Northern Cape
Department of Education

# MATHEMATICS- P1
# LAST PUSH

# TRIAL PAPERS
# PROVINCES-2024

maths

# ALGEBRA

## 1. EC

**QUESTION 1**

1.1 Solve for $x$:

1.1.1 $(2x-4)(x-1)=0$ (2)

1.1.2 $2x^2-3(x+2)=4$ (correct to TWO decimal places) (4)

1.1.3 $x^2+4x-21\leq 0$ (3)

1.1.4 $-\sqrt{x-1}=3-2x$ (4)

1.2 Solve simultaneously for $x$ and $y$:

$2x=1-y$ and $xy-x^2+y^2=5$ (6)

1.3 Given that:

- $f(x)=x^2+3x$
- $2x-[t(x)]^{\frac{1}{2}}=0$

For which values of $k$ will the equation $f(-x)+\dfrac{t(2k)}{4}=0$ have equal roots? (5)

**[24]**

## 2. FS

### QUESTION 1

1.1 Solve for $x$:

1.1.1 $(7-x)(10+x)=0$ (2)

1.1.2 $3x(2x+1)=1$ (correct to TWO decimal places) (4)

1.1.3 $6x^2+7x+2\geq 0$ (3)

1.1.4 $\sqrt{\sqrt{2x}+x}=2$ (5)

1.2 Solve simultaneously for $x$ and $y$:

$-2y+x=4$ and $x^2+xy-2y^2=0$ (5)

1.3 Given: $4^m=p\left(2^{2m-1}\right)+p$.

Show that for $p\neq 2$ the above equation can be written in the form $m=\frac{1}{2}\log_2\left(\frac{2p}{2-p}\right)$. (4)

**[23]**

## 3. GP

### QUESTION 1

1.1 Given: $2k=(x-5)(x-k)$, determine:

1.1.1 $k$ if $x=2$ (2)

1.1.2 $x$ if $k=2$ (4)

1.2 Solve for $x$:

1.2.1 $2x^2+3=8x$ (correct to TWO decimal places) (4)

1.2.2 $\sqrt{2(x+10)}-10=x-12$ (4)

1.2.3 $3^x(x-5)<0$ (2)

1.3 Solve the following equations simultaneously:

$\sqrt{3^x}.9^y=27$ and $x+4y^2=6$ (6)

1.4 The solutions of a quadratic equation are given by

$$x=\frac{-2\pm\sqrt{2p+5}}{7}.$$

State the value(s) of $p$ for which this equation will have:

1.4.1 Two equal solutions (1)

1.4.2 No real solutions (1)

**[24]**

## 4. KZN

### QUESTION 1

1.1 Solve for $x$:

1.1.1 $x^2 - 5x = 0$ (3)

1.1.2 $5x^2 + 2x - 6 = 0$ (answer correct to TWO decimal places) (3)

1.1.3 $2^{x+1} - 3.2^{x-1} + 2^x = 12$ (3)

1.1.4 $4x^2 + 12x + 9 > 0$ (3)

1.2 Solve simultaneously for $x$ and $y$:

$2x - y + 1 = 0$ and $x^2 + xy - y = 3x - 2$ (5)

1.3 Solve for $x$ in terms of $y$:

$(x+1)(x-3) = (y+1)(y-3)$, where $x \neq y$ (5)

**[22]**

## 5. LP

### QUESTION 1

1.1 Solve for $x$:

1.1.1 $x^2 - 3x + 2 = 0$ (3)

1.1.2 $3x^2 = -2 - 6x$ (Round off to TWO decimal digits) (4)

1.1.3 $2x - 1 = \sqrt{1 - x}$ (4)

1.1.4 $(x+3)(3-x) < 0$ (3)

1.2 Solve for $x$ and $y$ simultaneously:

$2x = y + 2$

$y - 2 = x^2 - 3x$ (6)

1.3 An athlete calculated that if he increases his current speed of $x$ km/h by 5 km/h, he can reduce his time (t) by 12 minutes. He will be participating in the City Marathon in Polokwane which is 72 km long.

Determine the value of $x$. (5)

**[25]**

## 6. MP

**QUESTION 1**

1.1 Solve for $x$:

1.1.1 $(2-x)(x+3)=0$ (2)

1.1.2 $3x^2-4x=5$ (4)

1.1.3 $\sqrt{5-x}-x=1$ (5)

1.1.4 $x(x-5)<0$ (2)

1.2 Solve for $x$ and $y$ simultaneously:

$-2y+x=-1$
$x^2-7-y^2=-y$ (6)

1.3 Prove that the roots of the following equation are non-real for all real values of $a$ and $b$, $a \neq 0$ and $b \neq 0$.
$a^2x^2+abx+b^2=0$ (3)

**[22]**

## 7. NC

**QUESTION 1**

**1.1** Solve for $x$:

1.1.1 $(x+2)^2-9=0$ (3)

1.1.2 $2x^2-3x-4=0$ (answers correct to TWO decimal places) (3)

1.1.3 $x^2-2x \leq 15$ (4)

1.1.4 $\sqrt{5^x}+\sqrt{5^{x+2}}=150$ (4)

**1.2** Solve for $x$ and $y$ simultaneously:
$x-y=3$ and $x^2-xy=2y^2+7$ (6)

**1.3** Given: $(a-b)(a+b)=2b(34a-b)$, $a>0$ and $b>0$

Determine $(a+b)^2$ in terms of $a$ and $b$. (4)
**[24]**

## 8. NW

### QUESTION 1

1.1 Solve for $x$:

1.1.1 $(2x-6)(x+5)=0$ (2)

1.1.2 $7x^2-11x+3=0$ (correct to TWO decimal places) (3)

1.1.3 $x^2 \geq 5x$ (4)

1.1.4 $3\sqrt{x+12}-x=8$ (5)

1.2 Solve for $x$ and $y$ simultaneously:

$2y=5+x$ and $y^2+3xy = 2x^2+50$ (6)

1.3 Determine the value of: $\dfrac{\left(2^{2p-1}\right)^3}{\sqrt{7^k}}$ if $2^{6p}=81$ and $7^k=729$ (4)

**[24]**

## 9. WC

1.1 Solve for $x$:

1.1.1 $(x-1)(2x-6)=0$ (2)

1.1.2 $x^2-7x-7=0$ (answers correct to TWO decimal places) (3)

1.1.3 $6x^2+7x>5$ (4)

1.1.4 $1=\frac{-6}{\sqrt{x+2}}+\sqrt{x+2}$ (5)

1.2 Solve for $x$ and $y$ simultaneously:

$6y+2x=4$ and $x^2+xy=4$ (6)

1.3 Simplify, without the use of a calculator:

$\sqrt{3}.\sqrt{48}-\frac{4^{x+1}}{2^{2x}}$ (3)

1.4 Given: $f(x)=3(x-1)^2+5$ and $g(x)=3$

1.4.1 Is it possible for the graphs of $f$ and $g$ to intersect? Give a reason for your answer. (2)

1.4.2 Determine the value(s) of $k$ for which $f(x)=g(x)-k$ has TWO unequal real roots. (4)

**[29]**

# NUMBER PATTERNS

# LINEAR & QUADRATIC PATTERNS

## EC

**QUESTION 2**

2.1 Given the quadratic number pattern: $-5; -4; -1; 4; \ldots$

2.1.1 Determine the $n^{th}$ term of the quadratic number pattern in the form $T_n = an^2 + bn + c$. (4)

2.1.2 Calculate the 35$^{th}$ term of the quadratic number pattern. (1)

2.1.3 Which TWO consecutive terms of the first differences sequence will have a product of 1 155? (4)

2.2 Given the arithmetic sequence: 60; 65; 70; ...

Calculate the value of $p$ for which $T_p = 430$. (3)

2.3 The sum of the first three terms of an increasing arithmetic series is 30 and the product of the same three terms is 510. Determine the values of $a$ and $d$, the first term and the common difference of the series respectively. (5)
**[17]**

## FS

**QUESTION 2**

Given: 0; 7 and 12 are the third, fourth and fifth terms of a quadratic number pattern.

2.1 Calculate the first term of the sequence. (2)

2.2 Determine an expression for the n$^{th}$ term of the pattern. (4)

2.3 Determine which term of the pattern will have the highest value. (3)
**[9]**

# GP

## QUESTION 2

2.1 Given the quadratic sequence: 0 ; 5 ; 14 ; ... ; 779 ; 860

2.1.1 Write down the value of the 4[th] term, $T_4$, of this sequence. (1)

2.1.2 Determine an expression for the $n^{th}$ term of this sequence. (4)

2.1.3 Calculate the number of terms in the sequence. (3)

2.2 Determine the sum of the whole numbers between 100 and 1 000 which are divisible by 11. (5)

**[13]**

# KZN

## QUESTION 2

2.1 The first three terms of a quadratic sequence are given:

3; 8; 15; ......

2.1.1 Determine the general term $T_n$ of the sequence. (4)

2.1.2 Is 1700 a term in this sequence? Motivate your answer, using calculations. (4)

2.2 Evaluate:

$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + \text{---------} + 399^2 - 400^2$ (5)

**[13]**

# LP

## QUESTION 2

2.1 The 4th term of an arithmetic sequence is 5 and the 14th term is 15.

2.1.1 Calculate the common difference. (4)

2.1.2 Determine the general term $(T_n)$ of the sequence. (2)

2.1.3 Calculate the sum of the first 22 terms. (2)

2.2 A quadratic pattern has the following properties:

$T_1 = x$

$T_2 = 7$

$T_4 = 7x$

$T_3 - T_2 = 6$

Determine the value of $x$. (4)

**[12]**

# MP

## QUESTION 2

2.1 Consider the following quadratic sequence: $6\ ;\ x\ ;\ 26\ ;\ 45\ ;\ y\ ;\ \ldots$
Determine the values of $x$ and $y$. (6)

2.2 Given the following series: $220 + 213 + 206 + \ldots - 11$

2.2.1 Calculate the sum of the series. (5)

2.2.2 Write the series in sigma-notation. (3)

2.3 A ball is dropped from a height of 15 m. It bounces back and loses 10% of its previous height on each bounce. Show that the total distance the ball will bounce cannot exceed 290m. (4)

2.4 Given : $25\left(\frac{1-t}{3}\right)+5\left(\frac{1-t}{3}\right)^2+\left(\frac{1-t}{3}\right)^3+\ldots\ldots\ldots$

2.4.1 For which value(s) of $t$ will the series converge? (3)

2.4.2 If $t = 15$, calculate the sum to infinity of the series if it exists. (4)

2.5 The sum of the first $n$ terms of a sequence is $S_n = 2^{n-5} + 3$.
Determine the 70th term. Leave your answer in the form $a.b^p$ where $a$, $b$ and $p$ are all integers. (4)

**[29]**

## NC

### QUESTION 2

2.1 Given the arithmetic sequence: 1 ; 5 ; 9 ; ... ; 181

2.1.1 Determine the $n$th term of the sequence. (2)

2.1.2 Calculate the:

a) number of terms in the sequence (2)

b) sum of the series $13 + 17 + ... + 181$ (3)

2.2 The general term of a quadratic sequence is $T_n = an^2 - 5n + c$

If it is given that $T_2 = T_1 + 1$ and $T_6 = 48$, calculate the values of $a$ and $c$. (4)

**[11]**

## NW

### QUESTION 2

Consider the linear pattern: 4; 10; 16; …

2.1 Write down the value of the following term of the pattern. (1)

2.2 Determine the value of the $50^{th}$ term of this pattern. (2)

2.3 A quadratic sequence is defined as: $P_k = \sum_{n=0}^{k-1}(6n-2)$

2.3.1 Show that the first 3 terms of the quadratic sequence are given by: $-2; 2; 12; ...$ (3)

2.3.2 Determine the general term ($P_k$) of the quadratic sequence. Write your answer in the form $P_k = ak^2 + bk + c$. (4)

2.3.3 Determine the value of the $50^{th}$ term of this quadratic sequence. (2)

2.3.4 The number of terms that must be added to $P_{50}$ to form $P_q$ is $m$. The difference between $P_{50}$ and $P_q$ of the quadratic sequence is $7\,920 + m$. Determine $m$. (5)

**[17]**

**WC**

**QUESTION 2**

2.1 On the first day Brett played an online game for 12 minutes. The second day he played for 12 minutes longer than the previous day. He kept on increasing the time each day.

The record of the time he spent per day is given in the table below:

| DAY | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| TIME | 12 | 24 | 40 | $a$ | $b$ |

If the time spent playing the game forms a quadratic sequence, determine:

2.1.1 the values of $a$ and $b$. (2)

2.1.2 a formula for the time spent on the $n^{th}$ day. (4)

2.1.3 on which day he played for 312 minutes. (4)

# ARITHMETIC & GEOMETRIC SERIES

## EC

**QUESTION 3.**

3.1 An infinite geometric series has a first term of 2 and constant ratio of $\frac{1}{3}$.

3.1.1 Calculate the next two terms. (1)

3.1.2 Calculate the value of $S_\infty$. (2)

3.2 Determine the value of $m$ if:

$$\sum_{k=3}^{m} 8(2)^{k-1} = 131\ 040$$

(5)

**[8]**

## FS

**QUESTION 3**

3.1 Given the arithmetic sequence: $10x + 6; 2x + 4; 4x - 8$

3.1.1 Determine the value of $x$. (2)

3.1.2 Determine the $10^{th}$ term of the sequence. (3)

3.1.3 Determine the sum of the first 99 terms of the sequence. (2)

3.2 Consider the infinite geometric series:
$2(k-5) + 2(k-5)^2 + 2(k-5)^3 + \cdots$

3.2.1 For which value(s) of $k$, is the series convergent? (3)

3.2.2 If $k = 4\frac{1}{2}$, calculate $S_\infty$ (3)

3.3 Three numbers are in the ratio $1:3:10$. If 20 is subtracted from the third number, the numbers form a geometric sequence. Determine the three numbers. (4)

**[17]**

## GP

### QUESTION 3

3.1 Given the geometric sequence: $8(x-2)^2\ ;\ 4(x-2)^3\ ;\ 2(x-2)^4\ ;\ \ldots\ x \neq 2$

3.1.1 Determine the value(s) of $x$ where the sequence converges. (3)

3.1.2 Determine the sum to infinity of the series if $x = 2{,}5$. (4)

3.2 Given: $\sum_{k=3}^{12} 3(-2)^{k-2}$

3.2.1 How many terms are there in this series? (1)

3.2.2 Calculate the sum of the series. (3)

[11]

## KZN

### QUESTION 3

3.1 The first three terms of a geometric sequence are given: $81;\ m;\ \frac{m}{3};\ \ldots\ldots$

3.1.1 Determine the value of $m$. (2)

3.1.2 Calculate: $\sum_{t=1}^{9} 81\left(\frac{1}{3}\right)^{t-1}$ (4)

3.2 Given the arithmetic sequence: $\frac{12}{5};\ 3;\ \frac{18}{5};\ \ldots\ldots\ldots\ldots\ldots\ldots\ ;\frac{333}{5}$

3.2.1 Calculate the number of terms in this sequence. (4)

3.2.2 How many terms of this sequence are integers? (4)

[14]

# LP

## QUESTION 3

Given the geometric series: $2+\frac{2}{3}+\frac{2}{9}+$..................

3.1 Determine the sum to infinity. (3)

3.2 Show that the sum of the first $n$ terms of the series is given by $3-3\left(\frac{1}{3}\right)^n$. (3)

3.3 Calculate the smallest value of $n$ for which the sum of the first $n$ terms is greater than $2,99$. (5)

**[11]**

# NC

## QUESTION 3

3.1 Consider $\sum_{k=0}^{p}\left(3^{5-k}\right)=\frac{1093}{3}$

3.1.1 Write down the first three terms of the series. (1)

3.1.2 Does the series converge? Motivate your answer. (2)

3.1.3 Calculate the value of $p$. (4)

3.1.4 Calculate: $\sum_{k=1}^{\infty}\left(3^{5-k}\right)$ (2)

3.2
- The first term of a geometric series is 9.
- The ratio of the sum of the first four terms to the sum of the first two terms is 13 : 9.

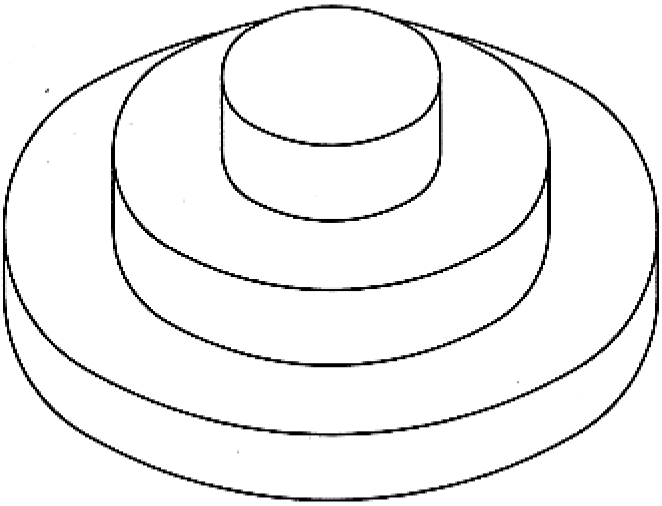
Calculate the common ratio (r) of the series. (5)

**[14]**

## NW

### QUESTION 3

A sightseeing point is built by placing concrete cylinders with a height of 0,2 m on top of one another. The radius of each consecutive cylinder is $\frac{4}{5}$ of the previous cylinder. The radius of the cylinder at the bottom is 15 m.



3.1 John is standing on the 17th cylinder. Calculate John's height above the ground. (1)

3.2 Calculate the volume of the 17th cylinder. (3)

3.3 Calculate the volume of concrete that will be used to fill the first 17 cylinders. (4)
**[8]**

## WC

2.2 Given: $S_\infty = \sum_{k=1}^{\infty} 4p^{1-k}$ , $p \neq 0$

2.2.1 Calculate the value of $p$, if it is given that $S_\infty = 3$. (4)

2.2.2 Is this series convergent? Explain your answer. (2)

2.3 An arithmetic series has a common difference of 3. The third and sixth terms of the series are respectively $\frac{3x-4}{2}$ and $\frac{-3x-10}{2}$.

2.3.1 Show that the value of $x = -4$. (2)

2.3.2 Hence, calculate the first term of the series. (3)

2.3.3 Determine the sum of the second set of 30 terms of this series. (4)
**[25]**

# FUNCTIONS

## 1. EC

## HYPERBOLA

**QUESTION 4**

Consider the function: $f(x)=\dfrac{-1}{x+5}-2$

4.1 Write down the equations of asymptotes of $f$. (2)

4.2 Determine the coordinates of the $x$-intercept of $f$. (2)

4.3 Determine the coordinates of the $y$-intercept of $f$. (2)

4.4 Sketch the graph of $f$, show clearly all asymptotes and intercepts with the axes. (3)

4.5 Determine the equation of the axis of symmetry that has a gradient of $-1$. (2)

**[11]**

## FS

**QUESTION 4**

Given the functions defined by: $f(x)=-\frac{2}{x}-1$ and $g(x)=k^x$.
The point $(1;3)$ lies on $g$.

4.1 Determine the value of $k$. (2)

4.2 Write down the equations of the asymptotes of $f$. (2)

4.3 Write down the equation of $g^{-1}$, the inverse of g. (2)

4.4 Determine the $x$-intercept(s) of $f$. (2)

4.5 Draw neat sketches of $f$ and $g^{-1}$ on the same system of the axes, clearly indicating all asymptotes and intercepts with the axes. (5)

4.6 For which values of $x$ will the axis of symmetry of $f$, which has a negative gradient, intersect with the graph of $f$? (4)

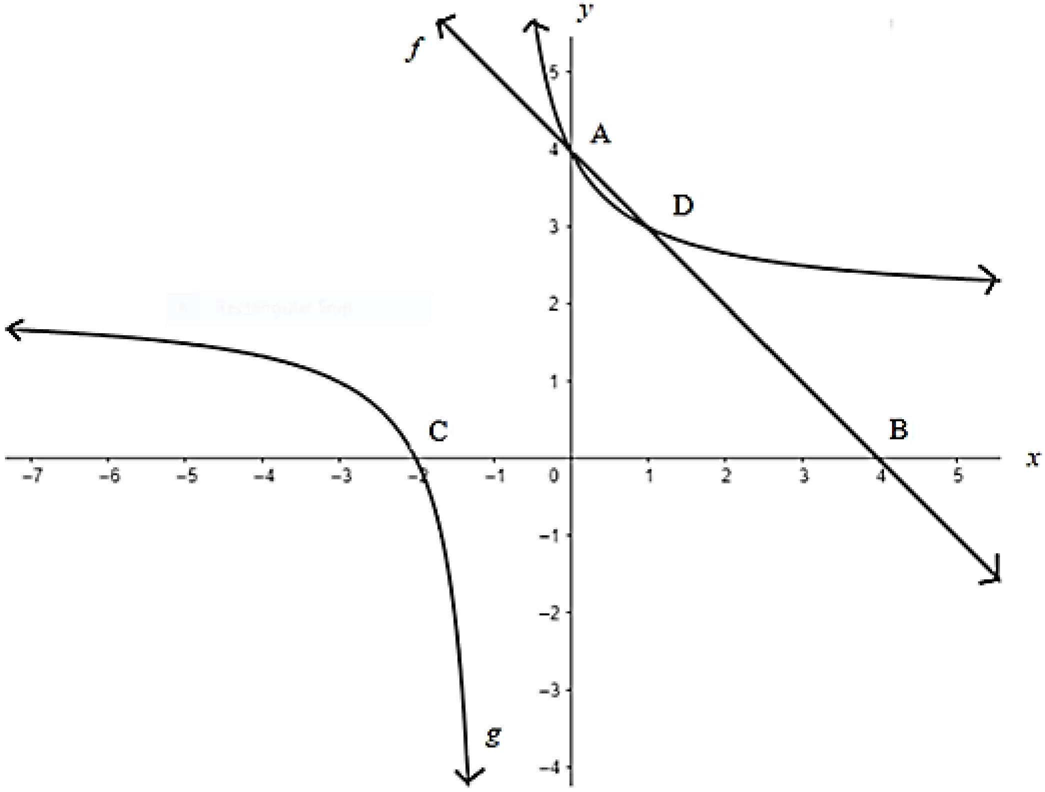
4.7 Consider the graph of $h$, where $h(x)=g^{-1}(x+2)$. Determine the value(s) of $x$ for which $g^{-1}(x+2)\le 1$. (2)

**[19]**

**GP**

**QUESTION 6**

Sketched below are the graphs of $f(x) = -x + 4$ and $g(x) = \dfrac{2}{x+1} + 2$.




6.1 Write down the domain of $g$. (1)

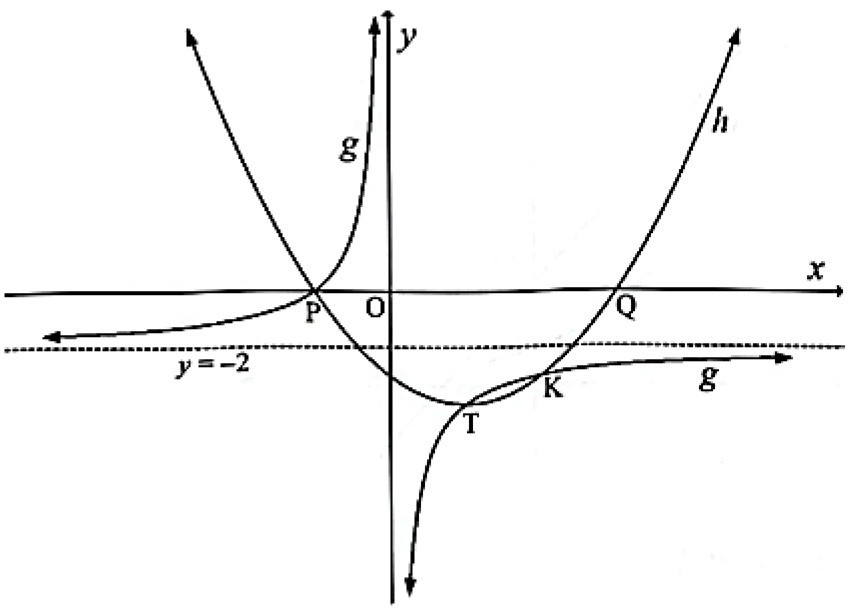
6.2 Write down the equations of the asymptotes of $g$. (2)

6.3 Calculate the coordinates of point D, a point of intersection of $g$ and $f$. (5)
**[8]**

**KZN**

## QUESTION 4

The graphs of $g(x)=\frac{a}{x}+q$ and $h(x)=x^2-2x-3$ are drawn. The graph of $h$ cuts the $x$-axis at P and Q. The two graphs intersect at points P, T and K. T is the turning point of $h$. The line $y=-2$ is the asymptote of $g$.




| | | |
|---|---|---|
| 4.1 | Calculate the coordinates of P and Q. | (3) |
| 4.2 | Calculate the coordinates of T. | (3) |
| 4.3 | Write down the equation of the vertical asymptote of $g$. | (1) |
| 4.4 | Determine the equation of $g$. | (3) |
| 4.5 | Is $g$ a function? Motivate your answer. | (2) |
| 4.6 | Calculate the coordinates of K. | (5) |
| 4.7 | For which values of $x$ will $g(x) > h(x)$? | (3) |

**[20]**

**LP**

4.2 Given: $f(x)=\frac{2}{x+2}+2$

4.2.1 Draw a sketch graph of $f$ clearly showing the intercepts and the asymptotes. (4)

4.2.2 Determine for which values of $x$ will: $\frac{2}{x+2}\geq -2$ (2)

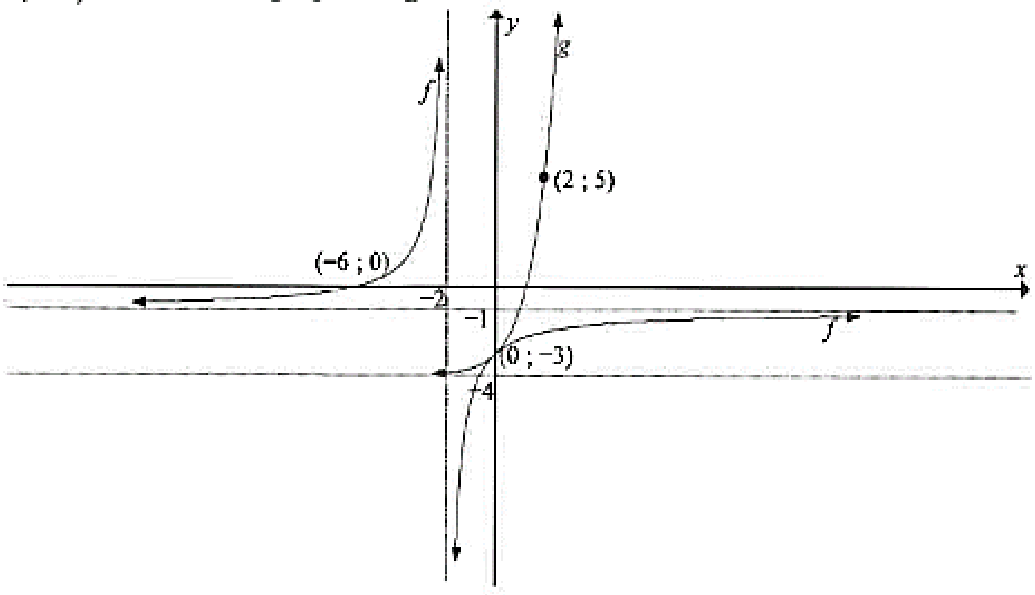
4.2.3 Determine the equation of the axes of symmetry for which the gradient is negative. (2)

**[27]**

**MP**

**QUESTION 3**

3.1 The sketch below shows the graph of $f(x)=\frac{a}{x+p}+q$ and $g(x)=b^x+c$. The $x$-intercept is at $(-6;0)$, and the $y$-intercepts of $f$ and $g$ is at $(0;-3)$. The point $(2;5)$ lies on the graph of $g$.



3.1 For which value(s) of $x$ is $f(x) = g(x)$. (1)

3.2 Write down the equation of the asymptotes of $f$. (2)

3.3 Hence determine the equation of $g$. (4)

3.4 Write down the range of $f$. (2)

3.5 Determine the equation of $k$ if $k(x)$ is the reflection of $g$ along the $x$-axis followed by a translation 4 units down. (2)

3.6 Determine the equation of $p(x)$, the axis of symmetry of $f$, if $p'(x) < 0$. (3)

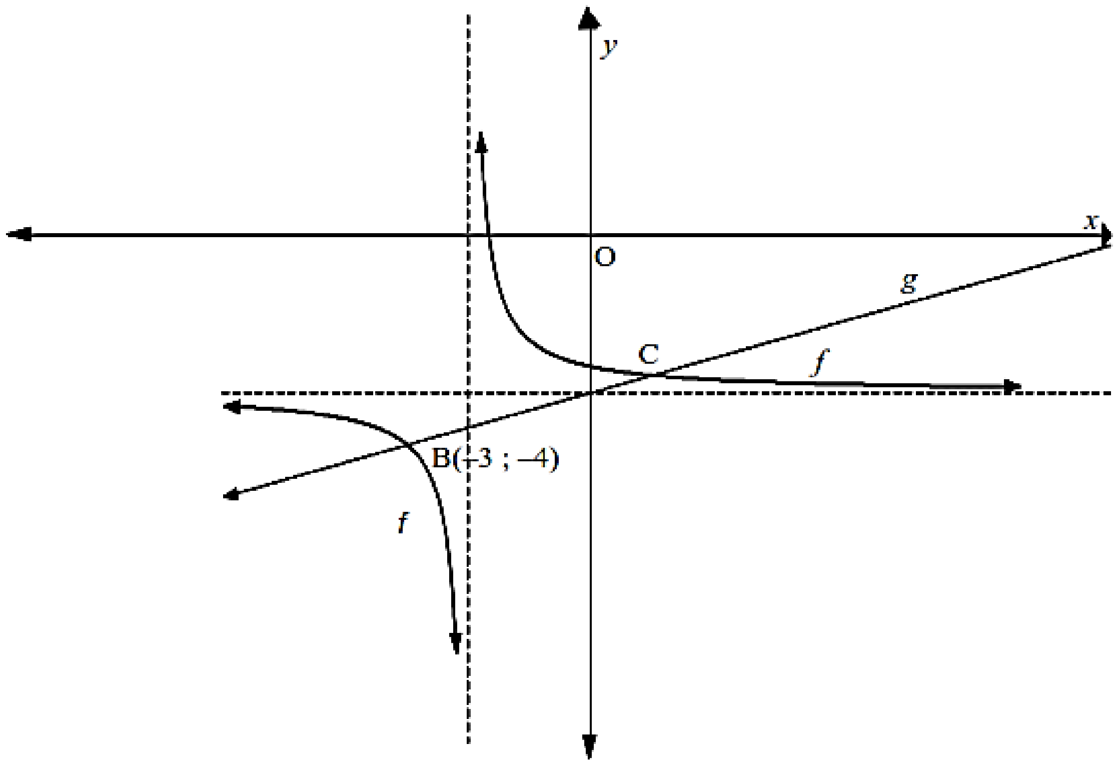
3.7 For which values of $x$ is $x.g'(x) \geq 0$? (2)

**[16]**

**NC**

## QUESTION 5

The graphs of $f(x) = \frac{a}{x+2} + q$ and $g(x) = mx - 3$ are drawn below. The graphs of $f$ and $g$ intersect at B(–3 ; –4) and C. The horizontal asymptote o $f$ goes through the $y$- intercept of $g$.



5.1 Write down the equations of the asymptotes of $f$. (2)

5.2 Determine the values of $a$ and $m$. (4)

5.3 Calculate the coordinates of C. (5)

5.4 Determine for which values of $x$ will $f(x) \leq g(x)$. (3)

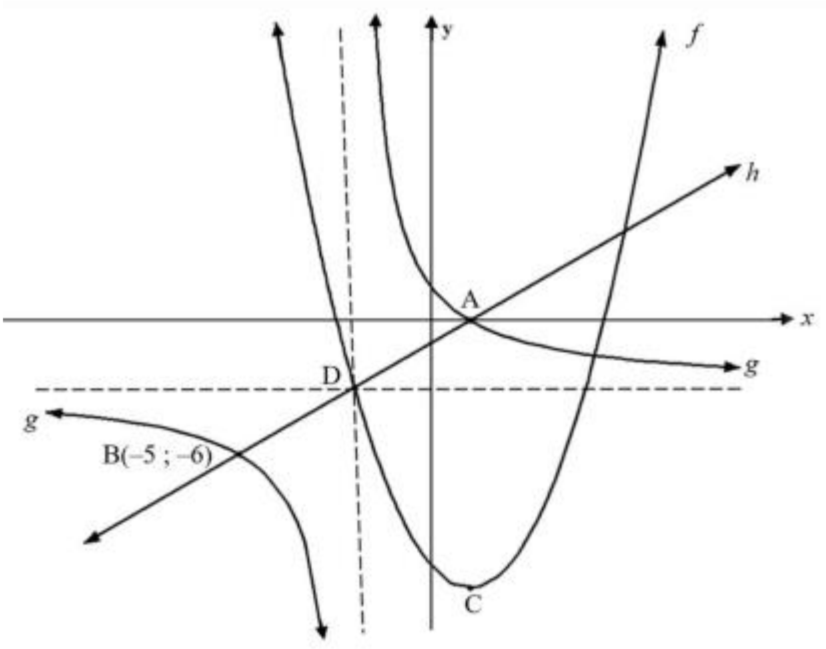
**[14]**

**NW**

## QUESTION 5

The graphs of $g(x) = \dfrac{a}{x+r} + t$ and $f(x) = (x+p)^2 + q$ are sketched below.

- The line $h(x) = x - 1$ is an axis of symmetry of $g$.
- Point A is the $x$-intercept of g.
- A and B($-5$ ; $-6$) are the points of intersection of $g$ and $h$.
- The axis of symmetry of $f$ intersects the $x$-axis at A.
- C is the turning point of $f$.
- D, a point on $f$, is the point of intersection of the asymptotes of $g$.



5.1 Determine the coordinates of A. (2)

5.2 Show that the coordinates of D is given by: D$(-2; -3)$ (2)

5.3 Determine the equation of $g$. (3)

5.4 Show that the equation of $f$ is: $f(x) = x^2 - 2x - 11$. (3)

5.5 Determine the $x$-intercepts of $f$. (3)

5.6 For which values of $x$ will $f'(x).f(x) < 0$? (2)

5.7 Calculate the maximum value of $\dfrac{f''(x)}{f(x)+14}$. (3)

5.8 For which value(s) of $m$ will $(x+m)^2 - 2(x+m) - 11 = x - 1$ have TWO different negative roots? (7)

**[25]**

# WC

## QUESTION 3

Given: $f(x) = \dfrac{2}{x-3} - 1$ and $g(x) = x - 3$

3.1 Write down the domain of $f$. (1)

3.2 Calculate the $x$-intercept of $f$. (2)

3.3 Draw the graphs of $f$ and $g$ on the same system of axes. Clearly show all the intercepts with the axes as well as the asymptotes. (5)

3.4 Calculate the value(s) of $x$ for which $f(x) \geq g(x)$ if $x \geq 3$. (4)

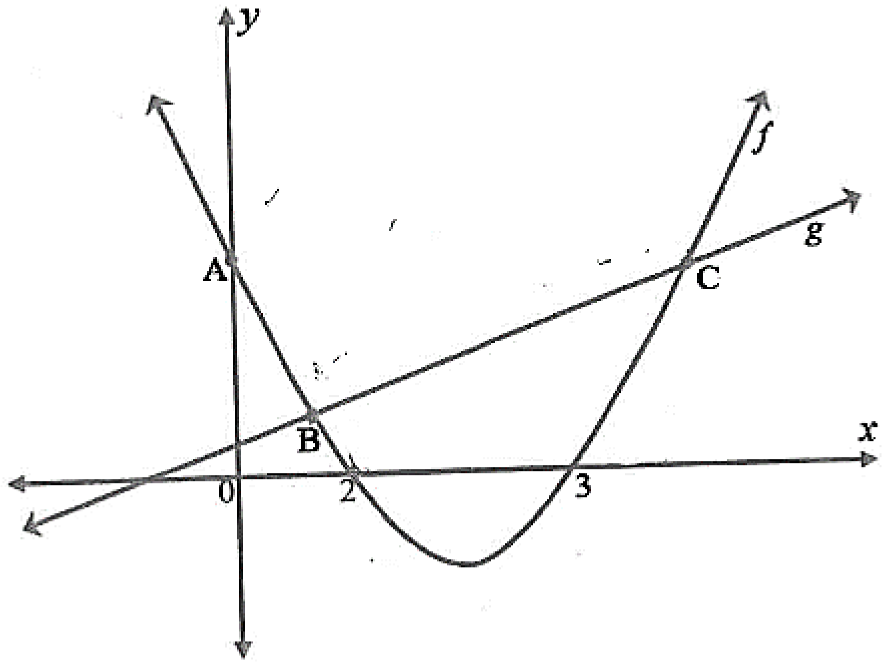
**[12]**

# PARABOLA & STRAIGHT LINE

# EC

## QUESTION 5

The graphs of $f(x)=x^2-5x+6$ and $g(x)=x+1$ are drawn below. B and C are points of intersection of $f$ and $g$. The graph of $f$ has $x$-intercepts at $(2;0)$ and $(3;0)$ and a $y$-intercept at A.




5.1 Determine the equation of the axis of symmetry of $f$. (2)

5.2 Calculate the coordinates of B and C. (4)

5.3 PQ is the vertical distance between the graphs $g$ and $f$ between B and C. Determine the maximum length of PQ. (4)

5.4 Determine the range of $t(x)$ if $f(x)-2=t(x)$. (2)

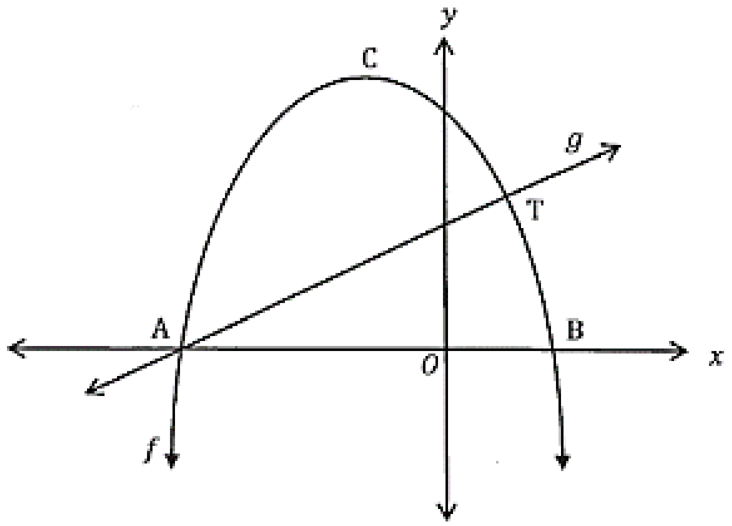
5.5 For which values of $x$ is $f(x).g'(x)<0$? (2)

**[14]**

**FS**

**QUESTION 5**

The sketch shows the graphs of the functions $f(x) = -x^2 - x + 12$ and $g(x) = x + 4$.
A and B are the $x$-intercepts of $f$, while C is the turning point of $f$. The functions intersect at A and T.




5.1 Determine the $x$-coordinates of A and B. (3)

5.2 Determine the range of $f(x)$. (3)

5.3 Determine the maximum value of $k(x) = 3^{f(x)-12}$ (2)

5.4 Determine for which value(s) of $x$ will $x.f(x) > 0$. (2)

5.5 For which real value(s) of $k$ will $-x^2 - x + 12 = k$ have two negative unequal roots? (2)

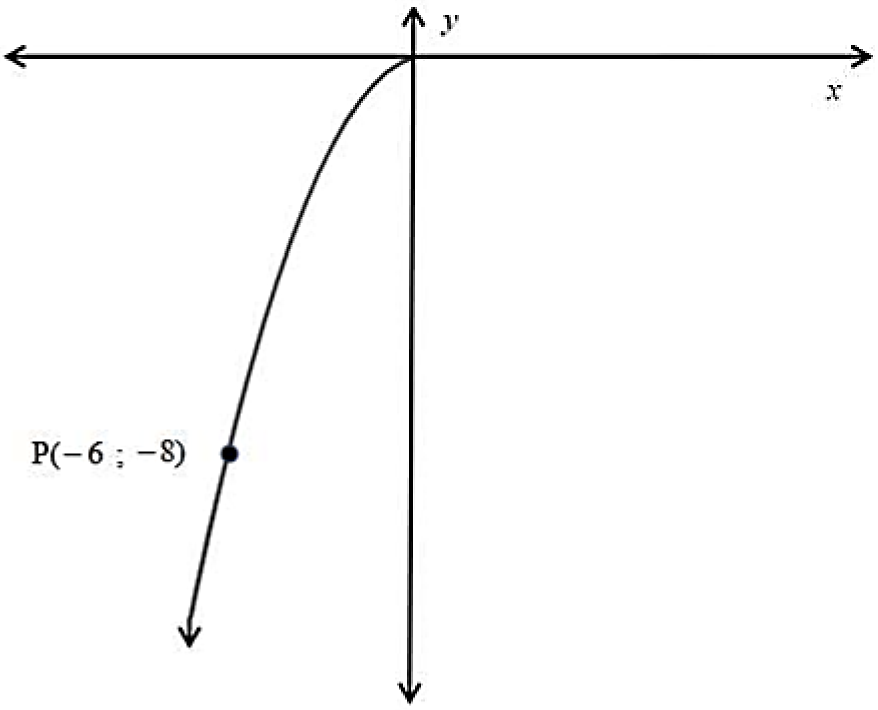
5.6 Write down the equation of $h$ in the form $h(x) = a(x+p)^2 + q$ if $h$ is the reflection of $f$ in the straight line $x = 1$. (2)

**[14]**

**GP**

## QUESTION 4

The graph of $f(x) = ax^2$, $x \leq 0$, is sketched below. The point P$(-6\ ;\ -8)$ lies on the graph of $f$.




4.1 Calculate the value of $a$. (2)

4.2 Determine the equation of $f^{-1}$, in the form $y = \ldots$ (2)

4.3 Write down the range of $f^{-1}$. (1)

4.4 Sketch the graph of $f^{-1}$. Indicate the coordinates of any point on the graph different to $(0\ ;\ 0)$. (2)

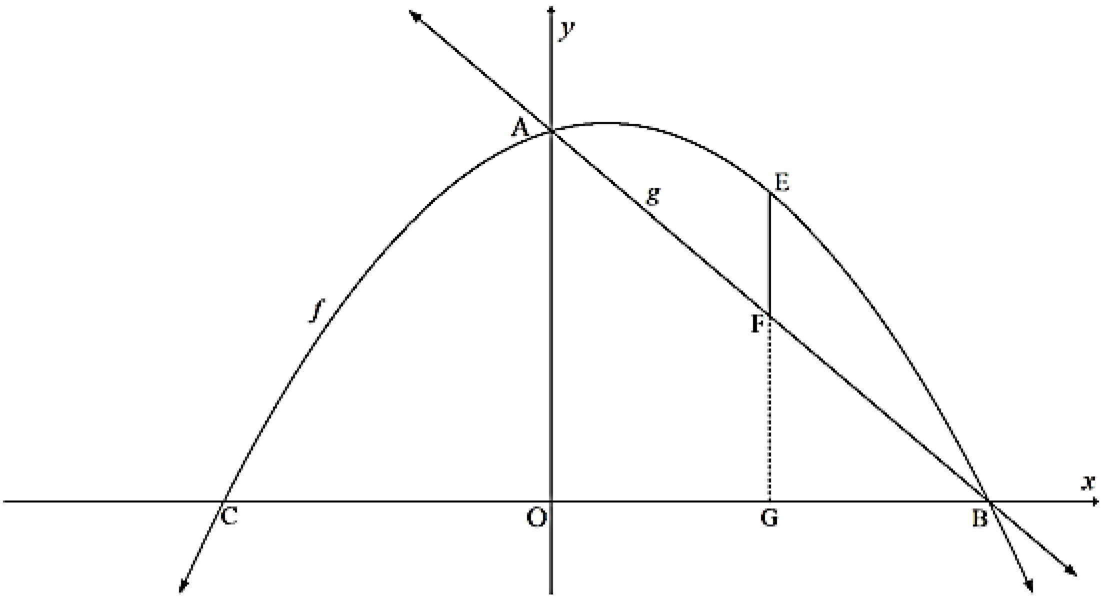
4.5 The graph of $f$ is reflected across the line $y = x$, and thereafter it is reflected across the $x$-axis.

Determine the equation of the new function in the form $y = \ldots$ (2)

**[9]**

**LP**

4.1 The graphs of $f(x) = -x^2 + x + 12$ and $g(x) = mx + k$ are sketched below. B and C are $x$-intercepts and A is the $y$-intercept. E is a point on $f$ and F is a point on $g$. EG is parallel to the y-axis.




4.1.1 Determine the coordinates of B. (2)

4.1.2 Calculate the values of $m$ and $k$. (3)

4.1.3 If OG $= 2$ units, calculate :

(a) The length of EF. (2)

(b) The area of AOGF. (3)

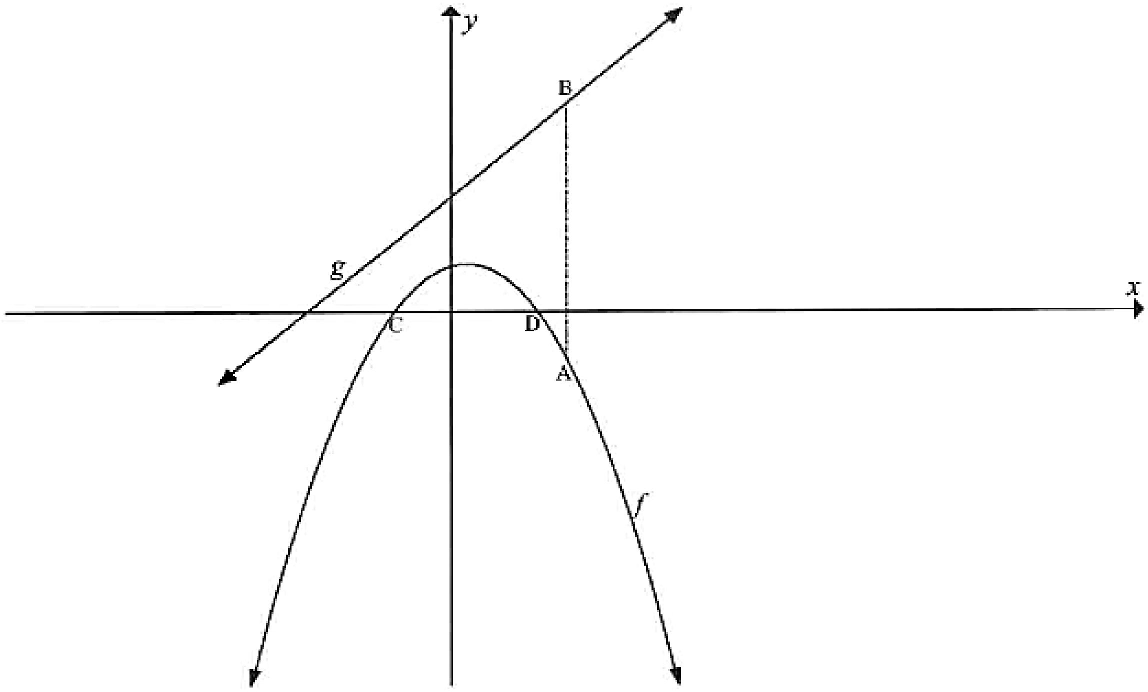
4.1.4 Determine the coordinates of the point of intersection of $g$ and the tangent of $f$ at C. (6)

4.1.5 Determine which value(s) of $x$ will $\frac{f(x)}{f'(x)} < 0$? (3)

**MP**

**QUESTION 4**

The graphs of $f(x) = -x^2 + x + 6$ and $g(x) = 3x + 10$ are drawn below. C and D are the $x$-intercepts of $f$. A is a point on $f$ and B is a point on $g$ such that AB is parallel to the $y$-axis.




4.1 Calculate the coordinates of the turning point of $f$. (3)

4.2 Calculate the distance of CD. (2)

4.3 Calculate the vertical distance of AB in terms of $x$. (2)

4.4 Calculate the smallest distance of AB between $f$ and $g$. (4)

4.5 Write the values of $x$ for which $f(x).g(x) > 0$ (3)

4.6 If $f(x) + 2 = k$ has two distinct roots, determine the value(s) of $k$. (3)

**[17]**

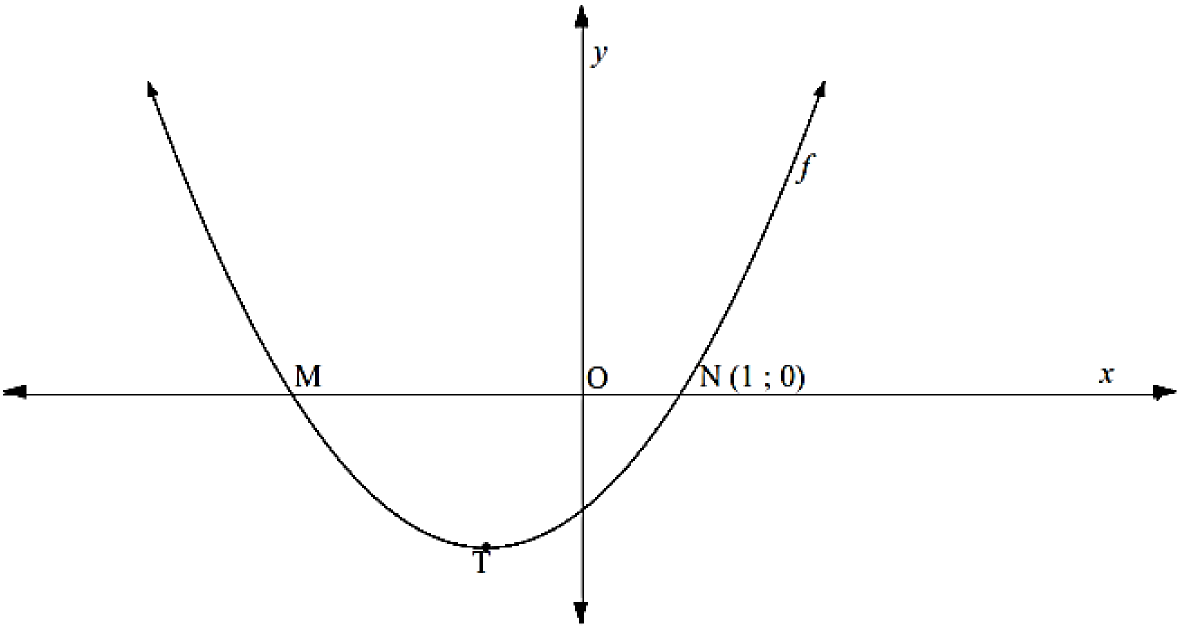
**NC**

**QUESTION 4**

Sketched below is the graph of $f(x)=(x+1)^2+q$.

M and N (1 ; 0) are the $x$-intercepts of $f$.

T is the turning point of $f$.




4.1 Write down the coordinates of M. (2)

4.2 Calculate the coordinates of T. (2)

4.3 Consider: $g(x) = -f(x) - 5$

4.3.1 Determine the equation of $g$ in the form $y = ax^2 + bx + c$ (2)

4.3.2 Describe the nature of the roots of $g$. (2)

4.4 For which value(s) of $x$ will $f(x-4) < -3$? (3)

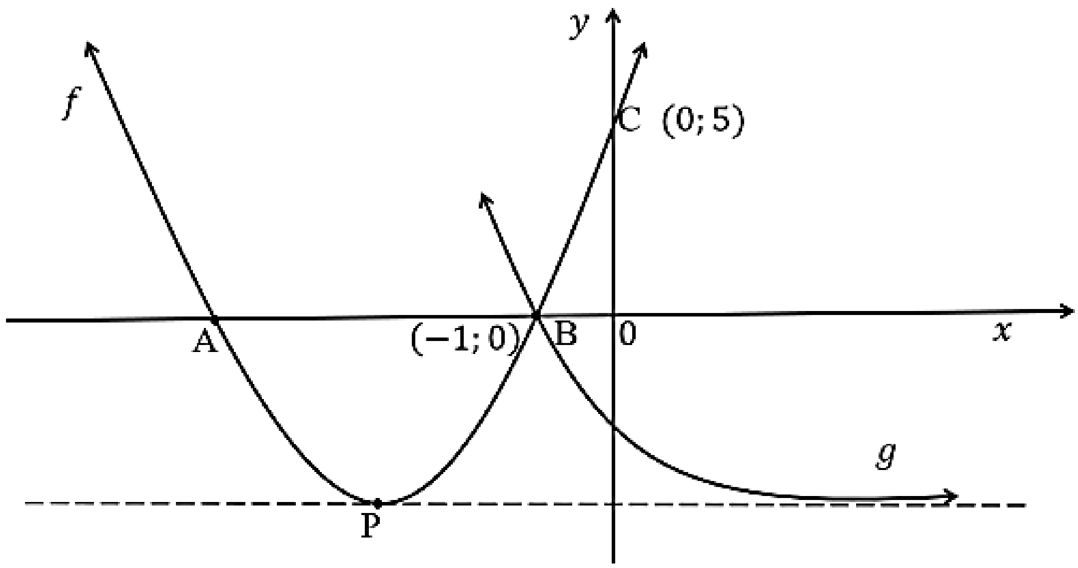
**[11]**

**WC**

## QUESTION 4

The graphs of the functions $f(x) = a(x+3)^2 + q$ and $g(x) = m^{-x} + k$ are drawn below.

- A and B$(-1;0)$ are the $x$-intercepts of $f$.
- C$(0;5)$ is the $y$-intercept of $f$.
- P is the turning point of $f$.
- The graphs intersect at B$(-1;0)$.
- The asymptote of $g$ is a tangent to $f$ at the turning point P of $f$.



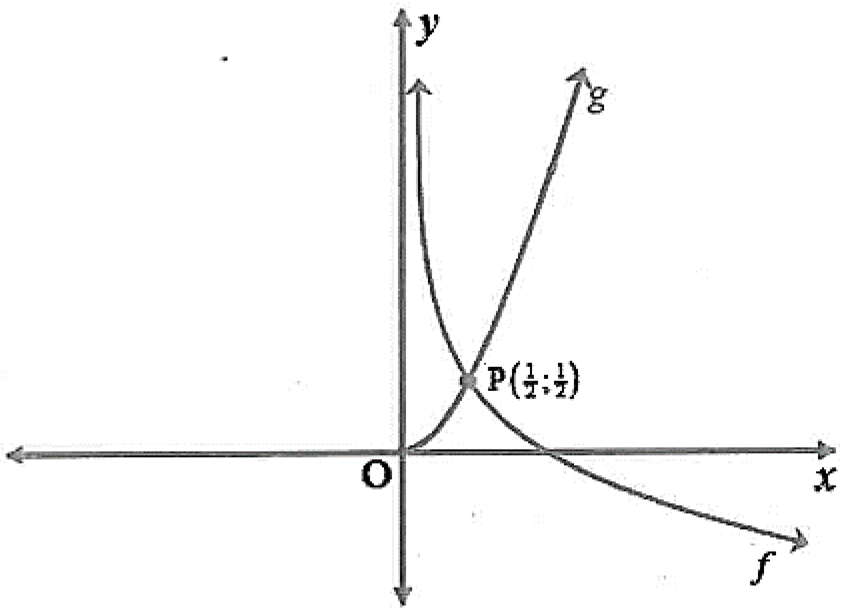

| | | |
|---|---|---|
| 4.1 | Write down the equation of the axis of symmetry of the parabola $f$. | (1) |
| 4.2 | Determine the equation of $f$ in the form $y = ax^2 + bx + c$. | (4) |
| 4.3 | Hence, show that the equation of the asymptote of $g$ is given by $y = -4$. Clearly show all your calculations. | (1) |
| 4.4 | Determine the value of $x$ for which $f'(x).g(x) > 0$ | (2) |
| 4.5 | The graph $g$ is shifted 4 units upwards to give a new function $h$. Determine the equation of $h^{-1}$, the inverse of $h$, in the form $y =$ ............... | (4) |
| | | **[12]** |

# EXPONENTIAL GRAPH

# EC

## QUESTION 6

The diagram below shows the graphs of $f(x) = -\log_c x$ and $g(x) = dx^2$ ; $x \geq 0$.
The point P$\left(\frac{1}{2};\frac{1}{2}\right)$ is the point of intersection of the graphs $f$ and $g$.




| | | |
|---|---|---|
| 6.1 | Calculate the values of $c$ and $d$. | (3) |
| 6.2 | Determine: | |

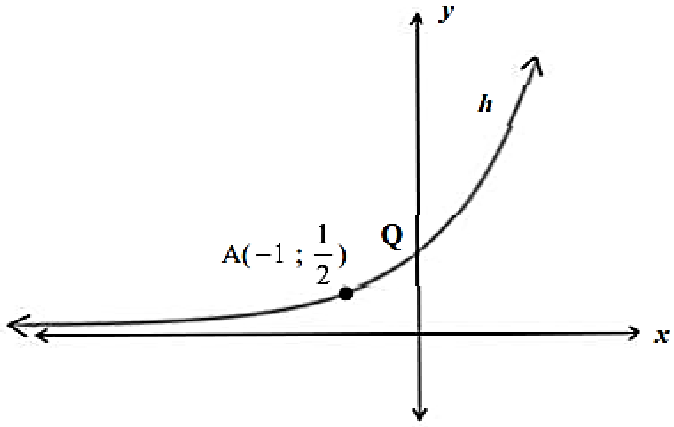
| | | |
|---|---|---|
| 6.2.1 | The equation of $g^{-1}(x)$ in the form $y = ...$ | (2) |
| 6.2.2 | The equation of $h^{-1}(x)$ in the form $y = ...$, if $h$ is a reflection of $f$ in the $x$-axis | (2) |
| 6.2.3 | The $x$-values for which $h^{-1}(x) > 0$ | (1) |
| | | **[8]** |

# GP

## QUESTION 5

5.1 The point P(2 ; $\sqrt{3}$) lies in the Cartesian plane. Determine the coordinates of the image of point P if P is rotated about the origin through $90^0$ in an anti-clockwise direction. (2)

5.2 The graph of $h(x) = a^x$ is sketched below. $A\left(-1\ ;\ \frac{1}{2}\right)$ is a point on the graph of $h$.




5.2.1 Substantiate why the coordinates of Q, the $y$-intercept of $h$, are (0 ; 1). (2)

5.2.2 Calculate the value of $a$. (2)

5.2.3 Write down the equation of the inverse function, $h^{-1}$ in the form $y = \ldots$ (2)

5.2.4 Draw a sketch graph of $h^{-1}$. Indicate the coordinates of TWO points that lie on this graph. (3)

5.2.5 Read off from your graph the values of $x$ for which $\log_2 x > -1$. (2)

5.2.6 If $g(x) = (100).3^x$, determine the values of $x$ for which $h(x) = g(x)$. (3)

5.3 The price ($p$), in Rands per unit, of EACH item in a consignment of $q$ items, is given by $p = \log\left(10 + \frac{q}{2}\right)$.

5.3.1 Calculate the value of $p$ and the total price of the consignment when the consignment has 1 980 items. (3)

5.3.2 Determine the number of items in the consignment when the price of each item is R2. (2)

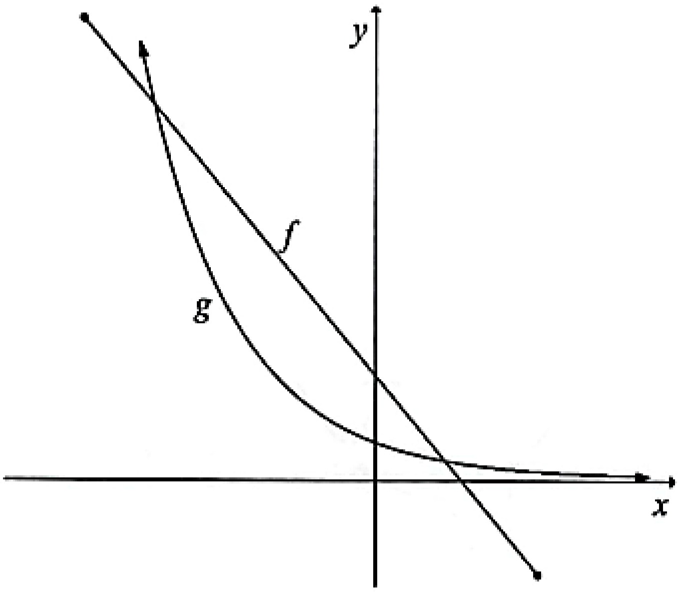
**[21]**

## KZN

### QUESTION 5

The following graphs are drawn below:

- $f(x) = -2x + 3$ for $-4 \leq x \leq 3$; and
- $g(x) = 2^{-x}$.



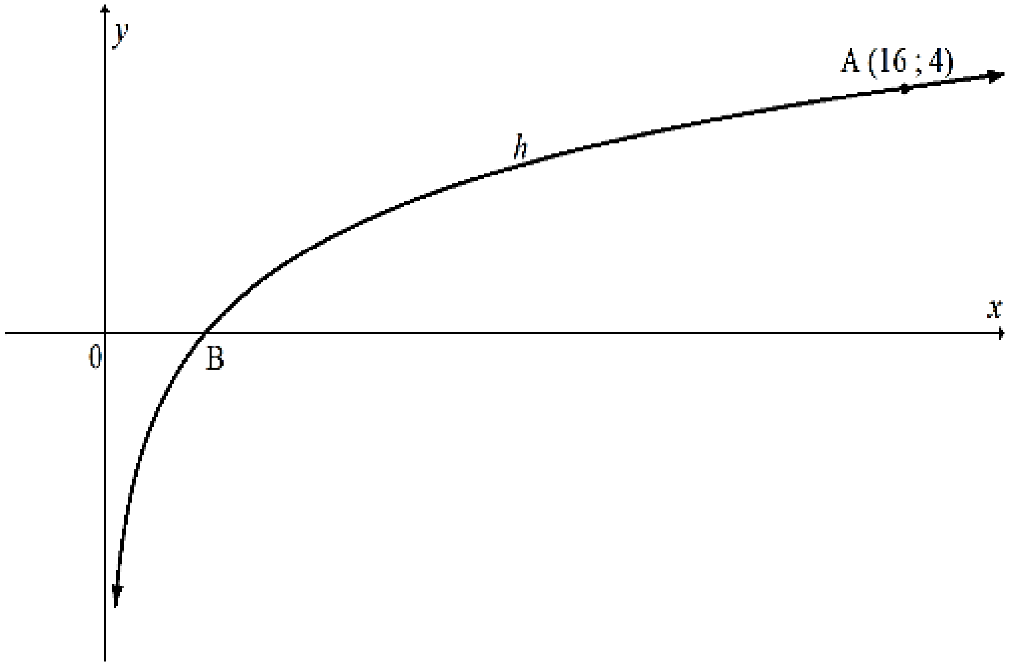

| | | |
|---|---|---|
| 5.1 | Calculate the $x$-intercept of $f$. | (2) |
| 5.2 | For which values of $x$ is $-2^{-x+1}.x + 6.2^{-x-1} < 0$? | (4) |
| 5.3 | A new graph $p$ is formed by reflecting $g$ in the line $y = x$. Write down the equation of $p$ in the form $y = ...$ | (2) |
| 5.4 | Write down the range of $f^{-1}$, the inverse of $f$. | (2) |
| 5.5 | Determine the coordinates of the point of intersection between $f$ and $f^{-1}$. | (3) |

**[13]**

**LP**

## QUESTION 5

The figure below shows the graph of $h(x) = \log_a x$. Point A$(16 ; 4)$ lies on the curve and B is the $x$-intercept of $h$.




| | | |
|---|---|---|
| 5.1 | Calculate the value of $a$. | (3) |
| 5.2 | Write down the coordinates of B. | (1) |
| 5.3 | Determine the equation of $h^{-1}$, in the form $h^{-1}(x) = ..........$ | (2) |
| 5.4 | Write down the range of $h^{-1}$. | (1) |
| | | **[7]** |

## NC

### QUESTION 6

Given: $g(x)=\log_{\frac{1}{3}} x$

6.1 Write down the domain of $g$. (2)

6.2 Calculate the $x$-intercept of $g$. (2)

6.3 Draw the graph of $g^{-1}$, indicating the intercept(s) with the axes as well as the asymptote. (2)

6.4 $P(a\,;-1)$ is a point on $g$; calculate the value of $a$. (2)

6.5 For which values of $x$ will $1<g^{-1}(x)<3$ ? (2)

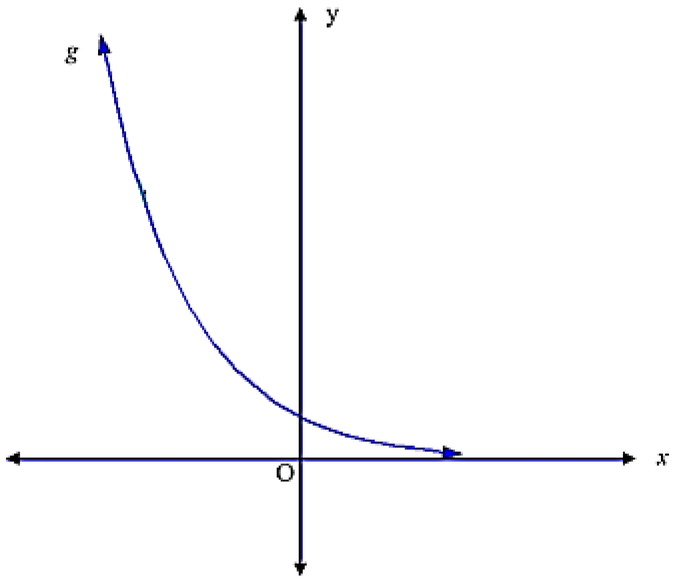
**[10]**

## NW

### QUESTION 4

Sketched below is the graph of $g(x)=\left(\frac{2}{5}\right)^x$.

- A(p ; 0,59) is the point of intersection of g($x$) and $g^{-1}(x)$.
- B(– 2 ; $q$) is a point on $g(x)$.



4.1 Calculate the value of $q$. (2)

4.2 Write down the equation of $g^{-1}(x)$ in the form $y = ...$ (2)

4.3 Write down the domain of $y = g^{-1}(x)$. (2)

4.4 For which values of $x$ will: $g(x) \leq g^{-1}(x)$ . (2)

4.5 Describe the translation from $g$ to $k(x) = \left(\frac{5}{2}\right)^{-x+2} - \frac{5}{2}$. (3)

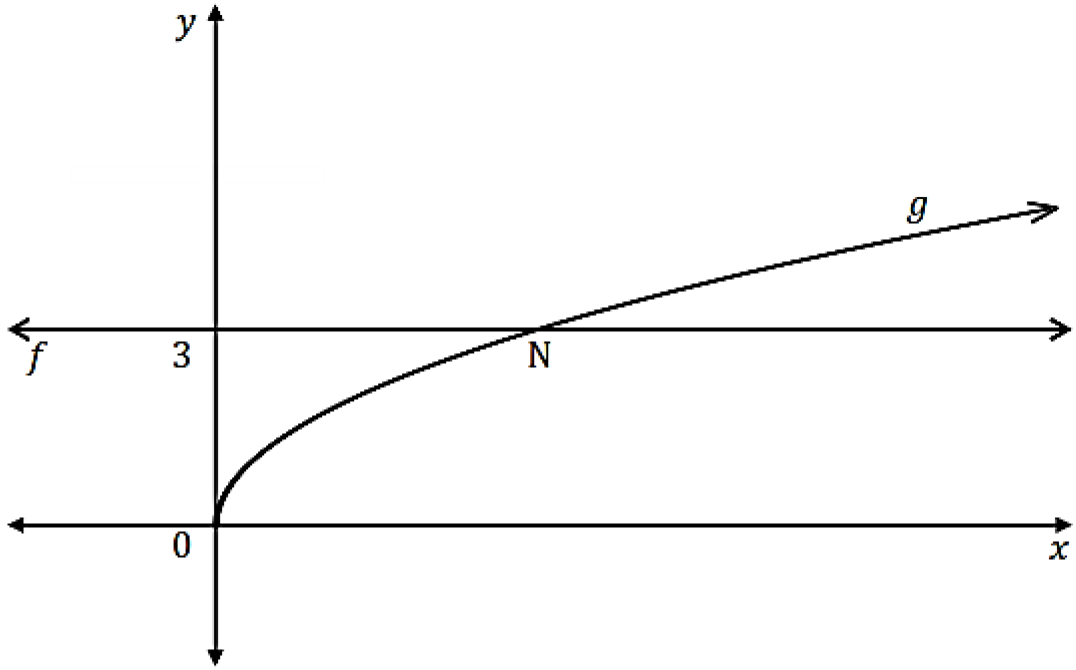
**[11]**

## WC

## QUESTION 5

In the diagram, the graphs of $f(x) = 3$ and $g(x) = \sqrt{x}$ for $x \geq 0$ are given.

- The $y$-intercept of $f$ is $(0; 3)$ and
- N is the point of intersection of the graphs.




5.1 Give the range of $g^{-1}$. (1)

5.2 Determine the coordinates of N. (2)

5.3 Determine the equation of the reflection of $g$ in the line $y = x$ in the form $y =$…….. (3)

**[6]**

# FINANCE

## 1. EC

### QUESTION 7

7.1 A car that is worth R180 000, depreciates at 13% p.a. compounded annually on the reducing balance method. Calculate the value of the car in 6 years. (3)

7.2 Lumi opened a 15-year savings plan account that pays interest at 8% per annum compounded monthly. She saves an amount of R900 every month for the first 10 years. Her first payment was at the end of the first month. For the last 5 years of her savings plan she managed to increase her monthly payments to R1 300.

Calculate the value of her savings at the end of the savings period. (5)

7.3 Mr Leanya bought a house for R850 000. He obtained a loan from the bank at an interest rate of 13% per annum compounded monthly to pay for the house. He agreed to pay monthly instalments of R9 958,39 for 20 years.

7.3.1 Calculate the balance of his loan immediately after his $75^{th}$ instalment. (3)

7.3.2 Mr Leanya experienced financial difficulties after his $75^{th}$ instalment and did not pay the $76^{th}$ to the $79^{th}$ instalments. At the end of the $80^{th}$ month he increased his monthly instalment so as to pay off the loan in the same time interval as planned initially.

Calculate the value of his new adjusted monthly instalment. (5)
**[16]**

## 2. FS

### QUESTION 6

6.1 The value of a vehicle worth R150 000 depreciate at 13% p.a. Calculate the value of the vehicle in 6 years if depreciation is calculated on the reducing-balance method. (3)

6.2 A loan of R300 000 is taken out at an interest rate of 5,3% p.a. compounded quarterly. The loan was taken out on the $1^{st}$ of March 2016. The first payment was made on 1 December 2016 and is repaid in 72 equal quarterly payments.

6.2.1 What is the outstanding balance of the loan on 1 September 2016? (3)

6.2.2 Determine the quarterly repayments required to pay back the loan. (3)

6.3 Gert landed a job which remunerated him R27 562,50 quarterly. He then decided to open an investment account and deposit 11% of his salary at the end of every quarter into the investment account, earning an interest of 7,9% p.a. compounded monthly for eight years.

What amount will be in the account at the end of eight years? (6)
**[15]**

## 3. GP

### QUESTION 7

7.1 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years? (3)

7.2 Micaela buys furniture to the value of R10 000. She borrows the money on 1 February 2023 from a financial institution that charges interest at a rate of 9,5% *p.a.* compounded monthly. Micaela agrees to pay monthly instalments of R450. The loan agreement allows Micaela to start paying equal monthly instalments from 01 August 2023.

7.2.1 Calculate the total amount owing to the financial institution on 1 July 2023. (3)

7.2.2 How many months will it take Micaela to pay back the loan? (4)

7.2.3 What is the balance of the loan immediately after Micaela has made the 25$^{th}$ payment? (3)

**[13]**

## 4. KZN

### QUESTION 6

6.1 Siphokazi invested R6 500 for 4 years at an interest rate of $r$ % p.a., compounded quarterly. At the end of this period, she received R13 460. Calculate $r$, correct to ONE decimal place. (4)

6.2 Terence has been planning to go on an overseas tour during December 2024. He needs R65 000 for this tour. Starting from 31 July 2023, he has been depositing R$x$ in a savings account at the end of each month. He will continue doing this until 30 November 2024, at which time there will be enough money in the account.

Terence will withdraw all the money in the savings account on 30 November 2024, immediately after depositing the last R$x$.

Calculate the value of $x$, if interest was calculated at 8% p.a., compounded monthly. (3)

6.3 Mrs Naidoo plans to buy a house. She will need a bank loan for R650 000. The bank charges interest at 11% p.a., compounded monthly, and will require her to pay a monthly instalment of R7 000.

6.3.1 How many instalments of R7 000 will Mrs Naidoo have to pay? (4)

6.3.2 Calculate the final payment that Mrs Naidoo will have to pay to settle the loan. (4)

**[15]**

## 5. LP

### QUESTION 6

Thapelo and Mahlatse invest their inheritance of R200 000 at 11, 5% p.a compounded quarterly. After 10 years, they use their return on their investment to build a house at a cost of R1 850 000. They borrow the balance needed to build the house from a bank. The bank grants them a loan at an interest rate of 12% p.a compounded monthly. They must repay the loan back over a period of 25 years.

| | | |
|---|---|---|
| 6.1 | Calculate the effective interest rate. | (3) |
| 6.2 | Calculate the value of the investment after 10 years. | (2) |
| 6.3 | Determine the loan amount needed to finish building the house. | (2) |
| 6.4 | Calculate their monthly payment on the loan. | (4) |
| 6.5 | Calculate the balance on the loan after 15 years. | (3) |
| 6.6 | Determine how much interest they will pay on the loan in the 25 years' time. | (2) |
| | | **[16]** |

## 6. MP

### QUESTION 5

5.1 How long will it take an item to depreciate to one quarter of its initial value if it does so at a rate of 11, 84% p.a. on the reducing balance-method? (3)

5.2 Daniel wishes to purchase a bike for R72 000. He takes out a loan at a rate of 9,8% per annum compounded monthly. Calculate:

5.2.1 the monthly instalment if the loan is to be paid back over 5 years. He makes his first payment one month after the loan is granted. (4)

5.2.2 the outstanding balance after 3,5 years. (3)

5.2.3 the amount that would be saved by settling the loan after 3,5 years instead of 5 years. (3)

5.3 After making the $6^{th}$-payment, Kgosi has an outstanding balance of R793749,25 from a loan of R800 000. He then missed the $7^{th}$, $8^{th}$ and $9^{th}$ payment due to financial difficulties. He resumed payments at the end of $10^{th}$ month onwards. Calculate his increased monthly payments in order to settle the loan within the stipulated 20 years, if interest was 10,25% per annum, compounded monthly. (5)

**[18]**

# 7. NC

## QUESTION 7

7.1 On the 1 April 2019, Lebere deposited R5 000 into a savings account at an interest rate of $r$% per annum, compounded quarterly.

7.1.1 On 1 April 2023, the amount in the savings account was R12 980. Calculate $r$. (3)

7.1.2 He used R7 580 of the money to buy a new laptop. The value of the laptop depreciated at a rate of 22% per annum, according to the reducing balance method. After how many years will its book value be R1 707? (3)

7.2 Andy takes up a loan of R850 000. The loan is repaid over 20 years at 8,5% p.a., compounded monthly.

7.2.1 Calculate the value of Andy's monthly instalment. (4)

7.2.2 Calculate how much Andy will still owe the bank after his $26^{th}$ payment (3)

7.2.3 Hence, calculate the total interest Andy will have paid over the first 26 months (2)

**[15]**

# 8. NW

## QUESTION 6

Frits deposited R3 000 into a savings account at the end of January 2004. He continued to make monthly deposits of R3 000 at the end of each month up to the end of December 2023. The savings account earned interest of 7,5% per annum, compounded monthly.

6.1 Calculate how much money will be in the account on 31 December 2023. (4)

6.2 Two years after Frits opened the savings account, he decided to invest R$x$ of his bonus each year at the end of the year to boost his savings account. He made his last deposit of R$x$ two years before 31 December 2023.

6.2.1 Calculate the yearly effective interest rate on his investment. Give your answer correct to 4 decimal places. (3)

6.2.2 Calculate his yearly deposit of R$x$ if he wants R3 500 000 in his savings account on 31 December 2023. (6)

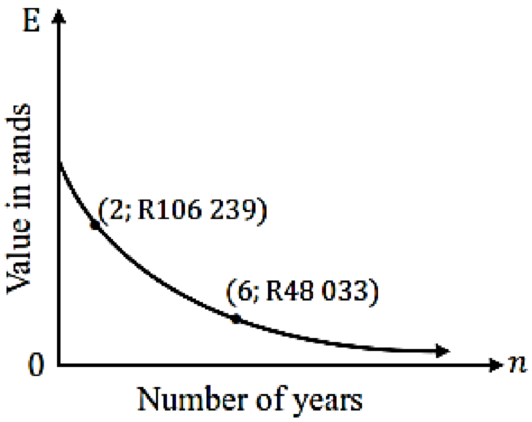
**[13]**

## 9. WC

### QUESTION 6

6.1 Determine how long it will take for an investment of R20 000 to grow to R45 000 in an account earning interest at 7,5% p.a., compounded monthly. Give your answer to the nearest month. (3)

6.2 The depreciation of office equipment is represented by the graph below.
E is the value of the equipment in Rands and $n$ is the number of years the equipment is being used.




6.2.1 What does the $y$-intercept of the graph indicate? (1)

6.2.2 According to this model, will the value of the equipment ever become R0? Explain your answer. (1)

6.2.3 Calculate the annual rate of depreciation. (3)

6.3 Herman plans to purchase a house for R2 464 000. He will take out a loan for the full amount for 30 years at 10,2% p.a., compounded monthly.

6.3.1 Calculate what his instalments will be if he is required to repay the bank equal amounts at the end of each month. (4)

6.3.2 Instead of paying the bank-stipulated amount calculated in QUESTION 6.3.1, Herman decides to repay the loan by R22 500 instead, at the end of every month. Calculate the balance outstanding on the loan immediately after he pays his $84^{th}$ payment. (3)

**[15]**

# CALCULUS

## 1. EC

**QUESTION 8**

8.1 Determine $f'(x)$ from first principles if $f(x) = x^2 - 3$. (4)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = -3x^2 + 7x$ (2)

8.2.2 $D_x\left[\frac{x^3 - 5x^2}{x^3} - \sqrt{x}\right]$ (4)

8.3 Suppose that $g(x)$ represents the rate of change of $h(x) = -x^3 - 3x^2 + 1$. Calculate the largest value of $g(x)$. (3)

**[13]**

## FS

**QUESTION 7**

7.1 Determine $f'(x)$ from first principles if it is given that $f(x) = -1 + 4x^2$. (5)

7.2 Determine:

7.2.1 $D_x[(2x^3 + 5)^2]$ (3)

7.2.2 $\frac{dy}{dx}$ if $y = 3x^4 - \frac{7}{x} + 2.\sqrt[3]{x^2}$ (4)

7.3 Determine the coordinates of the point on the curve of $y = 2x^2 + 3x + 1$, where the tangent at the point is perpendicular to $y + 5x = 4$. (4)

**[16]**

## GP

### QUESTION 8

8.1 If $f(x) = -2x^2 + 3x$, determine $f'(x)$ from first principles. (4)

8.2 Given: $f(x) = \frac{3x^2}{2} - 24\sqrt{x}$. Calculate $f'(9)$. (5)

8.3 A function $g(x) = ax^2 + \frac{b}{x}$ has a minimum value at $x = 4$. The function value at $x = 4$ is 96.

Calculate the values of $a$ and $b$. (6)

**[15]**

## KZN

### QUESTION 7

7.1 Given: $f(x) = -x^2 + x$. Determine $f'(x)$ from first principles. (5)

7.2 Determine the derivatives of the following:

7.2.1 $y = x^3\left(4 - x^{-3}\right)$ (2)

7.2.2 $f(x) = \frac{2x^2 + 3}{\sqrt{x}}$ (4)

**[11]**

## LP

### QUESTION 7

7.1 Determine $f'(x)$ from first principles if it given that $f(x) = 3x^2$. (5)

7.2 Determine:

7.2.1 $f'(x)$ if $f(x) = (x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ (3)

7.2.2 $D_x\left[\frac{x^3 + 2x^2 + x}{x + 1}\right]$ (4)

7.2.3 $\frac{dy}{dx}$ if $y = \sqrt[3]{x} - \frac{1}{3x}$ (4)

**[16]**

## MP

### QUESTION 6

6.1 Given $f(x) = -\frac{2}{x}$

Determine $f'(x)$ from first principles. (5)

6.2 Determine:

6.2.1 $\frac{dy}{dx}$ if $xy - 2y = x^2 - 4$ (3)

6.2.2 $D_x\left[\sqrt[5]{\frac{32}{x^3}}\right]$ (3)

**[11]**

## NC

### QUESTION 8

8.1 Determine $f'(x)$ from the first principles if $f(x) = 4 - 3x^2$. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = \frac{-4x^5 + 3x}{x^{-2}}$ (3)

8.2.2 $D_x\left(\sqrt{x^3} + b^2\right)$ (3)

8.3 The line $y = -4x + k$ is a tangent to $f(x) = \frac{9}{x} - 3x$ at $S(a\ ;\ b)$. Determine the value(s) of $a$. (4)

**[15]**

## NW

### QUESTION 7

7.1 Given: $f(x) = 5x^2 + 2x$

Determine $f'(x)$ from first principles. (5)

7.2 Determine $f'(x)$ if:

7.2.1 $f(x) = 5x^4 - x^3 + 2x$ (3)

7.2.2 $f(x) = \dfrac{8x^{\frac{1}{2}} + 4}{2x^3}$ (4)

7.3 If $y = 4x^2 - 3$ and $2xt = 7$ .

Determine $\dfrac{dy}{dt}$. (4)

**[16]**

## WC

### QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = -3x^2 + 7$. (5)

7.2 Determine $\dfrac{dy}{dx}$ for each of the following:

7.2.1 $y = 8\sqrt{x} - \sqrt{21}$ (3)

7.2.2 $x^2y = 2x^3 + 4$ ; $x \neq 0$ (4)

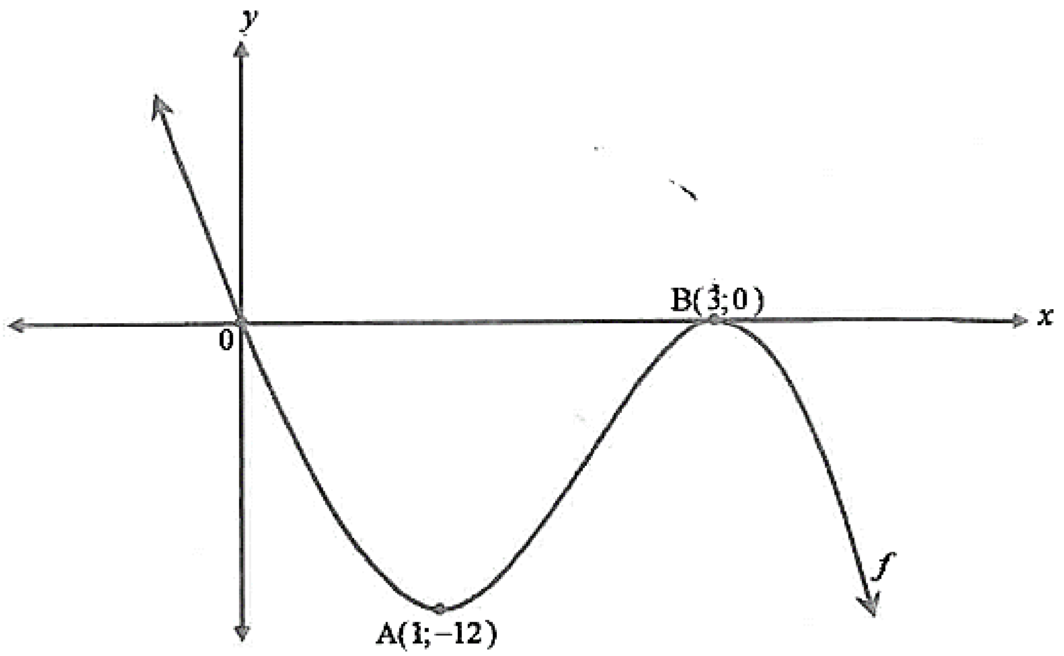
7.3 Calculate the value of the constant $h$ so that the curve of $y = hx + x^3$ will have a local maximum at $x = -1$. (3)

**[15]**

**EC**

**QUESTION 9**

9.1 The sketch below shows the graph of $f(x) = -3x^3 + mx^2 + nx$. The graph of $f$ passes through the origin and has a local minimum and a local maximum at A$(1;-12)$ and B$(3;0)$ respectively.




9.1.1 Show that $m = 18$ and $n = -27$ (5)

9.1.2 Explain the difference between $f(a)$ and $f'(a)$. (2)

9.1.3 $g(x)$ is the tangent to the curve of $f(x)$ at the point of inflection. Determine the equation of $h(x)$, the straight line that is perpendicular to $g(x)$ and passes through the origin. (5)

9.1.4 For which values of $x$ will $f''(x) > 0$? (2)

9.2 The function $t$ is defined by $t(x) = 2x^3 + bx + c$ and has the following properties.

- $t(-3) = t(3) = t(0) = 0$
- $t'(-1,5) = t'(1,5) = 0$

Use this information to draw a neatly labelled sketch graph of $t$, without solving for $b$ and $c$. (3)

**[17]**

**FS**

**QUESTION 8**

The equation of the cubic function $f$ is given as
$f(x) = -3x^3 + 15x^2 - 21x + 9$

8.1 Determine the $x$- and $y$-intercepts of the graph. (4)

8.2 Determine the coordinates of the turning points of $f$. (4)

8.3 Sketch the graph of $f$ clearly indicating the intercepts with the axes and the turning points. (4)

8.4 For which value(s) of:

8.4.1 $x$ is $f'(x) > 0$? (2)

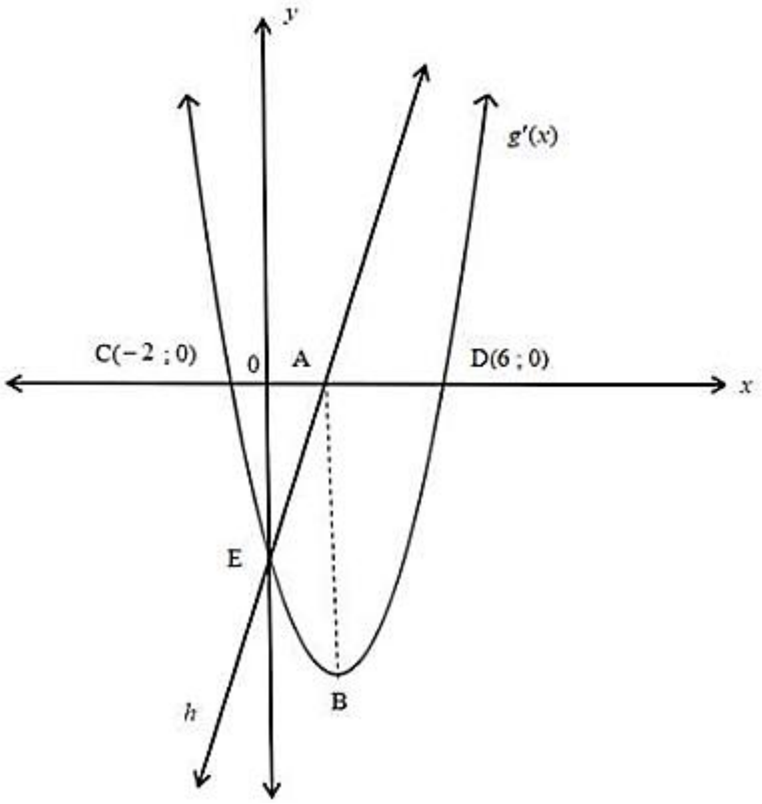
8.4.2 $k$ will $f(x) = k$ have exactly three different real roots? (2)
**[16]**

**GP**

**QUESTION 9**

9.1 The graphs of $g'(x) = ax^2 + bx + c$ and $h(x) = 2x - 4$ are sketched below.
The graph of $g'(x) = ax^2 + bx + c$ is the derivative graph of a cubic function $g$.

- The graphs of $h$ and $g'$ have a common $y$-intercept at point E.
- C($-2$ ; 0) and D(6 ; 0) are the $x$-intercepts of the graph of $g'$.
- Point A is the $x$-intercept of $h$ and point B is the turning point of $g'$.
- Line AB is parallel to the $y$-axis.



9.1.1 Write down the coordinates of point E. (1)

9.1.2 Determine the equation of the graph of $g'$ in the form $y = ax^2 + bx + c$. (4)

9.1.3 Write down the $x$-coordinates of the turning point of $g$. (2)

9.1.4 Write down the $x$-coordinate of the point of inflection of the graph of $g$. (1)

9.1.5 Explain why $g$ has a local maximum at $x = -2$. (2)

9.2 Given: $h(x) = 4x^3 + 5x$

Substantiate whether it is possible to draw a tangent to the graph of $h$ that has a negative gradient. (2)

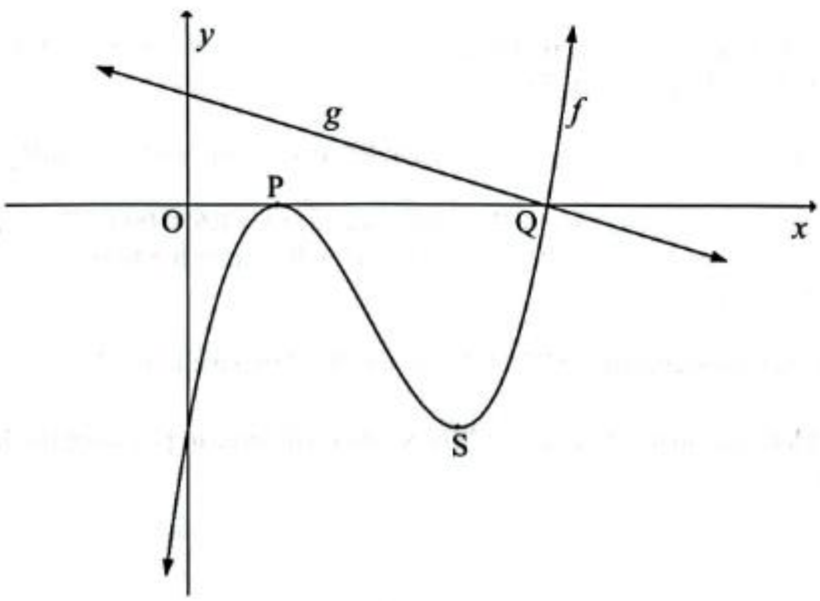
**[12]**

## KZN

## QUESTION 8

The graphs of $f(x) = (x-1)^2(x+n)$ and $g(x) = -\frac{1}{2}x + 2$ are drawn below.

- P and Q are the $x$-intercepts of $f$.
- P and S are the turning points of $f$.
- $g$ passes through Q.



| | | |
|---|---|---|
| 8.1 | Calculate the coordinates of Q. | (2) |
| 8.2 | Hence, write down the value of $n$. | (1) |
| 8.3 | Calculate the length of PQ. | (2) |
| 8.4 | Calculate the coordinates of S. | (5) |
| 8.5 | Describe the concavity of $f$ at $x=0$. | (1) |
| 8.6 | Given: $h(x)=-\frac{1}{2}x+k$.<br>For which values of $x$ will $h$ be a tangent to $f$? | (5) |
| | | **[16]** |

## LP

**QUESTION 8**

Given: $f(x)=2x(x^2-9x+24)$

| | | |
|---|---|---|
| 8.1 | Show that $P(3\ ;\ 36)$ is a point on the graph of $f$. | (2) |
| 8.2 | Calculate the coordinates of the turning points of the graph of $f$. | (4) |
| 8.3 | Draw a neat sketch graph of $f$. Indicate the coordinates of any intercepts with the axes and of the turning points | (3) |
| 8.4 | Determine the value(s) of $k$ for which $2x(x^2-9x+24)=k$ has three unequal roots. | (2) |
| 8.5 | Determine the maximum value of $f(x)$ if $x \in [0\ ;\ 5]$. | (1) |
| | | **[12]** |

## MP

**QUESTION 7**

Given: $f(x)=1-125x^3$

7.1 Determine the coordinates of the:

7.1.1 stationary point(s) (2)

7.1.2 point of inflection if it exists. (2)

7.2 Draw the graph of $f$. Show all the intercepts with the axes as well as stationary points if any. (3)

7.3 Give the values of $x$ for which the graph is concave up. (1)

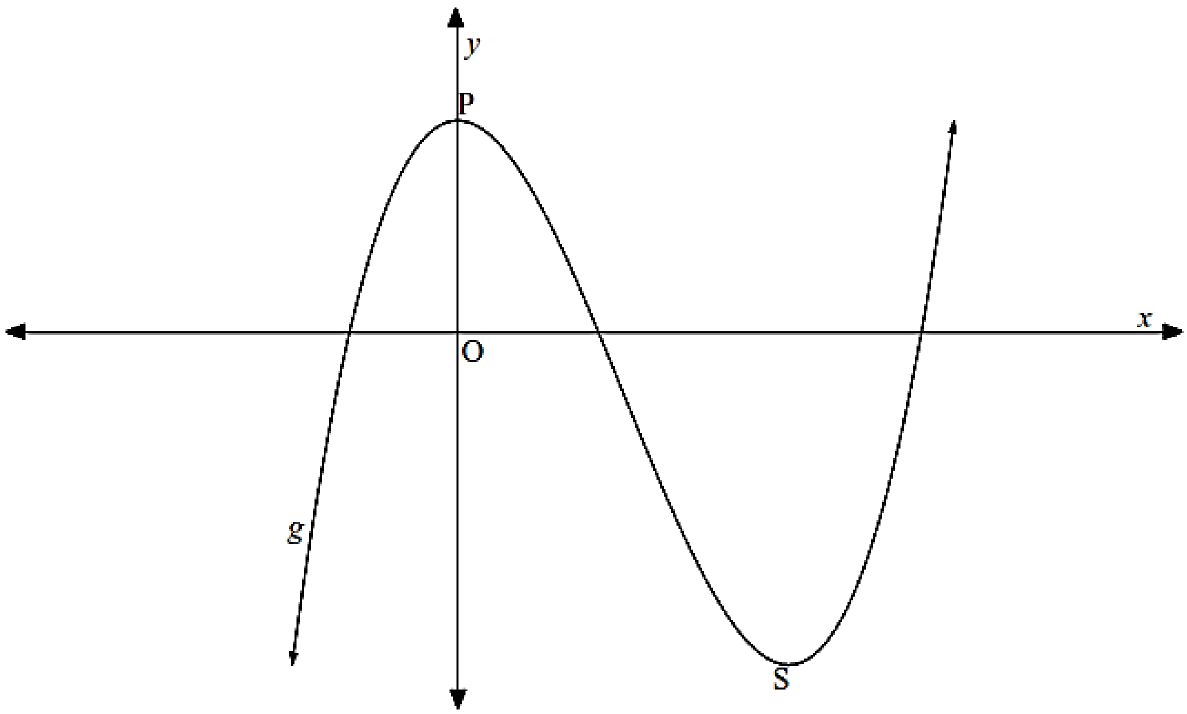
7.4 Determine the equation of the tangent to the graph at $x=\frac{1}{10}$. (4)

**[12]**

## NC

**QUESTION 9**

In the diagram, the graph of $g(x) = 2x^3 - 12x^2 + 25$ is drawn. P and S are the turning points of $g$.



9.1 Calculate the coordinates of P and S. (5)

9.2 For which value(s) of $x$ will:

9.2.1 $g$ concave downwards? (3)

9.2.2 $g^{/}(x) > 0$ ? (2)

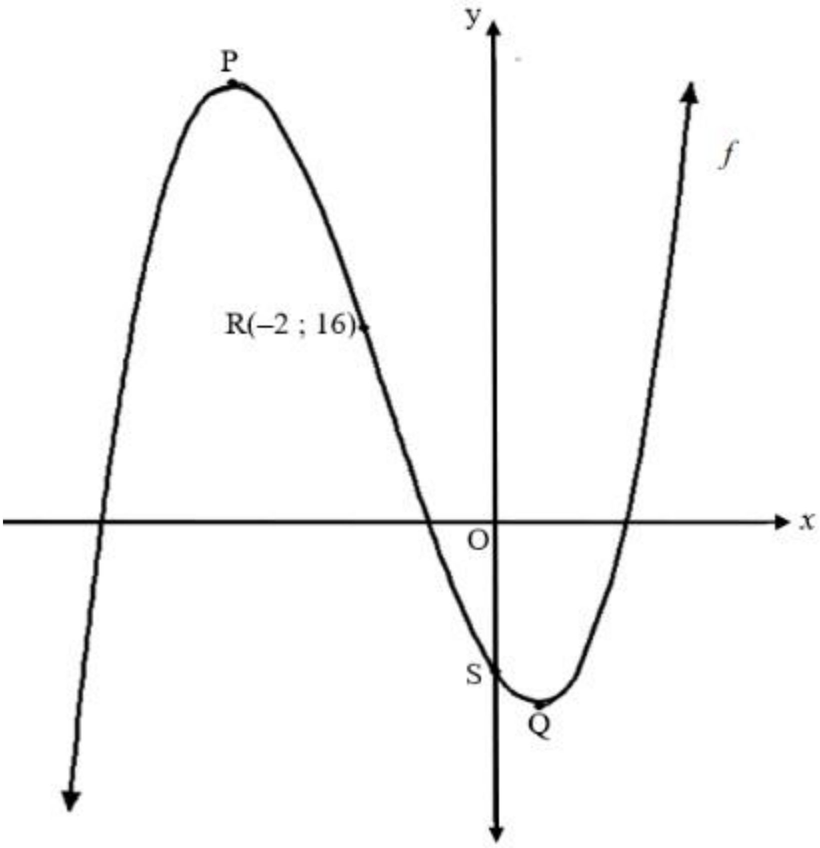
9.3 T(1 ; 15) is a point on $g$. Determine the equation of the tangent to $g$ at the point T in the form $y = mx + c$. (3)

9.4 For which values of $k$ will $2x^3 - 12x + 25 - k = 0$ have two unequal roots? (2)
**[15]**

**NW**

**QUESTION 8**

The graph of $f(x) = x^3 + ax^2 + bx + c$ is drawn below. The line $g(x) = -16x + k$ is a tangent to $f$ at R(– 2 ; 16). Graph $f$ is concave up at $x > -\frac{5}{3}$. P and Q are the turning points of $f$. S is the $y$-intercept of $f$.



8.1 Show that $a = 5$; $b = -8$ and $c = -12$. (5)

8.2 Determine the coordinates of P and Q. (3)

8.3 Sketch the graph of $f'$. Clearly indicate the $x$-intercepts and the $x$-value of the turning point(s). (3)

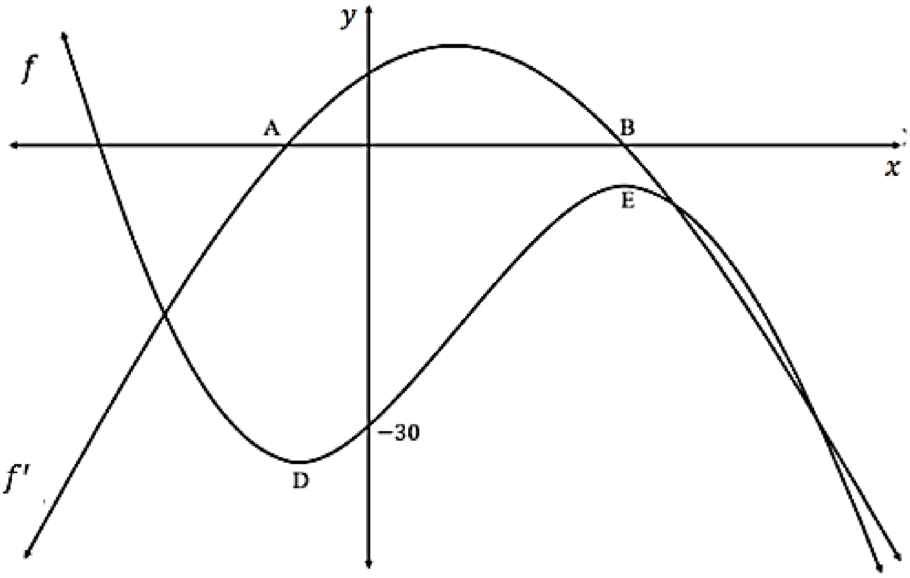
**[11]**

## WC

### QUESTION 8

The diagram below shows the curves of $f(x) = ax^3 + bx^2 + cx + d$ and $f'(x) = -3x^2 + 6x + 9$

- The graph of $f'(x) = -3x^2 + 6x + 9$ intersects the $x$-axis at A and B.
- D and E are the stationary points of the cubic graph $f(x) = ax^3 + bx^2 + cx + d$.
- $-30$ is the $y$-intercept of $f$.




8.1 Determine the $x$-coordinates of D and E. Show all your calculations. (3)

8.2 Determine the equation of $f$. (5)

8.3 Determine the value(s) of $x$ for which $f$ is increasing. (2)

8.4 For which values of $x$ is the graph of $f$ concave down? (3)

**[13]**

## EC

### QUESTION 10

The number of scripts marked by a certain marker was tracked at a marking centre $t$ days after marking started, and is represented by the function, $S(t) = -3t^2 + 30t$, $1 \leq t \leq 10, t \in Z$, where $S(t)$ is measured in scripts per day.

10.1 Determine the number of scripts that were marked by the marker on the third day. (2)

10.2 On which day will the marker reach the maximum number of scripts marked per day? (3)

10.3 The total number of scripts that a marker had to mark for the 10 days was 500. Did this marker reach the quota? Support your answer with calculations. (2)
[7]

## FS

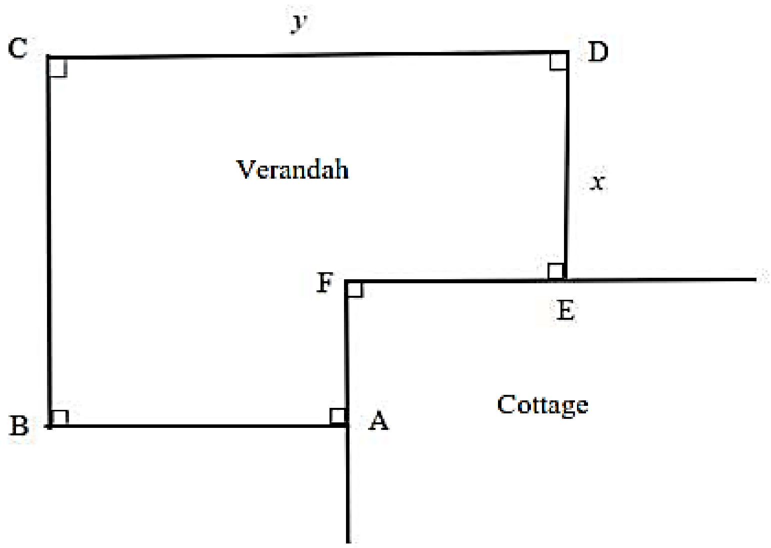
### QUESTION 9

At a price of R400 per bag, 200 bags are sold. For every R20 increase in the price of the bags, four fewer bags are sold. At what price must the bags be sold to maximise the profit? [7]

## GP

### QUESTION 10

The diagram below shows the plan for a verandah which is to be built onto the corner of a cottage. A railing ABCDE is to be constructed around the four edges of the verandah.
It is given that AB = DE = $x$ and BC = CD = $y$, and the length of the railing must be 30 metres.
Calculate the value of $x$ and $y$ for which the veranda will have a maximum area.



## KZN

### QUESTION 9

In February 2024, Bafana Bafana played in the semi-finals of the AFCON tournament. During the match, their striker Tebogo Mokoena kicked the soccer ball vertically upwards into the air and its motion was represented by the equation: $h(t)=1+20t-5t^2$, where $h$ is the height of the ball above the ground in metres, and $t$ is the time in seconds after the ball was kicked.

9.1 Determine the maximum height of the ball above the ground. (5)

9.2 How long will it take for the ball to hit the ground? (3)

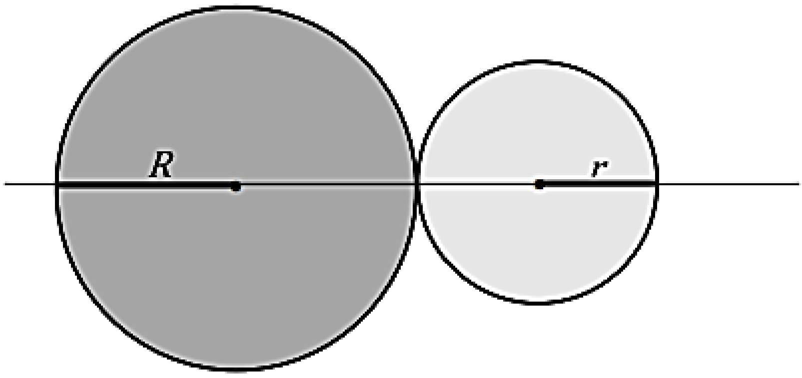
9.3 Determine the velocity of the ball 1,5 seconds after he has kicked it. (2)

**[10]**

## LP

### QUESTION 9

Mashudu Business Enterprise has asked you to design an advertising disc that consists of two circles and has the shape shown in the figure below. The larger circle has radius $R$ and the smaller circle has radius $r$.The values of $R$ and $r$ must vary, and $R+r=200$ mm. To minimise costs, Mashudu Business Enterprise has also stated that the area of the shape must be a minimum.




9.1 Show that the area(A), of the figure is given by:
$A=2\pi(R^2-200R+20\,000)$ (3)

9.2 Determine the values of $R$ and $r$ if the area, of the figure is a minimum. (4)

9.3 Hence, explain why the shape suggested by the company is not possible if you want to maintain a minimum area. (2)

**[9]**

## MP

### QUESTION 8

The radius of the base of a cylindrical cold drink can is $x$ cm, and its volume 440 cm³.
$A = \pi r^2, C = 2\pi r, V = \pi r^2 h$

8.1 Determine the height of the can in terms of $x$. (2)

8.2 Show that the area of the material needed to manufacture the can is $2\pi x^2 + \frac{880}{x}$. (2)

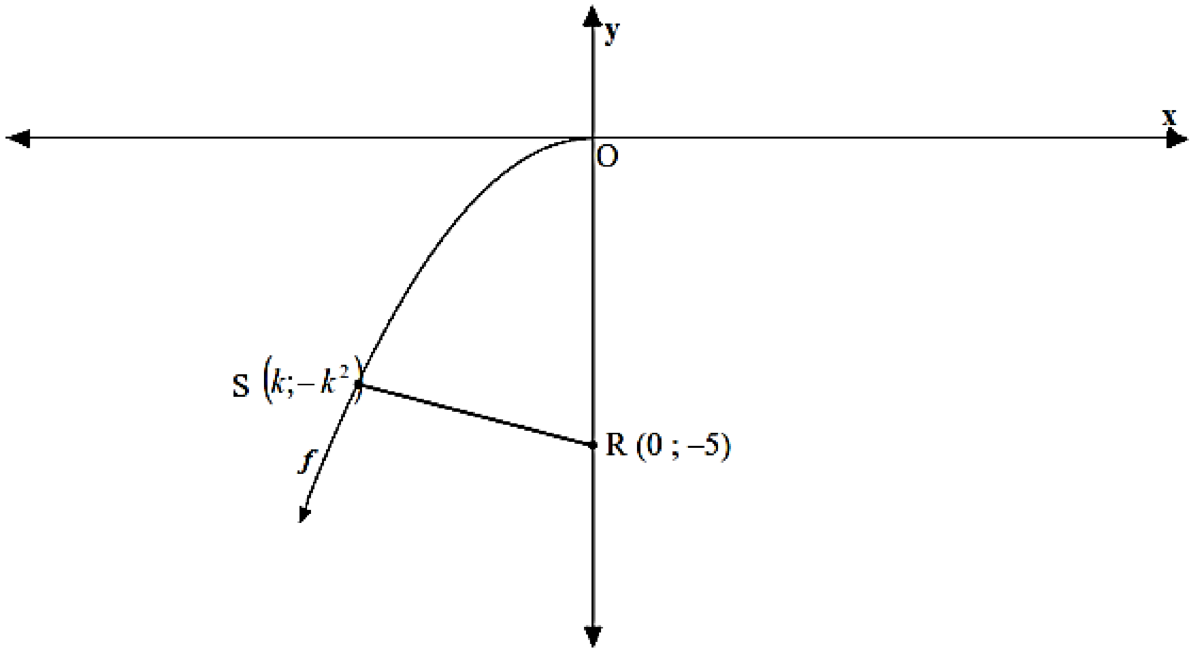
8.3 Determine the value of $x$ (correct to two decimals) for which the least amount of material is needed to manufacture such a can. (4)

**[8]**

## NC

### QUESTION 10

The graph of $f(x) = -x^2$, $x \leq 0$ is drawn below. R (0 ; –5) is a point on the $y$-axis and $S(k\,;-k^2)$ is a point on $f$. RS is drawn.




10.1 Determine the distance RS in terms of $k$, in simplest form. (2)

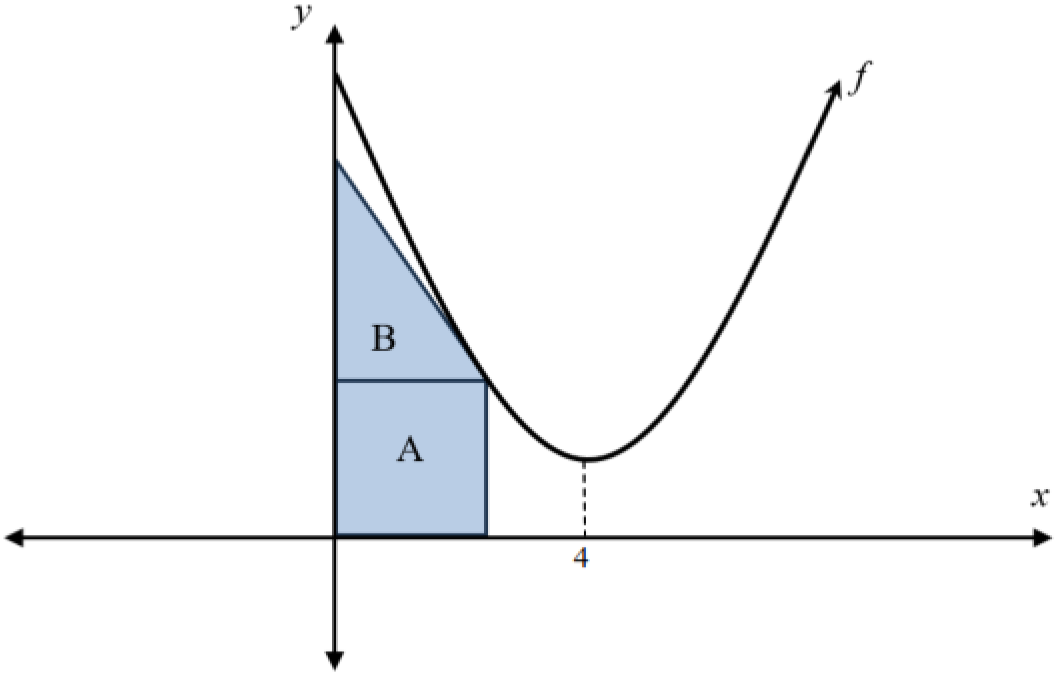
10.2 Calculate the value of $k$ such that the distance RS is a minimum. (4)

**[6]**

**NW**

## QUESTION 9

A river boards the farm of a farmer, which is represented by the equation $f(x) = x^2 - 8x + 17$. A tarred road is represented by the $x$- and $y$-axes and a border fence at $x = 4$.




9.1 Show that the area of the rectangular field (shaded area **A**), is given by: (1)

$A(x) = x^3 - 8x^2 + 17x$.

9.2 Determine the area of the largest rectangular field that the farmer can fence in (shaded area **A**). (5)

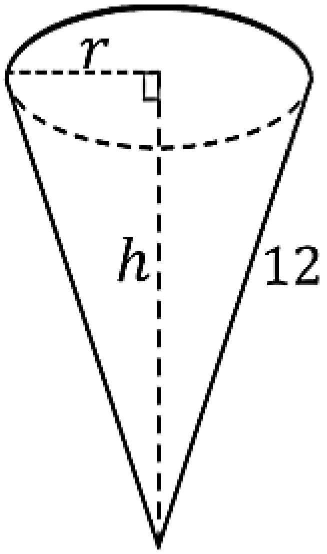
9.3 He decided to include an additional triangular field (shaded area **B**). Determine the largest area of the triangular field, if the base of area B is the same as the base of area A.
(Note: The fence should only touch the river and not cross it.) (4)
**[10]**

**WC**

**QUESTION 9**

The flowerpot below is in the shape of a cone.

- $r$ is the radius of the base and $h$ is the perpendicular height of the cone.
- The slant height of the cone is 12 cm.




| Formulae for volume: | | | |
|---|---|---|---|
| $V = lbh$ | $V = \pi r^2 h$ | $V = \frac{1}{3}\pi r^2 h$ | $V = \frac{4}{3}\pi r^3$ |

9.1 Show that the volume of the water needed to fill the entire flowerpot can be expressed as:

$V = 48\pi h - \frac{1}{3}\pi h^3$ (3)

9.2 The gardener wants to maximize the volume of water in the flowerpot. Determine the value of $h$ for which the volume is a maximum. (3)

**[6]**

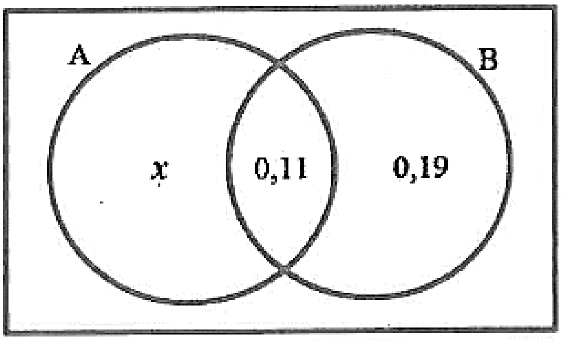
# PROBABILITY

## VENN-DIAGRAM

## EC

**QUESTION 11**

11.1 Two events A and B are shown on the Venn diagram below.
It is given that P[not (A or B)] = 0,41.




Determine:

11.1.1 The value of $x$ and hence P(A) (2)

11.1.2 P (A or not B) (2)

## CONTINGENCY TABLE

## EC

11.2 The results for the soccer club, City Brothers FC's 30 games during the 2022–2023 season are shown below.

| | HOME GAME | AWAY GAME | TOTAL |
|---|---|---|---|
| WINS | 3 | 4 | 7 |
| LOSSES | 7 | 7 | 14 |
| DRAWS | 5 | $a$ | 9 |
| TOTAL | 15 | 15 | 30 |

11.2.1 Write down the value of $a$. (1)

11.2.2 What is the probability that in a randomly selected match City Brothers FC was the losing team? (1)

11.2.3 Are the events 'winning a game' and 'playing at the home ground' independent? Justify your answer with calculations. (3)
[9]

# KZN

## QUESTION 10

10.1 During the Covid-19 pandemic, researchers conducted many studies to test the effectiveness of various vaccines. The table below shows data of one of those studies.

| | TESTED COVID-19 POSITIVE | TESTED COVID-19 NEGATIVE | TOTAL |
|---|---|---|---|
| MALE | 27 | 189 | 216 |
| FEMALE | 81 | 567 | 648 |
| TOTAL | 108 | 756 | 864 |

10.1.1 Calculate the probability that a randomly selected participant is female. (1)

10.1.2 Is the probability of testing positive for Covid-19 independent of gender? Show ALL calculations to motivate your answer. (4)

# WC

## QUESTION 10

10.1 A survey was conducted about the broadcasting of the Olympic games on the television. 150 males and 100 females were interviewed to establish if they liked the broadcasting or not. The table below shows some of the results:

| | MALE | FEMALE | TOTAL |
|---|---|---|---|
| Like the broadcasting | 60 | 70 | 130 |
| Did not like the broadcasting | (a) | 30 | (b) |
| TOTAL | 150 | 100 | 250 |

10.1.1 Calculate the values of (a) and (b). (2)

10.1.2 Calculate the probability that if a person is chosen randomly that he will be a male that liked the broadcasting. (1)

10.1.3 Determine whether a person's preference for the broadcasting is independent of the person's gender. Support your answer with appropriate calculations. (4)

10.2 Three friends from England, Mark, John and Michael, are to swim in the 100 m butterfly race which has eight swimmers in total. The eight swimmers line up one to a lane and the lanes are numbered 1 to 8.

10.2.1 Write down the total possible number of arrangements at the starting blocks. (1)

10.2.2 Determine the probability that Mark will be in lane 1, John in lane 2 and Michael in lane 3. (3)

# TREE DIAGRAMS

## GP

### QUESTION 12

12.1 The data obtained from a city's police department indicates that of all motor vehicles reported stolen, 80% were stolen by syndicates to be sold off, and 20% were stolen by individuals for personal use.

Of the vehicles presumed stolen by syndicates:

- 24% were recovered within 48 hours
- 16% were recovered after 48 hours
- 60% were never recovered

Of those vehicles presumed stolen by individuals:

- 38% were recovered within 48 hours
- 58% were recovered after 48 hours
- 4% were never recovered

12.1.1 Draw a tree diagram for the given information above. (3)

12.1.2 Calculate the probability that if a vehicle was stolen in this city, it would be stolen by a syndicate and recovered within 48 hours. (2)

12.1.3 Calculate the probability that a vehicle stolen in this city will not be recovered. (3)

12.2 You have to choose a password for your new "Facebook" profile. The password must be in the format: $\psi\psi\psi@@$ where $\psi$ is any digit (0's are not allowed) and @ is any vowel (a ; e ; i ; o ; u). You may repeat any digit, but you may not repeat a vowel.

How many passwords can be formed? (3)

**[11]**

## KZN

10.3 Tracey has 10 sweets in a bag. Some are green and some are red. She picks a sweet from the bag, takes note of the colour, and then puts it back into the bag. She does this four times.
How many green sweets are there in the bag, if the probability that she picks at least one green sweet is 97,44%? (4)

**[16]**

## LP

### QUESTION 10

10.1 Given $P(A) = 0,45$ and $P(B) = 0,25$.

Determine $P(A \text{ or } B)$ if A and B are mutually exclusive events. (2)

10.2 A survey was conducted in one of the schools in Limpopo province with a population of 50 educators. 30 of the educators indicated that they each own a car. Two educators were randomly selected one after the other without repetition.

10.2.1 Represent the given information on a tree diagram. Clearly indicate the possible outcomes of the event. (3)

10.2.2 Find the probability that only one of the educators selected owns a car. (3)

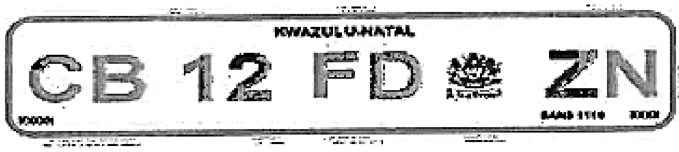
10.2.3 Find the probability that the two educators selected, each owns a car. (2)

# COUNTING PRINCIPLES

## EC

**QUESTION 12**

The province of KwaZulu-Natal has introduced a new number plate system starting from December 2023. The new number plate code consists of two letters, two digits and then two letters. The system is using the digits, 0–9 and the letters of the alphabet excluding the vowels. Below is an example of this new number plate. Note that all number plates come with the ZN suffix which is independent from the code.




[Source: KZN Provincial Gazette 2614-new number plates for KZN]

12.1 How many number plate codes are possible with the new system, if digits and letters may not be repeated? (2)

12.2 Calculate the probability that a number plate code will start with a letter of the alphabet that is before letter G, with the first digit being a composite number and that the last digit is a factor of 4. Digits and letters may not be repeated. (4)
**[6]**

## KZN

10.2 Towards the end of 2023, the KZN Traffic Department introduced a new number plate system for cars in the province.

Each new number plate consists of:

- two letters of the alphabet;
- followed by two digits;
- followed by two letters of the alphabet; and
- ending with the letters ZN.

One example of such a number plate is: **RR 23 GB ZN**

All 26 letters of the alphabet, excluding Q and the 5 vowels, may be used.
Any two digits from 0 to 9 may be used.

10.2.1 How many different number plates can be made if letters and digits may be repeated? (2)

10.2.2 How many different number plates can be made if letters and digits may not be repeated? (2)

10.2.3 If letters and digits may not be repeated, what is the probability that a number plate of this form will start with the letters B and C in any order? (3)

## LP

10.3 You require a password for an online account. The password must have 3 numerical values, followed by 2 vowels.

10.3.1 How many passwords are possible if repetitions are allowed? (2)

10.3.2 Determine the number of possible passwords with the following conditions:

- If repetitions are not allowed,
- The password does not start with a zero and
- It ends with a vowel $a$. (3)

**[15]**

## MP

9.3 Every morning a father has to drop his children off at school on his way to work and pick them up again on his way home from work. There are 3 different roads from home to school and 5 different roads from school to work.

In how many ways can the father:

9.3.1 travel from his home to work? (1)

9.3.2 travel from his home to work and back to his home? (2)

9.3.3 travel from his home to work and back home if he does not use the same road twice? (2)

**[17]**

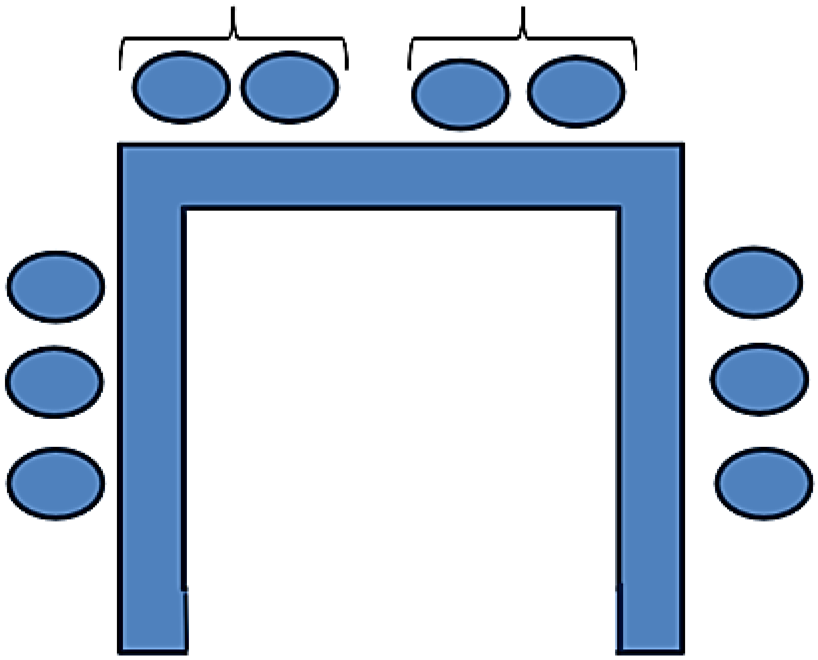
**NW**

**QUESTION 11**

Two learners (one boy and one girl) from each grade (grade 8, 9, 10, 11 and 12) are elected to form the grade representatives of the high school.

11.1 When meeting, they start and end with a prayer. In how many different ways can they select somebody to start and someone else to end the meeting with a prayer? (1)

11.2 When they meet, they sit at a u-shaped table with the two grade 12 members next to each other and the two grade 11 members next to one another at the top of the table. The rest of the members sit on the remaining chairs in any order.



Determine in how many different ways can the members be arranged along the u-shaped table? (Refer to the diagram.) (2)

10.3 They decide that during assembly on a Monday, they will be seated on the first chair of each row, for the first 10 rows. If they are randomly allocated seats, determine the probability that a boy will be seated in the first row, and another boy will be seated in the tenth row. (3)

**[6]**

**WC**

10.3 A company uses a coding system to identify its clients. Each code consists of two letters and a sequence of digits, for example:
AB 206 or CC 456789

- The letters are chosen from A, B, C, D and E.
  Letters may be repeated in the code.

- The digits 0 to 9 are used.
  NO digit may be repeated in the code.
  The digit part of the code may not begin with a zero (0)

10.3.1 How many different clients can be identified with a coding system made up of TWO letters and TWO digits? (2)

10.3.2 Determine the least number of digits that is required for a company to uniquely identify 680 000 clients using their coding system (4)

**[17]**

## ALL-

## DEPENDENT & MUTUAL INCLUSIVE EVENTS

## FS

**QUESTION 10**

10.1 A and B are two events in a sample space. $P(A \text{ or } B) = 0.8$ and $P(B) = 0.4$.

10.1.1 Determine the $P(A)$ if events A and B are mutually exclusive. (2)

10.1.2 Determine the $P(A)$ if events A and B are independent. (2)

10.2 A medical screening is performed to test for the presence of a disease in a population.
The test is not 100% accurate. 10% of those who have the disease will test negative, and 5% of those who don't have the disease will test positive. If the test is performed on the entire population and 6,7% test positive, what percentage of the population has the disease? (6)

10.3 In the Free State, license plates are designed with three letters of the alphabet, excluding vowels, next to one another and then any three digits from 0 to 9 next to one another. FS is constant in all Free State license plates, for example, DNV 295 FS. Letters and digits may be repeated in a license plate.

10.3.1 How many unique license plates will be available if the letters and the numbers are not repeated? (1)

10.3.2 Hence, determine the probability that the license plates will start with a 6, in keeping with the order above. (3)

**[14]**

## GP

**QUESTION 11**

Let A and B be two events in a sample space.

Suppose that $P(A) = 0{,}4$; $P(A \text{ or } B) = 0{,}7$ and $P(B) = k$.

11.1 For what value of $k$ are $A$ and $B$ mutually exclusive? (2)

11.2 For what value of $k$ are $A$ and $B$ independent? (3)

**[5]**

# MP

**QUESTION 9**

9.1 Events A and B are mutually exclusive. It is given that:

- $P(B) = 2P(A)$
- $P(A \text{ or } B) = 0{,}57$

Calculate $P(B)$. (4)

9.2 There are 5 loaves of brown bread (B) and 7 loaves of white (W) bread on a shelf at the local supermarket. Two clients, one followed by the other, each randomly select a loaf of bread from their shelf and put it their basket.

9.2.1 Determine the probability that the first client takes a loaf of white bread. (1)

9.2.2 Assume that the owner of the shop does not replace any of the loaves of bread on the shelf after a client has taken a loaf of bread.

Determine the probability that both clients take a loaf of brown bread. (3)

9.2.3 If the first client takes a loaf of white bread, the owner of the shop places a loaf of brown bread with the other loaves on the shelf. If the first client takes a loaf of brown bread, the owner of the shop places a loaf of white bread with the other loaves on the shelf.

Determine the probability that a loaf of white bread and a loaf of brown bread is sold to the two clients. (4)

**NC**

**QUESTION 11**

11.1 In his cupboard, Kevin has 2 pairs of trousers, a black one and a red one. He also has 3 T-shirts, one blue, one yellow and a white one.

11.1.1 He must decide what combination of trousers and T-shirt he will be wearing to town. What is the probability of him …

a) wearing red trousers? (1)

b) wearing black trousers with a white T-shirt? (1)

11.1.2 In how many ways can he hang these clothes on a rail in his cupboard? (2)

11.2 The probability that event A will occur is 0,5. The probability that event B will occur is 0,6 and the probability that neither of these events will occur is 0,1.

11.2.1 Calculate the probability that both events A and B will occur. (3)

11.2.2 Draw a complete Venn diagram to illustrate the above-mentioned. (2)

11.2.3 Use calculations to show that events A and B are not independent. (2)

11.3 A 3-digit number code is made up by using digits from 0 to 9. No digit may be repeated.

11.3.1 How many 3-digit number codes can be formed from these digits? (1)

11.3.2 Hence, determine the probability of 3-digit number codes formed which is greater than 600 and an even number. (3)

**[15]**

**NW**

**QUESTION 10**

Tom, Dic and Harry are friends and are learners at the same high school. All three are in the same mathematics class. Some days they are absent from the mathematics class. The probability that neither Tom nor Harry is absent from the mathematics class on a specific day, is 0,42. The probability that Tom is absent from the mathematics class on a randomly selected day is 0,40.

10.1 Calculate the probability that Tom or Harry will be absent from the mathematics class on a random selected day. (1)

10.2 The mathematics teacher was suspicious about the absenteeism of Tom and Harry from the mathematics class. He investigated and realised that their absenteeism is independent from one another.

Determine the probability that Tom and Harry will be absent from the mathematics class on the same day. (4)

10.3 Calculate the probability that only Tom will be absent from the mathematics class on a random selected day. (1)

10.4 The mathematics teacher finds out that the probability that Harry and Dic will be absent from the mathematics class on a random selected day is 0,16 and that the probability that Harry or Dic will be absent is 0,3.

Calculate the probability that Dic will be absent from the mathematics class on a randomly selected day. (2)

10.5 Will there be a day that Dic is absent from the Mathematics class, but Harry is present? (1)

**[9]**