JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

ANSWER BOOKLET

TEACHER

TERM 1

Patterns, Sequences and Series

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ICON DESCRIPTION

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BIBLIOGRAPHY	TERMINOLOGY	WORKED EXAMPLES	STEPS

TOPIC 1: Quadratic Pattern

Activity 1

QUESTION 3

3.3.1

5 ; 12 ; 29 ; 56

7 ; 17 ; 27

10 ; 10

93 ; 140

3.3.2

7 ; 17 ; 27

10 ; 10

$T_n = 10n - 3$

$= (10n - 2) - 1$

$= 2(5n - 1) - 1$

Since $10n - 2 = 2(5n - 1)$,

$10n - 2$ is even for any value of n.

Thus T_n is always odd, since for any value of n, T_n is always one less than an even number

QUESTION 4

4.1

3 ; 5 ; 8 ; 12

2 ; 3 ; 4

1 ; 1

$2a = 1$

$a = \frac{1}{2}$

$3\left(\frac{1}{2}\right) + b = 2$

$b = \frac{1}{2}$

$a + b + c = 3$

$\frac{1}{2} + \frac{1}{2} + c = 3$

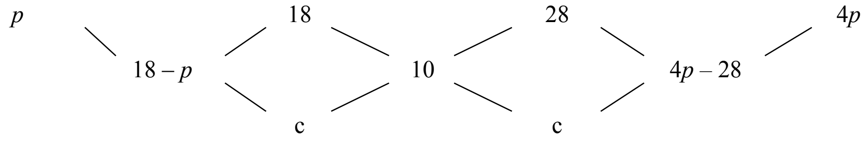
$c = 2$

$T_n = \frac{n^2}{2} + \frac{n}{2} + 2$

$T_{26} = \frac{26^2}{2} + \frac{26}{2} + 2$

$= 353$

4.2



$$10-(18-p)=4p-28-10$$
$$10-18+p=4p-28-10$$
$$3p=30$$
$$p=10$$

Activity 2	

QUESTION 7

7.1 29

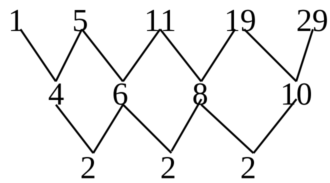
7.2
$$T_n = an^2 + bn + c$$
$$1 = a + b + c$$
$$\therefore c = 1 - a - b$$
$$5 = 4a + 2b + c$$
$$5 = 4a + 2b + 1 - a - b$$
$$4 = 3a + b$$
$$11 = 9a + 3b + c$$
$$11 = 9a + 3b + 1 - a - b$$
$$\therefore 10 = 8a + 2b$$

Solving (1) and (2) simultaneously.
Los (1) en (2) gelyktydig op.

$8 = 6a + 2b$ (1) x 2
$\underline{10 = 8a + 2b \quad (2)}$

$$\therefore 2 = 2a$$
$$\therefore a = 1$$
$$\therefore b = 1$$
$$\therefore c = -1$$
$$T_n = n^2 + n - 1$$

OR



2a = 2	3a + b = 4	a + b + c = 1
a = 1	3 + b = 4	1 + 1 + c = 1
	b = 1	c = -1

$T_n = n^2 + n - 1$

7.3 $T_n = n \quad + n - 1$ or/of $T_n = 100(101) - 1$
$\therefore T_{100} = 100^2 + 100 - 1$ $= 10\ 099$
$= 10\ 099$

Activity 3

3.1 45

3.2 $T_n = an^2 + bn + c$

Second difference of terms is 2.

$a = 1$

$3a + b = 7$

$3 + b = 7$

$b = 4$

$a + b + c = 5$

$1 + 4 + c = 5$

$c = 0$

$T_n = n^2 + 4n$

Activity 4

4.1 The first differences are 1; -1; -3; -5; ….

These form a linear pattern

$T_n = 1 + (n-1)(-2)$

$= 3 - 2n$

4.2 Between the 35th and 36th terms of the quadratic sequence lies the 35th first difference

35th first difference = 3 – 2(35)

= – 67

4.3 Second difference of terms is -2 .

$P_n = an^2 + bn + c$

$a = -1.$

$3a + b = 1$

$-3 + b = 1$

$b = 4$

$a + b + c = -3$

$-1 + 4 + c = -3$

$c = -6$

$P_n = -n^2 + 4n - 6$

4.4 $-n^2 + 4n - 6$

$= -(n-2)^2 + 4 - 6$

$= -(n-2)^2 - 2$

The function has a maximum-value of – 2 and therefore the pattern will never have positive values.

Activity 5

3.1 $\quad 4 \quad\quad 9 \quad\quad x \quad\quad 37$

$\quad\quad 5 \quad\quad x-9 \quad\quad 37-x$

$\quad\quad\quad x-14 \quad\quad 37-x-(x-9)$

First difference : $5;\ x-9;\ 37-x$

Second difference : $x-14;\ -2x+46$

$$x-14 = 46-2x$$

$$3x = 60$$

$$x = 20$$

3.2 $\quad 4 \quad\quad 9 \quad\quad 20 \quad\quad 37$

$\quad\quad 5 \quad\quad 11 \quad\quad 17$

$\quad\quad\quad 6 \quad\quad 6$

$2a = 6$

$a = 3$

$T_n = 3n^2 + bn + c$

$3 + b + c = 4 \qquad \ldots T_1$

$b + c = 1$

$12 + 2b + c = 9 \qquad \ldots T_2$

$2b + c = -3$

$\therefore 9 + b = 5$

$b = -4$

and

$c = 4 - (-1) = 5$

$\therefore T_n = 3n^2 - 4n + 5$

OR

$$2a = 6$$

$$a = 3$$

$$3a + b = 5$$

$$b = -4$$

$$a + b + c = 4$$

$$3 - 4 + c = 4$$

$$c = 5$$

$$T_n = 3n^2 - 4n + 5$$

Activity 6

4.1 The second, third, fourth and fifth terms are 1 ; – 6 ; T_4 and – 14

First differences are: – 7 ; $T_4 + 6$; $-14 - T_4$

So $T_4 + 6 + 7 = -14 - 2T_4 - 6$

$T_4 = -11$

d = – 11 + 6 + 7 = 2 or – 14 + 22 – 6 = 2

4.2

T_1 1 – 6

-9 -7

2

$T_1 = 10$

Activity 7

4.1 6 ; 6 ; 2 ; – 6 ; – 18 ;…

First difference: 0, – 4, – 8, – 12, …

Therefore the next term is:

– 18 – 16 = – 34

4.2

6 6 2 – 6

0 – 4 – 8

– 4 – 4

$2a = -4$

$a = -2$

$T_n = -2n^2 + bn + c$

$6 = -2 + b + c$

$8 = b + c \quad ...(i)$

$6 = -8 + 2b + c$

$14 = 2b + c \quad ...(ii)$

(ii) – (i): $6 = b$

$\therefore c = 2$

$T_n = -2n^2 + 6n + 2$

4.3

$-6838 = -2n^2 + 6n + 2$

$-2n^2 + 6n + 6840 = 0$

$n^2 - 3n - 3420 = 0$

$$n = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-3420)}}{2}$$

$$n = \frac{3 \pm 117}{2}$$

n = 60 or n = - 57

not possible

$\therefore T_{60} = -6838$

Activity 8

3.2 Let V be the volume of the first tank.

$\frac{V}{2};\frac{V}{4};\frac{V}{8}.......$

$$S_{19}=\frac{\frac{V}{2}\left[1-\left(\frac{1}{2}\right)^{19}\right]}{1-\frac{1}{2}}$$

$$=\frac{524287}{524288}V$$

$$=0{,}9999980927\ V$$

$$<V$$

Yes, the water will fill the first tank without spilling over.

OR

Let V be the volume of the first tank.

$\frac{V}{2};\frac{V}{4};\frac{V}{8}.......$

$$S_{19}=\frac{\frac{V}{2}\left[1-\left(\frac{1}{2}\right)^{19}\right]}{1-\frac{1}{2}}$$

$$=V\left[1-\left(\frac{1}{2}\right)^{19}\right]$$

$$<V\cdot 1$$

$$=V$$

3.3.1 $T_n=-2(n-5)^2+18$

Term 1 = – 14

Term 2 = 0

Term 3 = 10

3.3.2 Term 5 **OR** $n=5$ **OR** T_5

3.3.3

-14		0		10		
	14		10			
		– 4				

Second difference = – 4

3.3.4

$$-2(n-5)^2+18<-110$$

$$-2(n-5)^2+128<0$$

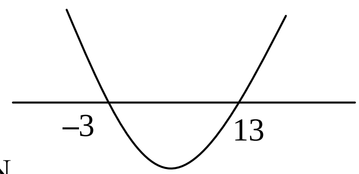
$$-2n^2+20n-50+128<0$$

$$-2n^2+20n+78<0$$

$$n^2-10n-39>0$$

$$(n-13)(n+3)>0$$

$n<-3$ or $n>13$



$n\geq 14\ ;\ n\in\mathbb{N}$ **OR** $n>13\ ;\ n\in\mathbb{N}$

Activity 9

2.1 $399 \; ; \; 360 \; ; \; 323 \; ; \; 288 \; ; \; 255$

$-39 \quad -37 \quad -35 \quad -33$

$2 \quad 2 \quad 2$

Let $T_n = an^2 + bn + c$

Then

$2a = 2$

$a = 1$

$T_1 = 399: \; a + b + c = 399; \; b + c = 398$

$T_2 = 360: \; 4a + 2b + c = 360; \; 2b + c = 356$

$b = -42$

$c = 440$

$T_n = n^2 - 42n + 440$

2.2 $n^2 - 42n + 440 = 0$

$(n-22)(n-20) = 0$

$n = 22$ and $n = 20$

both terms 22 and 20 have values of 0.

2.3 $n = \dfrac{-(-42)}{2(1)}$

$n = 21$

At the 21st term, the lowest value is obtained.

OR

$2n - 42 = 0$

$2n = 42$

$n = 21$

$\therefore$ At the 21st term, the lowest value is obtained.

OR

$T_n = (21-n)^2 - 1 \therefore$

For $n = 21, \; T_n = (21-n)^2 - 1 = (21-21)^2 - 1 = -1$

For $n = 21$, the lowest value $(= -1)$ is obtained.

Activity 10

2.1 39

2.2

3 9 17 27

6 8 10

2 2

Let $T_n = an^2 + bn + c$

Then

$2a = 2$

$a = 1$

$3a + b = 6$

$3(1) + b = 6$

$b = 3$

$a + b + c = 3$

$1 + 3 + c = 3$

$c = -1$

$T_n = n^2 + 3n - 1$

2.3

$n^2 + 3n - 1 > 269$

$n^2 + 3n - 270 > 0$

$(n + 18)(n - 15) > 0$

The first value of n is 16

The term is $16^2 + 3(16) - 1 = 303$

Activity 11

4.1

17 10 5 2

-7 -5 -3

2 2

$r = 1$

$s = 2$

4.2

$2a = 2$

$a = 1$

$3a + b = -7$

$\therefore 3(1) + b = -7$

$b = -10$

$\therefore a + b + c = 17$

$1 - 10 + c = 17$

$c = 26$

$\therefore d(n) = n^2 - 10n + 26$

4.3 $d(8) = (8)^2 - 10(8) + 26$

$= 10$ m

4.4 Since the distance from P is decreasing for $n < 5$ the athlete is moving towards P. Since the distance from P is increasing for $n > 5$, the athlete is moving away from P.

Activity 12

3.2 $T_{50} = 3 + (4 + 10 + 16 + \ldots \text{ to 49 terms})$

$$T_{50} = 3 + \frac{49}{2}\left[2(4) + (49 - 1)(6)\right]$$

$= 3 + 7252$

$= 7255$

TOPIC 2 : Arithmetic Sequence and Series

Activity 1

2.3.1

$$T_n = 20 + 3(n-1)$$
$$101 = 20 + (n-1)3$$
$$84 = 3n$$
$$n = 28$$

$23 + 29 + \ldots$ to 14 terms

$$= \frac{14}{2}[2(23) + (14-1)6] \quad \text{OR} \quad \frac{14}{2}[23 + 101]$$
$$= 868$$

Activity 2

2.3

$$S_n = a + [a+d] + [a+2d] + \ldots + [a+(n-2)d] + [a+(n-1)d]$$
$$S_n = [a+(n-1)d] + [a+(n-2)d] + \ldots + [a+d] + a$$
$$2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \ldots + [2a+(n-1)d] + [2a+(n-1)d]$$
$$= n[2a+(n-1)d]$$
$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Activity 3

2.1

$$T_2 - T_1 = T_3 - T_2$$
$$2x - (3x+1) = (3x-7) - 2x$$
$$2x - 3x - 1 = 3x - 7 - 2x$$
$$-x - 1 = x - 7$$
$$-2x = -6$$
$$x = 3$$

2.2.1

$$T_n = a + (n-1)d$$
$$T_{11} = 10 + (11-1)(-4)$$
$$= -30$$

OR

10; 6; 2; –2; –6; –10; –14; –18; –22; –26; –30 …

$\therefore T_{11} = -30$

2.2.2

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$-560 = \frac{n}{2}[2(10)+(n-1)(-4)]$$
$$-1120 = -4n^2 + 24n$$
$$4n^2 - 24n - 1120 = 0$$
$$n^2 - 6n - 280 = 0$$
$$(n-20)(n+14) = 0$$
$$n = 20 \quad \text{or} \quad -14$$
$$\therefore n = 20 \quad \text{only}$$

Activity 4

3.1 $(2p-3)-(1-p)=(p+5)-(2p-3)$

$$2p-3-1+p=p+5-2p+3$$
$$3p-4=-p+8$$
$$4p=12$$
$$p=3$$

3.2.1 First term = $1-p=1-3=-2$

3.2.2 $-2\ ;3\ ;8$

Common difference = 5

OR

$3p-4=3(3)-4=5$

3.3 After the first term -2, all the other terms end in either a 3 or an 8. Perfect squares never end in a 3 or an 8.

Activity 5

4.1

Term	Income	Expenses	Savings
1	120 000	90 000	30 000
2	132 000	105 000	27 000
3	144 000	120 000	24 000

30 000 + 27 000 + 24 000 + …+ 0.

4.2 Savings = Income – Expenses

Income in year n = 120 000 + 12 000(n – 1)

Expenses in year n = 90 000 + 15 000(n – 1)

$$120000+12000(n-1)=90000+15000(n-1)$$
$$30000+12000n-12000=15000n-15000$$
$$33000=3000n$$
$$n=11$$

$\therefore$ After 11 years.

OR

a = 30 000 d = –3000

$$T_n=30000+(n-1)(-3000)$$
$$0=30000-3000n+3000$$
$$3000n=33000$$
$$\therefore n=11$$

$\therefore$ After 11 years

4.3 $120000+12000(25-1)=90000+x(25-1)$

$$x=13250$$

Activity 6

2.1

$$T_n = a + (n-1)d$$
$$173 = -7 + (n-1)(4)$$
$$173 = -7 + 4n - 4$$
$$4n = 184$$
$$n = 46$$

2.2

$$S_n = \frac{n}{2}[a + l]$$
$$= \frac{46}{2}[-7 + 173]$$
$$= 23[166]$$
$$= 3\,818$$

Activity 7

2.2.1 16

2.2.2

$$T_n = -8 + 6(n-1)$$
$$148 = 6n - 14$$
$$6n = 162$$
$$n = 27$$

2.2.3

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$\frac{n}{2}[2(-8) + (n-1)(6)] > 10\,140$$
$$3n^2 - 11n > 10\,140$$
$$3n^2 - 11n - 10\,140 > 0$$
$$(3n + 169)(n - 60) > 0$$

When $n = 60$, $S_n = 10\,140$

Smallest $n = 61$

Activity 8

3.1.1

$$w-3;\ 2w-4;\ 23-w$$
$$(2w-4) - (w-3) = (23-w) - (2w-4)$$
$$w - 1 = 27 - 3w$$
$$4w = 28$$
$$w = 7$$

3.1.2 Sequence is: 4 ; 10 ; 16

First difference / *Eerste verskil* = 6

3.2

$$T_{50} = 3 + (4 + 10 + 16 + \ldots \text{ to 49 terms})$$
$$T_{50} = 3 + \frac{49}{2}[2(4) + (49-1)(6)]$$
$$= 3 + 7252$$
$$= 7255$$

OR

$$2a = 6$$
$$a = 3$$
$$3a + b = 4$$
$$3(3) + b = 4$$
$$b = -5$$
$$a + b + c = 3$$
$$3 - 5 + c = 3$$
$$c = 5$$
$$T_n = 3n^2 - 5n + 5$$
$$T_{50} = 3(50)^2 - 5(50) + 5$$
$$= 7255$$

Activity 9

2.1 20 ; 24 ; 28 ; 32 ; …
  4 4 4

$T_n = 20 + (n - 1)\,4$

$100 = 20 + 4n - 4$

$4n = 84$

$n = 21$

On the 21st day she will cycle 100 km.

OR

$T_n = 4n + 16$

$100 = 4n + 16$

$4n = 84$

$n = 21$

On the 21st day she will cycle 100 km.

OR

$100 = 20 + 80$

$\quad = 20 + 4(21 - 1)$

$\therefore n = 21$

2.2 $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{14} = \frac{14}{2}[2(20) + (14-1)4]$

$= 644 \text{ km}$

2.3 No.
It will not be humanly possible to just keep on increasing the distance covered indefinitely. For example: $T_{1\,000} = 4\,016$ km in one day.

TOPIC 3 : Geometric Sequence and Series, and Sum to infinity

Activity 1

3.1 $T_n = (8x^2)\left(\frac{x}{2}\right)^{n-1}$

3.2 $ratio = \frac{x}{2}$

$-1 < \frac{x}{2} < 1$

$-2 < x < 2$

3.3 $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{8x^2}{1-\frac{x}{2}}$

$S_\infty = \frac{8\left(\frac{3}{2}\right)^2}{1-\frac{1}{2}\left(\frac{3}{2}\right)}$

$S_\infty = 72$

Activity 2

4.1

$$S = a + ar + ar^2 + \ldots + ar^{n-1}$$

$$rS = ar + ar^2 + \ldots + ar^{n-1} + ar^n$$

$$S - rS = a - ar^n$$

$$S(1-r) = a(1-r^n)$$

$$S = \frac{a(1-r^n)}{1-r}$$

4.2.1 $15 \; ; \; 5 \; ; \; \frac{5}{3} \; ; \; \ldots$

$r = \frac{5}{15} = \frac{1}{3}$

The series converges because $-1 < r < 1$

4.2.2 $S_\infty = \frac{15}{1-\frac{1}{3}}$

$= \frac{45}{2}$

Activity 3

2.2.1 $5x \; ; \; x^2 \; ; \; \frac{x^3}{5} \; ; \; \ldots$

$r = \frac{x}{5}$

$-1 < \frac{x}{5} < 1$

$-5 < x < 5$

Answer can be written as $x \in (-5 \; ; \; 5)$

2.2.2 $r = \frac{2}{5}$ and $a = 10$

$S_\infty = \frac{10}{1-\frac{2}{5}}$

$= \frac{50}{3}$ or $16,67$

Activity 4

5.1 First year: 150

Second year: 150 + 18 = 168

Third year: $168 + \frac{8}{9}(18) = 184$

Growth = $18\left(\frac{8}{9}\right)^{n-2}$ after n years

17^{th} year growth is $18\left(\frac{8}{9}\right)^{17-2} = 3{,}08$ cm

	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5	Yr 6	Yr 7	Yr 8	Yr 9
Ht	150	168	184	198,2	210,84	222,07	232,06	240,94	248,83
Inc		18	16	14,2	12,64	11,23	9,99	8,88	7,89
	Yr 10	**Yr 11**	**Yr 12**	**Yr 13**	**Yr 14**	**Yr 15**	**Yr 16**	**Yr 17**	
Ht	255,84	262,08	267,62	272,55	276,93	280,82	284,28	287,36	
Inc	7,01	6,24	5,54	4,93	4,38	3,89	3,46	3,08	

5.2 Height after 10 years

$$= 150 + \frac{18\left(1 - \left(\frac{8}{9}\right)^{9}\right)}{1 - \frac{8}{9}}$$

= 150 + 105,8768146 …

= 255,88 cm

5.3 Max height = 150 + sum to infinity

$$= 150 + \frac{18}{1 - \frac{8}{9}}$$

= 150 cm + 162 cm

= 312 cm

The tree will never reach a height of more than 312 cm.

Activity 5

3.1.1 $T_n = ar^{n-1}$

$$= 27\left(\frac{1}{3}\right)^{n-1}$$

3.1.2 $-1 < r < 1$ or $|r| < 1$

OR

The common ratio (r) is $\frac{1}{3}$ which is between –1 and 1.

OR

$-1 < \frac{1}{3} < 1$

3.1.3

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{27}{1 - \frac{1}{3}}$$

$$= \frac{81}{2}$$ or 40,5 or 41

Activity 6

3.2 $a = 3 \; ; \; r = \frac{1}{3}$

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{3}{1-\frac{1}{3}}$$
$$= \frac{9}{2}$$

Activity 7

5.1 Area of unshaded square = 1- $\frac{1}{16}$

$$= \frac{15}{16}$$

5.2 Sum of the unshaded areas of the first seven squares

$$= (1-1) + \left(1-\frac{1}{4}\right) + \left(1-\frac{1}{4^2}\right) + \cdots + \left(1-\frac{1}{4^6}\right)$$
$$= 7 - \left(1 + \frac{1}{4} + \frac{1}{4^2} + \cdots + \frac{1}{4^6}\right)$$
$$= 7 - \left(\frac{1\left(1-\left(\frac{1}{4}\right)^7\right)}{1-\frac{1}{4}}\right)$$
$$= 7 - 1{,}333251953...$$
$$= 5{,}666748047...$$
$$= 5{,}67$$

Activity 8

3.1 $S_\infty = 8 + \frac{8}{\sqrt{2}} + ...$

$r = \frac{1}{\sqrt{2}}$ and

$$s_\infty = \frac{a}{1-r}$$
$$= \frac{8}{1-\frac{1}{\sqrt{2}}}$$
$$= \frac{8\sqrt{2}}{\sqrt{2}-1}$$
$$= \frac{8\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$
$$= 8\sqrt{2}\sqrt{2} + 8\sqrt{2}$$
$$= 16 + 8\sqrt{2}$$

Activity 9

2.1.1

$T_3 = 20$ and $T_4 = 40$

$r = \frac{T_4}{T_3} = 2$

2.1.2

$T_n = ar^{n-1}$

$20 = a.2^{3-1}$

$a = 5$

$T_n = 5.2^{n-1}$

Activity 10

3.1.1

$r = -\frac{1}{2}$

$T_4 = 1\left(-\frac{1}{2}\right)$

$= -\frac{1}{2}$

3.1.2

$T_n = 4\left(-\frac{1}{2}\right)^{n-1}$

$\frac{1}{64} = 4\left(-\frac{1}{2}\right)^{n-1}$

$\frac{1}{256} = \left(-\frac{1}{2}\right)^{n-1}$

$\left(-\frac{1}{2}\right)^{8} = \left(-\frac{1}{2}\right)^{n-1}$

$8 = n - 1$

$n = 9$

3.1.3

$S_\infty = \frac{a}{1-r}$

$= \frac{4}{1-\left(-\frac{1}{2}\right)}$

$= \frac{8}{3}$

3.2 For a geometric sequence:

$\frac{x+1}{1} = \frac{x-3}{x+1}$

$x^2 + 2x + 1 = x - 3$

$x^2 + x + 4 = 0$

$x = \frac{-1 \pm \sqrt{1-4(1)(4)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{-15}}{2}$

Solution is non-real.
There is no x-value that makes the sequence geometric.

Activity 11

2.1.1

$$r = -\frac{32}{64} = -\frac{1}{2}$$

$$p = 256\left(-\frac{1}{2}\right)$$

$$p = -128$$

2.1.2

$$S_n = \frac{a\left[1-r^n\right]}{1-r}$$

$$S_8 = \frac{256\left[1-\left(-\frac{1}{2}\right)^8\right]}{1+\frac{1}{2}}$$

$$= \frac{512}{3}\left(\frac{255}{256}\right)$$

$$= 170$$

2.1.3 $-1 < r < 1$

2.1.4

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{256}{1-\left(-\frac{1}{2}\right)}$$

$$= \frac{512}{3}$$

$$= 170{,}67$$

Activity 12

4.1

$$S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$$

$$a = p\left[1-\left(\frac{1}{2}\right)^1\right]$$

$$= \frac{p}{2}$$

$$r = \frac{1}{2}$$

$$\therefore 10 = \frac{\frac{p}{2}}{1-\frac{1}{2}}$$

$$5 = \frac{p}{2}$$

$$p = 10$$

4.2

$$r = \frac{1}{2}$$

$$\frac{a}{1-\frac{1}{2}} = 10$$

$$a = 5$$

$$T_2 = ar = \frac{5}{2}$$

TOPIC 4 : Sigma Notation

Activity 1

3.1 $-1 + 2 + 5 + \ldots$

OR

$-1 ; 2 ; 5$

3.2

$$S_n = -1 + 2 + 5 + 8 + \ldots \text{to } 100 \text{ terms}$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{100} = \frac{100}{2}\left[2(-1) + (100-1)(3)\right]$$

$$= 50[-2 + 297]$$

$$= 14\ 750$$

Activity 2

2.1

$$\sum_{n=1}^{20} 3^{n-2}$$

$$= \frac{1}{3} + 1 + 3 + \ldots \text{ to 20 terms}$$

$$= \frac{\frac{1}{3}\left(3^{20} - 1\right)}{3 - 1} \quad ; \quad r = 3; \ n = 20$$

$$= \frac{3^{20} - 1}{6}$$

$$= 581130733{,}33 \quad \text{OR} \quad 581130733\frac{1}{3} \quad \text{OR} \quad 581130733{,}3$$

Activity 3

2.2

$$P = \sum_{k=1}^{13} 3^{k-5}$$

$$= 3^{1-5} + 3^{2-5} + 3^{3-5} + \ldots + 3^{13-5}$$

$$= 3^{-4} + 3^{-3} + 3^{-2} + \ldots + 3^{8}$$

$$= \frac{3^{-4}\left(3^{13} - 1\right)}{3 - 1}$$

$$= 9841{,}49 \quad \text{or} \quad 9841\frac{40}{81} \quad \text{or} \quad \frac{797161}{81}$$

Activity 4	

3.2.1 $5 + 15 + 45 + \ldots + T_{20}$

$= \sum_{n=1}^{20} 5(3)^{n-1}$

3.2.2 $5 + 15 + 45 + \ldots + T_{20}$

$= \frac{5(3^{20} - 1)}{3 - 1}$

$= 8\ 716\ 961\ 000$

Activity 5	

2.3 $\sum_{n=1}^{46} (4n - 11)$

Activity 6	

2.3 $\sum_{k=1}^{30} (3k + 5)$

$a = 8 \quad n = 30 \quad d = 3$

$\sum_{k=1}^{30} (3k + 5) = \frac{30}{2}[2(8) + 29(3)]$

$= 15(103)$

$= 1545$

TOPIC 5 : Mixed Patterns

Activity 1

2.1.1 $\frac{1}{16}$; 13

2.1.2 $\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...to\ 25\ terms\right)$ $\quad$ $(4+7+10+13+...to\ 25\ terms)$

$\frac{a(r^n-1)}{r-1}$ $\quad$ $=\frac{n}{2}[2a+(n-1)d]$

$=\frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25}-1\right)}{\frac{1}{2}-1}$ $\quad$ $=\frac{25}{2}[2(4)+24(3)]$

= 0,9999999 $\quad$ = 1 000

$S_{50}=1001{,}00$

Activity 2

4.3.1 $S_{24}=2^{24+2}-4$

$=67108860$

4.3.2 $S_{24}=2^{24+2}-4=67108860$

$S_{23}=2^{23+2}-4=33554428$

$T_{24}=33554432$

4.3.3 $T_n=S_{n+1}-S_n$

$=2^{n+2}-2^{n+1}$

$=2\times 2^{n+1}-2^{n+1}$

$= \ ^{n+1}$

Activity 3

2.1.1 $T_n=5+(n-1)(4)$

$=4n+1$

2.1.2 $T_n=5(25)^{n-1}$

2.2 The sequence is 1 ; 1 + d ; 1 + 2d ; 1 + 3d; …. (AP)

and 1 ; r ; r^2 ; r^3 ; …. (GP)

$\therefore 1+d=r$ and $d=r-1$

But $1+2d=r^2$

$1+2(r-1)=r^2$		$r^2=1+2d$
$r^2-2r+1=0$		$(1+d)^2=1+2d$
$(r-1)^2=0$	OR	$1+2d+d^2=1+2d$
$r=1$		$d^2=0$
		$d=0$
		$r=1$

$\therefore d=0$

$\therefore$ the one and only such sequence is 1 ; 1; 1 ; …..

Nomsa is correct.

Activity 4

3.1 21; 24

3.2

$T_{2k} = 3.2^{k-1}$

and so $T_{52} = 3.2^{26-1} = 100663296$

$T_{2k-1} = 3 + 6(k-1) = 6k - 3$

and so $T_{51} = 6(26) - 3 = 153$

$$T_{52} - T_{51} = 100663296 - 153$$
$$= 100663143$$

3.3 For all $n \in \mathbf{N}$, $n = 2k$ or $n = 2k - 1$ for some $k \in \mathbf{N}$

If $n = 2k$:

$T_n = T_{2k} = 3.2^{k-1}$

If $n = 2k - 1$:

$$T_n = T_{2k-1}$$
$$= 6k - 3$$
$$= 3(2k - 1)$$

In either case, T_n has a factor of 3,
so is divisible by 3.

Activity 5

3.1 Jacob calculated that the sequence is geometric or exponential.
Vusi calculated that the sequence is quadratic.

3.2.1 $T_n = 3^n$

OR

$T_n = 3.3^{n-1}$

3.2.2

3   9   27   57

6   18   30

12   12

$2a = 12$ $\quad$ $3a + b = 6$ $\quad$ $a + b + c = 3$

$a = 6$ $\quad$ $18 + b = 6$ $\quad$ $6 - 12 + c = 3$

$b = -12$ $\quad$ $c = 9$

$T_n = 6n^2 - 12n + 9$

Activity 6

2.2.1 $\frac{-7}{125}$

2.2.2 $T_n = \frac{2+(n-1)(-3)}{(1).5^{n-1}}$

$T_n = \frac{5-3n}{5^{n-1}}$

2.2.3 $T_n = \frac{5-3n}{5^{n-1}}$

$T_{500} = \frac{5-3(500)}{5^{499}}$

$= \frac{-1495}{5^{499}}$

2.2.4 $5-3n < -59$

$-3n < -64$

$n > 21,333...$

$n = 22$

EXCTRACTS FROM PAST PAPERS

ACTIVITY 1

2.1	$\frac{90}{x}=\frac{81}{90}$ $81x=8\,100$ $x=100$ **OR/OF** $x=90\times\frac{10}{9}$ $x=100$	✓ $\frac{90}{x}=\frac{81}{90}$ ✓ answer (2) **OR/OF** ✓ $\frac{10}{9}$ ✓ answer (2)
2.2	$S_n=\frac{a(1-r^n)}{1-r}$ $S_n=\frac{100(1-(0,9)^n)}{1-0,9}$ $S_n=\frac{100(1-(0,9)^n)}{0,1}$ $\therefore S_n=1\,000(1-(0,9)^n)$	✓ $r=0,9$ ✓ substitution into correct formula (2)
2.3	$S_\infty=\frac{a}{1-r}$ $S_\infty=\frac{100}{1-\frac{9}{10}}$ $S_\infty=1000$ **OR/OF** $S_\infty=\lim_{n\to\infty}\left[1\,000\left(1-(0,9)^n\right)\right]$ $S_\infty=1000$	✓ substitution ✓ answer (2) **OR/OF** ✓ $S_\infty=\lim_{n\to\infty}\left[1\,000\left(1-(0,9)^n\right)\right]$ ✓ answer (2)
		[6]

ACTIVITY 2

QUESTION 3

3.1	-82	✓ answer (1)
3.2	$-145\,;-122\,;-101\,;\ldots$ 23 21 -2 -2 $2a=-2 \quad \therefore\ a=-1$ $3a+b=23 \quad \therefore\ 3(-1)+b=23 \quad \therefore b=26$ $a+b+c=-145 \ \therefore\ -1+26+c=-145 \ \therefore\ c=-170$ $\therefore T_n=-n^2+26n-170$ **OR/OF** $2a=-2 \quad \therefore\ a=-1$ $c=-145+(-2)-23=-170$ $\therefore T_n=-n^2+bn-170$ $-145=-1+b-170$ $b=26$ $\therefore T_n=-n^2+26n-170$	✓ $2a=-2$ ✓ $3(-1)+b=23$ ✓ $-1+26+c=-145$ (3) **OR/OF** ✓ $2a=-2$ ✓ $c=-145+(-2)-23$ ✓ $-145=-1+b-170$ (3)
3.3	$T_n=bn+c$ $T_n=-2n+25$ or $T_n=a+(n-1)d$ $=23+(n-1)(-2)$ $=25-2n$ $-2n+25=-121$ $-2n=-146$ $n=73$ Between T_{73} and T_{74} **OR/*OF*** $T_{n+1}-T_n=-(n+1)^2+26(n+1)-170-(-n^2+26n-170)$ $-121=-2n+25$ $n=73$ Between T_{73} and T_{74}	✓ $T_n=-2n+25$ ✓ $T_n=-121$ ✓ $n=73$ ✓answer (4) **OR/*OF*** ✓ $T_n=-2n+25$ ✓ $T_n=-121$ ✓ $n=73$ ✓ answer (4)

3.4	$n=\frac{-b}{2a}=\frac{-26}{2(1)}=13$ $T_{13}=-1$ $\therefore$ add 2 **OR/*OF*** $T'_n=-2n+26=0$ $n=13$ $T_{13}=-(13)^2+26(13)-170=-1$ $\therefore$ add 2	✓13 ✓ $T_{13}=-1$ ✓ add 2 (3) **OR/*OF*** ✓13 ✓ $T_{13}=-1$ ✓ add 2 (3)
		[11]

ACTIVITY 3

QUESTION/*VRAAG* 4

4.1	$a=5$ and/*en* $d=2$ $T_{51}=5+(51-1)(2)$ $=105$	✓ a and d ✓ substitution into correct formula ✓ answer (3)
4.2	$S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{51}=\frac{51}{2}[2(5)+(51-1)2]$ $=2\ 805$ or/*of* $S_{51}=\frac{51}{2}[5+105]$ $=2\ 805$	✓ substitution into correct formula ✓ answer (2)
4.3	$\sum_{n=1}^{5\,000}(2n+3)=5+7+9+....+10\ 003$	✓ expansion (1)

4.4	$T_1 = -3 \qquad T_{4999} = -2(4\,999) - 1 = -9\,999$ $\therefore \sum_{n=1}^{5000}(2n+3) + \sum_{n=1}^{4999}(-2n-1)$ $= (5 + 7 + 9 + \ldots + 9\,999 + 10\,001 + 10\,003) +$ $(-3 - 5 - 7 - 9 - \ldots - 9\,999)$ $= 10\,001 + 10\,003 - 3$ $= 20\,001$ **OR/OF** $S_{4999} = \frac{4\,999}{2}\left[2(-3) + (4\,999-1)(-2)\right] = -24\,999\,999$ $S_{5000} = \frac{5\,000}{2}\left((2)(5) + (5\,000-1)(2)\right) = 25\,020\,000$ $\sum_{n=1}^{5000}(2n+3) + \sum_{n=1}^{4\,999}(-2n-1) = 25\,020\,000 - 24\,999\,999$ $= 20\,001$	✓ $T_1 = -3$ ✓ $T_{4999} = -9\,999$ ✓both expansions ✓ answer (A) (4) **OR/OF** ✓ $T_1 = -3$ ✓ $S_{4999} = -24\,999\,999$ ✓ $S_{5000} = 25\,020\,000$ ✓ answer (A) (4)
		[10]

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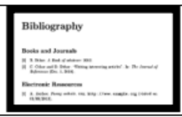
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