JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

ACTIVITY MANUAL SOLUTIONS

GRADE 12

Probability

CONTENTS **PAGE**

PART 1

QUESTION 1

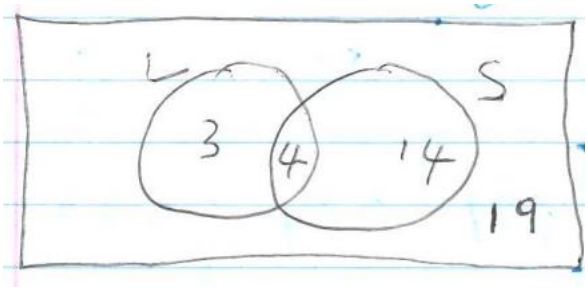
1.1.1 Inclusive events A and B

1.1.2 Sample space elements OR
Not A/ $A^{/}$

1.2 B

1.3.1 19

1.3.2

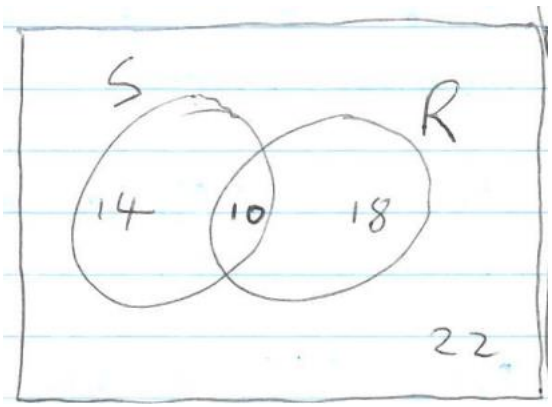


1.3.3 (a) $\frac{3+4+14}{40} = \frac{21}{40}$

(b) $\frac{14}{40}$

QUESTION 2

2.1.1



S= Soccer
R=Rugby

2.1.2 (a) $\frac{10}{64}$

(b) $\frac{14+10+8}{64} = \frac{42}{64}$

2.1.3 No, P(S and R) $\neq 0$

QUESTION 3

3.1 No, $P(\text{W and T}) \neq 0$

3.2 $P(\text{T and W}) = 0{,}14$

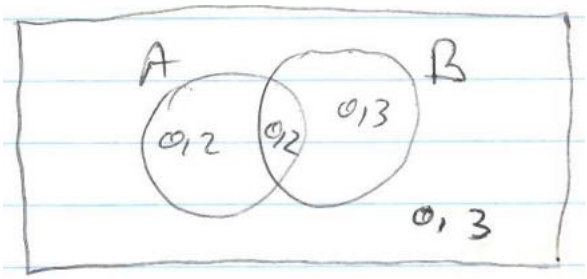
$P(\text{T}) \times P(W) = 0{,}35 \times 0{,}4 = 0{,}14$

$P(\text{T}) \times P(W) = P(\text{W and T})$

W and T are independent

QUESTION 4

4.1



4.1.1 $0{,}2 + 0{,}2 + 0{,}3 = 0{,}7$

4.1.2 0,3

QUESTION 5

5.1.1 $a = 5$

$b = 4$

$c = 8$

$d = 1$

$e = 6$

5.1.2 6 learners

5.1.3 $\frac{4}{33}$

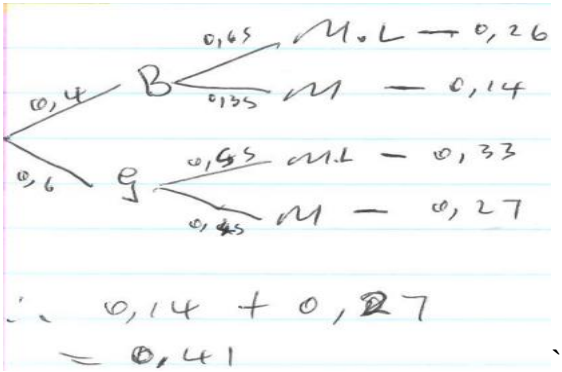
5.1.4 $\frac{C+A+2+3+B+4}{33} = \frac{8+5+2+3+4+4}{33} = \frac{26}{33} = 0{,}79$

OR

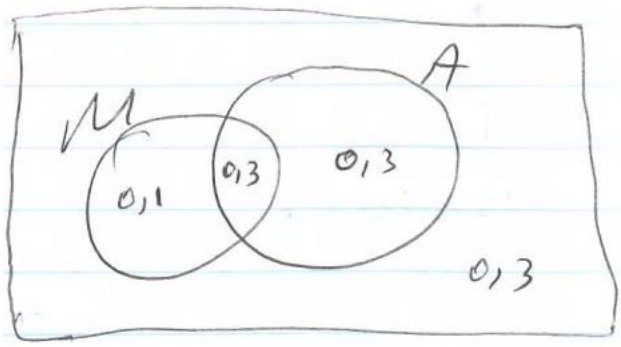
$P(HorN) = P(H) + P(N) - P(H\ and\ N)$

$= \frac{26}{33} + \frac{13}{33} - \frac{5}{33} = \frac{26}{33}$

5.2



5.3



$\therefore$ 0,3

QUESTION 6

6.1 5

6.2 No, $P(B\ and\ C) \neq 0$

6.3.1 $\frac{58}{240}$

6.3.3 $\frac{58+3+52+5}{240} = \frac{118}{240}$

6.3.4 $\frac{3+52+12+17+9+58}{240} = \frac{151}{240}$

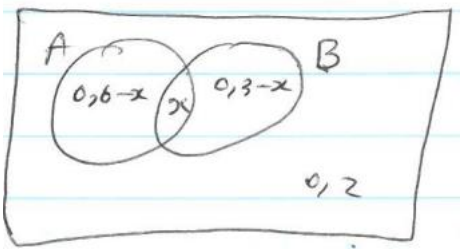
QUESTION 7

$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$
$0{,}428 = 0{,}12 + 0{,}35 - P(A\ and\ B)$
$P(A\ and\ B) = 0{,}042$
$P(A) \times P(B) = 0{,}12 \times 0{,}35 = 0{,}042$
$P(A\ or\ B) = P(A) \times P(B)$
A and B are independent

QUESTION 8

8.1 $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$
$0{,}8 = 0{,}6 + 0{,}3 - P(A\ and\ B)$
$P(A\ and\ B) = 0{,}1$
A and B are not mutually exclusive

OR



$0{,}6 - x + x + 0{,}3 - x = 0{,}8$

$-x = -0.1$

$x = 0.1$
$P(A\ and\ B) = 0.1$
A and B are not mutually exclusive

QUESTION 9

9.1 $c = 84 - 75 = 9$
$a = 17 - 8 = 9$
$b = 40 - (10 + 8 + 9) = 13$

$d + e + 9 + 8 = 34$
$d = 17 - e \ldots\ldots\ldots\ldots\ldots\ldots (1)$

$d + f + 8 + 10 = 41$
$d = 23 - f \ldots\ldots\ldots\ldots\ldots\ldots (2)$
(1)=(2) : $17 - e = 23 - f$

$f + d + e = 75 - (10 + 8 + 9 + 13)$
$f + d + e = 35 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$
$substitute\ (1) into\ (3)$
$f + 17 - e + c = 35$
$f = 18$
$d = 23 - f$
$d = 23 - 18 = 5$
$d = 17 - e$
$e = 12$

9.2 $\frac{d+a+10+8}{84} = \frac{5+9+10+8}{84} = \frac{32}{84}$

QUESTION 10

10.1 $P(W\ and\ N) = P(W) \times P(N)$
$0{,}1 = 0{,}5x + 0{,}05$
$0{,}05 = 0{,}5x$
$x = 0{,}1$
$x + 0{,}1 + 0{,}4 + y = 1$
$0{,}1 + 0{,}1 + 0{,}4 + y = 1$
$y = 0{,}4$

PART 2

QUESTION 1

1.1 $P(M) = \frac{1731}{2201}$

$P(N) = \frac{1490}{2201}$

$P(M) \times P(N) = \frac{1731}{2201} \times \frac{1490}{2201} = 0{,}53$

$P(M \text{ and } N) = \frac{1364}{2201} = 0{,}62$

$P(M \text{ and } N) \neq P(M) \times P(N)$

$\therefore \text{events are not independent}$

QUESTION 2

2.1.1 (a) $\frac{1832}{2646}$

(b) $\frac{460}{2646}$

(c) $\frac{340+14}{2646} = \frac{354}{2646}$

2.1.2 $P(x < 3200) \times P(A1)$

$\frac{960}{2646} \times \frac{1832}{2646} = 0{,}24$

$P(x < 3200 \text{ and } A1) = \frac{500}{2646} = 0{,}19$

$0{,}24 \neq 0{,}19$

$\therefore \text{not independent}$

2.1.3 $P(x < 3200 \text{ and } A1) = \frac{500}{1832} = 0{,}27$

$P(x < 3200 \text{ and } A2) = \frac{460}{814} = 0{,}57$

$\therefore \text{person from area 2 earning less than } R3200$

QUESTION 3

3.1.1 $\frac{16}{100}$

3.1.2 $\frac{30}{37}$

QUESTION 4

4.1 $a = 73$

$b = 42$

$c = 107$

$d = 68$

4.2 $\frac{72}{175}$

4.3 $P(A < 40 \text{ and liked the movie}) = \frac{107}{175} \times \frac{102}{175} = 0{,}36$

$P(A < 40) \times P(\text{liked movie}) = \frac{107}{175} \times \frac{102}{175} = 0{,}36$

$P(A < 40 \text{ and liked movie}) \neq P(A < 40) \times P(\text{liked movie})$

$\therefore$ *events not independent*

QUESTION 5

	DO NOT PLAY SPORT	PLAY SPORT	TOTAL
Male	51	69	120
Female	49	67	116
Total	100	136	236

5.1.1	$P(\text{male}) = \frac{120}{236}$ $= \frac{30}{59}$ $= 0{,}51\,(0{,}508474...)$	✓ 120 ✓ 236 (2)
5.1.2	$P(\text{female and plays sport})$ $= \frac{67}{236}$ $= 0{,}28\,(0{,}2838983051...)$	✓ 67 ✓ 236 (2)
5.2	No. From the table, $P(\text{male and do not play sport}) = \frac{51}{236}$, which is greater than zero. Since the probability of the intersection of these two events is greater than zero, these events are not mutually exclusive.	✓ No ✓ probability of intersection greater than zero (2)
5.3	$P(\text{male}) = \frac{120}{236}$ $P(\text{NS}) = \frac{100}{236}$ $P(\text{male}) \times P(NS) = \frac{120}{236} \times \frac{100}{236}$ $= \frac{750}{3\,481}$ $= 0{,}22 \quad (0{,}215455...)$ $P(\text{male and NS}) = \frac{51}{236}$ $= 0{,}22 \quad (0{,}2161016949...)$ So, $P(\text{male}) \times P(NS) = P(\text{male and NS})$ Therefore the events 'male' and 'do not play sport' are independent (correct to TWO decimal places). **OR** The events are not independent as there is a discrepancy from the third decimal place.	✓ $\frac{100}{236}$ ✓ $\frac{750}{3\,481}$ ✓ $\frac{51}{236}$ ✓ are independent (4) **[10]**

QUESTION 6

6.1 $b = 319$
$a = 450$
$c = 298$
$d = 748$

6.2 $\frac{298}{1530}$

6.3 F=Female Skydiver
B=Broken a limb
$P(F) \times P(B) = \frac{748}{1530} \times \frac{619}{1530} = 0{,}20$
$P(F \text{ and } B) = \frac{450}{1530} = 0{,}29$
$P(F) \times P(B) \neq P(F \text{ and } B)$
$\therefore$ *not independent*

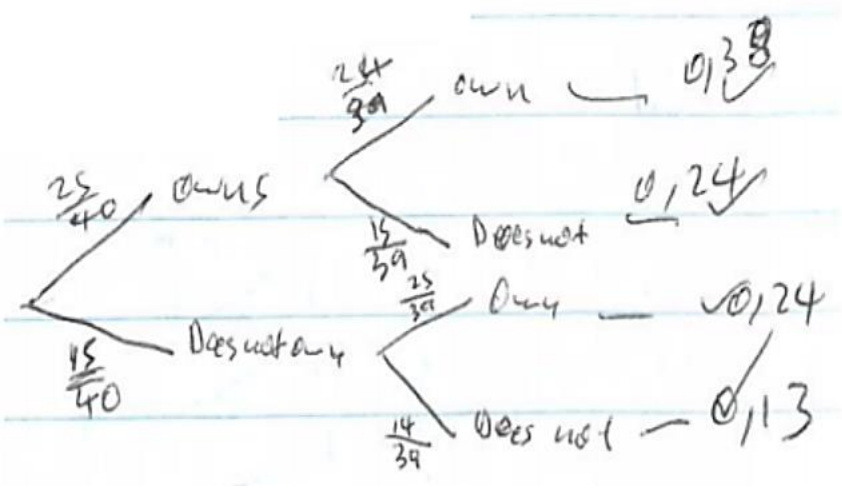
QUESTION 7

7.1.1	P(students receiving financial aid) $= \frac{6\,101}{10\,730}$ $= 0{,}57$ Answer only: Full marks	$\frac{6\,101}{10\,730}$ ✓ numerator ✓ denominator (2)
7.1.2	P(postgraduate not receiving financial aid) $= \frac{731}{10370}$ $= 0{,}068$ Answer only: Full marks Also accept: $\frac{731}{2610}$	$\frac{731}{10370}$ ✓ denominator ✓ numerator (2)
7.1.3	P(undergraduate receiving financial aid) $= \frac{4\,222}{10370}$ $= 0{,}39$ Answer only: Full marks Also accept: $\frac{4\,222}{8\,120}$	$\frac{4\,222}{10370}$ ✓ numerator ✓ denominator (2)
7.2	Let UG be the event of being an undergraduate and RF be the event of receiving financial aid. P(UG and RF) $= \frac{4\,222}{10\,730}$ $= 0{,}39$ P(UG) × P(RF) $= \frac{8\,120}{10\,730} \times \frac{6\,101}{10\,730}$ $= 0{,}43$ OR $= 0{,}76 \times 0{,}57$ $= 0{,}4332$ P(UG and RF) ≠ P(UG) × P(RF) The event of being an undergraduate and receiving financial aid are NOT independent.	✓ P(UG and RF) ✓ $\frac{4\,222}{10\,730} \times \frac{6\,101}{10\,730}$ ✓ P(UG and RF) ≠ P(UG) × P(RF) ✓ conclusion (4) [10]

PART 3

QUESTION 1

1.1.1



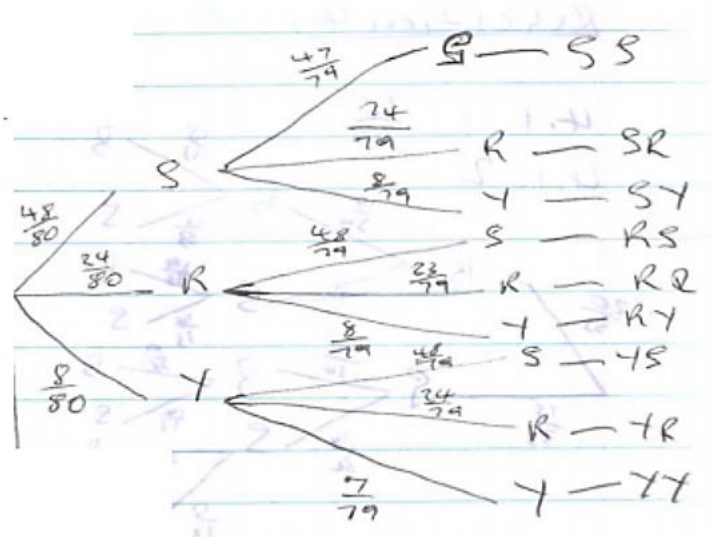
1.1.2 *Own and Doesnt own OR Doesnt own and Own*

$0{,}24 + 0{,}24 = 0{,}48$

QUESTION 2

2.1 $80 - \left(\frac{3}{5} \times 80 + 0{,}1 \times 80\right) = 24$

2.2

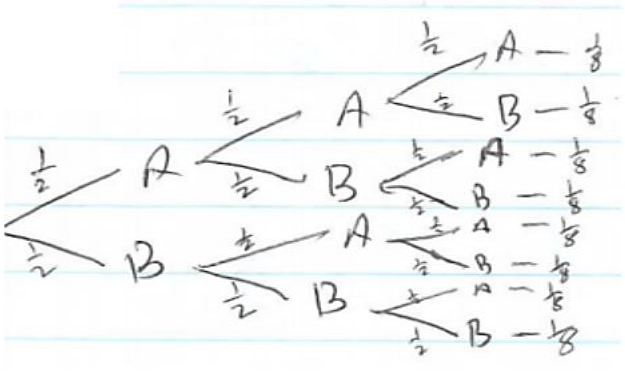


2.3 *GY or YG*

$\frac{48}{80} \times \frac{8}{79} + \frac{8}{80} \times \frac{48}{79} = 0{,}12$

QUESTION 3

3.1



3.2 $P(B \text{ and } B) = \frac{1}{8}$

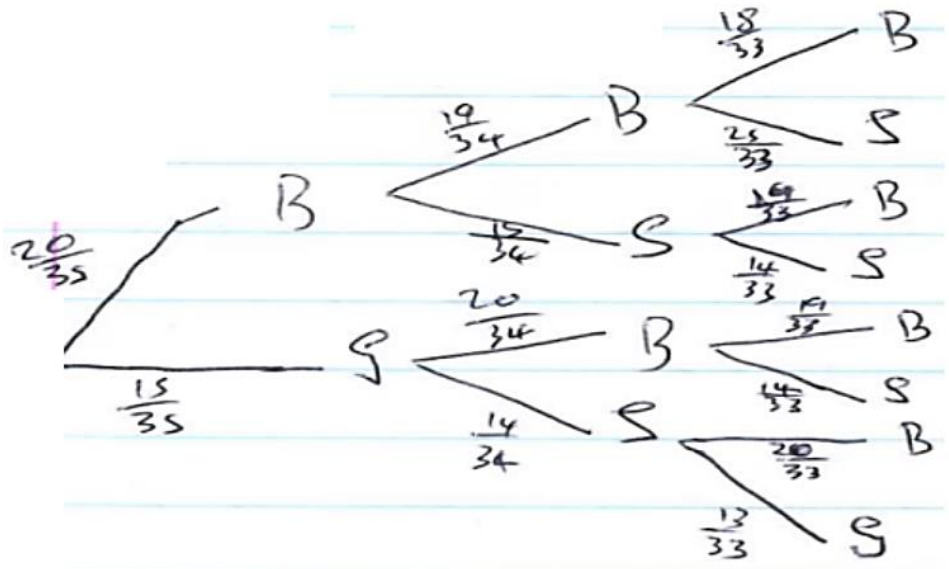
3.3 $P(AAB) OR\ P(ABA) OR\ P(BAA)$
$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

3.4 $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16} = 0{,}25$

QUESTION 4

4.1.1 $\frac{20}{35}$

4.1.2

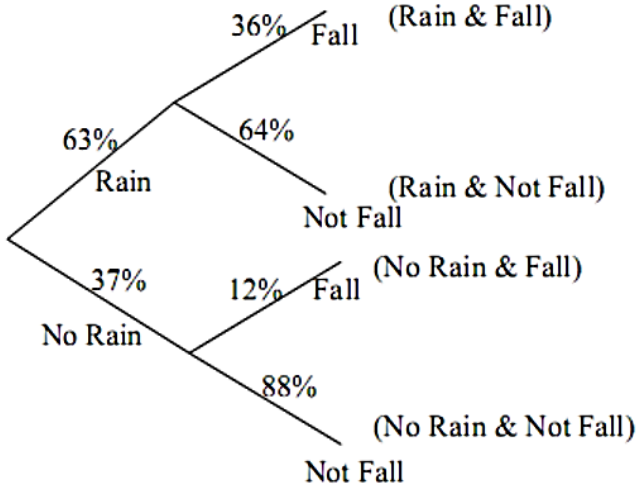


4.1.3 $P(B, G, B) = \frac{15}{35} \times \frac{14}{34} \times \frac{19}{33} = 0{,}15$

4.1.4 $P(G, G, G) = \frac{15}{35} \times \frac{14}{34} \times \frac{13}{33} = 0{,}07$

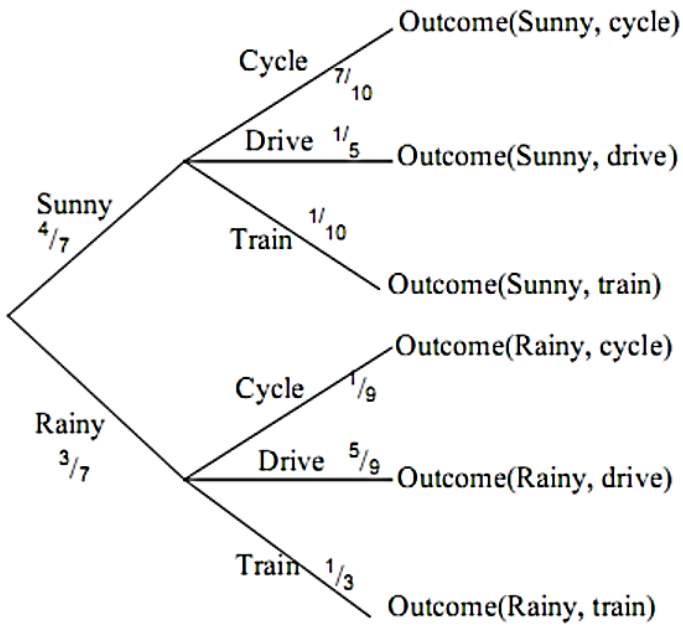
4.1.5 $1 - P(G, G, G) = 1 - 0{,}07 = 0{,}93$

QUESTION 5

5.1		✓✓ structure of the tree diagram ✓ 63% Rain ✓ 36% Fall ✓ 64% Not fall ✓ 88% Not Fall (6)
5.2	P(Not Fall) $= \left(\frac{37}{100} \times \frac{88}{100}\right) + \left(\frac{63}{100} \times \frac{64}{100}\right)$ $= \frac{407}{1250} + \frac{252}{625}$ $= \frac{911}{1250}$ $= 0{,}7288$	✓ $\frac{37}{100} \times \frac{88}{100}$ ✓ $\frac{63}{100} \times \frac{64}{100}$ ✓ answer (3)
5.3	P(Dry & Fall) $= \frac{37}{100} \times \frac{12}{100}$ $= \frac{111}{2500}$ $= 0{,}0444$	✓ $\frac{37}{100} \times \frac{12}{100}$ ✓ answer (2) **[11]**

QUESTION 6

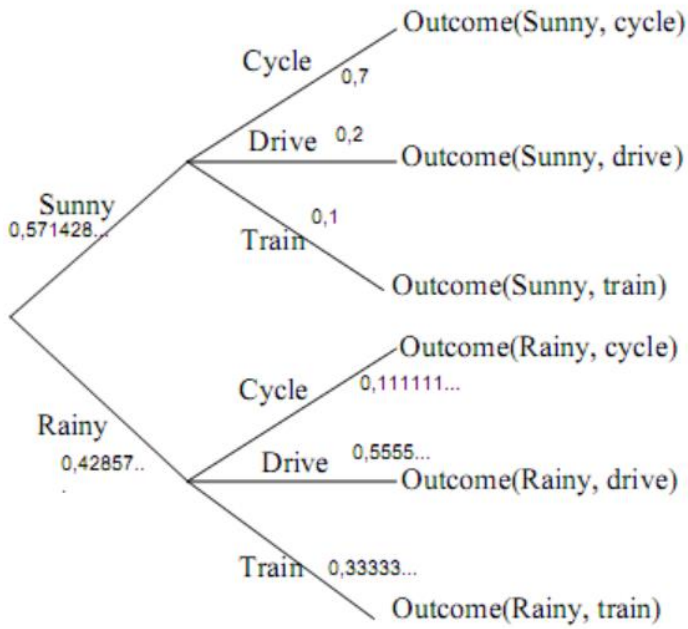
6.1



✓Sunny branch
✓Rainybranch
✓ cycle, drive, train branches on both weather types
✓ probabilities listed
✓ outcomes listed

(5)

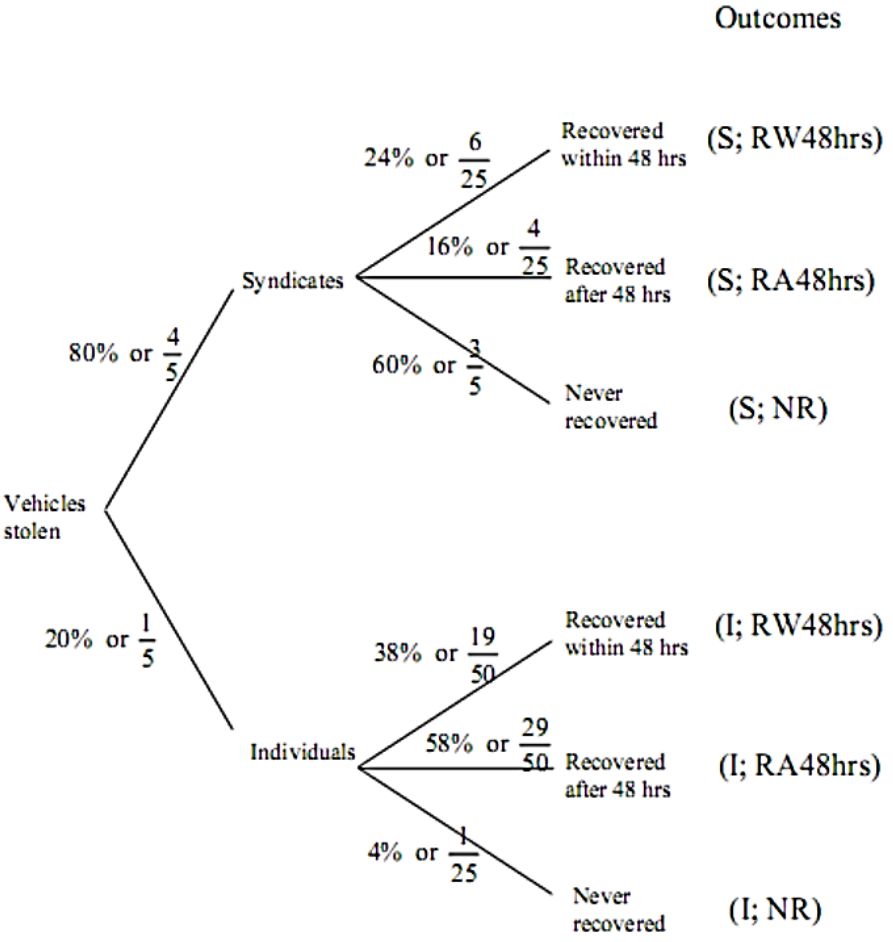
OR



6.2.1	P(Rainy, Cycle) $=\frac{3}{7}\times\frac{1}{9}$ $=\frac{1}{21}$ **OR** P(Rainy, Cycle) $=0,428...\times 0,1111...$ $=0,04761904762$ $\approx 0,05$ or 4,76% **Note:** If $\frac{3}{7}+\frac{1}{9}$ then 0 marks	✓$\frac{3}{7}\times\frac{1}{9}$ ✓ answerin any form (must be from multiplication) (2)
6.2.2	P(Train) $=\frac{4}{7}\times 0,1+\frac{3}{7}\times\frac{1}{3}$ $=\frac{1}{5}$ $=0,2$ $=20\%$ **OR** P(Train) $=\frac{4}{7}\times 0,1+\frac{3}{7}\times\frac{1}{3}$ $=0,05714...+0,1428...$ $=\frac{1}{5}$ $=0,2$ $=20\%$	✓$\frac{4}{7}\times 0,1$ and $\frac{3}{7}\times\frac{1}{3}$ ✓ addition ✓ answer (in any form) (3)
6.3	P(Drive) $=\frac{4}{7}\times 0,2+\frac{3}{7}\times\frac{5}{9}$ $=\frac{37}{105}$ $=0,35238...$ Vusi drives for $\frac{37}{105}\times 245 = 87$ days (86,333...) Accept: 86 days **OR** P(Drive) $=\frac{4}{7}\times 0,2\times 245+\frac{3}{7}\times\frac{5}{9}\times 245$ $=28+58,333$ $=87$ days (86,333...) Accept: 86 days	✓$\frac{4}{7}\times 0,2$ and $\frac{3}{7}\times\frac{5}{9}$ ✓ addition ✓$\frac{37}{105}$ ✓ answer (4) ✓$\frac{4}{7}\times 0,2$ and $\frac{3}{7}\times\frac{5}{9}$ ✓ addition ✓ $28+58,333$ ✓ answer (4) **[14]**

QUESTION 7

7.1



✓ First level
✓✓ Second level, Syndicates branch
Labels must be on (Recovered within 48 hrs; Recovered after 48 hrs; Never recovered)
✓✓ Second level, Individuals branch
Labels must be on (Recovered within 48 hrs; Recovered after 48 hrs; Never recovered)

Penalty 1 for every mistake in the second level branch

No values but tree diagram correct: 4 / 5

7.2 $P(S; RW48hrs) = \frac{80}{100} \times \frac{24}{100} = \frac{1\,920}{10\,000} = 0,192 = 19,2\%$ (0,19) (5)

OR

$P(S; RW48hrs) = \frac{4}{5} \times \frac{6}{25} = \frac{24}{125}$

✓ probability
✓ answer (2)

Penalty 1 for giving correct to 1 decimal place
Accept 0,19 and 0,192 or with more decimal places

7.3 P(stolen and not recovered) =

$\left(\frac{80}{100} \times \frac{60}{100}\right) + \left(\frac{20}{100} \times \frac{4}{100}\right) = 0,488 = 48,8\%$ (0,49)

✓ $\frac{12}{25}$ or 0,48
✓ $\frac{1}{125}$ or 0,08
✓ answer (3)

OR

P(stolen and not recovered) =

$\left(\frac{4}{5} \times \frac{3}{5}\right) + \left(\frac{1}{5} \times \frac{1}{25}\right) = \frac{12}{25} + \frac{1}{125} = \frac{61}{125}$

[10]

PART 4

QUESTION 1

1.1	Number of arrangements $= 7!$ $= 5040$	✓7 ✓7! (2)
1.2	Number of arrangements $= 5!$ $= 120$	✓5 ✓5! (2)
1.3	Number of arrangements $= 3! \times 5!$ $= 720$	✓ 3! ✓ 5! ✓ answer (3) [7]

QUESTION 2

2.1 $7! = 5040$
2.2 $2 \times 1 \times 5 \times 4 \times 3 \times 1 \times 2 \times 1 = 5! = 120$
2.3 $3! \times 5! = 720$

QUESTION 3

3.1 $12! = 429001600$
3.2 $9!$
3.3 $3! \times 4! = 144$

QUESTION 4

4.1 $11! = 39916800$
4.2 $7! \times 4! \times 2! = 241920$
4.3 $7 \times 9! \times 4$
$P(event) = \frac{14}{55} = 0{,}25$

QUESTION5

5.1

$= 7 \times 7 \times 7$
$= 7^3$
$= 343$

5.2

$= 7 \times 6 \times 5$
$= 210$

OR

$$\frac{7!}{4!}$$
$= 210$

5.3

$= 4 \times 7 \times 2 - 1$
$= 55$

OR

$14 \times 4 = 56$
$56 - 1 = 55$

QUESTION 6

6.1 $7! = 5040$

6.2 $1 \times 5 \times 4 \times 3 \times 2 \times 1 \times 1$
$\therefore 1 \times 5! \times 1 = 120$

6.3 $4! \times 4! = 576$

QUESTION 7

7.1.1	Number of PIN codes $= 10 \times 10 \times 10 \times 10 \times 10$ $= 10^5$ $= 100\ 000$	✓10 ✓ answer (2)
7.1.2	Number of PIN codes $= 10 \times 9 \times 8 \times 7 \times 6$ $= 30\ 240$ **OR** Number of PIN codes $= \frac{10!}{5!}$ $= 30\ 240$	✓ multiplication ✓ answer (2) ✓ $\frac{10!}{5!}$ ✓ answer (2)
7.2	Number of PINs that DO NOT contain 9s $= 9 \times 9 \times 9 \times 9 \times 9$ $= 59\ 049$ P(at least one 9) $= 1 - P(\text{no } 9s)$ $= 1 - \frac{59049}{100000}$ $= 0{,}41$ **OR** Number of PINs that DO NOT contain 9s $= 9 \times 9 \times 9 \times 9 \times 9$ $= 59\ 049$ Number of PINs that contain AT LEAST one 9 $= 100\ 000 - 59\ 049$ $= 40\ 951$ P(at least one 9) $= \frac{40951}{100000}$ $= 0{,}41$	✓ 9 ✓ 59 049 ✓ $1 - \frac{59049}{100000}$ ✓ answer (4) ✓ 9 ✓ 59 049 ✓ 40951 ✓ answer (4) **[8]**

QUESTION 8

8.1 $21 \times 21 \times 21 \times 10 \times 10 \times 10 = 21^3 \times 10^3 = 9261000$

8.2 $event = 1 \times 21 \times 21 \times 10 \times 10 \times 10$

$\frac{21^2 \times 10^3}{21^3 \times 10^3} = \frac{1}{21} = 0{,}05$

8.3 $\left(\frac{21^3 \times 9^2 \times 1}{21^3 \times 10^3}\right) \times 3 = \frac{243}{1000} = 0{,}24$

8.4 $21 \times 20 \times 19 \times 10 \times 9 \times 8 = 5745600$

QUESTION 9

9.1	Number of ways $= 8 \times 8$ $= 64$ If learner writes all the numbers out and then counts then, full marks Answer will be 11 12 13 14 15 16 17 18 21 22 23 24 25 26 27 28 31 32 33 34 35 36 37 38 41 42 43 44 45 46 47 48 51 52 53 54 55 56 57 58 61 62 63 64 65 66 67 68 71 72 73 74 75 76 77 78 81 82 83 84 85 86 87 88 64 ways to write a number	✓✓ answer (2) If candidate writes 8×7: 1/2
9.2	Number of ways for a 4-digit number $= 8 \times 7 \times 6 \times 5$ $= 1\,680$ Answer only: 3 / 3 **OR** Number of ways for a 4-digit number $= \frac{8!}{(8-4)!}$ $= \frac{8!}{4!}$ $= 1680$	✓ multiplication rule ✓ $8 \times 7 \times 6 \times 5$ ✓ answer (3) ✓✓ $\frac{8!}{(8-4)!}$ or $\frac{8!}{4!}$ ✓ 1680
9.3	Numbers between 4 000 and 5 000 $= 1 \times 8 \times 8 \times 8$ $= 512$ Answer only: 3 / 3 If leave answer as: $1 \times 8 \times 8 \times 8$ OR $8 \times 8 \times 8$: 2 / 3	✓ 1 ✓ 8^3 ✓ answer (3) **[8]**