education

Department:
Education
North West Provincial Government
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2
SEPTEMBER 2024

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, 1 information sheet and an answer book of 20 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.

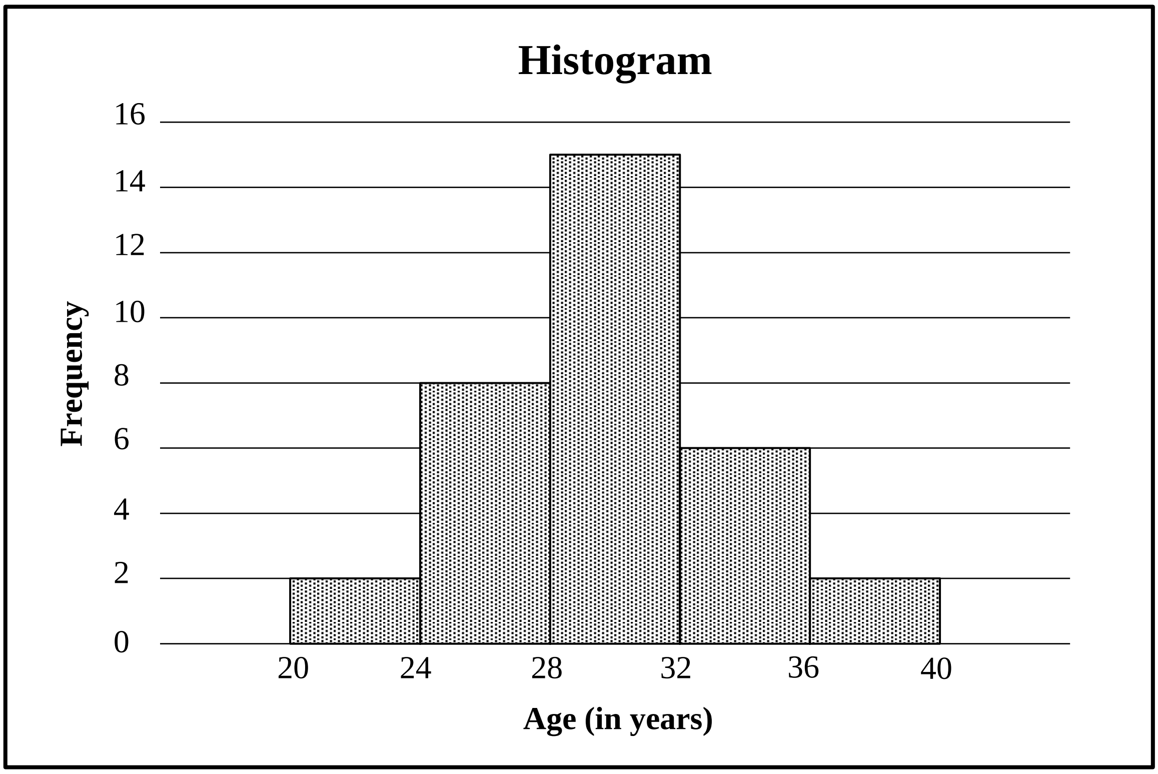
7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

During the Rugby World Cup of 2023, the ages (in years) of the players of the Springbok rugby squad were recorded. The data is represented in the histogram below.



1.1 How many players were in this rugby squad? (1)

1.2 Calculate the estimated mean age of these rugby players. (2)

1.3 Use the histogram to:

1.3.1 Complete the cumulative frequency column in the ANSWER BOOK (2)

1.3.2 Draw an ogive (cumulative frequency graph) of the above data on the grid that is provided in the ANSWER BOOK (3)

1.4 Write down the estimated median of the above data. (2)

1.5 It was discovered that the frequency of the age data for k player(s) in the modal age interval was recorded incorrectly. The mistake is corrected and the frequency of TWO other intervals are increased. The number of players in the squad remains unchanged. Determine the minimum value of k, if the data of the new histogram is symmetrical. (3)

[13]

QUESTION 2

Mrs Mochini wants to use mathematical modelling to predict the final results of her grade 12 Mathematics learners. She decides to use the Preparatory and Final Mathematics examination results of the previous year to help her in developing such a possible model.

She records in the table below, 10 learners' previous year's results (in %) as follows:

Preparatory examination (x)	38	65	78	23	67	93	39	83	51	66
Final examination (y)	57	72	81	27	59	94	41	85	54	79

2.1 Determine the equation of the least squares regression line. (3)

2.2 A learner obtained 46% for the Preparatory examination.

2.2.1 Calculate the possible final examination results that Mrs Mochine can expect from this learner. (2)

2.2.2 Is the answer in QUESTION 2.2.1 a good indication of the expected final examination result? Motivate your answer. (2)

2.3 The point $(\bar{x}\ ;\ q)$ lies on the regression line of QUESTION 2.1.

Only ONE of the options below correctly reflects the value of q.
Write only the letter of the correct option as your answer.

A $\sqrt{\bar{x}}$

B $\dfrac{\sum y}{10}$

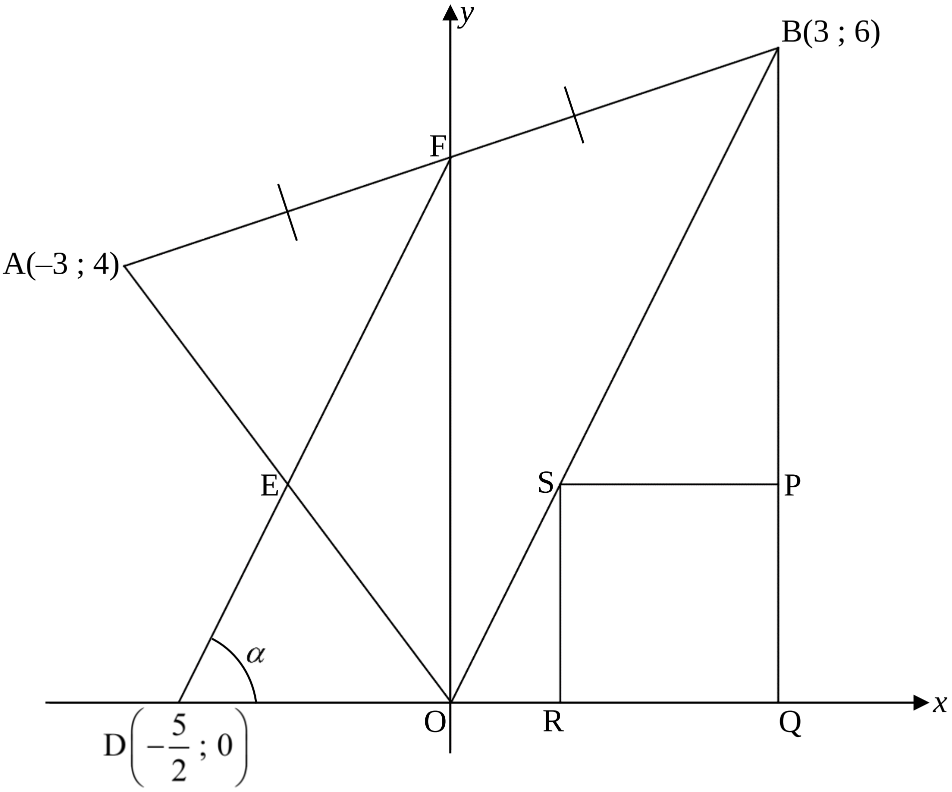
C σ_x

D σ_y (1)

[8]

QUESTION 3

In the diagram, A(–3 ; 4), B(3 ; 6) and O (origin) are vertices of ΔABO. F is the midpoint of AB and is joined with $D\left(-\frac{5}{2}\ ;\ 0\right)$. The angle of inclination of FD is α. The lines AO and DF intersect at E. A quadrilateral PQRS is drawn with QR on the x-axis and S is a point on OB. The side QP is produced to B.



3.1 Calculate the:

3.1.1 Coordinates of F (2)

3.1.2 Gradient of DF (2)

3.1.3 Size of α (2)

3.2 Write down the equation of OB. (1)

3.3 Give a reason why DF || OB. (1)

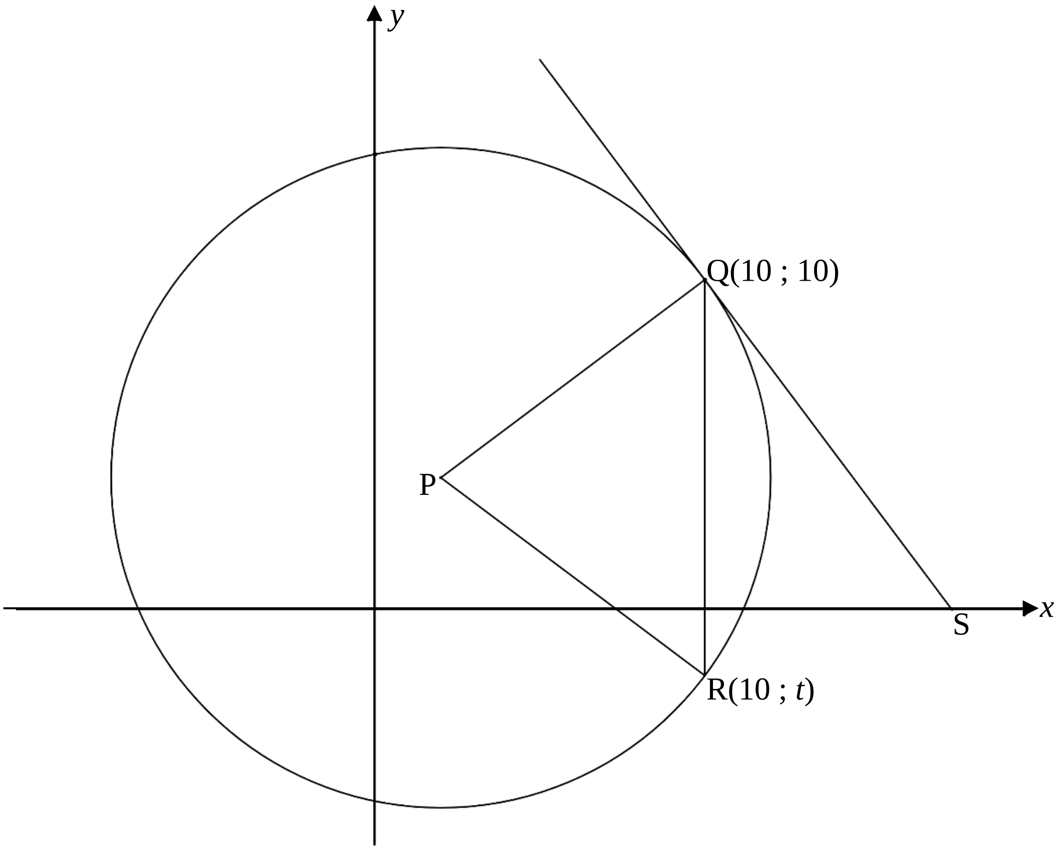
3.4 It is given that PQRS is a square with an area of $9x^2$ squared units.
Calculate the coordinates of S. (6)

3.5 Prove that EDOS forms a parallelogram. (4)

[18]

QUESTION 4

In the diagram below, the circle with centre P has the equation $x^2 - 4x + y^2 - 8y = 80$. QS is a tangent that touches the circle at Q and intersects the *x*-axis at S. The point R(10 ; *t*) lies on the circle.



4.1 Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (2)

4.2 Write down the:

4.2.1 Coordinates of P (2)

4.2.2 Equation of QR (1)

4.3 Determine the equation of the tangent QS. (5)

4.4 Calculate the size of $R\hat{Q}S$. (3)

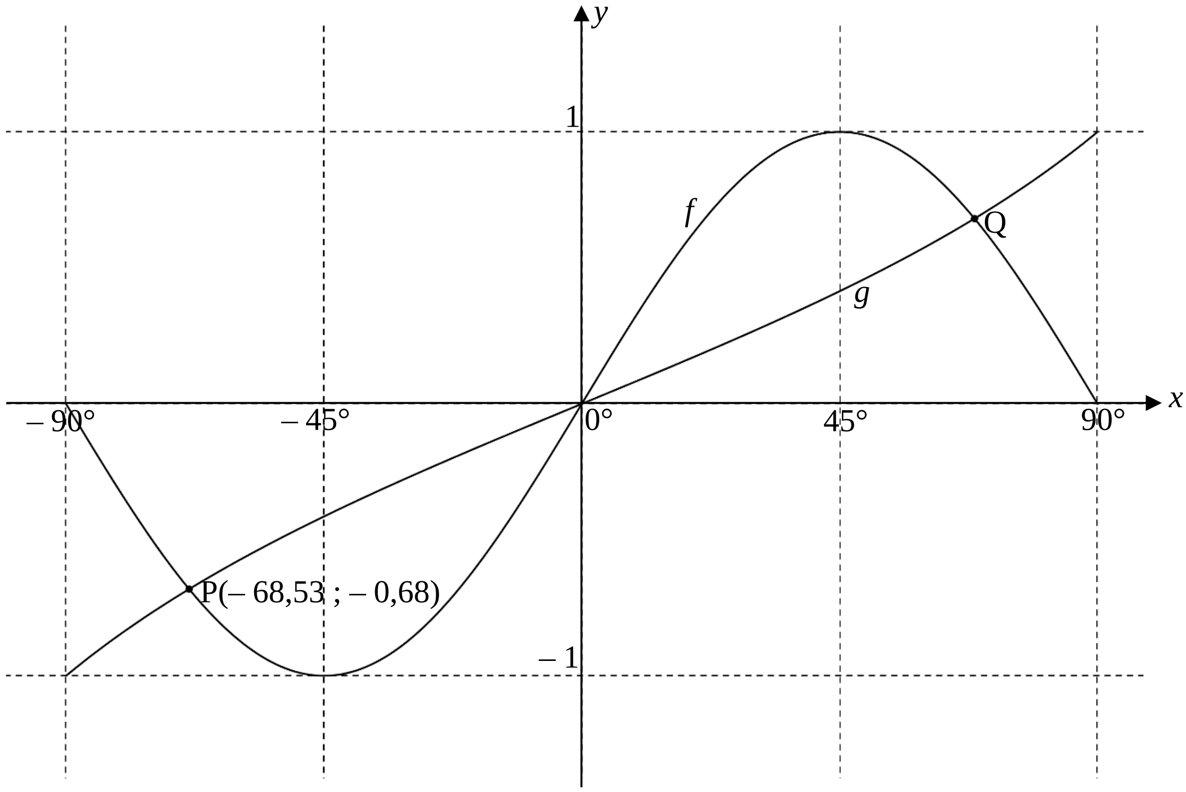
4.5 Calculate the area of Δ PQR. (4)

4.6 A function *h* is formed by restricting the range of the circle to $y \geq 4$.
If $\sum_{x=-8}^{2} h(x) = k$, determine the value of $\sum_{x=3}^{12} h(x)$ in terms of *k*. (4)

[21]

QUESTION 5

In the diagram below, the graphs of $f(x) = a\sin 2x$ and $g(x) = \tan bx$ for $x \in [-90° ; 90°]$ are drawn. P(– 68,53 ; – 0,68) and Q are points of intersection of f and g.



5.1 Write down the:

5.1.1 Value of a (1)

5.1.2 Coordinates of Q (2)

5.1.3 x values of the turning points of h, if $h(x) = f(x + 30°)$ (2)

5.1.4 Value(s) of x where $-0,68 < g(x) \leq 1$ (2)

5.1.5 Value of m where $f(x + m) = -\cos 2x$ (2)

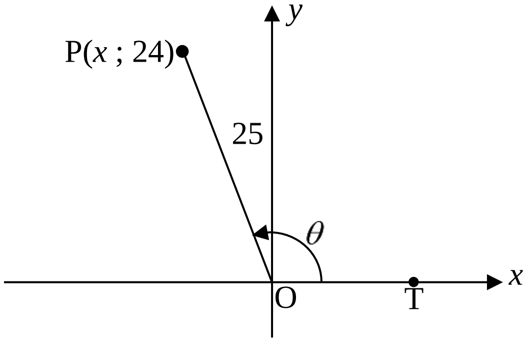
5.1.6 Value of b (1)

5.2 For which value(s) of x, in the given interval, will $x \, . \sqrt{g(x) - f(x)} > 0$? (2)

[12]

QUESTION 6

6.1 In the diagram below, the point P(x ; 24) is 25 units from the origin O. T is a point on the x-axis such that $\hat{TOP} = \theta$.



6.1.1 Calculate the value of x. (2)

6.1.2 **Without using a calculator**, determine the value of $\tan(360° - \theta)$ (2)

6.1.3 Calculate the size of $\hat{POT}$ (2)

6.2 **Without using a calculator**, calculate the value of the following expressions:

6.2.1 $\sin 20° + \cos 120°.\tan 405° + \cos 110°$ (4)

6.2.2 $\dfrac{(\sqrt{2}\cos 15° + 1)(\sqrt{2}\cos 15° - 1)\sin(-2x)}{4\sin x\cos x}$ (4)

6.3 If $\sin(x+y).\cos(x+y) = t$, express the following in terms of t:

$4\cos(90° - 2y).\cos 2x + 4\sin 2x.\cos(360° + 2y)$ (5)

6.4 Given: $\sin^2 x + \cos^2 x + \tan^2 x$

6.4.1 Prove that $\sin^2 x + \cos^2 x + \tan^2 x = \dfrac{1}{\cos^2 x}$ (3)

6.4.2 Attie, a grade 12 learner, argues that $\sqrt{\sin^2 x + \cos^2 x + \tan^2 x} \neq \dfrac{1}{\cos x}$, if $x \in (180° ; 270°)$.
Is Attie's argument correct? Motivate your answer. (2)

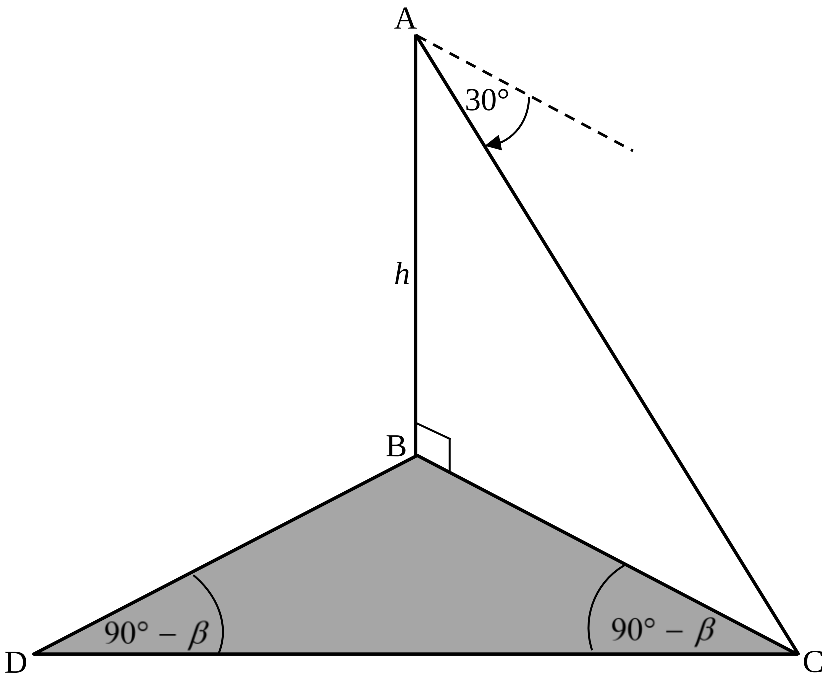
6.5 Given: $2^{2\sin^2 x} - 5.2^{\cos 2x} = -3$, $x \in (0° ; 90°)$

Show, **without using a calculator**, that $\sin x = \dfrac{1}{\sqrt{2}}$. (6)

[30]

QUESTION 7

In the diagram, D, B and C lie in the same horizontal plane. AB is a vertical tower. The angle of depression from A to C is 30°. $B\hat{C}D = B\hat{D}C = 90° - \beta$ and AB = h metres.



7.1 Determine the length of BC in terms of h. (2)

7.2 Write down the size of $D\hat{B}C$ in terms of β. (1)

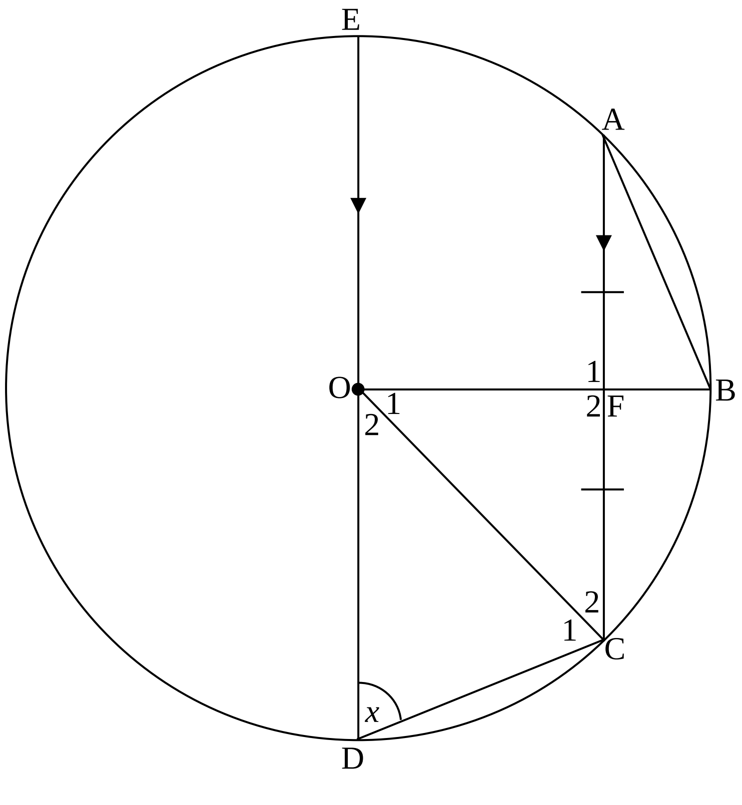
7.3 Show that $DC = \sqrt{12}\,h\sin\beta$ (5)

[8]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

DE is the diameter of the circle having centre O. Points A, B and C lie on the circle.
$AC \parallel DE$, AF = FC and $O\hat{D}C = x$.



8.1 Calculate the size of $\hat{F}_2$. (2)

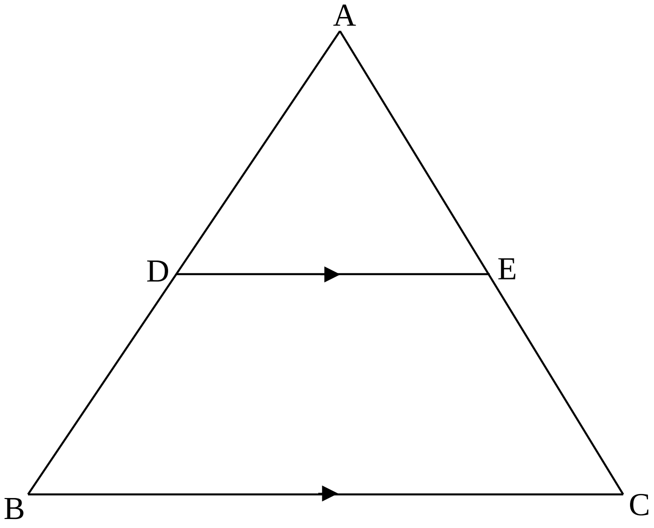
8.2 Determine in terms of *x*, giving reasons, the size of the following angles:

8.2.1 $\hat{O}_2$ (2)

8.2.2 $\hat{B}$ (6)
[10]

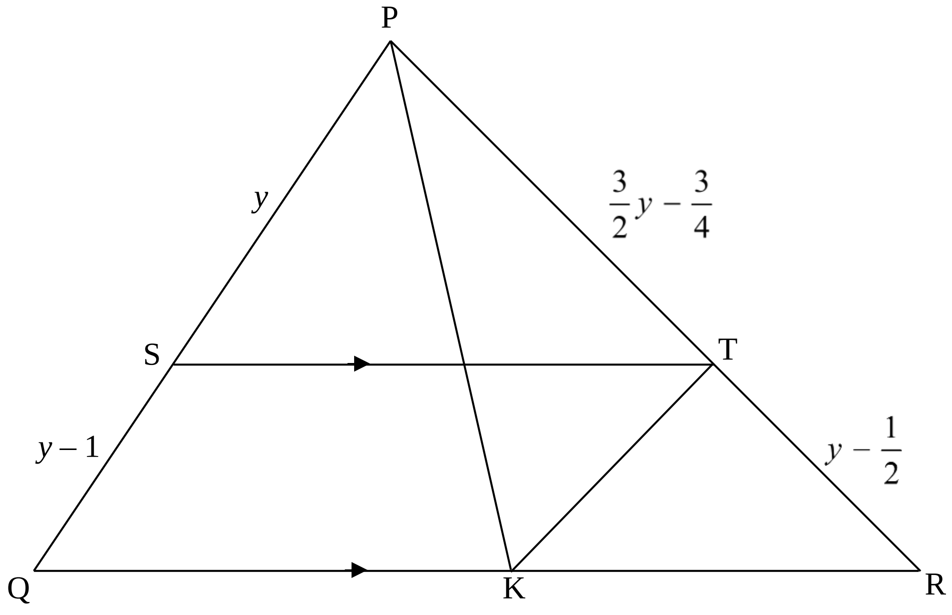
QUESTION 9

9.1 In the diagram is Δ ABC with DE$\parallel$BC.



Prove the theorem that states that $\frac{AD}{DB} = \frac{AE}{EC}$. (5)

9.2 In the diagram is Δ PQR with ST$\parallel$QR. K is a point on QR and is joined to T and P.
$PS = y$, $SQ = y - 1$, $TR = y - \frac{1}{2}$ and $PT = \frac{3}{2}y - \frac{3}{4}$ units.



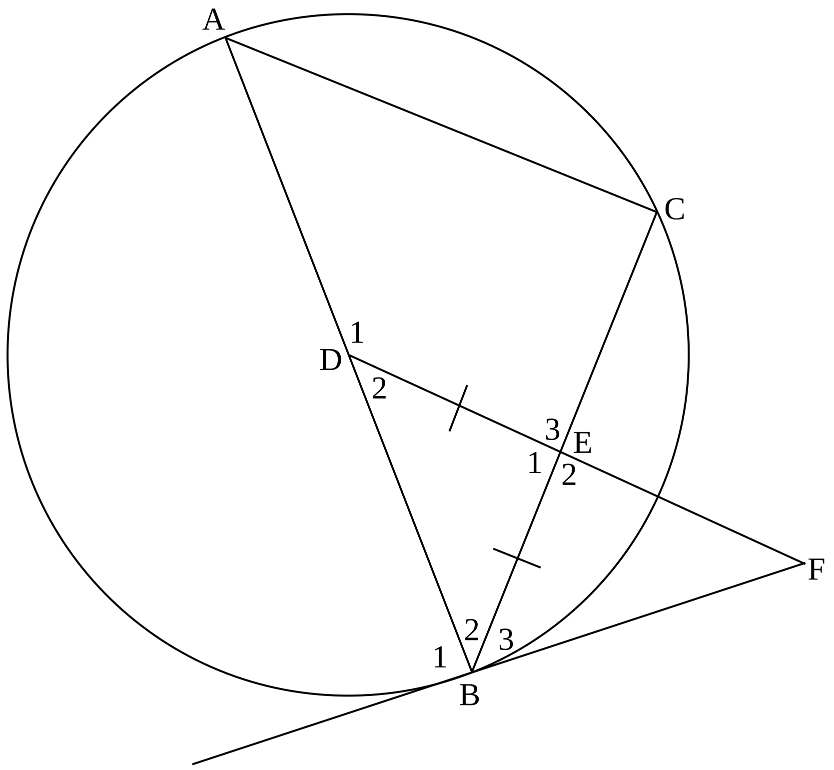
9.2.1 Calculate, with reasons, the value of y. (5)

9.2.2 If $P\hat{K}T = Q$, prove that PSKT is a cyclic quadrilateral. (2)

[12]

QUESTION 10

In the diagram, D is the centre of the circle. BF is a tangent that touches the circle at B. C lies on the circle to form chords AC and BC. D is joined with F and BE = DE.



10.1 Prove that:

10.1.1 $\hat{C} = A\hat{B}F$ (4)

10.1.2 $DF.BC = AB.BD$ (4)

10.1.3 E is the midpoint of the circle that passes through the points D, B and F. (4)

10.2 Prove that: $\frac{1}{BC^2 - AB^2} = -\frac{AD.FB}{EF.AC^3}$ (6)

[18]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{n}$ $\quad$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

$In\ \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$