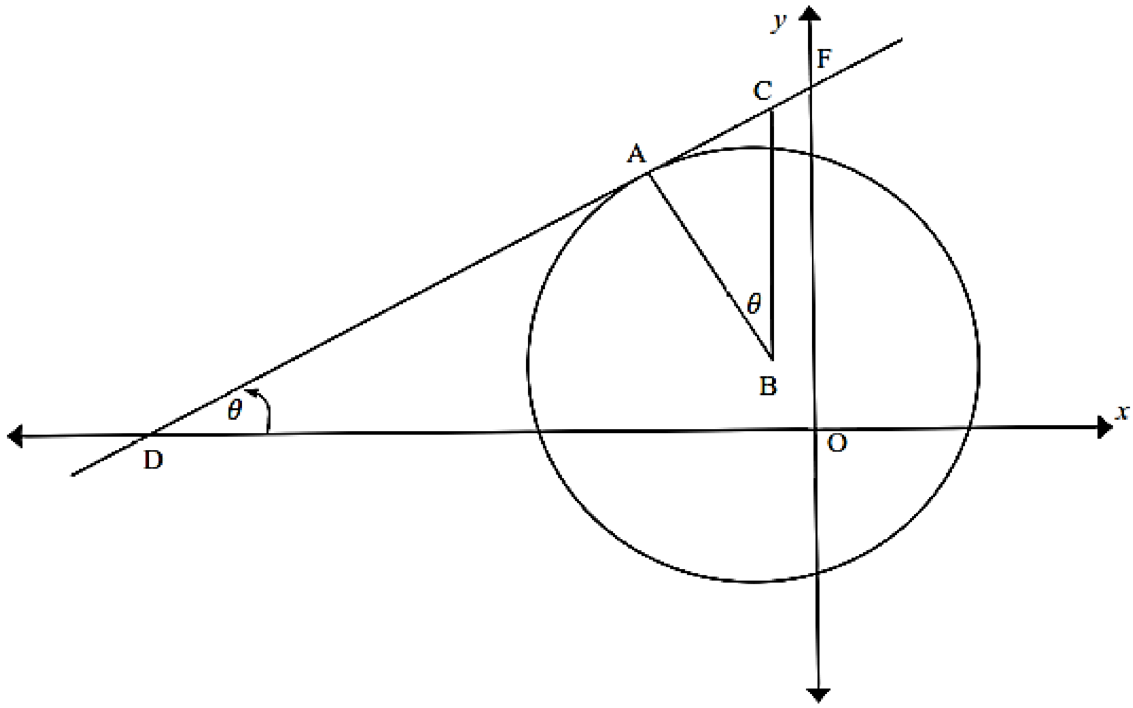
4.4.2 Calculate the numerical value(s) of k (5)

[22]

GP

QUESTION 4

In the diagram, the equation of the circle centred at B is given by $(x+1)^2+(y-1)^2=20$.
DF is a tangent to the circle at A with D and F, the x- and y-intercepts respectively. $C(-1;6)$ is a point on DF with BC parallel to the y-axis. $C\hat{B}A = A\hat{D}O = \theta$.



4.1	Write down the coordinates of B.	
		(1)
4.2	Show that $AC=\sqrt{5}$.	
		(3)

4.3	Write down the value of $\tan\theta$.	(1)
4.4	Show that the equation of AB is given by $y = -2x - 1$.	(3)

	Determine the coordinates of A.	(4)

4.6	Calculate the ratio of the area of ΔABC to the area of ΔODF. Simplify your answer.	
		(6)
		[18]

KZN

QUESTION 4

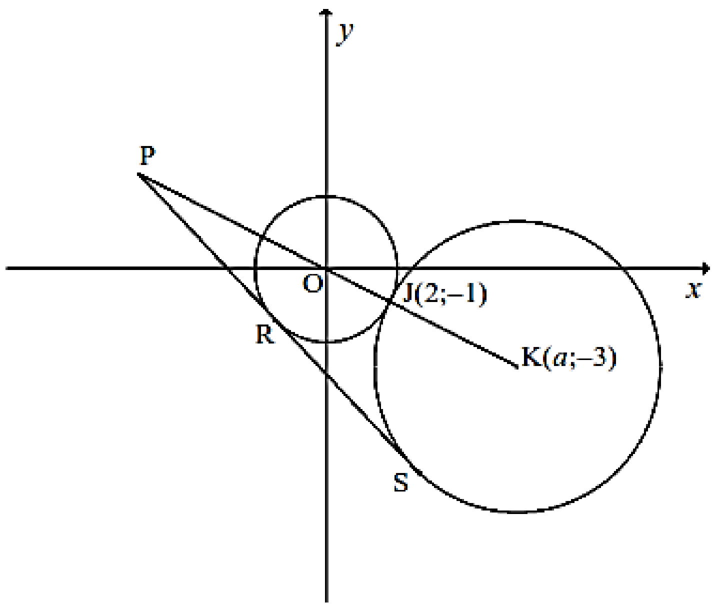
4.1 The diagram below shows two circles touching at $J(2;-1)$.

The smaller circle has its centre at the origin and the bigger circle has centre $K(a;-3)$.

The length of the radius of the bigger circle is TWICE the length of the radius of the smaller circle.

SR is a tangent to both circles, touching the bigger circle at S and the smaller circle at R.

KO and SR are both produced to intersect in point P.



4.1.1 Calculate the length of the radius of the smaller circle. (2)

4.1.2 Show that $a = 6$. (3)

4.1.3 Determine the equation of the bigger circle. (2)

4.1.4 Does the point $(10;-4)$ lie outside, inside or on the bigger circle? (3)

4.1.5 Calculate the length of PS. (5)

4.2 The length of the diameter of the circle with equation $x^2 - 4x + y^2 + 5y = -d$ is 24. Determine:

4.2.1 the coordinates of the centre of the circle. (4)

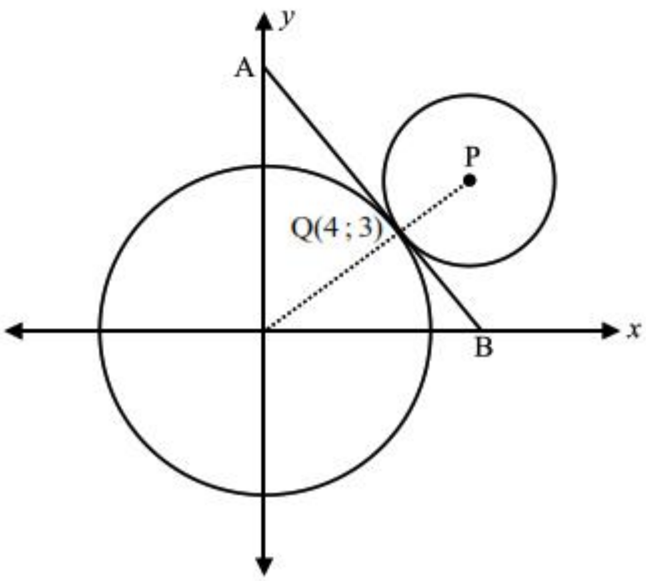
4.2.2 the value of d. (3)

[22]

MP

QUESTION 4

Two circles in the diagram below represent two interlocking gears, which touch at the point Q(4 ; 3). The circles have the following equations: $x^2 + y^2 = 25$ and $x^2 - 12x + y^2 - 9y + 50 = 0$



4.1 Show that the coordinates of P are $(6\,;4\frac{1}{2})$. (3)

4.2 Determine the equation of the common tangent AB. (4)

4.3 If the larger gear makes one full revolution, how many times will the smaller gear turn completely? (4)

4.4 Determine the area of ΔAOB (3)

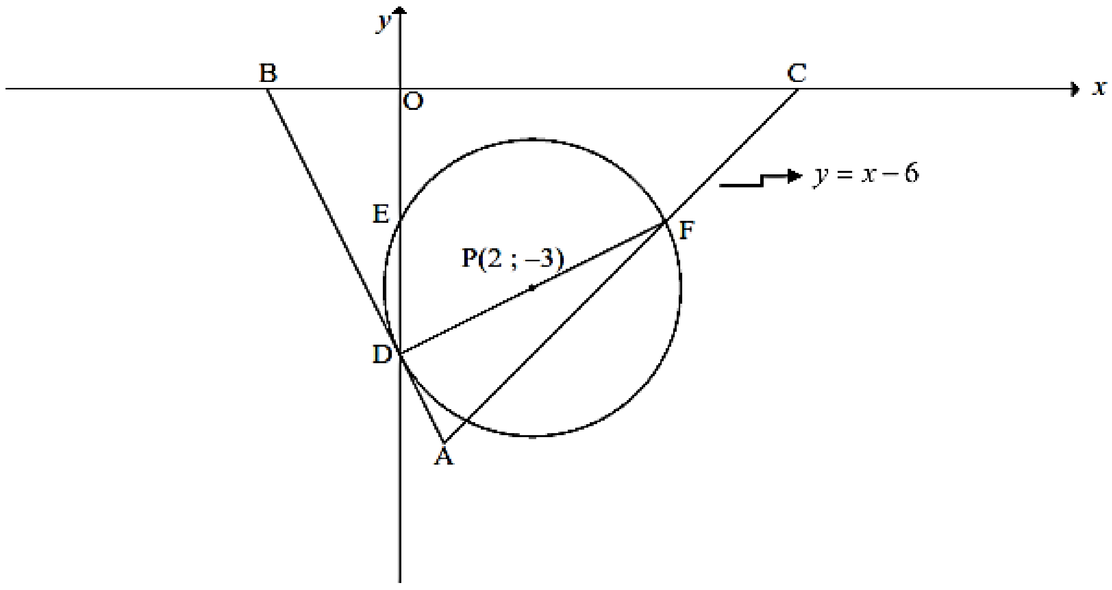
4.5 Another tangent to the circle with centre O, drawn from A, touches the circle at C. And C is the reflection of Q by the y axis.
Determine the length of CQ. (2)

[16]

NC

QUESTION 4

In the diagram, P(2 ; –3) is the centre of the circle with equation $x^2-4x+y^2+6y+8=0$, passing through D, E and F. D and E are y-intercepts of the circle. The equation of AC is $y=x-6$. AB is a tangent to the circle at D. B and C are points on the x-axis.



4.1 Calculate the coordinates of:

4.1.1 D and E (3)

4.1.2 F (2)

4.2 Write the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

4.3 Determine the equation of AB in the form $y = mx + c$. (4)

4.4 Calculate the:

4.4.1 size of $B\hat{D}E$ (3)

4.4.2 coordinates of A (4)

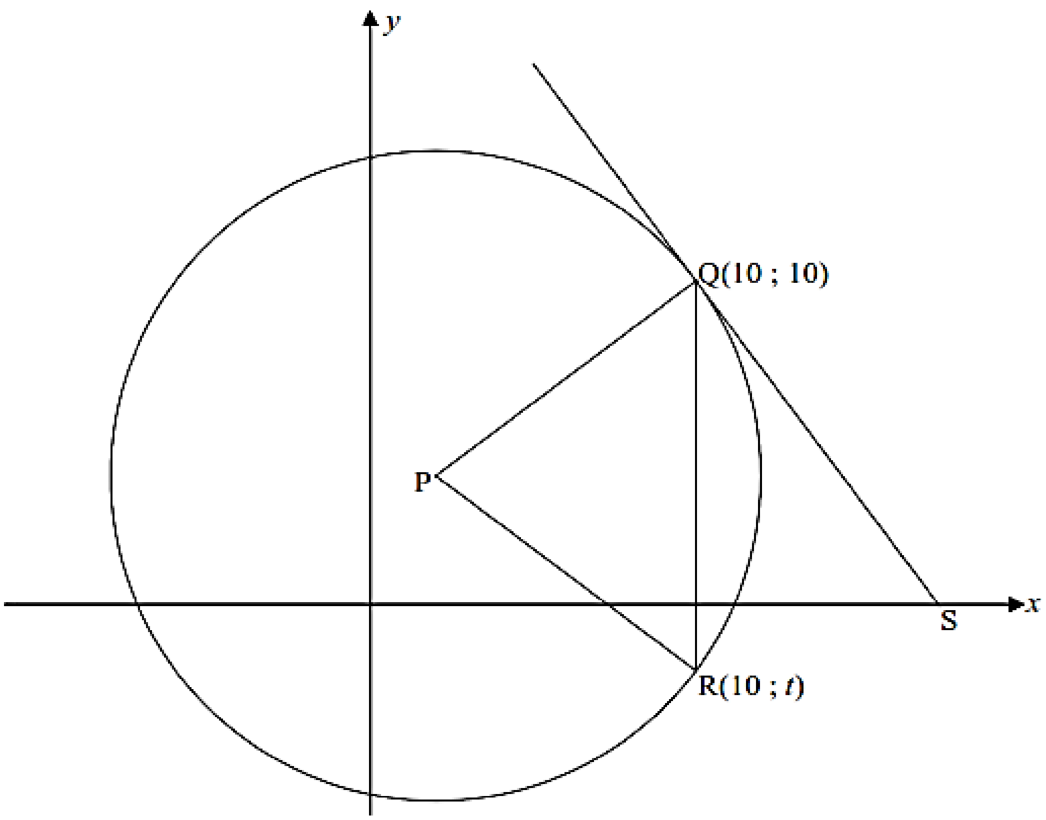
4.5 Circle P is translated such that the new centre has coordinates $(x + k\ ;\ -3)$. Determine all possible values of k for which the translated circle will never intersect the y-axis. (4)

[23]

NW

QUESTION 4

In the diagram below, the circle with centre P has the equation $x^2 - 4x + y^2 - 8y = 80$. QS is a tangent that touches the circle at Q and intersects the x-axis at S. The point R(10 ; t) lies on the circle.



4.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (2)

4.2 Write down the:

4.2.1 Coordinates of P (2)

4.2.2 Equation of QR (1)

4.3 Determine the equation of the tangent QS. (5)

4.4 Calculate the size of $\hat{RQS}$. (3)

4.5 Calculate the area of Δ PQR. (4)

4.6 A function h is formed by restricting the range of the circle to $y \geq 4$.

If $\sum_{x=-8}^{2} h(x)=k$, determine the value of $\sum_{x=3}^{12} h(x)$ in terms of k. (4)

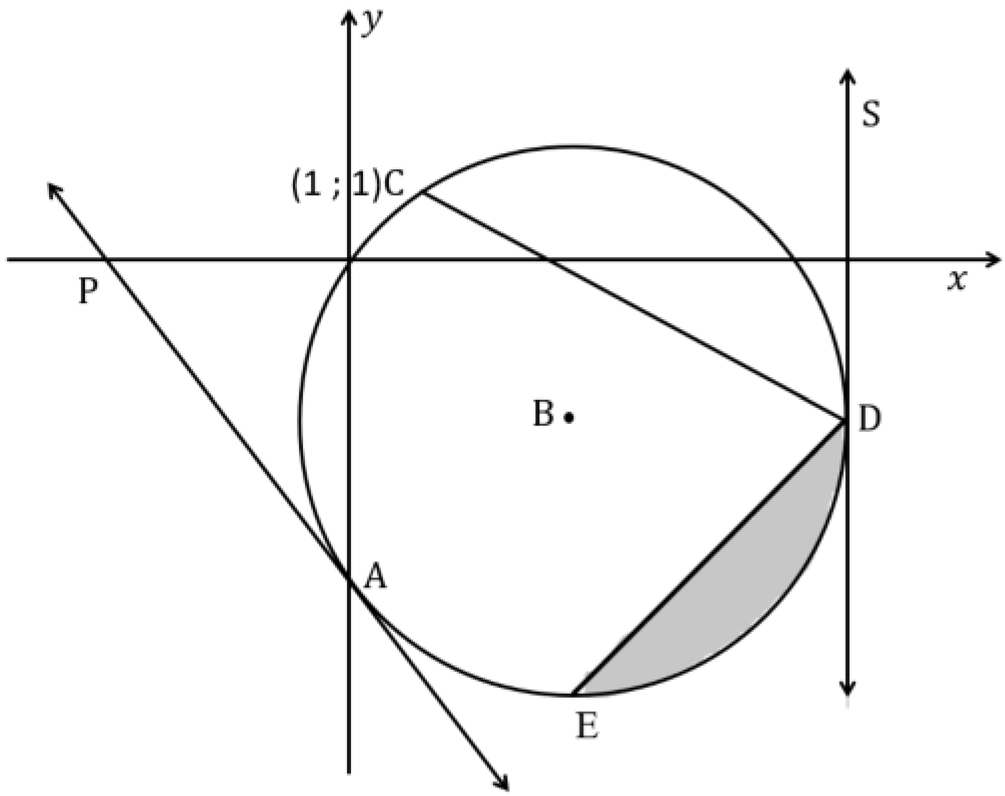
[21]

WC

QUESTION 4

In the diagram, the equation of the circle centred at B is given as: $x^2-8x+y^2+6y=0$

- DS and PA are tangents to the circle at D and A respectively.
- A is the y-intercept of the circle.
- DS is parallel to the y – axis.
- $C(1;1)$ is a point on the circumference of the circle.
- CD and DE are chords of the circle.
- BE, if drawn, is parallel to DS.
- The smaller segment of the circle on chord DE is shaded.



4.1 Show that the coordinates of B is $(4\,;\,-3)$. Show all your calculations. (3)

4.2 Determine:

4.2.1 the equation of the tangent DS. (2)

4.2.2 the length of chord CD. (3)

4.2.3 the equation of the tangent PA in the form $y = mx + c$. (5)

4.2.4 the area of the shaded segment. (3)

4.3 Another circle with center M and equation $(x+1)^2 + (y-2)^2 = 8$ is drawn. Determine, with an explanation, whether the two circles will intersect or not. (3)
[19]

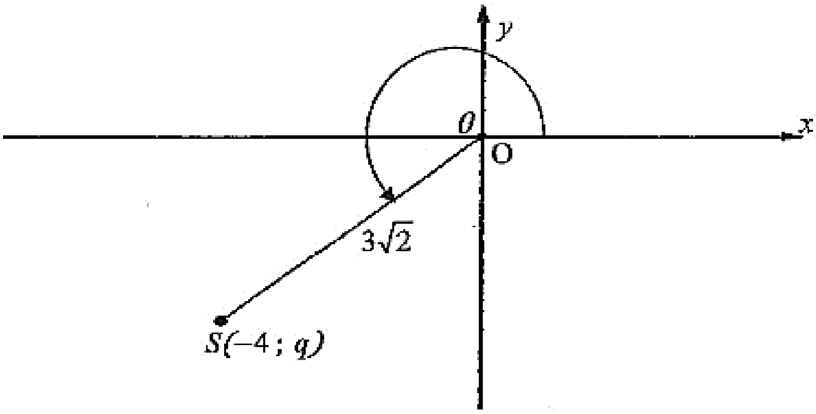
TRIGONOMETRY

A. WITH/ WITHOUT A SKETCH; IDENTITIES; PROOFS; GENERAL SOLUTION

EC

QUESTION 5

5.1 In the diagram below, point $S(-4\,;q)$ and reflex angle θ are shown. O is the point at the origin. $OS = 3\sqrt{2}$.



Without using a calculator, determine the value of:

5.1.1 q (2)

5.1.2 $\sin(\theta + 45°)$ (4)

5.1.3 $\cos(2\theta - 360°)$ (4)

5.2 Simplify the following without using a calculator:

$$\frac{\sin(90^\circ-\theta).\cos 480^\circ+\cos(180^\circ-\theta)}{\cos\theta.\sin 150^\circ-\tan 180^\circ}$$ (5)

5.3 Prove that $\frac{\cos x}{\sin 2x}-\frac{\cos 2x}{2\sin x}=\sin x$ (5)

5.4 Given: $\frac{\cos 60^\circ}{\sin x}-\frac{\sin 60^\circ}{\cos x}=2$

5.4.1 Show that the equation $\frac{\cos 60^\circ}{\sin x}-\frac{\sin 60^\circ}{\cos x}=2$ can be written as $\cos(x+60^\circ)=\cos(90^\circ-2x)$ (3)

5.4.2 Hence, or otherwise, determine the general solution of $\frac{\cos 60^\circ}{\sin x}-\frac{\sin 60^\circ}{\cos x}=2$ (4)

5.5 Given that $\cos 22{,}5^\circ=\frac{a}{c}$ and $a^2+b^2=c^2$.

With the aid of a diagram, or otherwise, show that $\frac{2ab}{c^2}=\frac{\sqrt{2}}{2}$. (5)

[32]

FS

QUESTION 5

5.1 If $\sqrt{5}\sin\theta + 2 = 0$ and $\theta \in [90°; 270°]$

Determine **without the use of a calculator** the value of the following:

5.1.1 $\tan\theta$ (2)

5.1.2 $\cos 2\theta$ (3)

5.2 **Without the use of a calculator,** simplify the following expression to ONE trigonometric ratio:

$$2\cos^2 15° - 1 + \frac{2\sin 140°}{\cos 310°}$$ (5)

5.3 If $\sin\frac{x}{2} = p$, express $\sin x - 1$ in terms of p. (4)

5.4 Prove the following:

$$\frac{3\sin x + 2\sin 2x}{2 + 3\cos x + 2\cos 2x} = \tan x$$ (4)

5.5 Show that: $\frac{\sin x + \cos x}{\sin x - \cos x} = \frac{p+t}{p-t}$ if $\tan x = \frac{p}{t}$ (5)

[23]

GP

QUESTION 5

5.1	Simplify $\dfrac{\sin^2(180^\circ+x).\sin(-x)}{-\sin(90^\circ+x).\tan x}-1$ to a single trigonometric term.	
		(6)
5.2	Given: $\cos(A-B)=\cos A\cos B+\sin A\sin B$	
5.2.1	Use the above identity to deduce that $\sin(A+B)=\sin A\cos B+\cos A\sin B$.	
		(3)
5.2.2	**Without using a calculator,** simplify the following: $\cos 420^\circ \cos 15^\circ + \sin 300^\circ \cos 105^\circ$	
		(5)

5.3	Given: $\tan^2 x\left(\frac{1}{\tan^2 x}-1\right)$	
5.3.1	Prove that $\tan^2 x\left(\frac{1}{\tan^2 x}-1\right)=\frac{\cos 2x}{\cos^2 x}$.	
		(3)
5.3.2	For what value(s) of x in the interval $x \in (0° ; 180°)$ is $\tan^2 x\left(\frac{1}{\tan^2 x}-1\right)$ undefined?	
		(1)

5.4	Determine the general solution of the equation $\cos 2x = \cos x$.	
		(6)

5.5	Given that $\sin\theta = \frac{1}{2}$, calculate the numerical value of $\sin 3\theta$, **without using a calculator.**	
		(7)
		[31]

KZN

QUESTION 5

5.1 If $\tan 58^\circ = n$, determine the following in terms of n without using a calculator.

5.1.1 $\sin 58^\circ$ (3)

5.1.2 $\sin 296^\circ$ (4)

5.1.3 $\cos 2^\circ$ (3)

5.2 Given the following identity:

$$\frac{1-\cos 2x}{\sin 2x} = \tan x$$

5.2.1 Prove the identity. (3)

5.2.2 Use the identity to determine the value of $\tan 15^\circ$ in its simplest form. **No calculator may be used.** (3)

5.3 Simplify to a single trigonometric ratio:

$$\sin(360^\circ + x).\cos(90^\circ + x) - \frac{\sin x}{\cos(-x).\tan(360^\circ - x)}$$ (6)

5.4 Determine the general solution of: $\cos 2x - \frac{1}{3} = \frac{1}{3}\sin x$ (6)

5.5 For which values of k will $\sin(2x + 30^\circ) + k = 3$ have no solution? (5)

[33]

MP

QUESTION 5

5.1 If $\cos 21° = p$ determine the following in terms of p.

5.1.1 $\tan 201°$ (3)

5.1.2 $\sin 42°$ (3)

5.1.3 $\cos 51°$ (3)

5.2 Simplify : $\dfrac{\sin 210°.\cos 510°}{\cos 315°.\sin(-135°)}$ (7)

5.3 Prove the identity:

$$\frac{\cos\theta - \cos 2\theta + 2}{3\sin\theta - \sin 2\theta} = \frac{1+\cos\theta}{\sin\theta}$$ (5)

5.4 Determine the general solution of $\sin\theta \sin\dfrac{3\theta}{2} + \cos\dfrac{3\theta}{2}\cos\theta = -\dfrac{\sqrt{3}}{2}$. (4)

5.5 Given: $\sin\theta.\cos\beta = -1$

5.5.1 Write down the maximum and minimum value of $\cos\beta$ (1)

5.5.2 Solve for $\theta \in [0°;270°]$ and $\beta \in [-180°;90°]$. (4)

[30]

NC

QUESTION 5

5.1 Simplify the following expression **without using a calculator.**

$\sin(90^\circ - x).\tan(360^\circ - x) - 2\sin(180^\circ + x)$ (5)

5.2 Calculate the value of the following expression **without using a calculator**.

$\sin(23^\circ + x)\cos(7^\circ - x) + \cos(23^\circ + x)\sin(7^\circ - x)$ (3)

5.3 If $\sin 2\theta = \frac{-4\sqrt{2}}{9}$, and $2\theta \in [90°; 270°]$, determine the value of the following, **without using a calculator**:

5.3.1 $\cos 2\theta$ (3)

5.3.2 $\sin \theta$ (4)

5.4 Determine the general solution of $2\sin\alpha\cos\alpha = 2\cos^2\alpha - 2\sin^2\alpha$ (7)

5.5 Consider: $\frac{\cos 2x + \cos^2 x + 3\sin^2 x}{2 - 2\sin^2 x} = \frac{1}{\cos^2 x}$

5.5.1 Prove the identity. (4)

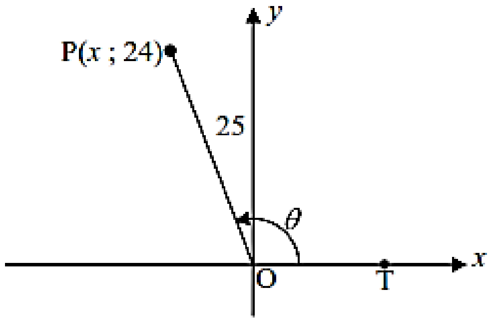
5.5.2 For which value(s) of x, $x \in [0°; 360°]$, will the identity be undefined? (2)

[28]

NW

QUESTION 6

6.1 In the diagram below, the point P(x ; 24) is 25 units from the origin O. T is a point on the x-axis such that $\text{T}\hat{\text{O}}\text{P} = \theta$.



6.1.1 Calculate the value of x. (2)

6.1.2 **Without using a calculator,** determine the value of $\tan(360° - \theta)$ (2)

6.1.3 Calculate the size of $\text{P}\hat{\text{O}}\text{T}$ (2)

6.2 **Without using a calculator,** calculate the value of the following expressions:

6.2.1 $\sin 20° + \cos 120°.\tan 405° + \cos 110°$ (4)

6.2.2 $\dfrac{\left(\sqrt{2}\cos 15° + 1\right)\left(\sqrt{2}\cos 15° - 1\right)\sin(-2x)}{4\sin x \cos x}$ (4)

6.3 If $\sin(x + y).\cos(x + y) = t$, express the following in terms of t:

$4\cos(90° - 2y).\cos 2x + 4\sin 2x.\cos(360° + 2y)$ (5)

6.4 Given: $\sin^2 x + \cos^2 x + \tan^2 x$

6.4.1 Prove that $\sin^2 x + \cos^2 x + \tan^2 x = \dfrac{1}{\cos^2 x}$ (3)

6.4.2 Attie, a grade 12 learner, argues that $\sqrt{\sin^2 x + \cos^2 x + \tan^2 x} \neq \dfrac{1}{\cos x}$, if $x \in (180° ; 270°)$.

Is Attie's argument correct? Motivate your answer. (2)

6.5 Given: $2^{2\sin^2 x} - 5.2^{\cos 2x} = -3$, $x \in (0° ; 90°)$

Show, **without using a calculator,** that $\sin x = \dfrac{1}{\sqrt{2}}$. (6)

[30]

WC

QUESTION 5

5.1 **Without using a calculator,** simplify the following expression completely:

$$\cos(-\theta) \times \sin(90° - \theta) \times (1 + \tan^2 \theta)$$ (5)

5.2 Given: $\cos(A + B) = \cos A.\cos B - \sin A.\sin B$

5.2.1 Use the formula for $\cos(A + B)$ to derive a formula for $\sin(A - B)$. (4)

5.2.2 **Without using a calculator,** show that:

$$\sin(x + 63°).\cos(x + 378°) + \cos(x + 63°).\cos(x + 108°) = \frac{1}{\sqrt{2}}$$ (5)

5.3 Given that $\sin 10° = \sqrt{k}$

Without using a calculator, write each of the following in terms of k:

5.3.1 $\sin 190°$ (2)

5.3.2 $\cos 20°$ (3)

5.3.3 $\cos 50°$ (4)

5.4 Given:

$$\frac{1}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)} - \frac{(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)}$$

5.4.1 Simplify the expression above to a single trigonometry ratio. (5)

5.4.2 Determine the general solution of θ for which this expression will be undefined? (3)

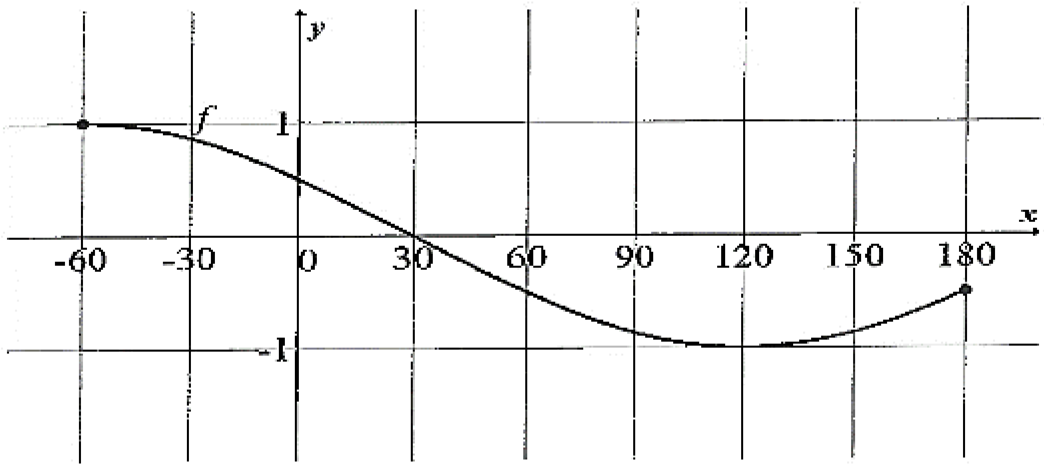
[31]

B. GRAPHS

EC

QUESTION 6

The graph of $f(x) = -\sin(x - 30°)$ is drawn in the interval of $x \in [-60°; 180°]$.



Use the graph to answer the following questions.

6.1 Write down the period of f. (1)

6.2 Write down the minimum value of f. (1)

6.3 Determine the range of $f(x) + 1$. (2)

6.4 For which values of x is the graph of f increasing, where $x \in [-60°; 180°]$? (2)

6.5 The graph of f is shifted $60°$ to the right and then reflected in the x-axis to form a new graph of g. Determine the equation of g in its simplest form. (3)

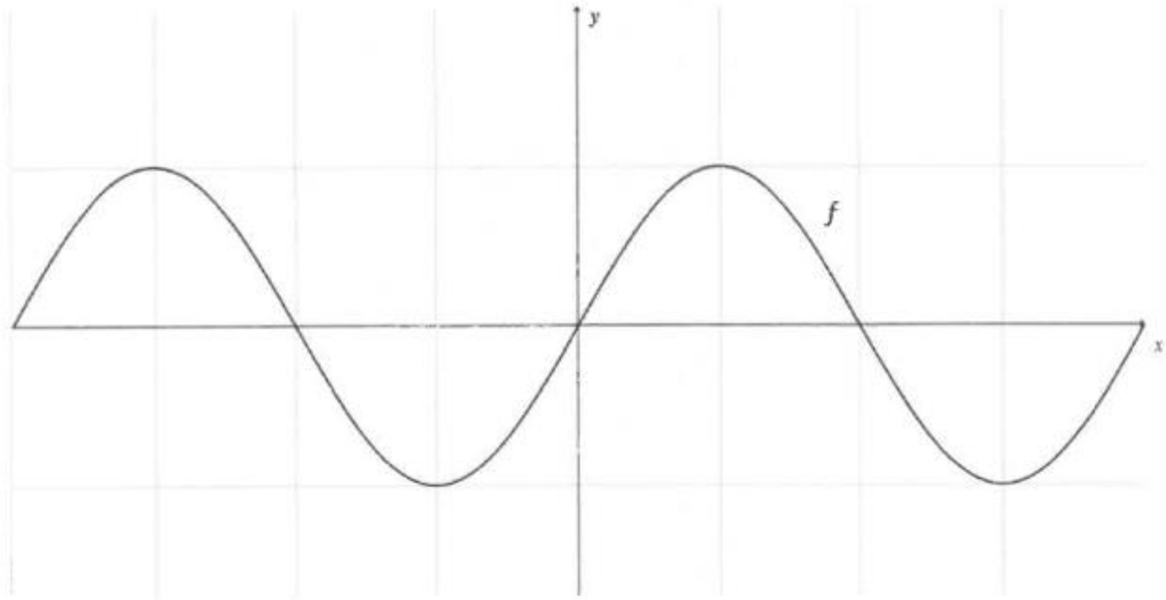
6.6 Draw the graph of g on the same set of axes. Clearly show the intercepts with the axis and the turning points in the interval of $x \in [-60°; 180°]$. (3)

[12]

FS

QUESTION 6

In the diagram below, the graph of $f(x) = \sin 2x$ is drawn for the interval $x \in [-180° ; 180°]$.



6.1 Sketch the graph of $g(x) = \cos(x - 45°)$ on the same set of axes in the answer book. (3)

6.2 Determine the values of x in the interval $x \in [0°; 180°]$ for which:

6.2.1 $f(x) = g(x)$ (7)

6.2.2 $f(x + 30°) = g(x + 30°)$ (2)

6.2.3 $f(x) > g(x)$ (2)

6.3 Write down the period of g. (1)

6.4 Show how the graphs of f and g can be used to solve

$\sqrt{2} \sin 2x = \cos x + \sin x$. (3)

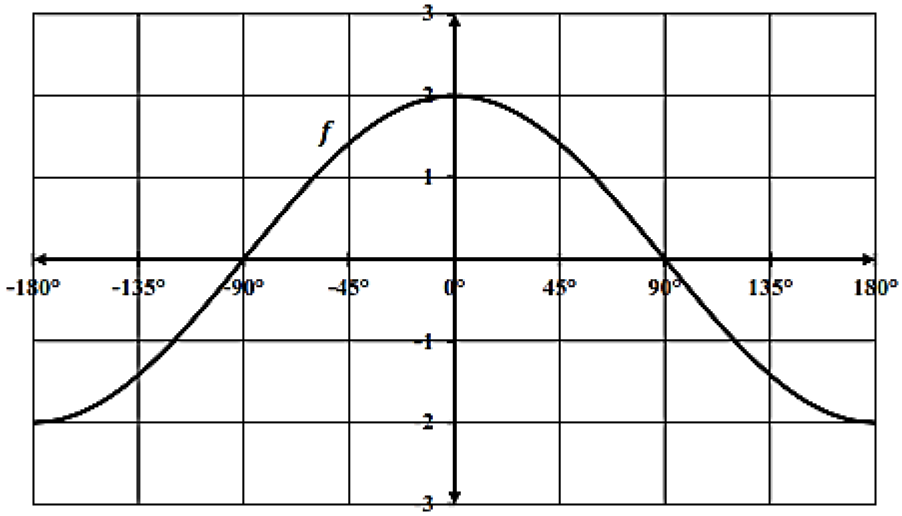
[18]

GP

QUESTION 6

In the diagram below, the graph of $f(x) = 2\cos x$ is drawn for the interval $x \in [-180°;180°]$.

6.1	On the grid below, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180°\ ;\ 180°]$. Clearly show all intercepts with the axes and the turning point(s) of the graph.	



(3)

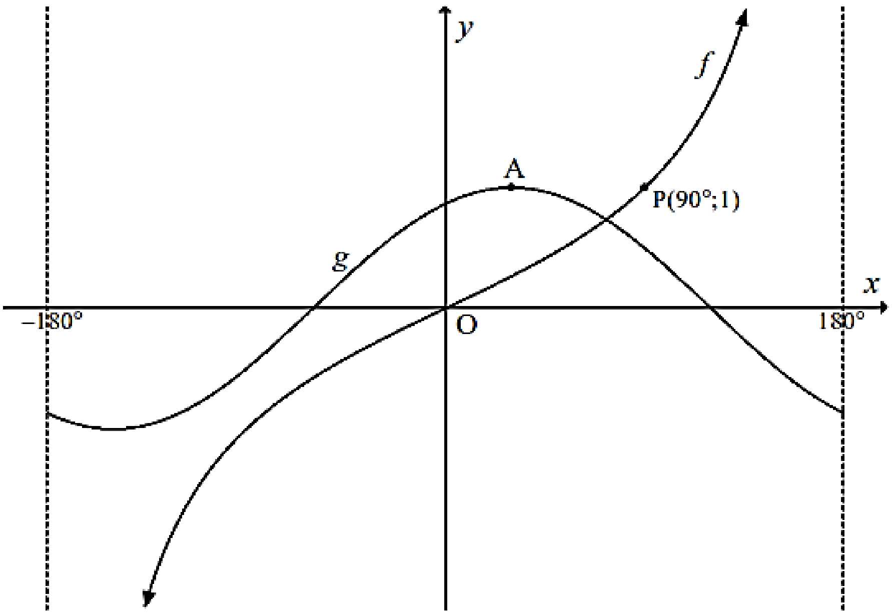
6.2	Write down the period of $2g(x+10°)$.	(1)
6.3	Use the graphs to determine the value(s) of x in the interval $x \in [0°;180°]$ for which $f(x).g(x) \leq 0$.	(2)
6.4	Write down the maximum value of $f(x) - g(x)$.	(1)

6.5	Determine the range of $y = 2^{2\cos x + 2}$	
		(2)
		[9]

KZN

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan bx$ and $g(x) = \cos(x - 30°)$ are drawn on the same system of axes for $-180° \leq x \leq 180°$.
The point $P(90° ; 1)$ lies on f.



Use the diagram to answer the following questions:

6.1	Determine the value of b.	(1)
6.2	Write down the period of g.	(1)
6.3	Write down the coordinates of A, a turning point of g.	(2)
6.4	Write down the equation(s) of the asymptote(s) of $y = \tan b(x + 20°)$ for $x \in [-180°;180°]$.	(1)
6.5	Determine the range of h if $h(x) = 2g(x) - 1$.	(2)
		[7]

MP

QUESTION 6

Given that $y = f(x) = 2\cos x$ and $y = g(x) = \sin(x + 30^\circ)$:

6.1 Sketch the graphs of f and g on the ANSWER BOOK on the same set of axes for $x \in [-180^\circ; 180^\circ]$. (6)

6.2 Read the following answers from your graph and show where these answers have been read:

6.2.1 Write down the period of f. (1)

6.2.2 Determine one value of x for which $f(x) - g(x) = 1{,}5$. (1)

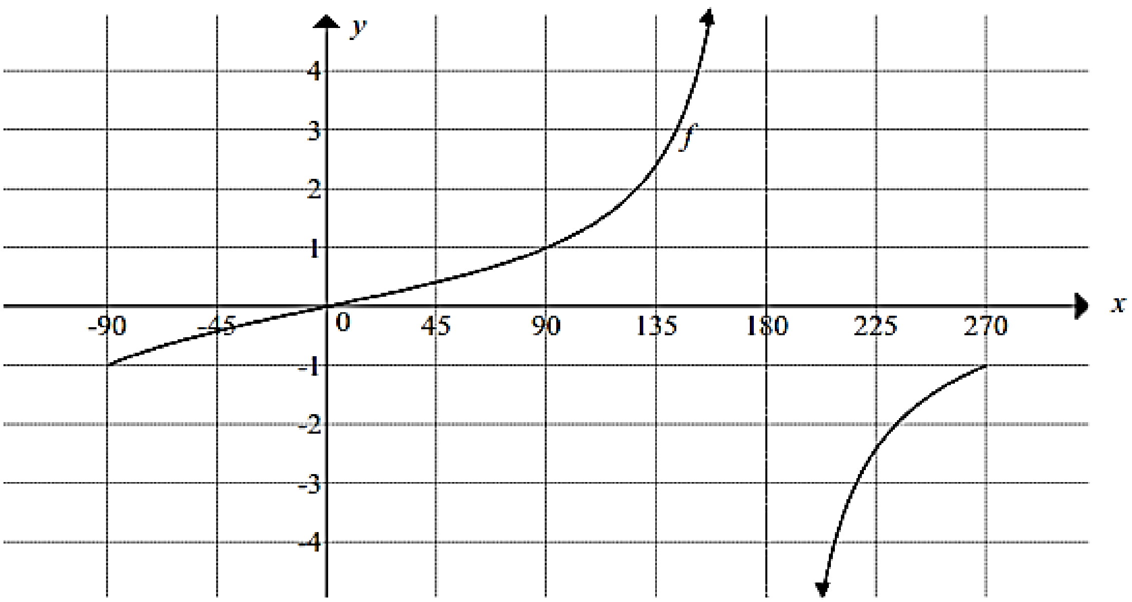
6.2.3 Determine the positive values of x for which $2\sin(x + 30^\circ).\cos x < 0$ (2)

[10]

NC

QUESTION 6

The graph of $f(x) = \tan\frac{1}{2}x$ for the interval $x \in [-90^\circ; 270^\circ]$, is drawn below.

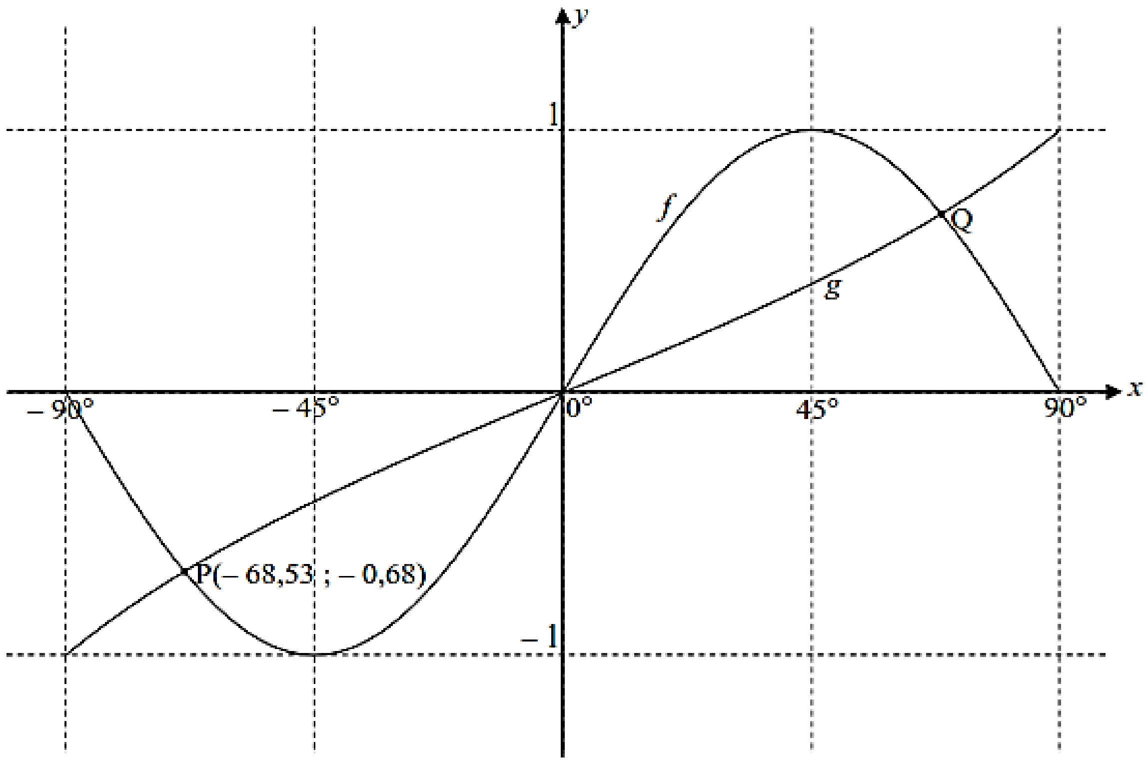


6.1	Write down the period of f.	**(1)**
6.2	Write down the equation of the asymptote of f in the interval $x \in [-90°; 270°]$.	**(1)**
6.3	On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = 2\cos x$ for the interval $x \in [-90°; 270°]$. Clearly show all intercepts with the axes, turning points and end points.	**(3)**
6.4	Give 2 values of x, in the interval $x \in [-90°; 270°]$, for which $g(x) - f(x) = 1$.	**(1)**
6.5	Write down the range of $g(x) - 3$ in the interval $x \in [-90°; 270°]$.	**(2)**
6.6	Determine the maximum value of $[5 - 2\sin(90° - x)]^2$ for $x \in R$.	**(3)**
		[11]

NW

QUESTION 5

In the diagram below, the graphs of $f(x) = a\sin 2x$ and $g(x) = \tan bx$ for $x \in [-90° ; 90°]$ are drawn. P(– 68,53 ; – 0,68) and Q are points of intersection of f and g.



5.1 Write down the:

5.1.1 Value of a (1)

5.1.2 Coordinates of Q (2)

5.1.3 x values of the turning points of h, if $h(x) = f(x + 30°)$ (2)

5.1.4 Value(s) of x where $-0,68 < g(x) \leq 1$ (2)

5.1.5 Value of m where $f(x + m) = -\cos 2x$ (2)

5.1.6 Value of b (1)

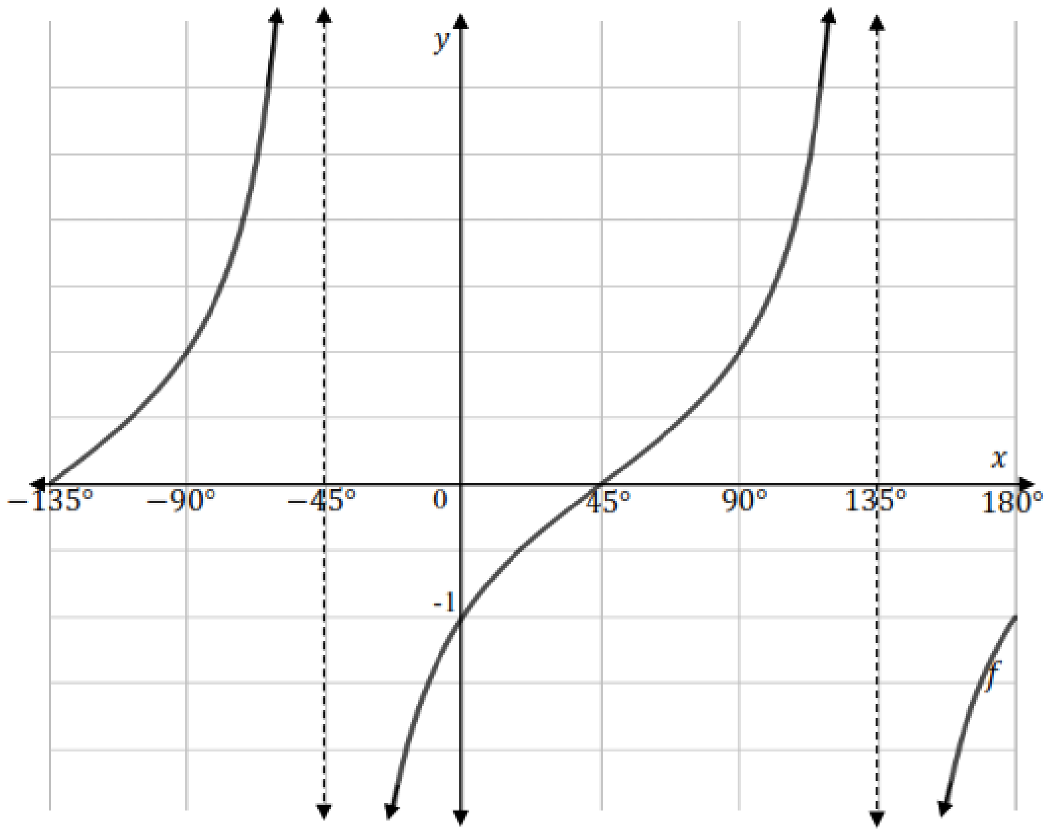
5.2 For which value(s) of x, in the given interval, will $x \cdot \sqrt{g(x) - f(x)} > 0$? (2)

[12]

WC

QUESTION 6

In the diagram, the graph of $f(x) = \tan(x + p°)$ is drawn for the interval $x \in [-135°; 180°]$ with asymptotes at $x = -45°$ and $x = 135°$.



6.1 Write down the value of p. (1)

6.2 Draw the graph of $g(x) = \sin 2x$ for the interval $x \in [-135° ; 180°]$ on the grid given in the ANSWER BOOK. Show ALL intercepts with axes, as well as the minimum and maximum points on the graph. (3)

6.3 Write down the period of g. (1)

6.4 The graph of g is shifted $45°$ to the left to form the graph of h. Determine the equation of h in the simplest form. (2)

6.5 Use the graph to determine the values of x in the interval $x \in [-135° ; 0°]$ for which:

6.5.1 $f(x) \leq -1$ (2)

6.5.2 $\sin x \cos x + 2 < 2$ (3)

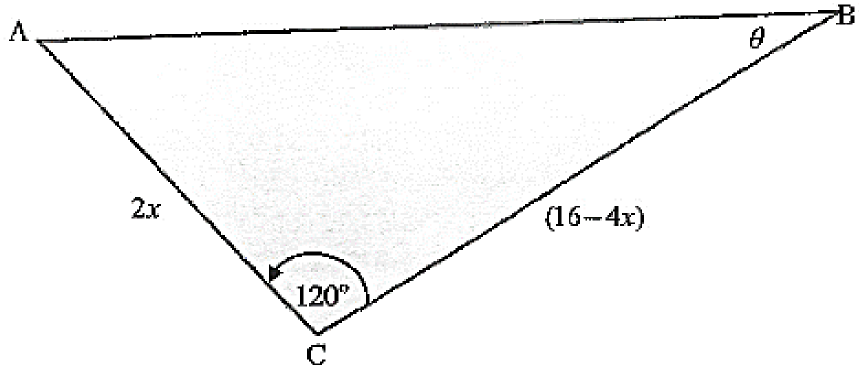
[12]

C. 2D & 3D(SINE-COSINE- & AREA RULE

EC

QUESTION 7

In ΔABC below, AC = $2x$, BC = $(16-4x)$, $\hat{C}=120^\circ$, $\hat{B}=\theta$.



7.1 Determine the area of ΔABC in terms of x, without using a calculator. (3)

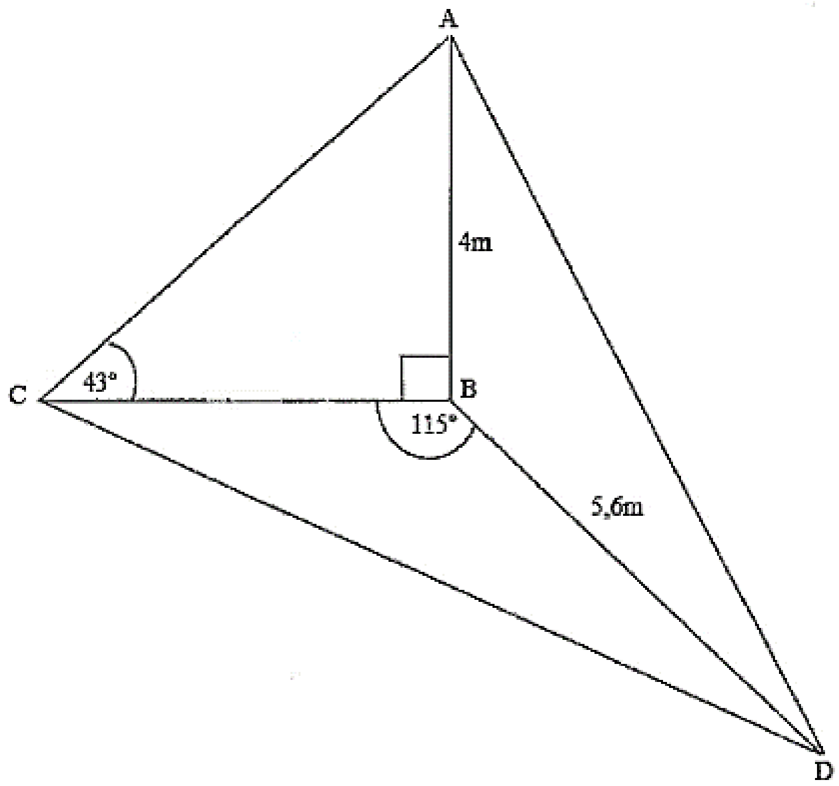
7.2 For which value(s) of x will the area of ΔABC be a maximum? (3)

[6]

FS

QUESTION 7

BCD are points in the same horizontal plane. AB is a vertical pole of length 4 m, BD = 5,6 m, $C\hat{B}D = 115°$ and the angle of elevation of A from C is 43°.



7.1 Calculate the length of CB (3)

7.2 Calculate the length of CD (3)

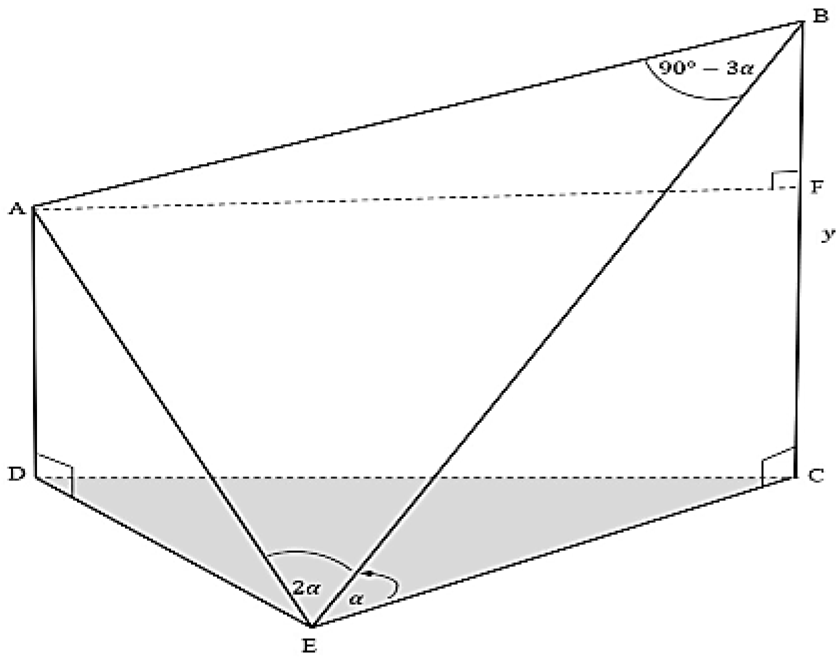
7.3 Determine the area of ΔBCD (3)

[9]

GP

QUESTION 7

The diagram below shows two vertical poles, AD and BC. Point E lies on the same horizontal plane as bases D and C of poles AD and BC.
$A\hat{E}B = 2\alpha$; $B\hat{E}C = \alpha$; $A\hat{B}E = 90° - 3\alpha$ and $BC = y$ metres



7.1	Determine BE in terms of α and y.	
		(2)

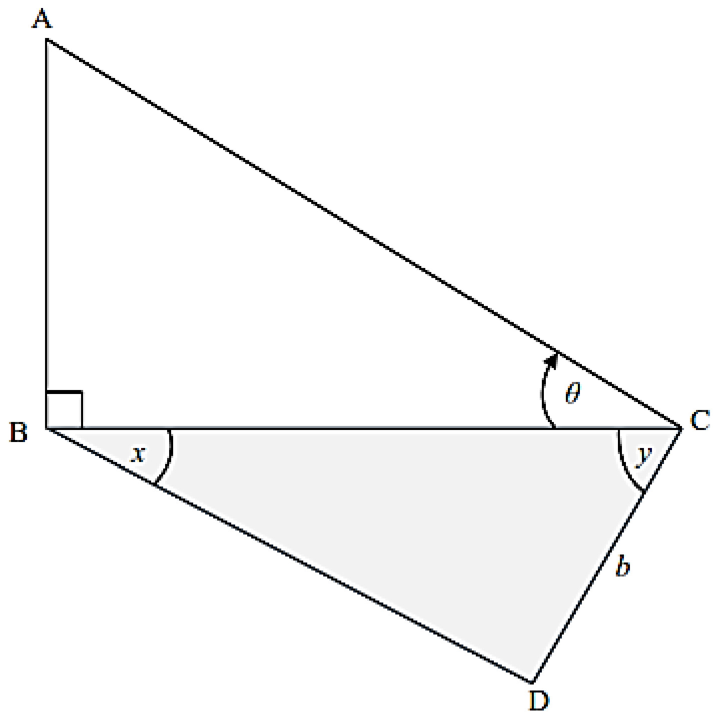
7.2	Show that AB = $2y$.	
		(5)

7.3	It is further given that $AF = \frac{7}{4} BC$. Determine $B\hat{A}F$, the angle of elevation of B from A. Give your answer to the nearest degree.	
		(3)
		[10]

KZN

QUESTION 7

In the diagram, B, C and D lie in the same horizontal plane.
BD = 2CD.
$C\hat{B}D = x$, $B\hat{C}D = y$ and CD = b meters.
AB is a vertical tower.
The angle of elevation of A from C is θ.



7.1 Show that $\sin y = 2\sin x$. (2)

7.2 Prove that $AB = b\tan\theta\sqrt{5+4\cos(x+y)}$ (7)

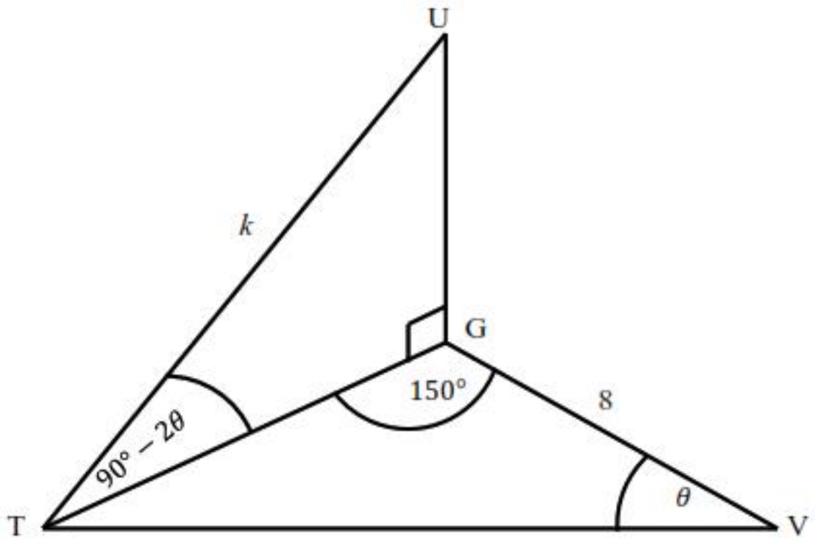
7.3 Hence, determine the height of the tower, rounded off to two decimal places, if:
$b = 54{,}8$ metres, $x = 31°$, $\theta = 42{,}6°$ and $y = 75{,}84°$. (2)

[11]

MP

QUESTION 7

A mouse on the ground at point T is looking up to an owl in a tree at point U and a cat to his right on the ground at point V. The angle of elevation from the mouse to the owl is $(90° - 2\theta)$. $TU = k\ units$, $GV = 8\ units$, $T\hat{G}V = 150°$ and $T\hat{V}G = \theta$.

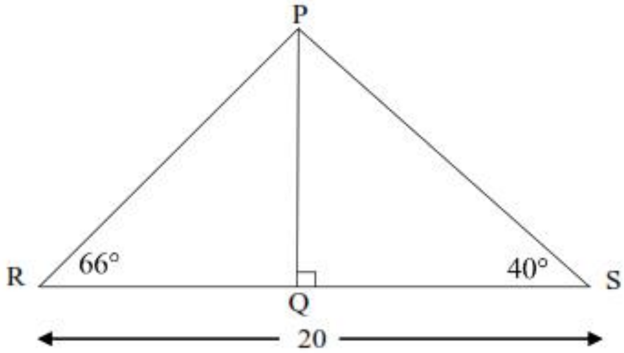


7.1	Write down the size of $T\hat{U}V$ in terms of θ	(1)
7.2	Show that $TG = k\sin 2\theta$.	(2)
7.3	Show that $TV = k\cos\theta$	(4)
7.4	Show that the area of $\Delta TGV = 2k\sin 2\theta$.	(2)
		[9]

NC

QUESTION 7

7.1 In the diagram below, ΔPRS is drawn with RS = 20 units, $\hat{R} = 66°$ and $\hat{S} = 40°$. Q is a point on RS such that $PQ \perp RS$.

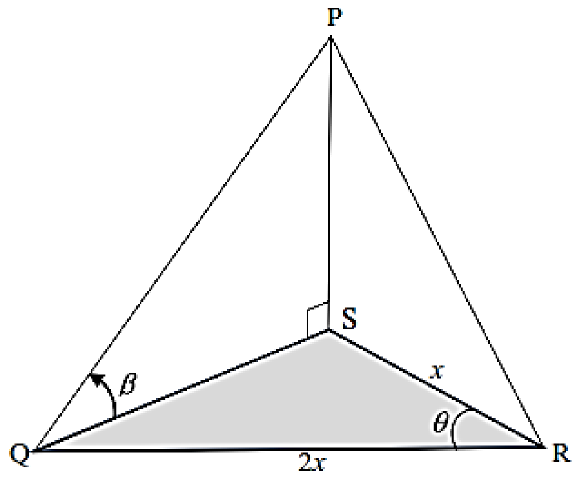


Calculate the length of:

7.1.1 PS (3)

7.1.2 PQ (2)

7.2 In the diagram, PS is a vertical flagpole. Q, R and S lie in the same horizontal plane. PQ and PR are two cables, anchored at Q and R. $P\hat{Q}S = \beta$ and $Q\hat{R}S = \theta$.
$SR = x$ and $QR = 2x$.



7.2.1 Show that $PQ = \frac{x\sqrt{5 - 4\cos\theta}}{\cos\beta}$ (4)

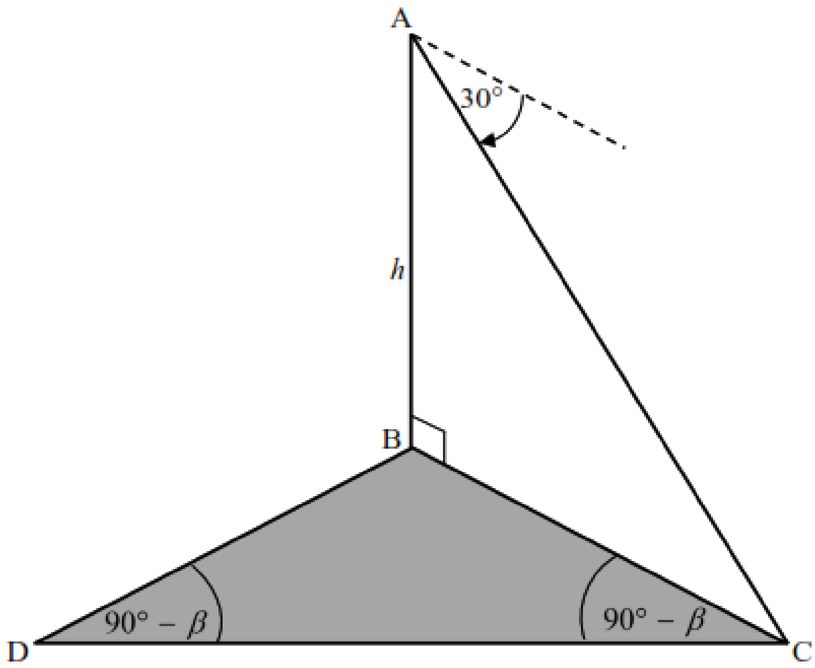
7.2.2 The area of ΔQRS is $57,36 m^2$ and $\theta = 35°$. Calculate the value of x. (2)

[11]

NW

QUESTION 7

In the diagram, D, B and C lie in the same horizontal plane. AB is a vertical tower. The angle of depression from A to C is 30°. $B\hat{C}D = B\hat{D}C = 90° - \beta$ and AB = h metres.



7.1 Determine the length of BC in terms of h. (2)

7.2 Write down the size of $D\hat{B}C$ in terms of β. (1)

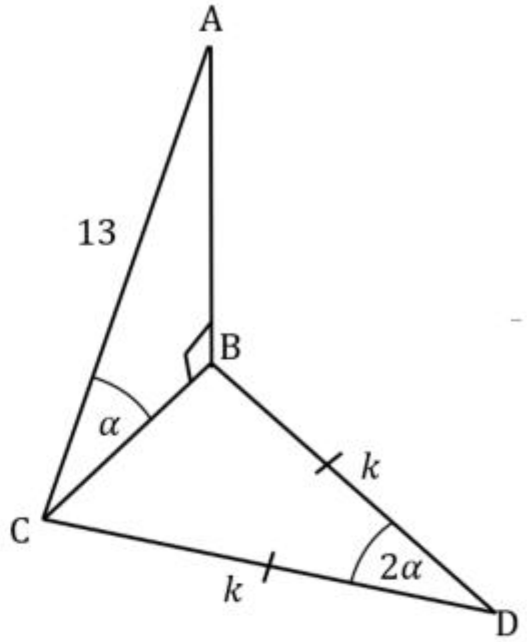
7.3 Show that $DC = \sqrt{12}\, h \sin\beta$ (5)

[8]

WC

QUESTION 7

In the diagram below, ABC is a vertical triangular wall on the floor CBD.
CA = 13 meters, CD = BD = k meters, $A\hat{C}B = \alpha$ and $B\hat{D}C = 2\alpha$



7.1 Show that:

7.1.1 $CB = 13\cos\alpha$ (1)

7.1.2 $k = \dfrac{13}{2\tan\alpha}$ (4)

7.2 Calculate the area of the floor ΔBCD if $\alpha = 26°$ (2)
[7]

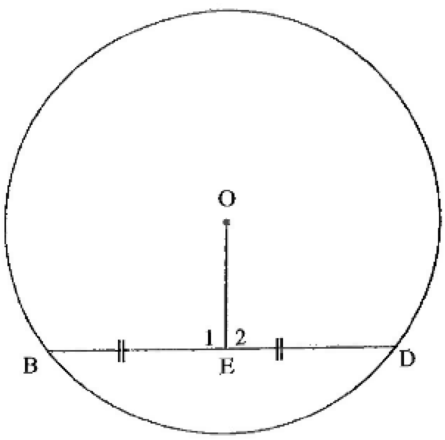
EUCLIDEAN GEOMETRY

A. PROOFS

EC

QUESTION 8

8.1 In the diagram below, O is the centre of the circle. BD is the chord of the circle. E is the midpoint of chord BD.



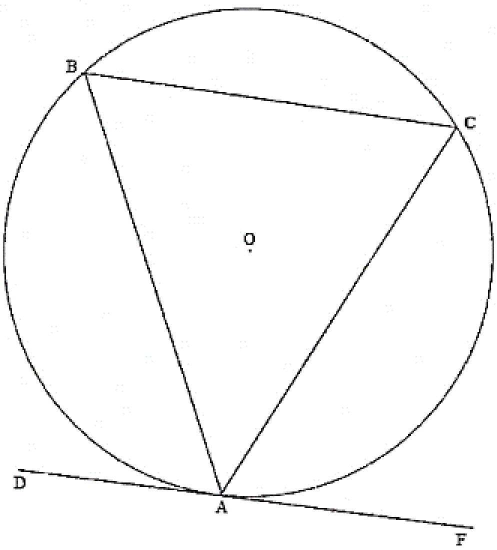
Use the diagram provided in the ANSWER BOOK to prove the theorem which states that: The line drawn from the centre of a circle that bisects a chord is perpendicular to the chord.

In other words, prove that: $OE \perp BD$. (5)

FS

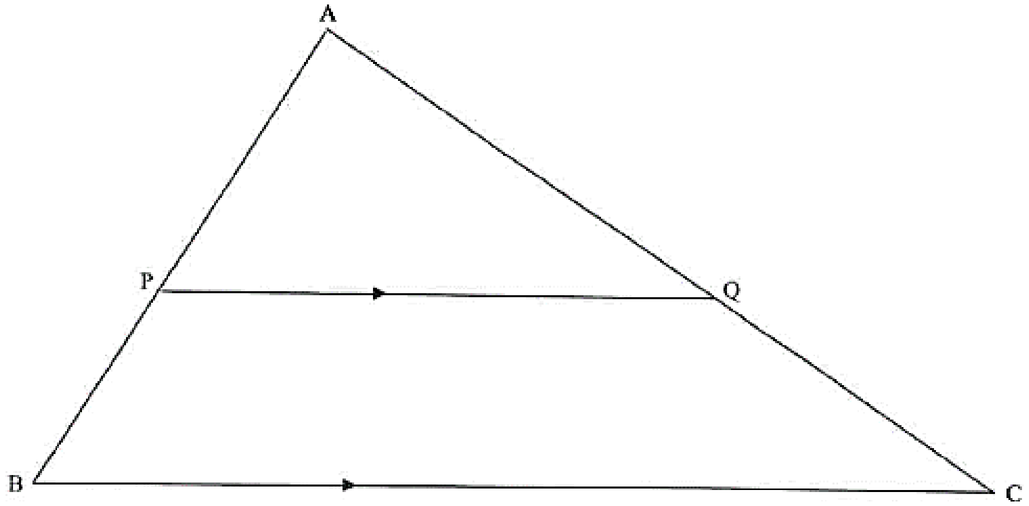
QUESTION 8

8.1 In the diagram, chords AB, BC and CA are drawn in the circle with centre O. DAF is a tangent to the circle at A.



Prove the theorem which states that $C\hat{A}F = A\hat{B}C$ (5)

9.2 In the diagram, ΔABC is drawn with PQ ∥ BC and AP : AB = 4 : 7



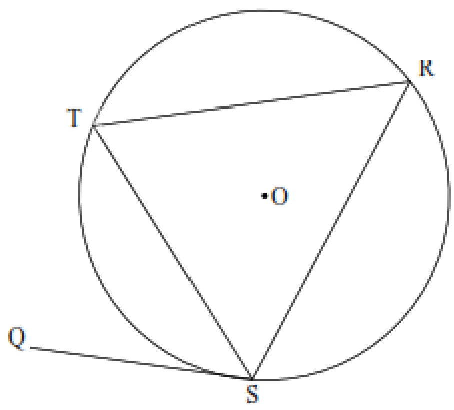
Determine $\dfrac{\text{Area } \Delta APQ}{\text{Area } \Delta ABC}$ (5)

[14]

GP

QUESTION 9

9.1 In the diagram below, O is the centre of the circle. Points S, T and R lie on the circle. QS is a tangent to the circle at S.

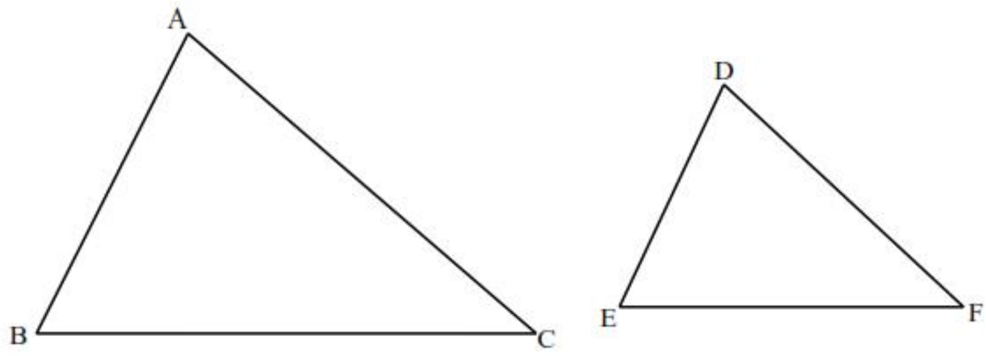


Use the diagram above to prove that the angle between the tangent and the chord is equal to the angle in the alternate segment, that is, prove that $Q\hat{S}T = \hat{R}$.	

KZN

QUESTION 10

10.1 In the diagram below, ΔABC and ΔDEF are drawn with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

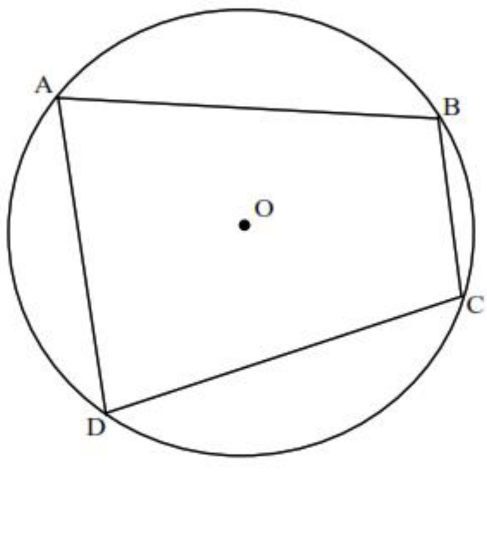


Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. $\frac{AB}{DE} = \frac{AC}{DF}$. (6)

MP

QUESTION 8

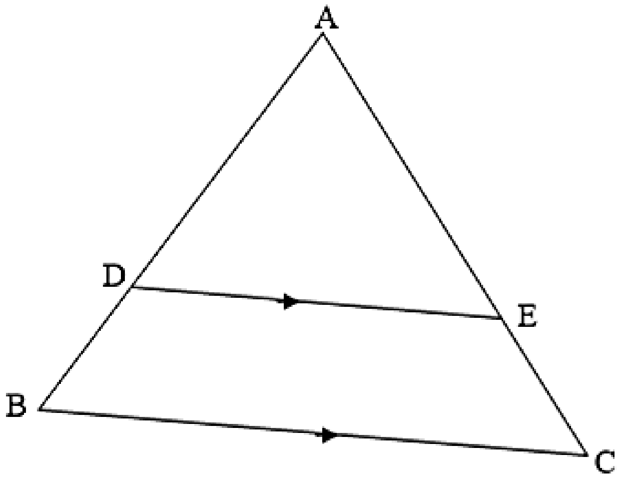
8.1 A, B, C and D are points on the circumference of the circle centre O.
Prove the theorem which states that $\hat{A} + \hat{C} = 180°$. (5)



NC

QUESTION 9

9.1 In the diagram, ΔABC is drawn. D and E are points on AB and AC respectively such that DE || BC.

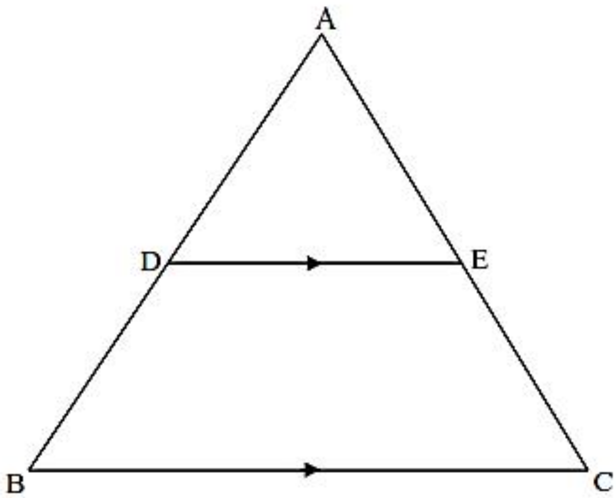


Use the diagram in the ANSWER BOOK to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e. prove that $\frac{AD}{DB} = \frac{AE}{EC}$. (5)

NW

QUESTION 9

9.1 In the diagram is Δ ABC with DE||BC.



Prove the theorem that states that $\frac{AD}{DB} = \frac{AE}{EC}$. (5)

WC

QUESTION 8

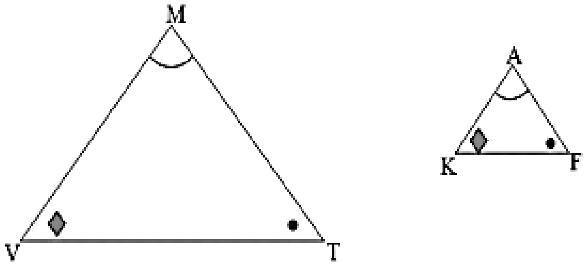
8.1 Complete the following theorem:

The angle at the centre of a circle is twice the angle ______________________ subtended by _________________.

. (2)

QUESTION 10

10.1 In the diagram, ΔMVT and ΔAKF are drawn such that $\hat{M} = \hat{A}$, $\hat{V} = \hat{K}$ and $\hat{T} = \hat{F}$.

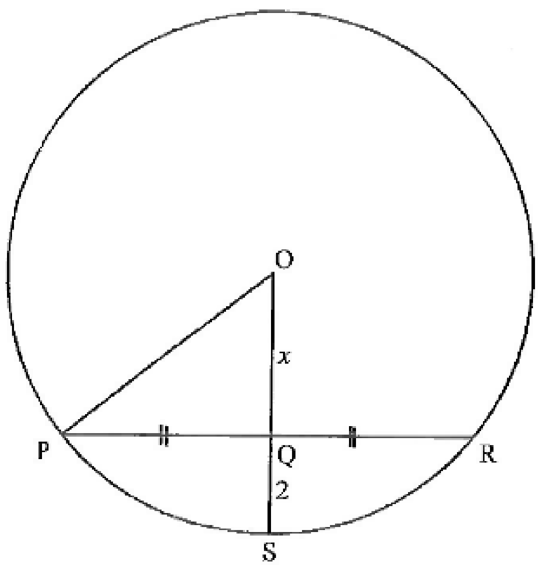


Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is prove that $\frac{MV}{AK} = \frac{MT}{AF}$.

B. GR11-CIRCLES

EC

8.2 In the diagram below, O is the centre of the circle. Q is the midpoint of chord PR. OQS is the radius of the circle. PR = 8 units, OQ = x units and QS = 2 units.



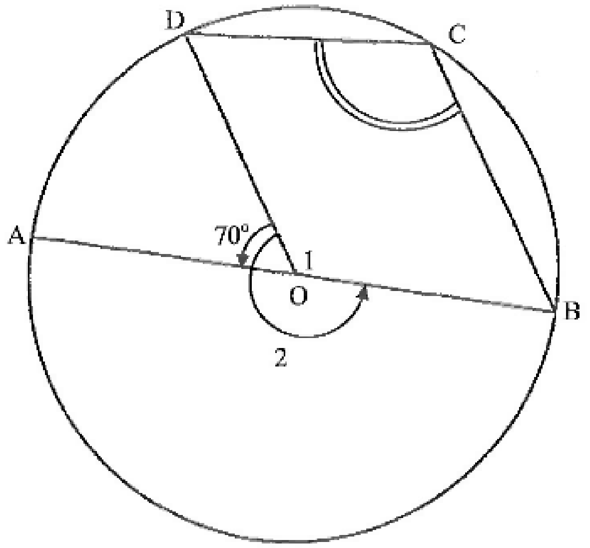
8.2.1 Determine, giving reasons, the size of $O\hat{Q}P$. (2)

8.2.2 Calculate the length of PO. (5)

[12]

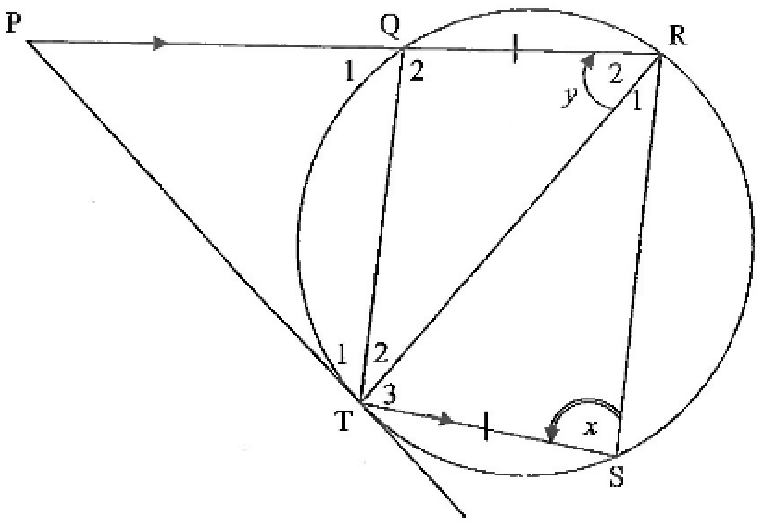
QUESTION 9

9.1 A, B, C and D are points on the circumference of the circle with centre O. AOB is the diameter of the circle. $A\hat{O}D = 70°$.



Calculate the size of $\hat{C}$, giving reasons. (5)

9.2 PT is a tangent to the circle at T. PR∥TS and PQR is a straight line. Q, R and S are points on the circumference of the circle. $\hat{R}_2 = y$ and $\hat{S} = x$. QR = TS.



9.2.1 Name, giving reasons, TWO other angles each equal to y. (4)

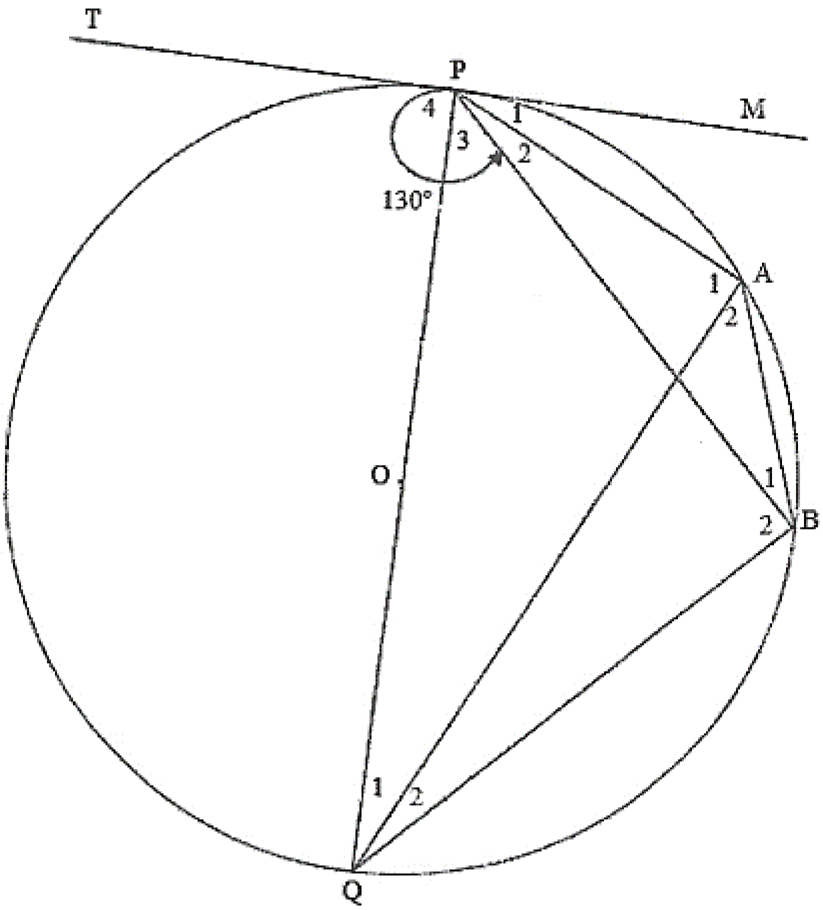
9.2.2 Determine, giving reason, another angle which is equal to $\hat{T}_2$. (2)

9.2.3 Prove that TR is the diameter of the circle. (4)

[15]

FS

8.2 In the diagram, TPM is a tangent to a circle with centre O and $T\hat{P}B = 130°$. POQ is a diameter and ABQP is a cyclic quadrilateral.



Determine with reasons the size of the following:

8.2.1 $Q\hat{P}B$ (3)

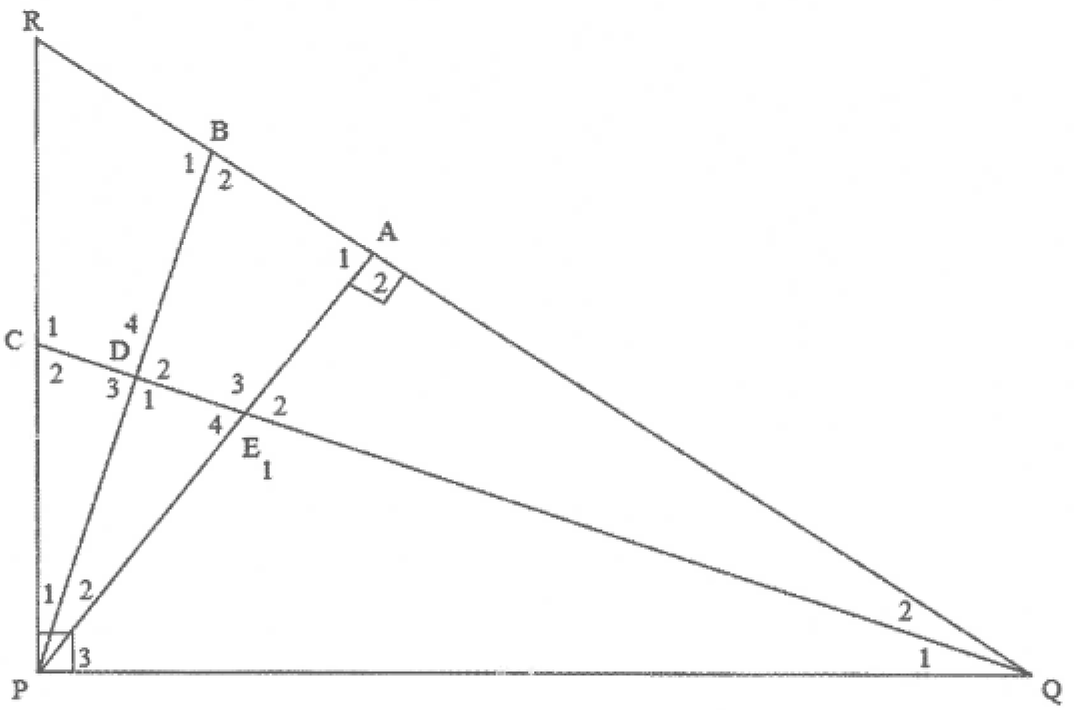
8.2.2 $P\hat{Q}B$ (3)

8.2.3 $P\hat{A}B$ (2)

[13]

QUESTION 9

9.1 In the diagram, QRP is a triangle with RP ⊥ QP and PA ⊥ QR. QC intersects PA and PB at E and D respectively. $\widehat{Q}_1 = \widehat{Q}_2$ and $P_1 = P_2$



Prove:

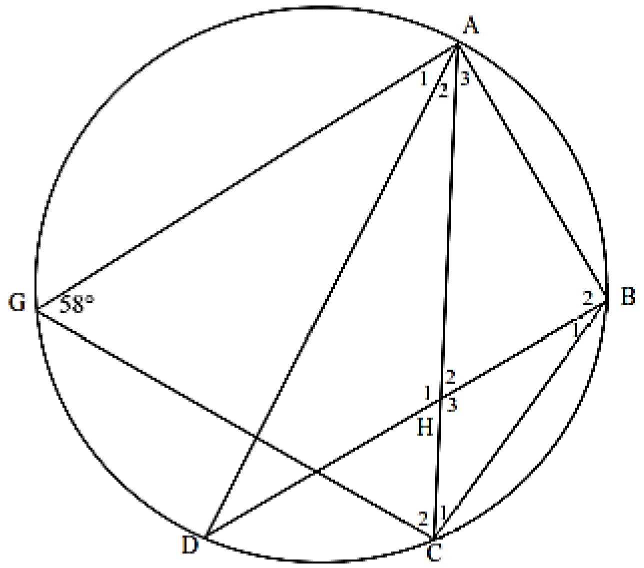
9.1.1 BP is a tangent to circle PQE (6)

9.1.2 ADPQ is a cyclic quadrilateral (3)

GP

QUESTION 8

In the diagram below, the circle passes through points A, B, C, D and G. AD is the diameter of the circle. BD and AC intersect at H and $A\hat{G}C = 58°$.



8.1 Determine, giving reasons, the size of the following angles:

8.1.1 $\hat{B}_2$

8.1.2 $\hat{B}_1$

8.1.3 $\hat{A}_2$

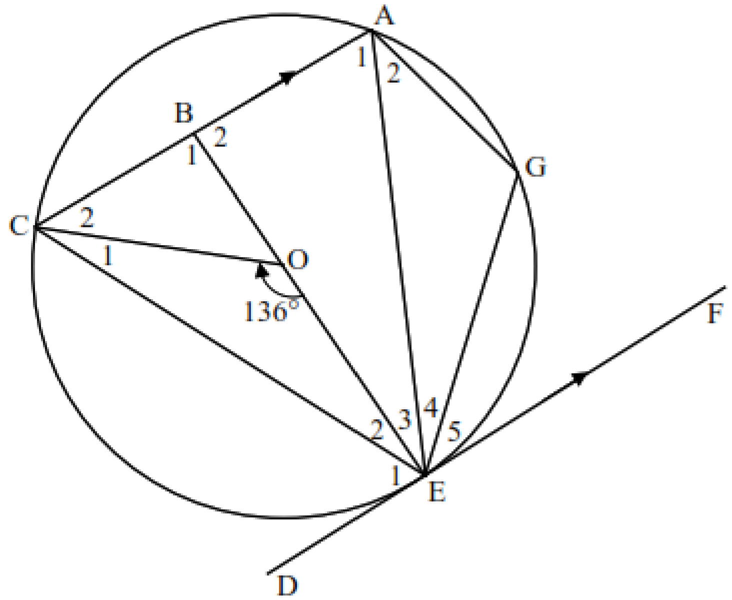
8.2	If it is given that AB = BC. Prove that AB is a tangent to the circle passing through A, H and D.	
		(3)
		[9]

KZN

QUESTION 8

In the diagram, A , C, E and G are points on the circumference of the circle with centre O. $C\hat{O}E = 136°$. DEF is a tangent to the circle at E, with DF||CA. BOE is a straight line, with B a point on AC. AE is drawn. AC = 14 units.

.



8.1 Calculate, with reasons, the size of each of the following:

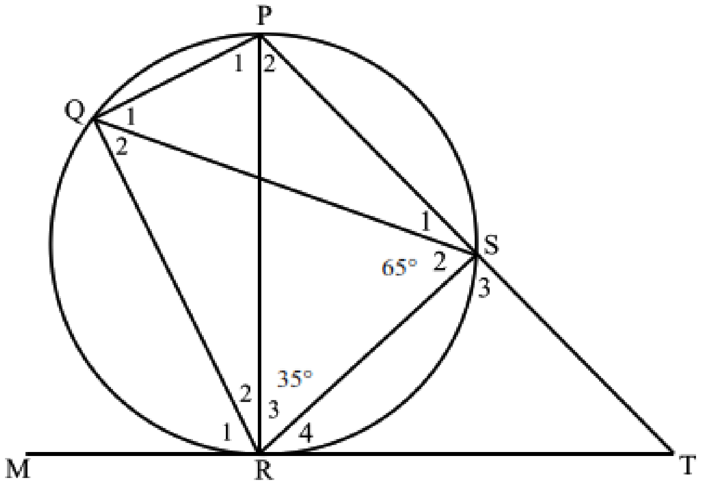
8.1.1	$\hat{A}_1$		(2)
8.1.2	$\hat{E}_1$		(2)
8.1.3	$B\hat{C}E$		(2)
8.1.4	$\hat{G}$		(2)

8.2 Calculate, with reasons, the length of AB. (5)

[13]

MP

8.2 In the diagram below, PQRS is a cyclic quadrilateral and PR is a diameter of the circle. The line MRT is a tangent to the circle at R. $\hat{S}_2 = 65°$ and $\hat{R}_3 = 35°$.



Determine the sizes of each of the following angles, with reasons:

8.2.1 $\hat{R}_1$ (1)

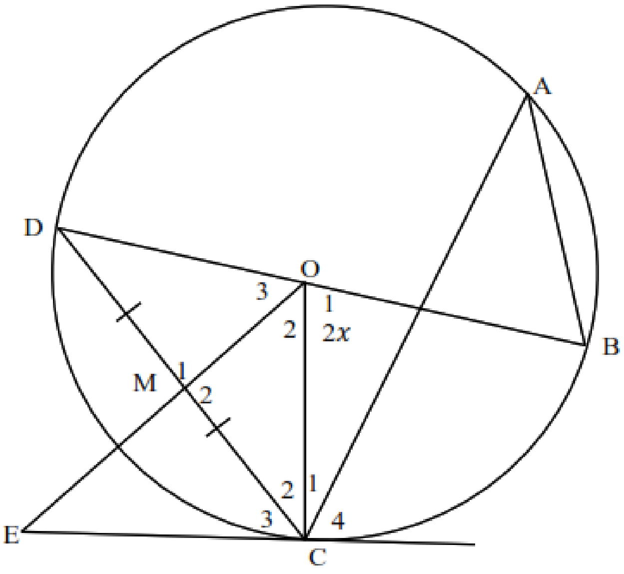
8.2.2 $\hat{R}_4$ (2)

8.2.3 $\hat{T}$ (3)

[11]

QUESTION 9

In the diagram, O is the centre of the circle and EC is a tangent to the circle at C. DM = MC and OME is a straight line. Let $\hat{O}_1 = 2x$.

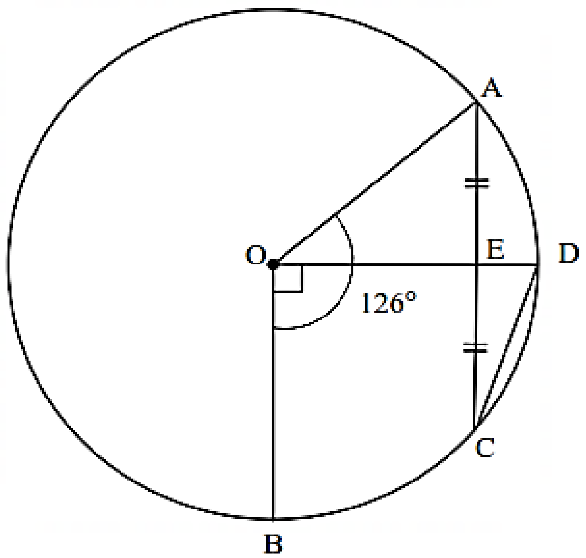


9.1	Write, with reasons, THREE angles equal to x.	(6)
9.2	Prove that $\hat{O}_2 = 90° - x$	(3)
9.3	Prove that EC is a tangent to the circle passing through points, M,C and O.	(4)
9.4	Prove that DOCE is a cyclic quadrilateral.	(3)
		[16]

NC

QUESTION 8

8.1 In the diagram, a circle with centre O is drawn. Radii OB, OD and OA are drawn such that $BO \perp OD$ and obtuse angle $A\hat{O}B = 126°$. OD bisects chord AC at E and CD is drawn.

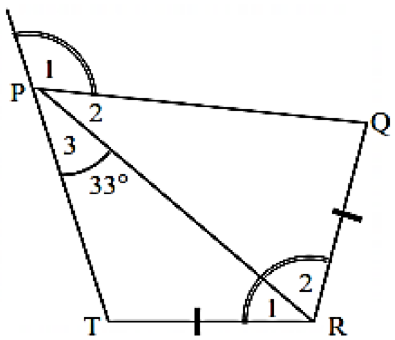


Calculate, giving reasons, the size of:

8.1.1 $O\hat{A}C$ (4)

8.1.2 $A\hat{C}D$ (2)

8.2 In the diagram below, quadrilateral PQRT is drawn with TR = RQ. TP is produced such that $\hat{P}_1 = T\hat{R}Q$. PR is drawn. $\hat{P}_3 = 33°$ and $\hat{R}_2 = 66°$.



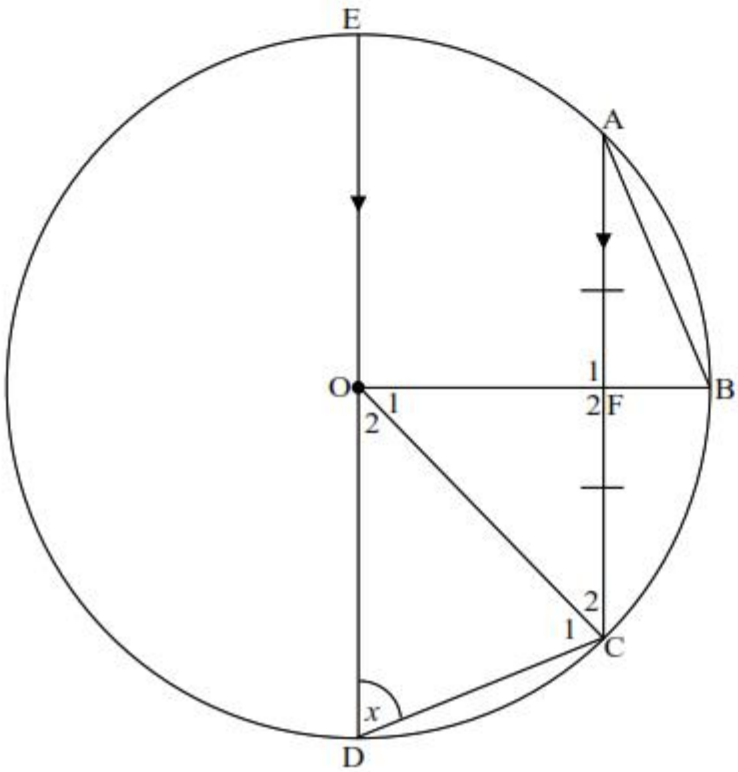
Calculate, giving reasons, the size of $\hat{T}$. (6)

[12]

NW

QUESTION 8

DE is the diameter of the circle having centre O. Points A, B and C lie on the circle. $AC \parallel DE$, $AF = FC$ and $O\hat{D}C = x$.



8.1 Calculate the size of $\hat{F}_2$. (2)

8.2 Determine in terms of x, giving reasons, the size of the following angles:

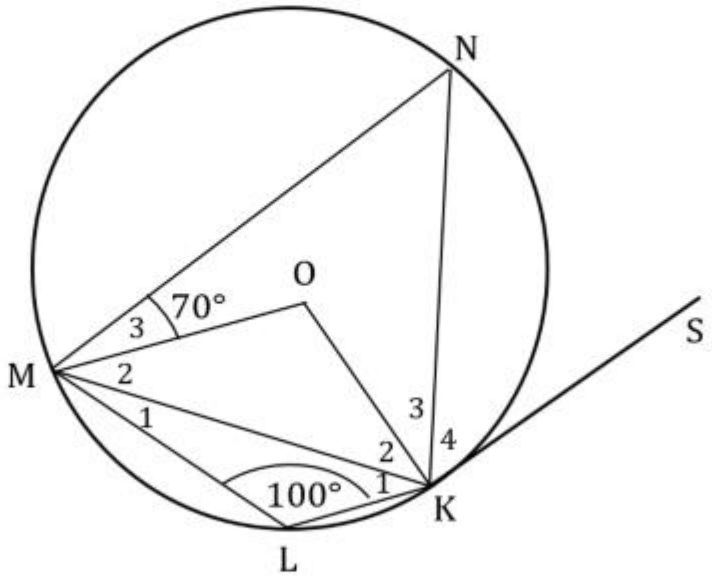
8.2.1 $\hat{O}_2$ (2)

8.2.2 $\hat{B}$ (6)

[10]

WC

8.2 In the diagram below, O is the centre of the circle.
K, L, M and N are points on the circumference of the circle.
KS is a tangent to the circle at K and $\hat{L} = 100°$



8.2.1 Determine, with reasons:

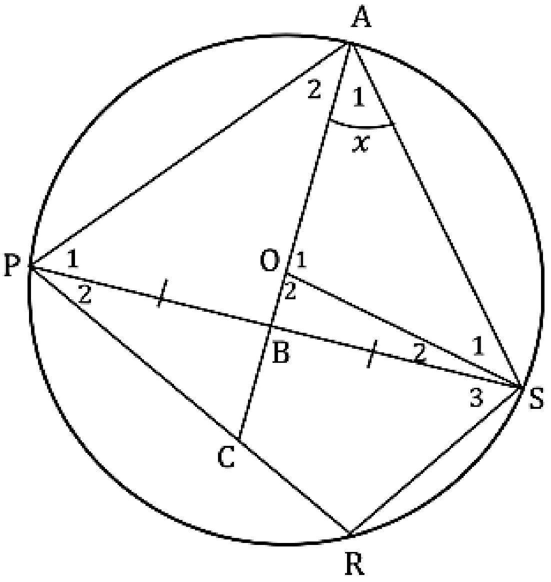
(a) the size of $\hat{N}$. (2)

(b) the size of $\hat{K}_2$. (3)

8.2.2 Prove, giving reasons, that: MN ∥ KS (4)
[11]

QUESTION 9

In the diagram below, O is the centre of the circle. P, R, S and A are points on the circumference of the circle. AOBC is a straight line such that B lies on PS with PB = BS and $\hat{A}_1 = x$



9.1 Determine, with reasons, $\hat{P}_1$ in terms of x. (4)

9.2 Prove, giving reasons, that:

9.2.1 AC bisects $P\hat{A}S$. (4)

9.2.2 COSR is a cyclic quadrilateral. (4)

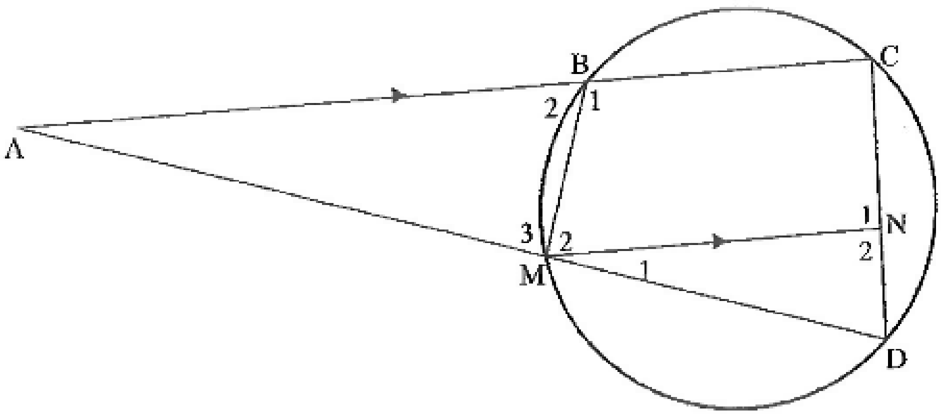
[12]

C. PROPORTIONALITY & SIMILARITY

EC

QUESTION 10

BCDM is a cyclic quadrilateral. Chords MD and BC are produced to meet at point A. N is a point on CD. AC ∥ MN and AM = CD.
AC = 36 units, AD = 48 units and BM = 24 units.

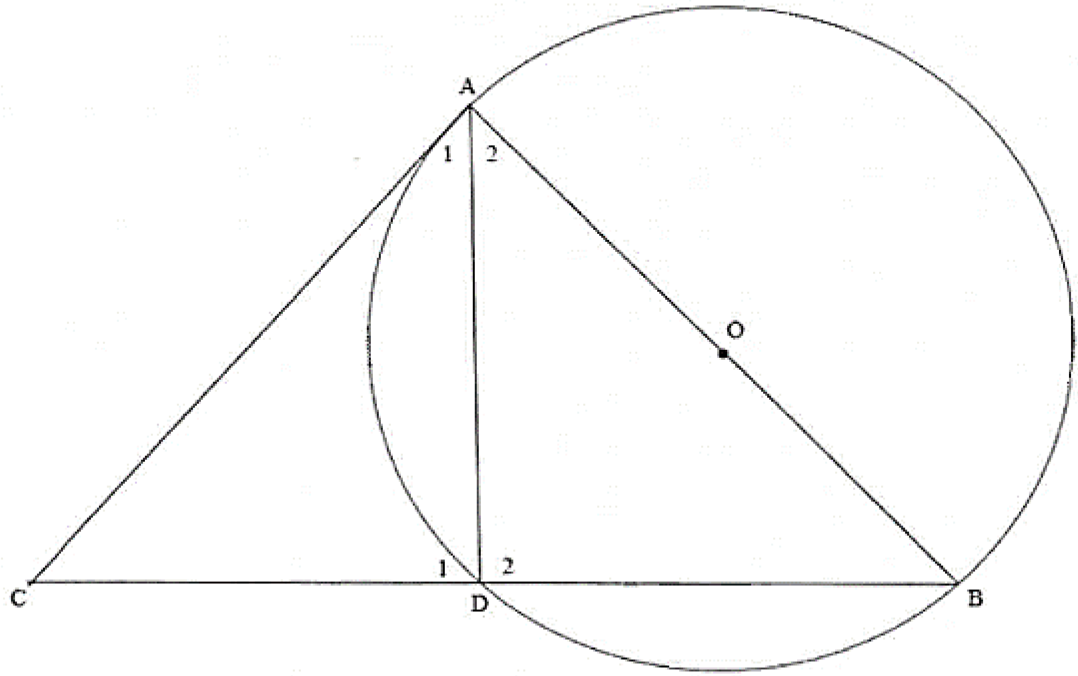


10.1	Prove that $\Delta ABM \parallel\!\mid \Delta ADC$.	(4)
10.2	Prove that $CD^2 = BM \times AC$.	(3)
10.3	Calculate the length of CN.	(6) [13]

FS

QUESTION 10

In the diagram, CA is a tangent to the circle O with diameter AOB. Points A, D and B are points on the circumference of the circle. BD is produced to C and AD intersects CB at D. It is further given that CD : DB = 2 : 3.



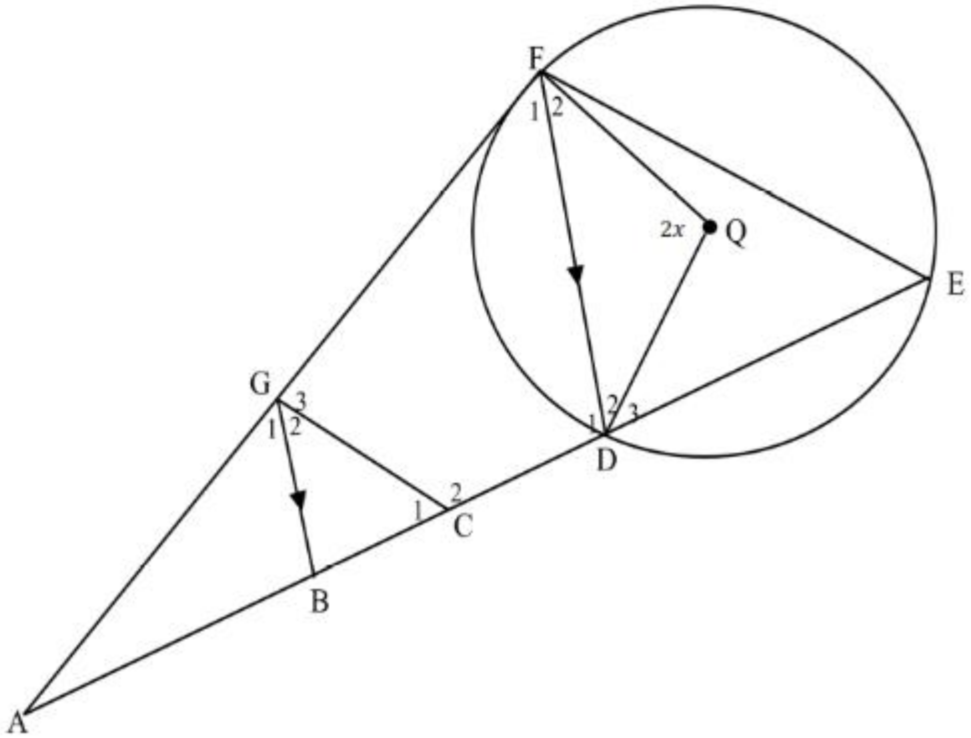
Prove:

10.1 $AD^2 = 6x^2$ (6)

10.2 $\sqrt{AD^2 + AC^2 + AB^2} = x\sqrt{31}$ (4)
[10]

GP

9.2 In the diagram below, D, E and F are points on the circle centred at Q. AGF is a tangent to the circle at F. ED is produced to meet the tangent at A. B and C are points on AE such that GB || FD. GC is joined. GCDF is a cyclic quadrilateral and $F\hat{Q}D = 2x$.



9.2.1	Give a reason why $\hat{E}$ is equal to x.	
		(1)

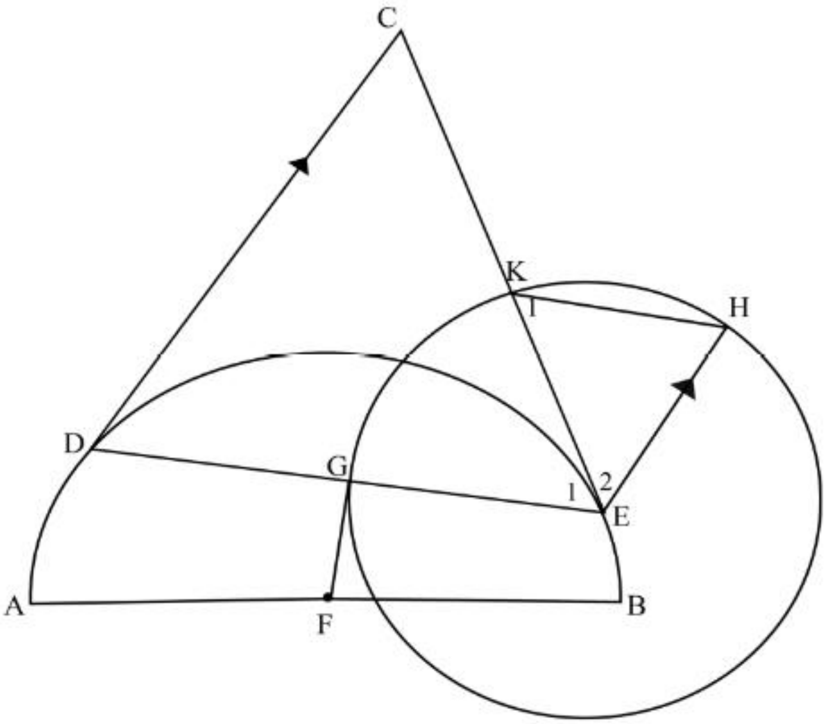
9.2.2	Prove that GC ∥ FE.	
		(3)

9.2.3	Prove that $\frac{AB}{BD}=\frac{AC}{CE}$.	
		(3)

9.2.4	It is further given that AB = 12 units, CE = 22,5 units and 5AG = 4GF. Determine the length of CD.	
		(5)
		[18]

QUESTION 10

In the diagram below, K, H and G are points on the circle and FG is a tangent to the circle with centre E. Semi-circle with centre F is drawn. Points D and E lie on the semi-circle. CD and CE are tangents to the semi-circle at D and E respectively. CD || HE.



10.1	Give a reason why DC = EC.	
		(1)
10.2	Prove that $\Delta DCE \parallel\!| \Delta HEK$.	
		(6)

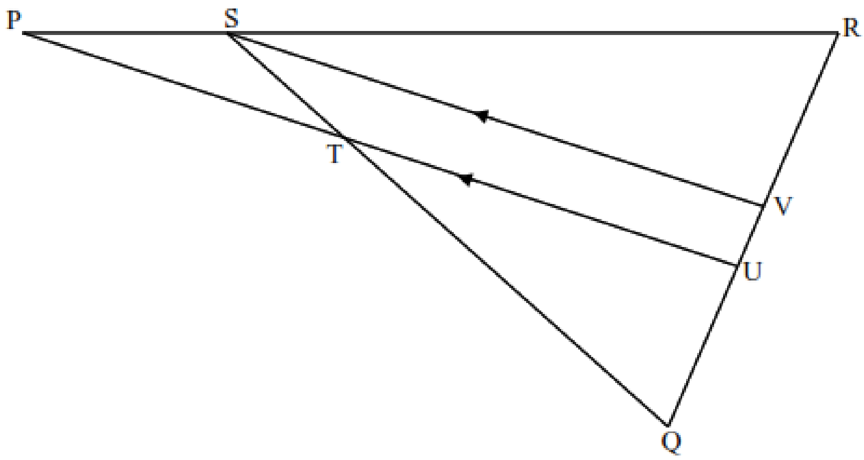
10.3	Prove that DG = GE.	
		(3)

10.4	Show that $2HE^2 = DC \times HK$.	
		(3)
		[13]

KZN

QUESTION 9

In the diagram, ΔQRS is a triangle with RS produced to P. U and V are points on QR such that PU||SV. PU intersects QS in T. $\frac{QU}{UR} = \frac{2}{3}$ and $\frac{QT}{TS} = \frac{5}{2}$.

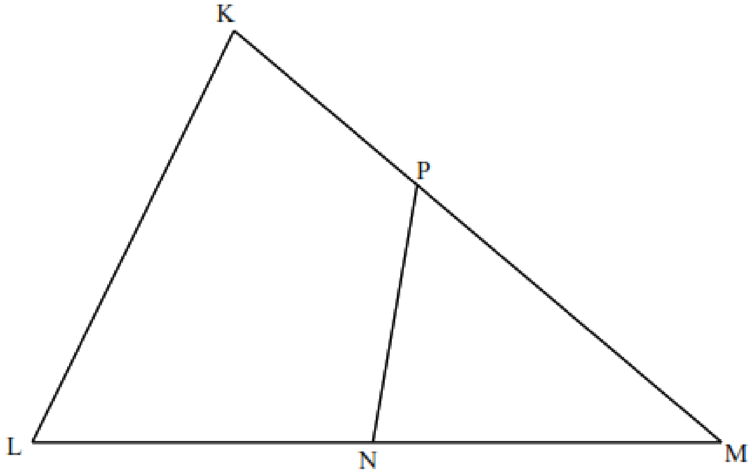


Calculate, giving reasons, the value of $\frac{PS}{SR}$. (6)

[6]

10.2 In the diagram below, ΔKLM is given. PN is drawn with P on KM and N on LM.

- LM = 30 units,
- KL = 25 units,
- KM = 40 units,
- MN = 16 units,
- PM = 12 units and
- PN = 10 units.

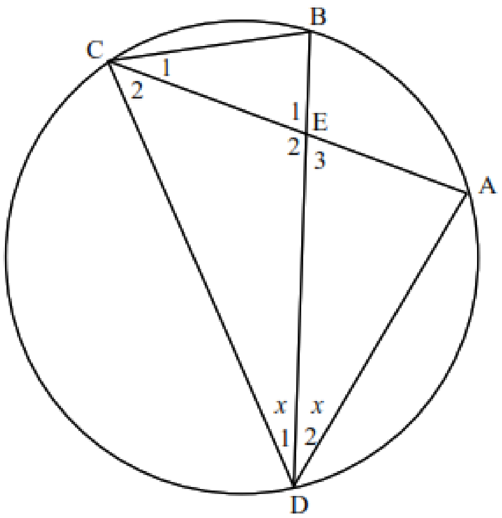


Prove that:

10.2.1 $\Delta MKL \,|||\, \Delta MNP$ (4)

10.2.2 KLNP is a cyclic quadrilateral. (3)

10.3 In the diagram A, B, C and D are points on a circle. BD bisects $A\hat{D}C$ and BD and AC intersect in E.
Let $\hat{D}_1 = \hat{D}_2 = x$.



Prove that:

10.3.1 $\dfrac{AD.CE}{DE} = BC$ (5)

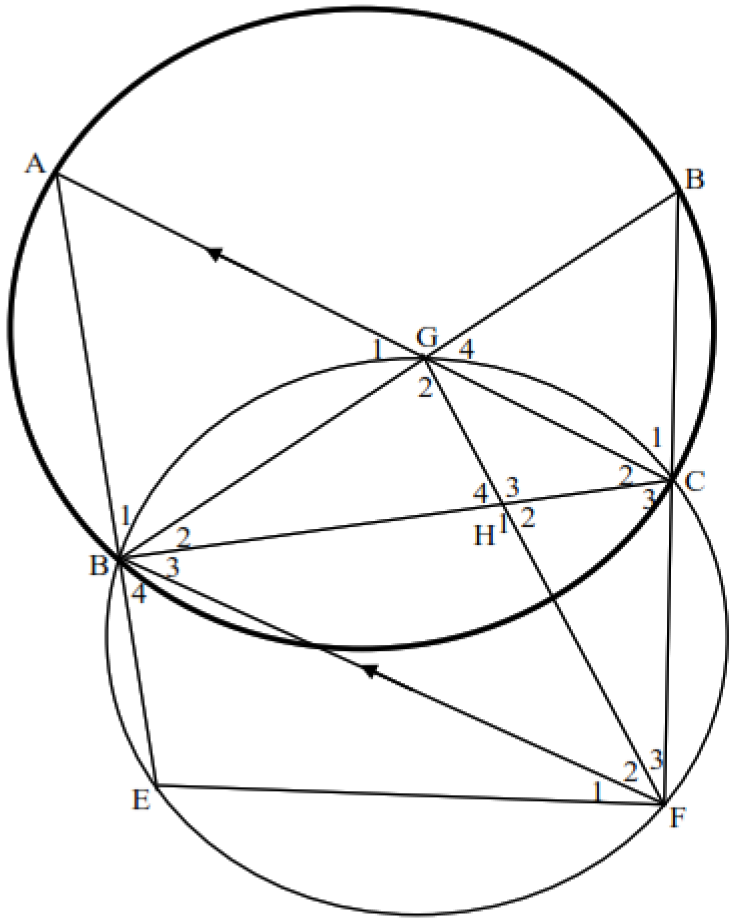
10.3.2 $AD.CD = DE^2 + DE.BE$ (5)

[23]

MP

QUESTION 10

In the diagram below, two circles ABCD and BEFCG intersect at B, C and G. AC || BF. AC and BD intersect at G. BC and FG intersect at H.



10.1 Complete the following reasons

10.1.1 $\hat{G}_1 = \hat{F}_2 + \hat{F}_3$ ………………………………………….. (1)

10.1.2 $B\hat{F}D = \hat{C}_1$ ………………………………………….. (1)

10.2 Prove that

10.2.1 BH = FH (4)

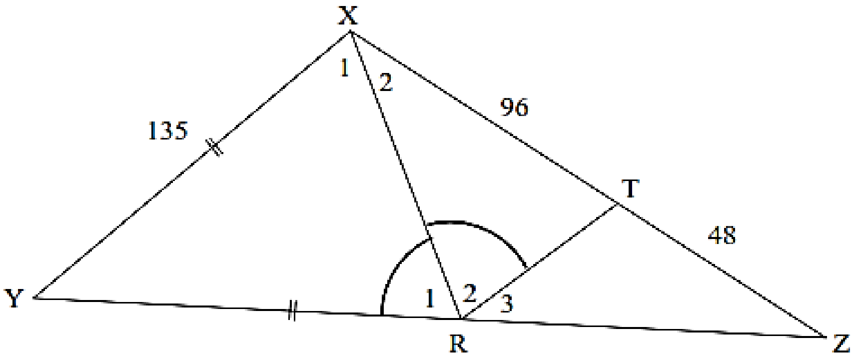
10.2.2 $\Delta BEF \; ||| \; \Delta DGF$ (3)

10.2.3 FH. BG = BH.FC (4)

[13]

NC

9.2 In the diagram, ΔXYZ is drawn. R and T are points on YZ and XZ respectively and XR and RT are drawn. XY = 135 units, XT = 96 units and TZ = 48 units. $\hat{R}_1 = \hat{R}_2$ and XY = YR.

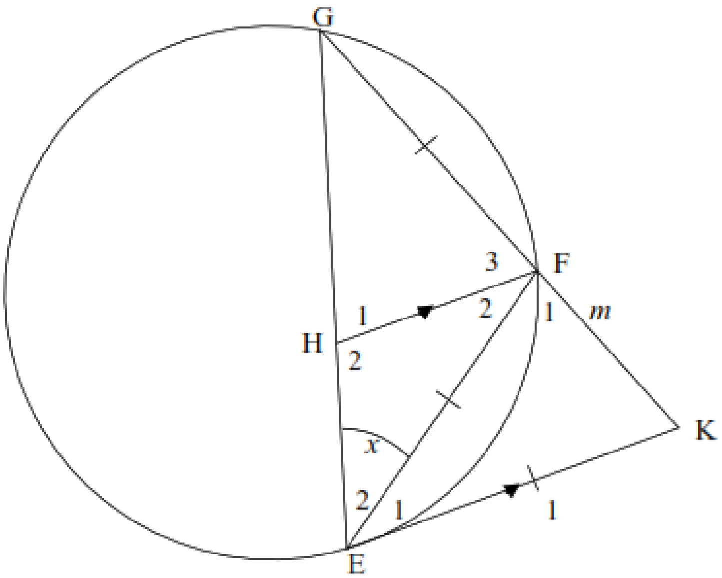


9.2.1 Prove, giving reasons, that RT || YX. (3)

9.2.2 Calculate, giving reasons, the length of RZ. (3)
[11]

QUESTION 10

In the diagram, E, F and G are points on the circle. KE is a tangent to the circle at E. KFG is a straight line. H is a point on chord GE such that HF || EK.
KE = EF = FG. KF = m units and KE = 1 unit. $\hat{E}_2 = x$.



10.1 Name, giving reasons, THREE other angles each equal to x. (4)

10.2 Express $\hat{K}$ in terms of x. (1)

10.3 Calculate, with reasons, the size of x. (3)

10.4 Prove that:

10.4.1 $\Delta KGE \parallel\!| \Delta KEF$ (3)

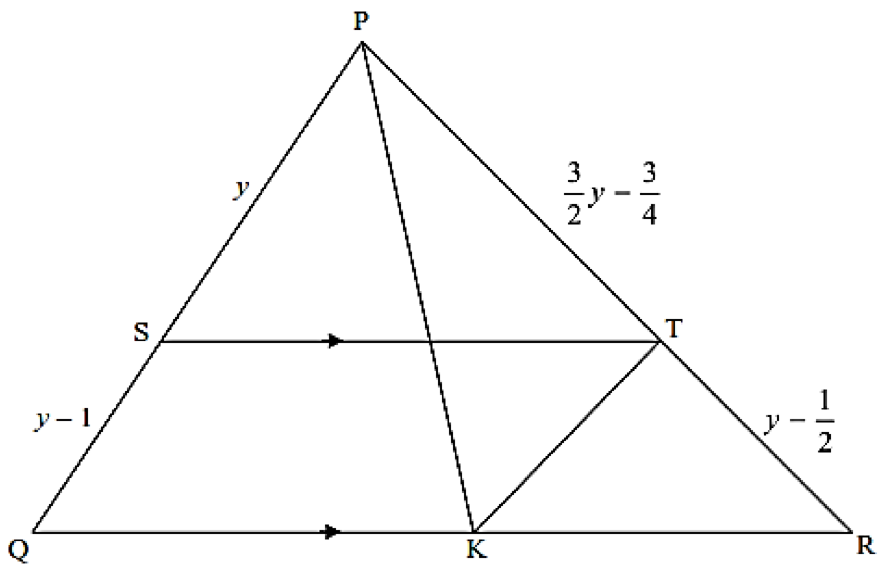
10.4.2 $m^2 + m - 1 = 0$ (2)

10.5 If it is given that GH = 2 units, calculate the length of HE, rounded off to 2 decimal places. (4)

[17]

NW

9.2 In the diagram is Δ PQR with $ST \parallel QR$. K is a point on QR and is joined to T and P. PS = y, SQ = $y - 1$, TR = $y - \frac{1}{2}$ and PT = $\frac{3}{2}y - \frac{3}{4}$ units.



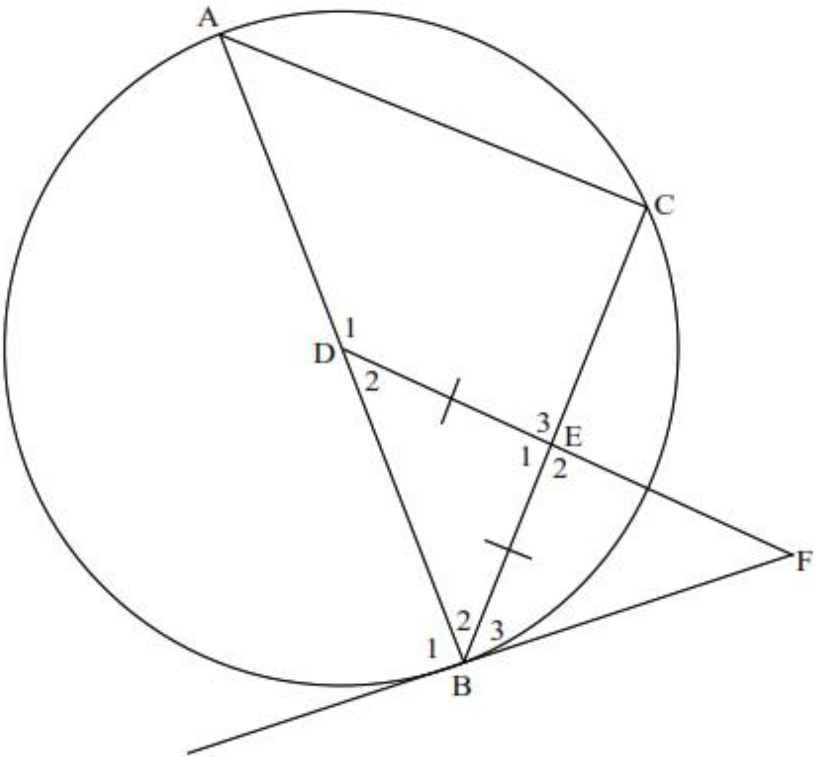
9.2.1 Calculate, with reasons, the value of y. (5)

9.2.2 If $P\hat{K}T = Q$, prove that PSKT is a cyclic quadrilateral. (2)

[12]

QUESTION 10

In the diagram, D is the centre of the circle. BF is a tangent that touches the circle at B. C lies on the circle to form chords AC and BC. D is joined with F and BE = DE.



10.1 Prove that:

10.1.1 $\hat{C} = A\hat{B}F$ (4)

10.1.2 $DF.BC = AB.BD$ (4)

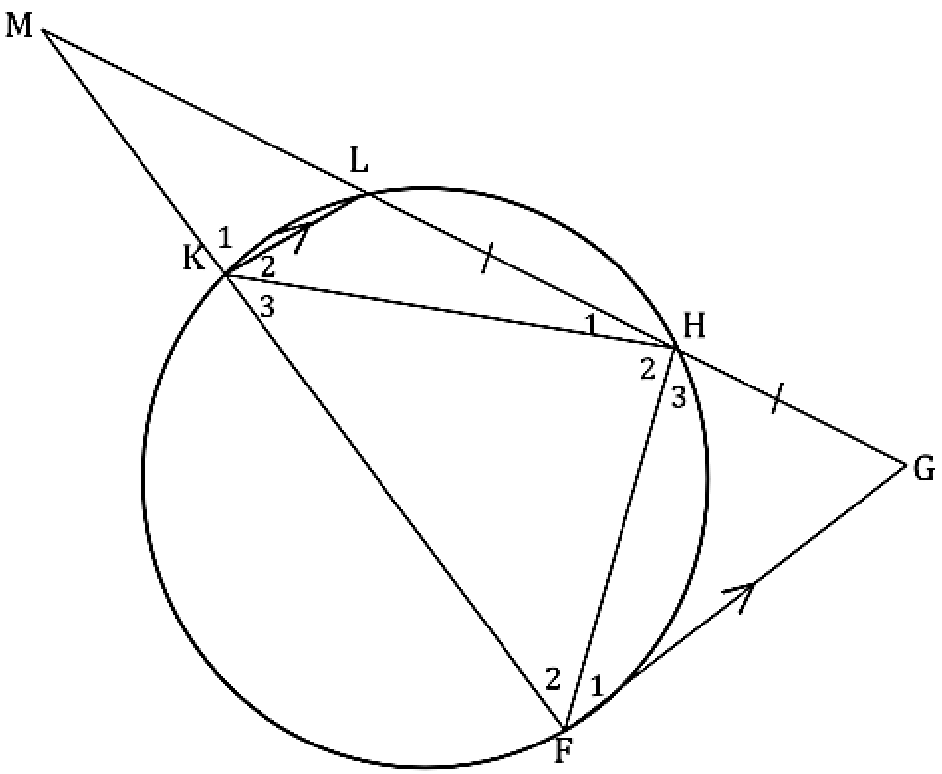
10.1.3 E is the midpoint of the circle that passes through the points D, B and F. (4)

10.2 Prove that: $\dfrac{1}{BC^2 - AB^2} = -\dfrac{AD.FB}{EF.AC^3}$ (6)

[18]

WC

10.2 In the diagram, HLKF is a cyclic quadrilateral. The chords HL and FK are produced to intersect at M. The line FG parallel to KL, meets MH produced at G.
$\frac{MK}{KF} = \frac{1}{2}$ and LH = HG.



10.2.1 Give a reason why $G\hat{F}M = L\hat{K}M$. (1)

10.2.2 Prove, giving reasons, that:

(a) $\Delta MFH \; ||| \; \Delta MGF$ (4)

(b) $\frac{MF}{ML} = \sqrt{6}$ (5)

[17]