JENN

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MATHEMATICS

GRADE 12

EXERCISES LEARNER MANUAL

Probability

Probability

Outline :

1. Revise:
- dependent and independent events;
- the product rule for independent events: P(A and B) = P(A) $\times$ P(B).
- the sum rule for mutually exclusive events A and B: $P(A \text{ or } B) = P(A) + P(B)$
- the identity:

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

- the complementary rule:

$P(\text{ not } A) = 1 - P(A)$

2. probability problems using Venn diagrams, trees, two-way contingency tables and other techniques (like the fundamental counting principle) to solve probability problems (where events are not necessarily independent).
3. Apply the fundamental counting principle to solve probability problems

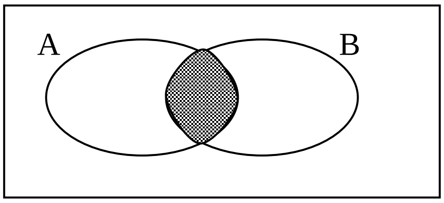
Contents

Part 1

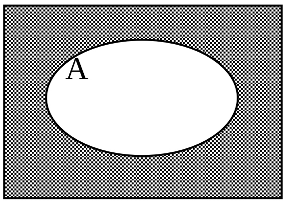
QUESTION 1

1.1 What expression BEST represents the shaded area of the following Venn diagrams?

1.1.1



1.1.2



1.2 State which of the following sets of events is mutually exclusive:

A Event 1: The learners in Grade 10 in the swimming team
Event 2: The learners in Grade 10 in the debating team

B Event 1: The learners in Grade 8
Event 2: The learners in Grade 12

C Event 1: The learners who take Mathematics in Grade 10
Event 2: The learners who take Physical Sciences in Grade 10

1.3 In a class of 40 learners the following information is TRUE:

- 7 learners are left-handed
- 18 learners play soccer
- 4 learners play soccer and are left-handed
- All 40 learners are either right-handed or left-handed

Let L be the set of all left-handed people and S be the set of all learners who play soccer.

1.3.1 How many learners in the class are right-handed and do NOT play soccer?

1.3.2 Draw a Venn diagram to represent the above information.

1.3.3 Determine the probability that a learner is:

(a) Left-handed or plays soccer

(b) Right-handed and plays soccer

QUESTION 2

2.1 At a certain school there are 64 boys in Grade 10. Their sport preferences are indicated below:

- 24 boys play soccer
- 28 boys play rugby
- 10 boys play both soccer and rugby
- 22 boys do not play soccer or rugby

2.1.1 Represent the information above in a Venn diagram.

2.1.2 Calculate the probability that a Grade 10 boy at the school, selected at random, plays:

(a) Soccer and rugby

(b) Soccer or rugby

2.1.3 Are the events a Grade 10 boy plays soccer at the school and a Grade 10 boy plays rugby at the school, mutually exclusive? Justify your answer.

QUESTION 3

Given: $P(W) = 0{,}4$
$P(T) = 0{,}35$
$P(T \text{ and } W) = 0{,}14$

3.1 Are the events W and T mutually exclusive? Give reasons for your answer.

3.2 Are the events W and T independent? Give reasons for your answer.

QUESTION 4

4.1 It is given that A and B are independent events. P(A) = 0,4 and P(B) = 0,5.

Use a Venn diagram and calculate:

4.1.1 P(A or B)

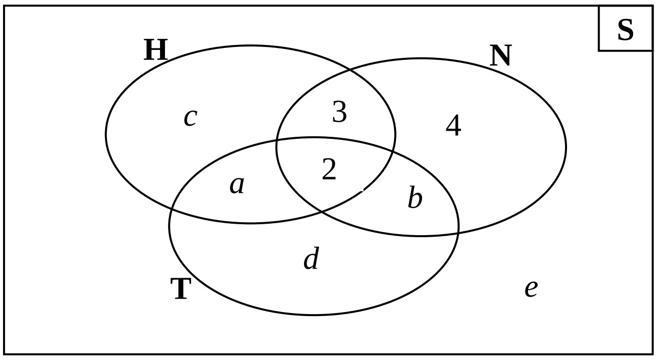
4.1.2 P(neither A or B)

QUESTION 5

5.1 A group of 33 learners was surveyed at a school. The following information from the survey is given:

- 2 learners play tennis, hockey and netball
- 5 learners play hockey and netball
- 7 learners play hockey and tennis
- 6 learners play tennis and netball
- A total of 18 learners play hockey
- A total of 12 learners play tennis
- 4 learners play netball ONLY

5.1.1 A Venn diagram representing the survey results is given below. Use the information provided to determine the values of a, b, c, d and e.



5.1.2 How many of these learners do not play any of the sports on the survey (that is netball, tennis or hockey)?

5.1.3 Write down the probability that a learner selected at random from this sample plays netball ONLY.

5.1.4 Determine the probability that a learner selected at random from this sample plays hockey or netball.

5.2 In all South African schools, EVERY learner must choose to do either Mathematics or Mathematical Literacy.

At a certain South African school, it is known that 60% of the learners are girls. The probability that a randomly chosen girl at the school does Mathematical Literacy is 55%. The probability that a randomly chosen boy at the school does Mathematical Literacy is 65%.

Determine the probability that a learner selected at random from this school does Mathematics.

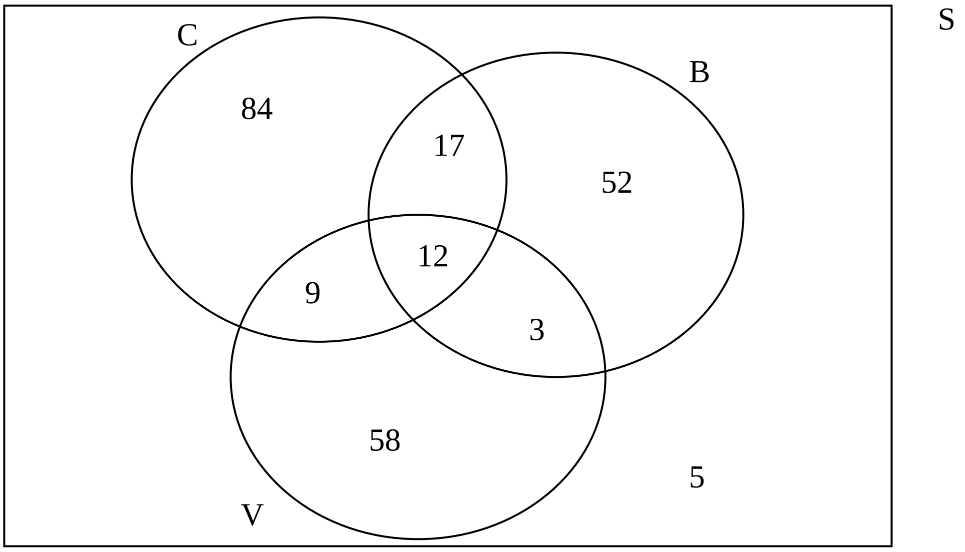
5.3 William writes a Mathematics examination and an Accounting examination.

He estimates that he has a 40% chance of passing the Mathematics examination.
He estimates that he has a 60% chance of passing the Accounting examination.
He estimates that he has a 30% chance of passing both.

Determine the probability that William will fail Mathematics and Accounting.

QUESTION 6

A survey was carried out with 240 customers who bought food from a fastfood outlet on a particular day. The outlet sells cheese burgers (C), bacon burgers (B) and vegetarian burgers (V). The Venn diagram below shows the number of customers who bought different types of burgers on the day.



6.1 How many customers did NOT buy burgers on the day?

6.2 Are events B and C mutually exclusive? Give a reason for your answer.

6.3 If a customer from this group is selected at random, determine the probability that he/she:

6.3.1 Bought only a vegetarian burger

6.3.2 Bought a cheese burger and a bacon burger

6.3.3 Did not buy a cheese burger

6.3.4 Bought a bacon burger or a vegetarian burger

QUESTION 7

Given: P(A) = 0,12
P(B) = 0,35
P(A or B) = 0,428

Determine whether events A and B are independent or not. Show ALL relevant calculations used in determining the answer.

QUESTION 8

8.1 Given: P(A) = 0,6
P(B) = 0,3
P(A or B) = 0,8 where A and B are two different events

Are the events A and B mutually exclusive? Justify your answer with appropriate calculations and/or a diagram.

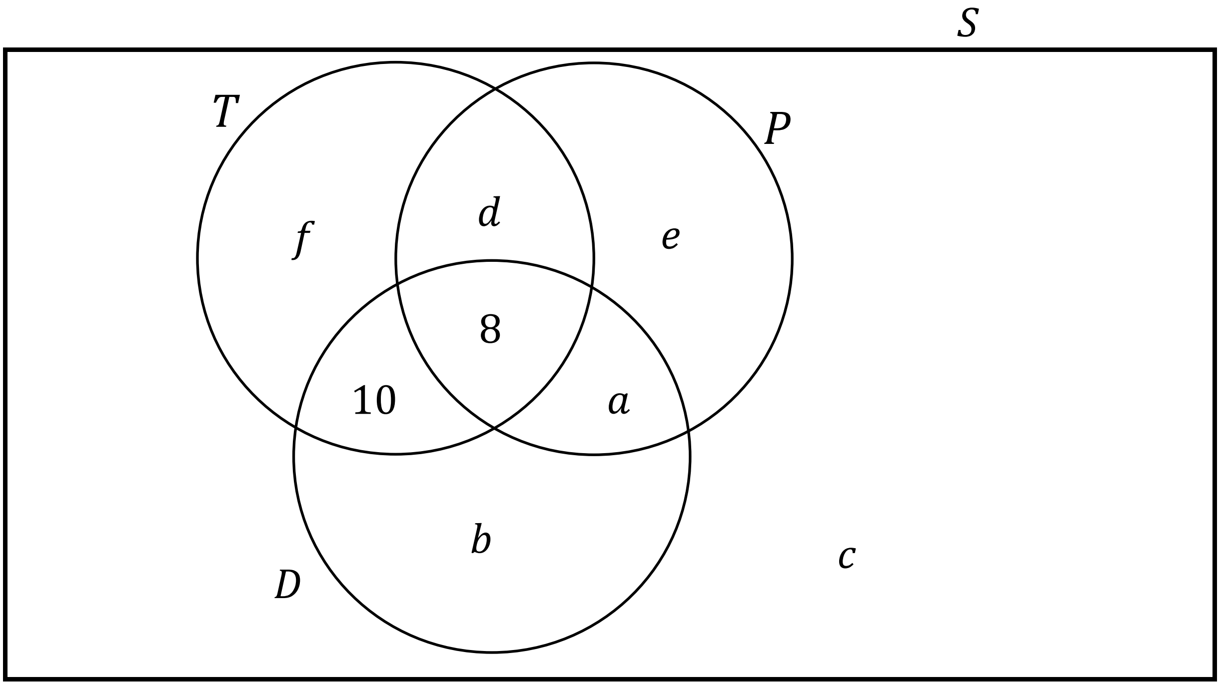
QUESTION 9

A survey regarding their favourite magazine(s) was conducted among 84 high school girls. Three magazines, namely *Teen Vogue* (T), *Drum* (D) and *People's Magazine* (P) were used in the survey.

The results are as follows:

- 41 read *Teen Vogue*.
- 34 read *People's Magazine.*
- 40 read *Drum*.
- 18 read *Teen Vogue* and *Drum*.
- 8 read all three magazines.
- 75 read at least one magazine.
- $n(P \text{ and } D) = 17$.

The Venn-diagram below shows the above information.

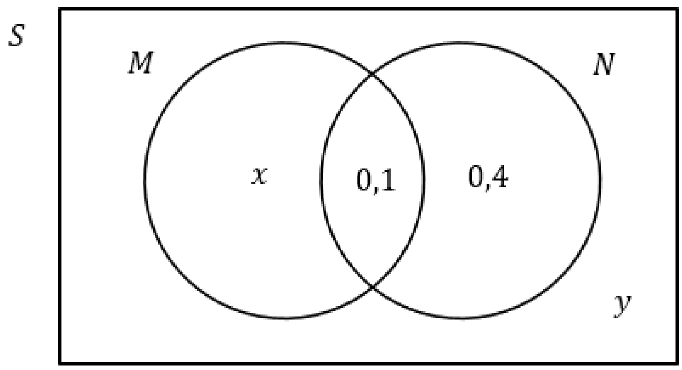


9.1 Determine the values of a, b, c, d, e and f.

9.2 Determine the probability that a randomly selected girl reads at least two of the three magazines.

QUESTION 10

10.1 The Venn-diagram below shows two independent events, M and N.



Determine the values of probabilities x and y. Show all calculations.

Part 2

QUESTION 1

1.1 The *Titanic* sank in 1912 without enough life boats for the passengers and crew. The contingency table below provides data on the passengers who were on board during the disaster. Use the information and determine, with reasons, whether the events M = {a passenger was male} and N = {a passenger did not survive} are independent or not.

***Titanic* survival data**

Gender

	Male	Female	Total
Yes	367	344	711
No	1 364	126	1 490
Total	1 731	470	2 201

QUESTION 2

2.1 The table below shows data on the monthly income of employed people in two residential areas. Representative samples were used in the collection of the data.

MONTHLY INCOME (IN RANDS)	AREA 1	AREA 2	TOTAL
$x < 3\ 200$	500	460	960
$3\ 200 \leq x < 25\ 600$	1 182	340	1 522
$x \geq 25\ 600$	150	14	164
Total	1 832	814	2 646

2.1.1 What is the probability that a person chosen randomly from the entire sample will be:

(a) From Area 1

(b) From Area 2 and earn less than R3 200 per month

(c) A person from Area 2 who earns more than or equal to R3 200

2.1.2 Prove that earning an income of less than R3 200 per month is not independent of the area in which a person resides.

2.1.3 Which is more likely: a person from Area 1 earning less than R3 200 or a person from Area 2 earning less than R3 200? Show calculations to support your answer.

QUESTION 3

3.1 The following contingency table shows information on the drivers' tests of 100 drivers tested at a test centre in Port Elizabeth.

	Male	**Female**	**Total**
Pass	30	47	77
Fail	7	16	23
Total	37	63	100

A driver is randomly selected from the 100 drivers.

3.1.1 Determine the probability that a female that failed is selected.

3.1.2 Determine the probability that the driver passed, given it is a male.

QUESTION 4

One hundred and seventy-five movie critics were invited to preview a new movie. After seeing the movie, a survey was conducted and the results were recorded in a two-way contingency table.

	Age < 40	**Age $\geq$ 40**	**Totals**
Liked the movie	65	37	102
Did not like the movie	b	31	a
Totals	c	d	175

4.1 Calculate the values of a, b, c and d in the contingency table.

4.2 A movie critic is selected at random. What is the probability that the critic was less than 40 years old and did not like the movie?

4.3 Are the events, age of the critic and preference for the movie, independent? Support your answer with the appropriate calculations.

QUESTION 5

The sports director at a school analysed data to determine how many learners play sport and what the gender of each learner is. The data is presented in the table below.

	DO NOT PLAY SPORT	**PLAY SPORT**	**TOTAL**
Male	51	69	120
Female	49	67	116
Total	100	136	236

5.1 Determine the probability that a learner, selected at random, is:

5.1.1 Male

5.1.2 Female and plays sport

5.2 Are the events 'male' and 'do not play sport' mutually exclusive? Use the values in the table to justify your answer.

5.3 Are the events 'male' and 'do not play sport' independent? Show ALL calculations to support your answer.

QUESTION 6

In a survey 1 530 skydivers were asked if they had broken a limb. The results of the survey were as follows:

	Broken a limb	Not broken a limb	TOTAL
Male	463	b	782
Female	a	c	d
TOTAL	913	617	1 530

6.1 Calculate the values of a, b, c and d.

6.2 Calculate the probability of choosing at random in the survey, a female skydiver who has not broken a limb.

6.3 Is being a female skydiver and having broken a limb independent? Use calculations, correct to TWO decimal places, to motivate your answer.

QUESTION 7

The data below was obtained from the financial aid office at a certain university.

	RECEIVING FINANCIAL AID	NOT RECEIVING FINANCIAL AID	TOTAL
Undergraduates	4 222	3 898	8 120
Postgraduates	1 879	731	2 610
TOTAL	6 101	4 629	10 730

7.1 Determine the probability that a student selected at random is ...

7.1.1 receiving financial aid.

7.1.2 a postgraduate student and not receiving financial aid.

7.1.3 an undergraduate student and receiving financial aid.

7.2 Are the events of being an undergraduate and receiving financial aid independent? Show ALL relevant workings to support your answer.

Part 3

QUESTION 1

1.1 During a survey, 25 out of the 40 learners in a class indicated that they own a cellphone. Two learners are selected at random from the class, the first not being replaced before the second one is selected.

1.1.1 Draw a tree diagram that shows the possible outcomes of the situation. Write the probabilities on the relevant branches.

1.1.2 What is the probability that of the two learners selected, one will own a cellphone and the other one not?

QUESTION 2

Paballo has a bag containing 80 marbles that are either green, yellow or red in colour. $\frac{3}{5}$ of the marbles are green and 10% of the marbles are yellow. Paballo picks TWO marbles out of the bag, one at a time and without replacing the first one.

2.1 How many red marbles are in the bag?

2.2 Draw a tree diagram to represent the above situation.

2.3 What is the probability that Paballo will choose a GREEN and a YELLOW marble?

QUESTION 3

Alfred and Barry have an equal chance of winning a point in a game.

3.1 Draw a tree diagram to represent the situation after a total of 3 points have been contested. Indicate on your diagram the probabilities and all the outcomes associated with each branch.

3.2 Calculate the probability that Barry would have won all 3 points.

3.3 Calculate the probability that Alfred would have won 2 points and Barry would have won 1 point of the 3 points contested.

3.4 Barry and Alfred play a fourth point. Calculate the probability that Alfred will win 3 of the 4 points contested.

QUESTION 4

4.1 There are 20 boys and 15 girls in a class. The teacher chooses individual learners at random to deliver a speech.

4.1.1 Calculate the probability that the first learner chosen is a boy.

4.1.2 Draw a tree diagram to represent the situation if the teacher chooses three learners, one after the other. Indicate on your diagram ALL possible outcomes.

4.1.3 Calculate the probability that a boy, then a girl and then another boy is chosen in that order.

4.1.4 Calculate the probability that all three learners chosen are girls.

QUESTION 5

The probability that it will rain on a given day is 63%. A child has a 12% chance of falling in dry weather and is three times as likely to fall in wet weather.

5.1 Draw a tree diagram to represent all outcomes of the above information.

5.2 What is the probability that a child will not fall on any given day?

5.3 What is the probability that a child will fall in dry weather?

QUESTION 6

During summer in a certain city in South Africa the probability of a sunny day is $\frac{4}{7}$ and the probability of a rainy day is $\frac{3}{7}$.

- If it is a sunny day, then the probability that Vusi cycles to work is $\frac{7}{10}$, the probability that Vusi drives to work is $\frac{1}{5}$ and the probability that Vusi takes the train to work is $\frac{1}{10}$.
- If it is a rainy day, then the probability that Vusi cycles to work is $\frac{1}{9}$, the probability that Vusi drives to work is $\frac{5}{9}$ and the probability that Vusi takes the train to work is $\frac{1}{3}$.

6.1 Draw a tree diagram to represent the above information. Indicate on your diagram the probabilities associated with each branch as well as all the outcomes.

6.2 For a day selected at random, what is the probability that:

6.2.1 It is rainy and Vusi will cycle to work

6.2.2 Vusi takes the train to work

6.3 If Vusi works 245 days in a year, on approximately how many occasions does he drive to work?

QUESTION 7

Figures obtained from a city's police department seem to indicate that of all the motor vehicles reported stolen, 80% were stolen by syndicates to be sold off and 20% were stolen by individual persons for their own use.

Of those vehicles presumed stolen by syndicates:

- 24% were recovered within 48 hours
- 16% were recovered after 48 hours
- 60% were never recovered

Of those vehicles presumed stolen by individual persons:

- 38% were recovered within 48 hours
- 58% were recovered after 48 hours
- 4% were never recovered

7.1 Draw a tree diagram for the above information.

7.2 Calculate the probability that if a vehicle were stolen in this city, it would be stolen by a syndicate and recovered within 48 hours.

7.3 Calculate the probability that a vehicle stolen in this city will not be recovered.

Part 4

QUESTION 1

Eight learners are seated on eight chairs in the front row at an assembly.

1.1 In how many different ways can these 8 learners be seated?

1.2 In how many different ways can the 8 learners be seated if 3 of the learners must sit together?

1.3 In how many different ways can the 8 learners be seated if 2 particular learners refuse to sit next to each other?

QUESTION 2

Consider the word: PRODUCT.

2.1 How many different arrangements are possible if all the letters are used?

2.2 How many different arrangements can be made if the first letter is T and the fifth letter is C?

2.3 How many different arrangements can be made if the letters R, O and D must follow each other, in any order?

QUESTION 3

Three items from four different departments of a major chain store will be featured in a one-page newspaper advertisement. The page layout for the advertisement is shown in the diagram below where one item will be placed in each block.

A	B	C
D	E	F
G	H	I
J	K	L

3.1 In how many different ways can all these items be arranged in the advertisement?

3.2 In how many different ways can these items be arranged if specific items are to be placed in blocks A, F and J?

3.3 In how many different ways can these items be arranged in the advertisement if items from the same department are grouped together in the same row?

QUESTION 4

There are 7 different shirts and 4 different pairs of trousers in a cupboard. The clothes have to be hung on the rail.

4.1 In how many different ways can the clothes be arranged on the rail?

4.2 In how many different ways can the clothes be arranged if all the shirts are to be hung next to each another and the pairs of trousers are to be hung next to each another on the rail?

4.3 What is the probability that a pair of trousers will hang at the beginning of the rail and a shirt will hang at the end of the rail?

QUESTION 5

The digits 0, 1, 2, 3, 4, 5 and 6 are used to make 3 digit codes.

5.1 How many unique codes are possible if digits can be repeated?

5.2 How many unique codes are possible if the digits cannot be repeated?

5.3 In the case where digits may be repeated, how many codes are numbers that are greater than 300 and exactly divisible by 5?

QUESTION 6

A South African band is planning a concert tour with performances in Durban, East London, Port Elizabeth, Cape Town, Bloemfontein, Johannesburg and Polokwane.

In how many different ways can they arrange their itinerary if:

6.1 There are no restrictions

6.2 The first performance must be in Cape Town and the last performance must be in Polokwane

6.3 The performances in the four coastal cities (the cities close to the sea or ocean) must be grouped together?

QUESTION 7

Every client of CASHSAVE Bank has a personal identity number (PIN) which is made up of 5 digits chosen from the digits 0 to 9.

7.1 How many personal identity numbers (PINs) can be made if:

7.1.1 Digits can be repeated

7.1.2 Digits cannot be repeated

7.2 Suppose that a PIN can be made up by selecting digits at random and that the digits can be repeated. What is the probability that such a PIN will contain at least one 9?

QUESTION 8

In Gauteng number plates are designed with 3 alphabetical letters, excluding the 5 vowels, next to one another and then any 3 digits, from 0 to 9, next to one another. The GP is constant in all Gauteng number plates, for example TTT 012 GP. Letters and digits may be repeated in a number plate.

8.1 How many unique number plates are available?

8.2 What is the probability that a car's number plate will start with a Y?

8.3 What is the probability that a car's number plate will contain only one 7?

8.4 How many unique number plates will be available if the letters and numbers are not repeated?

QUESTION 9

Consider the digits 1, 2, 3, 4, 5, 6, 7 and 8 and answer the following questions:

9.1 How many 2-digit numbers can be formed if repetition is allowed?

9.2 How many 4-digit numbers can be formed if repetition is NOT allowed?

9.3 How many numbers between 4 000 and 5 000 can be formed?

QUESTION 10

10.1 The Matric Dance Committee has decided on the menu below for the 2008 Matric Dance. A person attending the dance must choose only ONE item from each category, that is starters, main course and dessert.

MENU

STARTERS	MAIN COURSE	DESSERT
Crumbed Mushrooms	Fried Chicken	Ice-cream
Garlic Bread	Beef Bolognaise	Malva Pudding
Fish	Chicken Curry	
	Vegetable Curry	

10.1.1 How many different meal combinations can be chosen?

10.1.2 A particular person wishes to have chicken as his main course. How many different meal combinations does he have?

10.2 A photographer has placed six chairs in the front row of a studio. Three boys and three girls are to be seated in these chairs.

In how many different ways can they be seated if:

10.2.1 Any learner may be seated in any chair

10.2.2 Two particular learners wish to be seated next to each other

Annexure A: Information Sheet

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

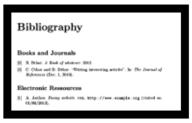
$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$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